Research Article

Analysis on the Vertical Additional Force of Shaft and Drainage Settlement Characteristics of Topsoil Containing Multiaquifers

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The vertical additional force induced by the drainage consolidation settlement of the topsoil is one of the main factors for shaft wall fracture. To date, the number of aquifers of topsoil rises with the depth increasing of shaft, which would lead to a more serious additional force effect. Thus, studying the vertical additional force law of the shaft when drainage settlement occurs in its surrounding topsoil containing multiaquifers is of great significance to predicting the shaft stress and guaranteeing shaft safety. In this study, mechanical analysis of the topsoil with the shaft crossing multiaquifer was carried out, and the settlement of each aquifer and aquiclude was calculated by separating the single-slope drainage consolidation and the double-slope drainage consolidation. Then, the calculation model of vertical additional force was established due to the settlement caused by the reaction of the additional force on the topsoil containing multiaquifers, and the calculation model of shaft wall stress was also developed. Verification of this model was conducted by comparing the filed measurement data of the shaft wall strain and the theoretical data calculated by the stress model. Finally, the effect laws of drainage velocity, central aquifer thickness, location, and number on the additional force were obtained and analysed. This paper is expected to provide theoretical support for predicting the additional force and shaft wall stress during its service time.

1. Introduction

As the strategic passage of a coal mine, the security of vertical shaft is of great significance in protecting the safety of personnel and normal production [1–4]. The additional force induced by drainage settlement of the topsoil has been proven to play an important role in the breakage of shafts [5, 6]. The failure mechanism is that the large-scale strata settlement of coal mining regions would destroy the hydraulic relation of topsoil and induce the continuous drainage, while this behaviour would gradually reduce the water level and pressure of surrounding aquifers of shaft and increase the effective stress of surrounding topsoil. Then, the drainage consolidation settlement of surrounding topsoil appears and results into the relative displacement of shaft and surrounding strata due to their differential deformation modulus. Thus, the vertical additional force derives from the negative friction force induced by the relative displacement of the shaft and surrounding strata [7]; thus, understanding the additional force laws is of great significance to predict the shaft stress and guarantee shaft safety.

A series of studies have focused on the theoretical analysis of additional forces. Hong [8] found that the appearance of additional force would lead to a serious tension strain of the inner edge of a thick-walled shaft, while the tensile strain of the shaft is much less than the compressive strain; thus, when the tension strain of the shaft exceeds its ultimate value, circumferential cracks may appear on the inner edge of the shaft. Chen [9] pointed out that the breakage of a shaft would be due to the negative friction force induced by the
relative displacement of the shaft and its surrounding strata based on the failure characteristics and geological conditions of the shaft, while the displacement is usually caused by the compression consolidation settlement of soil induced by coal mining and draining. Yang et al. [10–12] studied the influencing laws of each parameter on the additional force of the shaft during the drainage of topsoil based on elastic theory and verified these laws by a battery of physical similarity model experiments, which indicates that the expressions of additional forces can be applied to shaft design calculations. Zhou and Cheng [13] derived the distribution laws of the vertical additional force of a shaft subjected to dead weight, additional force, and ground pressure based on the spatial elasticity theory, field investigation, and physical similarity model experiments.

In addition, Lou and Su [14] figured out the calculation formula of the additional force induced by drainage consolidation settlement and built a mathematical model of the shaft wall stress based on the effective stress principle of soil mechanics and the following hypothesis: shear action occurs between the contact surface of the shaft wall and surrounding soil, which makes the soil in the contact zone enter the state of plastic failure and causes the relative displacement of the shaft and surrounding soil. Zhang et al. [15] studied the magnitude and variation rule of additional force based on field measurement data by optimizing back analysis. Liu et al. [16] found that the additional force is a constant value of 51.1 kPa according to the basic characteristics and specific parameters of 16 fractured shafts. The magnitude of the vertical additional force did not depend on the shaft depth but rather on the complex formation conditions. Ding [17] derived the calculation formula of shaft wall stress under different loading conditions and obtained a theoretical model of shaft wall stress and displacement in the cross section based on elastic theory. Xue et al. [18] obtained the variation laws and analytical solution of additional force by dividing the strata within a certain range of the shaft wall into the left and right symmetrical thin plate units through the mathematical element method and establishing the equation according to the bending deformation theory of thin plates.

These studies above provide a series of useful theoretical basis for additional force analysis. Following these theoretical studies, some numerical studies on the additional force of shafts were also conducted to investigate the effects of each factor on the additional force and shaft wall stress.

Lv and Cui [19] studied the stress and strain development laws of shaft walls in the whole period from the drainage occurrence of bottom aquifers to the fracture of shafts based on the 3D shaft wall model by numerical simulation software and discussed the fracture location, fracture morphology, and fracture development laws of shaft walls. Chen et al. [20] analysed the influence of different magnitudes of additional force by drainage settlement on the stress state, fracture location and style, fracture development, and position of the plastic zone of a shaft wall based on the 3D calculation model of a reinforced concrete shaft wall by FEM. Xu...
et al. [21] studied the relationships among the following parameters based on the numerical model by FEM: the water level drop of the aquifer, the relative displacement of the strata and shaft wall, and the shear stress of the stratum and shaft wall; the results show that the inner edge of the shaft wall is the most prone to fracture. He [22] pointed out that the temperature and drainage velocity also affect the additional force and studied these effect magnitudes through FLAC.

However, to date, most of the analytical solutions of the additional force were basically based on the assumption that there is no relative displacement between the shaft wall and the surrounding soil; meanwhile, multiaquifers were not considered. With the further development of the economy

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**Figure 2**: Schematic diagram of shaft and surrounding strata: (a) single bottom aquifer; (b) multiaquifers.
and society, the demand for coal rises up and leads to a dramatic increase in the depth of coal seam, which brings about one important problem: the number of central aquifers in topsoil increases. The large-scale strata settlement of coal mining regions would destroy the water-resisting function of central aquicludes to some extent, and a number of water flow channels connecting different aquifers appear in central aquicludes. The water level and pressure of each aquifer would decrease at the same time due to the existence of water flow channels. Then, the drainage in multi-aquifers would appear and result into the obvious consolidation settlement of surrounding topsoil of shaft. However, the amount of drainage settlement of all surrounding strata (aquifer or aquiclude) would be different due to their differential mechanical behaviours, which would lead to more serious and complicated vertical additional force and threaten the safety of shaft, as shown in Figure 1. Thus, in this study, a calculation model of vertical additional force was established by the settlement caused by the reaction of the additional force on the topsoil containing multi-aquifers, and the calculation model of shaft wall stress was also built. In addition, the effect laws of drainage velocity, central aquifer thickness, location, and numbers on the additional force were obtained and analysed.
2. Calculation Model of Settlement and Vertical Additional Force

2.1. The Mechanical Model of the Shaft with Surrounding Strata. Figure 2(a) shows the simplified mechanical model of vertical additional force and drainage settlement with a single bottom aquifer in the traditional study. Based on the basic assumptions below and according to the traditional mechanical model due to the pile foundation theory and the generalized shear displacement method, a new mechanical model was built for the vertical additional force and drainage settlement of topsoil containing multi-aquifer strata in this paper, which is shown in Figure 2(b) [23–30]. The basic assumptions are as follows: (1) the single-layer shaft model is used as representative of actual shaft wall, and the shaft wall, the depth and thickness of the strata, and the settlement deformation are all symmetric about the centerline of the shaft; (2) relative displacement between the shaft wall and surrounding strata will occur due to vertical additional forces; (3) the contact interface between the shaft wall and surrounding strata adopts a hyperbolic constitutive relationship (shown in Figure 3); (4) the strength criteria are taken on the shaft wall and each aquiclude and aquifer; (5) the interaction of the drainage effect between aquifers is ignored.

In Figure 2(a), $E_1$ is the compression modulus of soil (MPa), $E_p$ is the compression modulus of shaft wall (MPa), $E_d$ is the compression modulus of bottom aquifer (MPa), $\mu_1$ is the Poisson’s ratio of soil, $\mu_p$ is the Poisson’s ratio of shaft wall, $\mu_d$ is the Poisson’s ratio of bottom aquifer, $H$ is the total thickness of topsoil (m), $h$ is the thickness of bottom aquifer (m), $\rho_0$ is the density of soil (kg·m$^{-3}$), $g$ is the gravitational acceleration (N·kg$^{-1}$), $z$ is the vertical coordinate (depth, m), $r$ is the radius coordinate (m), $b$ is the outer radius of shaft (m), and $R$ is the radius of topsoil (m). In Figure 2(b), $E_g$ is the compression modulus of top aquiclude (MPa), $E_i$ is the compression modulus of central aquifer 1, 2, 3 ⋯ $j$ (MPa), $E_j$ is the compression modulus of central aquiclude 1, 2, 3 ⋯ $j$ (MPa), $\mu_g$ is the Poisson’s ratio of top aquiclude, $\mu_i$ is the Poisson’s ratio of central aquifer 1, 2, 3 ⋯ $j$, $\mu_j$ is the Poisson’s ratio of central aquiclude 1, 2, 3 ⋯ $j$, $v_i$ is the drainage velocity of the central aquifer (MPa·a$^{-1}$), $v_d$ is the drainage velocity of the bottom aquiclude (MPa·a$^{-1}$), $h_g$ is the thickness of top aquiclude (m), $h_i$ is the thickness of central aquifer 1, 2, 3 ⋯ $j$ (m), $h_j$ is the thickness of central aquiclude 1, 2, 3 ⋯ $j$ (m), and $h_d$ is the thickness of bottom aquifer (m).

$f_n(z)$ is the vertical additional force at depth $z$ (kPa), $f_{n,max}$ is the maximum vertical additional force (kPa), and $\omega(z)$ is the relative displacement of the shaft wall and surrounding strata at depth $z$ (mm).

2.2. The Amount of Settlement Caused by Drainage Consolidation. The whole settlement consists of drainage settlement and settlement induced by additional force, while drainage settlement includes the aquifer settlement and aquiclude settlement.

2.2.1. Aquifer Settlement. Aquifers are thought to consist mainly of sandy layers, then the drainage consolidation of the bottom aquifer and central aquifer can be recognized as a transient and uniform process; thus, the bottom aquifer settlement $w_d$ and central aquifer settlement $w_i$ can be
expressed by:

$$w_\ell = \frac{v_d t_h_d}{E_d},$$  \hspace{1cm} (1)$$

$$w_i = \frac{v_i t_h_i}{E_i}.$$  \hspace{1cm} (2)

2.2.2. Aquiclude Settlement. Aquicludes are thought to consist mainly of clay layers, then the drainage consolidation of aquicludes can be recognized as a gradual completion process. However, the drainage method of the top aquiclude belongs to the single-slope drainage case because the top surface is close to the ground and the bottom surface is close to the aquifer, which can be thought of as the unidimensional consolidation process, as shown in Figure 4.

$u_{\mu t}$ is the initial pore water pressure on the bottom surface of the top aquiclude (MPa).

According to the Terzaghi’s One-Dimensional Consolidation Theory:

$$\frac{\partial u(z, t)}{\partial t} = C_v \frac{\partial^2 u(z, t)}{\partial z^2},$$  \hspace{1cm} (3)

where $C_v$ is the consolidation compression index ($C_v = kE_s/\gamma_w$), $E_s$ is the compression modulus (MPa), $\gamma_w$ is the water
unit weight (kN·m$^{-3}$), and $k$ is the permeability coefficient (cm·s$^{-1}$).

The boundary conditions are:

$$
\begin{align*}
0 < t \leq \infty, & \quad z = 0, u = 0, \\
0 < t \leq \infty, & \quad z = h_g, \frac{\partial u}{\partial z} = 0.
\end{align*}
$$

(4)

The initial conditions are:

$$
t = 0, \quad 0 < z \leq h_g, u = \frac{z}{h_g} u_g.
$$

(5)

Thus, according to the separating variables method, the calculation equation of $w_g(t)$ can be expressed as:

$$
w_g(t) = \frac{1}{E_g} \left( \frac{u_g h_g}{2} - \frac{16 h_g^2}{\pi^3} \sum_{m=1}^{\infty} \frac{(-1)^{m-1} h_g}{m^3} e^{-T_v m^2 \pi^2/4} \right),
$$

(6)

where $T_v$ is the time factor.

The drainage method of the central aquiclude belongs to the double-slope drainage case because the top and bottom surfaces are both aquifers, which is still the unidimensional consolidation process, as shown in Figure 5.

$p_{j1}$ is the value of the pore water pressure on the top surface of the central aquiclude (MPa), $p_{j2}$ is the value of the pore water pressure on the bottom surface of the central aquiclude (MPa), the ratio of these two parameters is $\alpha_j = p_{j1}/p_{j2}$, and the value of the $p_{j1}$ and $p_{j2}$ are determined by the location of this central aquiclude. For example, there are one central aquifer and one central aquiclude in the topsoil, the total height of this topsoil is 200 m, the height of the bottom aquifer is 20 m, the height of central aquifer is 20 m, and the location of central aquifer is 0.5 H, then the top and bottom location of central aquiclude connecting the central aquifer and the bottom aquifer are 110 m and 180 m separately; thus, the $p_{j1}$ and $p_{j2}$ of this central aquiclude are 1.1 MPa and 1.8 MPa separately.

The boundary conditions are:

$$
\begin{align*}
0 < t \leq \infty, & \quad z = 0, u = 0, \\
0 < t \leq \infty, & \quad z = h_j, u = 0.
\end{align*}
$$

(7)

The initial conditions are:

$$
t = 0, \quad 0 < z \leq h_j, u = u_j = p_{j2} \left[ 1 + (\alpha_j - 1) \frac{h_j/2 - z}{h_j/2} \right],
$$

(8)

where $u_j$ is the initial pore water pressure of the central aquiclude (MPa).

According to the separating variables method, $w_j(t)$ can be expressed by:

$$
w_j(t) = \frac{1}{E_j} \left( p_{j2} h_j - \frac{2 h_j p_{j2}}{\pi^2} \sum_{m=1}^{\infty} \frac{2 \left[ 1 - (-1)^m \alpha_j \right]}{m^2} e^{-T_v m^2 \pi^2/4} \right),
$$

(9)
In summary, the total amount of settlement of all strata with \(i\) aquifers and \(j\) aquicludes (\(i, j = 1, 2, 3, \ldots\)) can be expressed by:

\[
\omega = \omega_d(t) + \omega_i(t) + \omega_g(t) + \omega_j(t) = \frac{v_d t h_d}{E_d} + \sum_{i=1}^{i} \frac{v_i t h_i}{E_i} \\
+ \frac{1}{E_g} \left( u_{p_0} h_g \frac{16 t u_{g2} h_g}{\pi^2} \sum_{m=1}^{\infty} \frac{(-1)^{(m-1)/2}}{m^2} e^{-T_{vg} m^2 \pi^2/4} \right) \\
+ \sum_{j=1}^{j} \frac{1}{\rho_j h_j} \left( b_j + \frac{2h_j \rho_j}{\pi^2} \sum_{m=1}^{\infty} \frac{2 \left[ 1 - (-1)^m \alpha_j \right]}{m^2} e^{-T_{vj} m^2 \pi^2/4} \right),
\]

where \(T_{vg}\) is the time factor of top aquiclude (\(T_{vg} = C_{vg} t h_g^2\)) and \(T_{vj}\) is the time factor of central aquiclude 1, 2, 3 \(\ldots\) \(j\) (\(T_{vj} = 4 C_{vj} t h_j^2\)).

2.3. The Amount of Settlement Caused by Vertical Additional Force. Due to the difference between the deformation modulus of the shaft wall and the surrounding strata, the drainage consolidation of surrounding strata would result in the appearance of vertical additional force, and the amount of corresponding settlement is related to the magnitude of the additional force. The simplified interaction model of settlement and vertical additional force is shown in Figure 6.

\(f_n\) is the vertical additional force (kPa), \(\Delta S = \omega(z)\), \(\omega\) is the settlement induced by drainage (mm), and \(\omega_j\) is the settlement induced by the additional force (mm).

Shear deformation will appear on the surrounding strata due to the interfacial resistance during drainage settlement. According to the shear displacement method, the vertical equilibrium equation of the surrounding strata can be expressed by:

\[
\frac{\partial \tau}{\partial r} + \frac{\tau}{r} + \frac{\partial \sigma_z}{\partial z} = 0.
\]

Generally, the increase in shear stress of the interface between the shaft and surrounding strata would be minimal when the shaft wall is subjected to the additional force; thus, this item \(\partial \sigma_z/\partial z\) can be ignored [31], and Equation (11) can
Figure 13: Continued.
(c) Relation between additional force and depth and time (2 central aquifer)

(d) Relation between additional force and depth and time (3 central aquifer)

FIGURE 13: Continued.
be simplified as:
\[ \frac{\partial \gamma}{\partial r} + \frac{\tau}{r} = 0. \quad (12) \]

Therefore, the transfer equation of shear stress in the radial direction is:
\[ \tau = \tau_0 \frac{r_0}{r}. \quad (13) \]

where \( \tau_0 \) is the additional stress of the vertical shaft wall surface and \( r_0 \) is the external radius of the vertical shaft wall.

Ignoring the influence of radial strain, the shear deformation can be expressed as:
\[ \gamma = \frac{\partial w}{\partial r}. \quad (14) \]

From the relationships of additional force and settlement in a previous study [32], the shear deformation of surrounding strata increases when the settlement grows, and the stress-strain relationship curve of the boundary of the shaft wall and surrounding strata is nonlinear. Thus, a hyperbolic constitutive relation is adopted in the relationship between the shear stress and shear strain of interface between the
(a) Relation between additional force and depth and time ($\nu = 0.004$ MPa/a)

(b) Relation between additional force and depth and time ($\nu = 0.008$ MPa/a)

Figure 14: Continued.
(c) Relation between additional force and depth and time ($\nu = 0.012$ MPa/a)

(d) Relation between additional force and depth and time ($\nu = 0.016$ MPa/a)

**Figure 14: Continued.**
(e) Relation between additional force and depth and time ($v = 0.020$ MPa/a)

(f) Relation between additional force and depth and time ($v = 0.024$ MPa/a)

**Figure 14:** Continued.
shaft and surrounding strata, as shown in Equation (15). Meanwhile, the relevant piecewise lines are also adopted to describe the strata disturbance on the constitutive relation of soil, as shown in Figure 7.

\[
\gamma = \frac{\tau}{G_0\left(1 - \left(\frac{R_f}{\tau_f}\right)\right)},
\]

(15)

\[
\frac{\partial w}{\partial r} = \frac{\tau}{G_0\left(1 - \left(\frac{R_f}{\tau_f}\right)\right)}.
\]

(16)

where \( \tau \) is the shear stress (MPa), \( \tau_f \) is the ultimate shear strength of soil (MPa), \( \gamma \) is the shear strain, \( G_0 \) is the initial shear modulus (MPa), and \( R_f \) is the breakage ratio (0.75−0.95).

The above equations can be obtained simultaneously:

Thus, the amount of settlement caused by the additional
(a) Relation between additional force and depth and time (central aquifer 0.2 H)

(b) Relation between additional force and depth and time (central aquifer 0.3 H)

Figure 15: Continued.
(c) Relation between additional force and depth and time (central aquifer 0.4 H)

(d) Relation between additional force and depth and time (central aquifer 0.5 H)

Figure 15: Continued.
Figure 15: Continued.
force can be expressed by:

\[ w_t = \frac{\tau_0 r_0}{G_0} \ln \left( \frac{\tau_f R_h - \tau_0 r_0 R_f}{\tau_f r_0 - \tau_0 r_0 R_f} \right) = \frac{\tau_0 r_0}{G_0} \ln \left( \frac{(R_h/r_0) - (\tau_0 R_f/\tau_f)}{1 - (\tau_0 R_f/\tau_f)} \right), \] (17)

where \( R_h \) is the influence scope of shear deformation.

Therefore, the calculation formula of the relative displacement of the shaft wall and surrounding strata \( \Delta S \) at depth \( z \) is shown as follows:

\[ \Delta S = w - w_t, \] (18)

where \( w \) is the settlement induced by drainage and \( w_t \) is the settlement induced by the additional force.

Based on the force analysis of a single pile while the surrounding strata are assumed to be elastic-plastic elements, the relationship between \( \Delta S \) and \( f_n(z) \) can be expressed by the hyperbolic constitutive relation for the shaft at depth \( z \) before the appearance of relative sliding of the shaft wall.
Figure 16: Continued.

(a) Relation between additional force and depth and time (central aquifer 10 m)

(b) Relation between additional force and depth and time (central aquifer 15 m)
(c) Relation between additional force and depth and time (central aquifer 20 m)

(d) Relation between additional force and depth and time (central aquifer 25 m)

**Figure 16**: Continued.
Figure 16: Continued.
and surrounding strata, and the expression formula is shown in Equation (19). When the relative sliding of the shaft wall and surrounding strata reaches the plastic sliding state, the additional force reaches its maximum.

\[
f_n(z) = \begin{cases} 
\frac{\Delta S}{a(z) + b(z)\Delta S} & f_n(z) < f_{n,max} \\
p' \tan \varphi + c & f_n(z) \geq f_{n,max}
\end{cases}
\] (19)

where \(a(z)\) is the inverse of the initial shear stiffness, \(b(z)\) is the inverse of the ultimate negative friction force (\(b(z) = 1/f_{n,max}\)), \(p'\) is the lateral pressure on the shaft (\(p' = 0.013z\)), \(\varphi\) is the internal friction angle of the interfacial soil, and \(c\) is the cohesion of the interfacial soil.

When \(f_n(z) \geq f_{n,max}\), according to the Mohr–Coulomb strength theory, the following formula can be obtained:

\[
b(z) = \frac{1}{f_{n,max}} = \frac{1}{p' \tan \varphi + c}, \quad (20)
\]

\[
a(z) = \frac{1}{G} = \frac{1}{p' \tan \varphi}, \quad (21)
\]
(a) Relation between additional force and depth and time (bottom aquifer 10 m)

(b) Relation between additional force and depth and time (bottom aquifer 15 m)

Figure 17: Continued.
Figure 17: Continued.
(e) Relation between additional force and depth and time (bottom aquifer 30 m)

(f) Relation between additional force and depth and time (bottom aquifer 35 m)

FIGURE 17: Continued.
where $\rho$ is the density of soil (g·cm$^{-3}$) and $v_s$ is the shear wave velocity of soil (m·s$^{-1}$).

2.4. Shaft Wall Stress Analysis. One unit shaft wall is taken, and a simplified mechanical model is established. The lateral horizontal pressure of the shaft wall is $p_z = -kz - p_0$, the lateral vertical pressure is the additional force $f_n(z) = p$, and the top pressure is the upper shaft dead weight $q$. The mechanical model is shown in Figure 8.

$a$ is the inner radius of shaft (m), $b$ is the outer radius of shaft (m), and $p_z$ is the lateral horizontal pressure of the shaft wall (MPa).

The calculation formula of the anisotropic stress component of the shaft wall subjected to the lateral ground stress and gravity stress can be expressed by:

$$
\begin{align*}
\sigma_r &= \frac{kb^2z}{b^2-a^2} + \frac{a^2b^2kz}{(b^2-a^2)r^2} - \frac{p_0 b^2}{b^2-a^2} + \frac{a^2b^2p_0}{b^2-a^2}, \\
\sigma_\theta &= \frac{kb^2z}{b^2-a^2} - \frac{a^2b^2kz}{(b^2-a^2)r^2} - \frac{p_0 b^2}{b^2-a^2} - \frac{a^2b^2p_0}{b^2-a^2}, \\
\sigma_z &= -q - \gamma z, \\
\tau_{rz} &= 0,
\end{align*}
$$

Figure 17: Relationship between the additional force and bottom aquifer thickness.
where $\sigma_r$, $\sigma_\theta$, and $\sigma_z$ are the radial, circumferential, and vertical stresses of the shaft wall, respectively, and $\tau_{rz}$ is the shear stress.

The calculation formula of the anisotropic stress component of the shaft wall subjected to the additional force induced by the settlement due to drainage consolidation can be expressed by:

$$\begin{align*}
\begin{cases}
\sigma_r &= 0, \\
\sigma_\theta &= 0, \\
\sigma_z &= \frac{-2h}{b^2-a^2} z, \\
\tau_{rz} &= \frac{bp}{b^2-a^2} \left( r - \frac{a^2}{r} \right).
\end{cases}
\end{align*} \tag{23}$$

Thus, the whole expression formula of the anisotropic stress component of the shaft wall is obtained by the summation of Equation (22) and Equation (23), and shown below:

$$\begin{align*}
\begin{cases}
\sigma_r &= -\frac{k}{b^2-a^2} \left( \frac{a^2 b^2 k}{(b^2-a^2)^2} + \frac{p_0 b^2}{b^2-a^2} + \frac{a^2 b^2 p_0}{(b^2-a^2)^2} \right), \\
\sigma_\theta &= -\frac{k}{b^2-a^2} \left( \frac{a^2 b^2 k}{(b^2-a^2)^2} - \frac{p_0 b^2}{b^2-a^2} + \frac{a^2 b^2 p_0}{(b^2-a^2)^2} \right), \\
\sigma_z &= \frac{2}{b^2-a^2} \frac{2b}{z}, \\
\tau_{rz} &= \frac{bp}{b^2-a^2} \left( r - \frac{a^2}{r} \right).
\end{cases}
\end{align*} \tag{24}$$

The strain value can be used to evaluate the safety of the shaft. The ultimate strain value of the shaft wall can be calculated according to the ultimate bearing capacity of the reinforced concrete normal section, and the relation curve of the stress and strain of concrete under compression satisfies the following conditions:

If $\varepsilon_c \leq \varepsilon_0$,

$$\sigma_c = f_c \left[ 1 - \left( 1 - \frac{\varepsilon_c}{\varepsilon_0} \right)^n \right]. \tag{25}$$

If $\varepsilon_0 \leq \varepsilon_c \leq \varepsilon_{cu}$,

$$\sigma_c = f_c \varepsilon_c, \tag{26}$$

$$n = 2 - \frac{1}{60} (f_{cu,k} - 50), \tag{27}$$

$$\varepsilon_0 = 0.002 + 0.5 (f_{cu,k} - 50) \times 10^{-5}, \tag{28}$$

where $\sigma_c$ is the compressive stress when the strain $\varepsilon = \varepsilon_c$ (MPa), $f_c$ is the design value of the axial compressive strength of concrete (MPa), $\varepsilon_0$ is the compressive strain when the strain reaches $f_c$, $f_{cu,k}$ is the standard value of the compressive strength of the concrete cube (MPa), and $n$ is the index (when $n > 2$, $n$ is 2).

Therefore, the strain value of the shaft wall can be obtained by theoretical calculation or field measurement data and compared with $\varepsilon_0$ to evaluate the mechanical condition of the shaft wall.

2.5. Example Verification

2.5.1. Shaft Wall and Surrounding Strata Parameters. The field measurement data of the damaged west ventilation shaft located in the Dongtan coal mine (Shandong Province, P.R. China) were obtained by a series of strainometers placed in the shaft wall during the shaft construction and then were taken as the model verification case of the calculation model of additional force and shaft wall stress. According to the measurement data, a simplified stratigraphic model of geological conditions around the west ventilation shaft is shown in Figure 9.

There are seven aquicludes and six aquifers in this model, and the basic parameters of these layers are listed in Table 1. The inner diameter of the shaft is 3 m, the outer diameter is 3.8 m, the concrete strength and elastic modulus of the shaft are designed as C35 and 31.5 GPa, Poisson’s ratio is 0.25, and the thickness of the overburden is 134.79 m.

2.5.2. Additional Force Analysis. The additional forces of the west ventilation shaft at different times along the depth were obtained by taking the parameters in Table 1 into the calculation formula in Equation (19), and the results are shown in Figure 10.

It can be found that the additional force changes little when drainage starts, and the initial difference value between the bottom and top surfaces of the topsoil is approximately 8 kPa. As the drainage continues over time, the additional force increases significantly, and this difference value increases to 40 kPa at 30 years (a). This result indicates that the additional force difference between the bottom and top surface of the topsoil would be more obvious for a shaft which was built in an earlier time, and collapse usually occurs in a shaft at the bottom surface of the topsoil.

Then, the anisotropic stress of the shaft wall was calculated by the additional force, and the theoretical value of the concrete strain was obtained and compared with the field measurement value.

The comparison of the theoretical value and measurement value at two locations at five times (0a, 2a, 5a, 8a, and 12a) is listed in Table 2.

All the theoretical and measurement values of the concrete strain are shown in Figure 11. A more satisfactory agreement of the theoretical and measurement values indicates the applicability and accuracy of the additional force calculation formula in this study.

2.5.3. Settlement Analysis. To verify the applicability of the calculation formula of settlement, the theoretical (calculated by Equations (10) and (17)) and measurement values of the settlement of this shaft were also obtained and compared, as shown in Table 3 and Figure 12. As seen from the table, the measurement value of settlement due to drainage is roughly
consistent with the theoretical value, and the maximum difference between the theoretical value and the measured value is approximately 4.926 mm from 2005a to 2019a, which can be accepted within the allowable range. This phenomenon also indicates the accuracy of the additional force formula. In addition, the possible reason for the errors in strain verification and settlement verification is as follows: (1) when the concrete strain gauge is applied to the actual project on site, it would be affected by many factors and the measurement value will produce errors; (2) deviation of parameter values in the calculation formula of the additional force, such as geotechnical parameters and hydrologic hole parameters, will lead to deviation of the calculation results.

3. The Effect of Various Factors on Vertical Additional Force

To study the effect of various factors on the vertical additional force, a simple model was established by MATLAB software, and the initial value of each parameter, such as the bottom aquifer thickness, drainage velocity, central aquifer thickness, central aquifer number, and location of the central aquifer, was set. The brief calculation process is as follows:

(1) Shaft wall and strata location definition

The drainage velocity of bottom and central aquifers ($v_i, v_c$) and the breakage ratio ($R_f$) are firstly designed. The number and location of topsoil (including the central aquifer, bottom aquifer, top aquiclude, and central aquiclude) and shaft are chosen.

(2) Shaft wall and strata parameters acquisition

The basic parameters ($r_0, r_i, G_0, r_0, E_y, E_i, E_d, E_t, h_y, h_i, h_d, v_i, v_c$, $c, f$) of each layer can be defined due to the location of each layer and shaft, while the relevant parameters ($a_j, b_j, P_2, \rho_2, C_{2,0}, C_{2,1}, \bar{a}, T_{2,0}, T_{2,1}, a(z), b(z)$) can be calculated by their definitions and the basic parameters above.

(3) Settlement calculation

The amount of settlement of central aquifer, bottom aquifer, top aquiclude, and central aquiclude ($w_i, w_d, w_g, w_j$) can be calculated by substituting the parameters of each layer into Equations (1), (2), (6), and (9) separately, then the settlement of topsoil induced by drainage ($w$) is obtained through Equation (10). The settlement of topsoil induce by additional force ($w_i$) can also be calculated by substituting the interface parameters between shaft and surrounding strata into Equation (17).

(4) Additional force calculation

The relative displacement of the shaft wall and surrounding strata ($\Delta S$) can be calculated through Equation (18), and then, the additional force ($f_a$) is finally obtained through Equation (19).

The parameters are chosen and set: (1) the thickness of topsoil $H = 200$ m, the inner radius of shaft $A = 3$ m, and the outer radius of shaft $B = 4$ m. (2) The surface soil section is divided into four types of soil layers: top aquiclude, central aquifer, central aquiclude, and bottom aquifer. The numbers of central aquifers and central aquicludes are equal. (3) The parameters of the bottom aquifer and central aquifer were taken as the same, i.e., $E_d = E_i = 50$ MPa and $v_y = v_c = 0.004 - 0.024$ MPa (typical parameters are 0.004 MPa). (4) The parameters of the top aquiclude and central aquiclude were taken as the same, i.e., $E_d = E_i = 80$ MPa. (5) The thickness of the bottom aquifer $h_y = 15-35$ m (a typical parameter is 20 m), and the thickness of the central aquifer is $h_i = 10-30$ m (a typical parameter is 20 m). (6) The location of the central aquifer ranges from 0.2 H to 0.7 H (a typical parameter is 0.5 H). (7) The central aquifer number ranges from 0 to 3 (a typical parameter is 1).

3.1. Effect of the Central Aquifer Number. There may be different numbers of central aquifers distributed in the layers surrounding the shaft. To understand the effect of the central aquifer number on the additional force, four kinds of conditions (0, 1, 2, and 3) were selected and set.

Figure 13 shows the relationship between the additional force of shaft and central aquifer number. It can be found that the additional force increases linearly with the increase of depth and service time of shaft when there is no central aquifer in the topsoil, while the maximum additional force is approximately 30.4 kPa. However, the additional force shows different increasing behaviours when the central aquifer appears. The maximum additional force shows a dramatic increase when the central aquifer number grows. For example, the maximum additional force ranges from 30.4 kPa to 113.2 kPa when one central aquifer exists, and this value increases to 272 kPa when there are three central aquifers in the topsoil. These phenomena indicate that the existence of central aquifer would lead to a greater settlement amount of topsoil during the drainage, which further result into larger additional force. In addition, the average enhancing magnitude of additional force due to the increase of central aquifer number gradually rises from 270% to 302% when this number ranges from 1 to 3. This result indicates that the effect of the central aquifer on the additional force would be enhanced by its number since greater settlement amount would be induced by more serious drainage conditions. Besides, the additional force increases with increasing depth and service time, while these two increasing rates both reduce. The probable reason for this phenomenon may be that the drainage settlement would lead to consolidation, while the settlement would continue to decrease; thus, the additional force would decrease following the decrease in settlement.

3.2. Effect of Drainage Velocity. Drainage is usually known as an important reason for topsoil settlement; thus, studying the effect of drainage velocity on the additional force is of great significance to enable the prediction of shaft breakage.
In this study, six kinds of drain velocities (0.004, 0.008, 0.012, 0.016, 0.020, and 0.024 MPa/a) were selected and set, and the drainage velocities of the bottom aquifer and central aquifer were the same.

Figure 14 presents the relationship between the additional force of shaft and drainage velocity. It can be observed that the drainage velocity also shows an important role in the increase of additional force, i.e., the greater the drained velocity is, the greater the additional force. For example, the maximum additional force varies from 113.5 kPa to 131.9 kPa when the drainage velocity increases from 0.004 MPa/a to 0.008 MPa/a, and this value finally increases to 200.5 kPa when the drainage velocity increases to 0.024 MPa/a. It may be due to that the total water level loss rises with the increase of drainage velocity, which induces greater compression-consolidation settlement of surrounding strata and then leads to larger additional force. Besides, the increasing rate of the additional force slightly reduces with increasing of drainage velocity. Although the amplification effect on the additional force induced by the increase of drainage velocity is smaller than that of the central aquifer number, this result indicates that the effect of drainage velocity on the additional force should be noticed highly. According to the results in Figure 14(g), the calculation formula of the additional force as a function of time and drainage velocity is shown below:

$$f(v, t) = 0.00097 + 0.1998v + 0.0105t + 0.3038vt - 0.000272t^2,$$

where the R-square value (determination coefficient) of the fitting formula is 0.9997.

3.3. Effect of the Location of Central Aquifer. The location of the central aquifer may transform with the change in geological conditions surrounding the shaft. Thus, to study the effect of the location of the central aquifer on the additional force, six kinds of depths of the central aquifer (0.2, 0.3, 0.4, 0.5, 0.6, and 0.7 H) were selected and set.

The relationship between the additional force and location of central aquifer is shown in Figure 15. The additional force increases when the central aquifer moves down. One possible reason for this phenomenon is that when the central aquifer is close to the bottom, the corresponding settlement of upper aquicludes increases, and the whole topsoil settlement grows, which causes an increase in the additional force. Besides, the average increasing rate of additional force also magnifies with the down movement of the central aquifer, which increases from 13.1% to 16.2% when the location ranges from 0.2 H to 0.7 H. This may be due to that when the central aquifer is close to the bottom aquifer, the codrainage effect would be enhanced and then leads to more serious settlement of topsoil. In addition, it can be found that the effect of the location of the central aquifer on the additional force is slightly lower than that of the drainage velocity and higher than that of the central aquifer thickness and bottom aquifer thickness shown in Figures 16 and 17. Thus, it is necessary to carefully investigate the geological situation of the site to determine the location of the central aquifer, and the pre-pumping method can also be carried out on the central aquifer.

A simple fitting model was also adopted according to the results in Figure 15(g). The calculation formula of the additional force as a function of time and location of the central aquifer is shown below:

$$f(H, t) = -13.144 + 5.1835H + 8.042t + 0.616Ht - 0.207t^2,$$

where the R-square value (determination coefficient) of the fitting formula is 0.9937 and $H$ represents the ratio of central aquifer depth to total soil thickness.

3.4. Effect of Central Aquifer Thickness. The central aquifer thickness would also influence the additional force by affecting the settlement. Thus, in this study, six thickness values (10, 15, 20, 25, 30, and 35 m) were selected and set to study the effect of central aquifer thickness on the additional force.

The relationship between the additional force and central aquifer thickness is presented in Figure 16. The results show that the increase in central aquifer thickness also slightly enhances the additional force. For instance, the maximum additional force transforms from 105.1 kPa to 125.1 kPa when the central aquifer thickness increases from 10 m to 35 m. The increase of central aquifer thickness provides more space for drainage settlement and further leads to greater additional force. The effect of central aquifer thickness on the additional force is relatively lower than that of the central aquifer number, drainage velocity, and location of central aquifer. Besides, the increment of central aquifer thickness is proportional to that of additional force, and the average increasing ratio of additional force is approximately 0.8. The existence of central aquifer plays a dominant role in the additional force increase of shaft, while the expansion of central aquifer thickness would play a supporting role through the comparison of the results in Figures 13 and 16. Additionally, to further exhibit the effect of the central aquifer thickness on the additional force, a simple fitting model was adopted according to the results in Figure 16(g). The calculation formula of the additional force as a function of time and central aquifer thickness is shown below:

$$f(h_c, t) = -0.07961 + 0.1631h_c + 10.11t + 0.04841h_c t - 0.2372t^2,$$

where the R-square value (determination coefficient) of the fitting formula is 0.9995.

3.5. Effect of Bottom Aquifer Thickness. The bottom aquifer thickness may transform with the change in geological conditions surrounding the shaft. Thus, to study the effect of the bottom aquifer thickness on the additional force, considering the actual measurement thickness data, six thickness values (10, 15, 20, 25, 30, and 35 m) were selected and set.

Figure 17 shows the relationship between the additional force and bottom aquifer thickness, and it can be observed...
that the increasing of bottom aquifer thickness enhances the additional force when other conditions are maintained. This may be because the settlement of surrounding topsoil is related to the bottom aquifer thickness, and the increasing of bottom aquifer thickness would lead to the growth of settlement, which strengthens the additional force. Besides, similar to the effect of central aquifer thickness, the increment of bottom aquifer thickness is also proportional to that of additional force, while the average increasing ratio of additional force is approximately 0.6. In addition, to further exhibit the effect of bottom aquifer thickness on the additional force, a simple fitting model was adopted according to the results in Figure 17(g). The calculation formula of the additional force as a function of time and bottom aquifer thickness is shown below:

$$f(h_d, t) = -0.9088 + 0.2168h_d + 10.5t + 0.02649h_d t - 0.2361t^2,$$

(32)

where the R-square value (determination coefficient) of the fitting formula is 0.9994.

4. Conclusion

In this paper, a calculation model of the vertical additional force and the shaft wall stress was built, which was also verified by the filed measurement data of a damaged shaft. The effects of various factors, such as central aquifer thickness and number, drainage velocity, and bottom aquifer thickness, on vertical additional force were obtained and concluded. The detailed conclusions are listed below:

1. First, a mechanical model of a shaft wall considering multi-aquifer drainage conditions is simplified and conducted, and the calculation formula of strata settlement is proposed by summarizing the settlement of each aquifer or aquiclue. Then, the calculation model of the additional force is obtained based on the theoretical analysis of settlement induced by drainage consolidation and vertical additional force using the pile foundation theory. In addition, the mechanical model of shaft wall stress is established, and the calculation formula of shaft wall stress under the action of dead weight, lateral pressure, and additional force is obtained. Verification of this model is conducted by comparing the measurement data of concrete strain of the damaged west ventilation shaft and the theoretical data calculated by the stress model.

2. The additional force increases with increasing depth and service time of shaft, while these two increasing rates both reduce. The additional force becomes greater when the central aquifer thickness and number, drainage velocity, and bottom aquifer thickness increases, while the down movement of the location of central aquifer can also enhance the additional force. The average enhancing magnitude of additional force rises when the central aquifer number increases and its location moves down, while the increment of central and bottom aquifer thickness is proportional to that of additional force. Combined with the comprehensive analysis of the effect of each factor on the additional force, it is concluded that the order of importance of the factors affecting the magnitude of the additional force is approximately as follows: central aquifer number > drainage velocity > location of central aquifer > central aquifer thickness > bottom aquifer thickness.

This study is aimed at providing a calculation model for additional force prediction of shaft induced by drainage settlement of surrounding topsoil containing multi-aquifers. The numerical results show the existence of central aquifers significantly affects the amplitude of additional force of shaft during drainage, while the order of importance of other factors affecting the magnitude of the additional force was concluded and analysed. This paper is expected to provide prediction and protection for the security of vertical shaft.

Data Availability

The (code) data used to support the findings of this study were supplied by Tao Han under license and so cannot be made freely available. Requests for access to these data should be made to Tao Han (hantaoctumt@163.com).

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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