Research Article

A Study on the Generation Mechanism and Development Process of Piping Based on the Theory of Muddy Water Seepage

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In the process of piping development, the sediment carried by seepage causes the change of porosity and then causes the change of permeability coefficient. Therefore, piping belongs to muddy water seepage (i.e., unstable seepage). Based on the theory of muddy water seepage, the new control equation of piping model is established. The generation mechanism and development process of piping for white fine sand and sand gravel are explored. The advantages for the new control equation of piping model are discussed. The results show that the new control equation of piping model is not only simpler in form but also easier to solve and can better describe the characteristics of piping generation, development, and stability. Compared with white fine sand (the initial relative density and porosity are smaller, and the initial permeability coefficient and critical gradient are larger), the porosity, permeability coefficient, seepage flow, and sediment volume of sandy gravel increase faster with time. When the porosity is the same, the permeability coefficient, seepage flow, and sediment volume of sand gravel are larger. The smaller the area affected by piping, the faster the increase of porosity, permeability coefficient, seepage flow, and sediment volume. When the porosity and permeability coefficient are the same, the seepage flow and sediment volume are smaller. The smaller the initial porosity and permeability coefficient, the later the generation, and development of piping. It can provide a reference for piping danger forecast and emergency response.

1. Introduction

The embankment is an important part of flood control engineering system in China, and it is also the last barrier against flood. However, piping is the main reason for the danger of dams during flood. The piping phenomenon was simulated by Jiang and Hu [1] via the discrete element method (DEM). The development process of piping in levee foundation was simulated by Zhou et al. [2, 3] using the coupling method of seepage pipe. This method is more reasonable in theory than the traditional finite element method and will not increase the calculation cost as much as the discrete element method. The influence of the length and permeability coefficient of piping area on the seepage field was analyzed by Yin [4] with the finite element method, but the damage for the front end of piping development was not involved. The generation mechanism of piping was studied by Ni et al. [5] from microlevel, the piping phenomenon was simulated by the particle flow program, and better results were achieved. Combined with the small-scale micromodel test, the occurrence and development process of piping was simulated by Zhou et al. [6] using the particle flow method based on bulk medium theory and considering the effect of fluid-solid coupling. Taking advantage of the flume model test, the erosion process of piping in the single and double layer of levee foundations were investigated, and the corresponding generalized mathematical models and numerical simulation methods were established by Liu et al. [7, 8]. The sediment transport balance equation of piping channel for the typical “binary embankment foundation” was established by Wu et al. [9]. Combined with the results of sand tank model test, the seepage field and the channel length with time were calculated and analyzed in the development process of piping. The capillary mathematical model of
piping and blockage of soil embankment on clay foundation were established by Khilar et al. [10], and the processes of deposition and diffusion for clay particles in soil pores were described. Meanwhile, the capillary model describing the occurrence and development of piping was established by Liu et al. [11] via analyzing the force of fine particles in skeleton pores. The unsteady seepage model test for the piping failure of embankments during flood peak and tidal amplitude variation was conducted by Mao et al. [12], the model similarity law and time scale calculation of sand tank model test were discussed, and the time reference value for the development of piping danger was gave. The generation mechanism of piping was deeply analyzed by Chen et al. [13], the corresponding calculation model was put forward and discussed, and the detailed discrimination conditions for the fine particles movement and mathematical simulation during piping were put forward. Subsequently, the failure process of piping for the multilayer embankment foundations of three kinds of sand layers was simulated by Chen et al. [14, 15] using the indoor sand tank model test and changing the grading composition of the underlying sand layer, and the influence of sand layers with different particle grading compositions on the generation and development process of piping was studied. The time lag effect of soil generating piping under the action of flood peak was proposed by Wei [16] via discussing the local piping phenomenon of earth rock dams during flood. In addition, the seepage characteristics of liquid nitrogen, gas, and methane in rock and soil were investigated by Xue et al. [17–19].

Overall, for the research on the generation and development mechanism of piping, most scholars carried out relevant research under the action of stable seepage, while the study of piping under the action of unstable seepage was relatively few. However, the main factors affecting the development of piping are porosity and permeability coefficient, which change with time. Therefore, the piping is actually an unstable seepage (muddy water seepage). It is considered that the sediment taken away in the process of muddy water seepage causes the change of porosity and then causes the change of permeability coefficient, and the increase of porosity and permeability coefficient further strengthens the ability of seepage to carry sediment, which is the process of mutual influence and interaction. In view of this, based on the Darcy’s law and combined with the theory of muddy water seepage proposed by Dang et al. [20], the new control equation of piping model is established via mathematical methods. The generation mechanism and development process of piping for white fine sand and sand gravel are explored. The advantages of the new control equation of piping model are discussed. The research results can provide a reference for piping danger forecast and emergency response.

2. Control Equation of Piping Model Based on the Theory of Muddy Water Seepage

Assuming that the seepage in saturated soils meets Darcy’s law, the relationship between seepage velocity and soil particle shape and size, liquid property, porosity, and hydraulic gradient is as follows [21]:

\[ v = \frac{\beta_\lambda}{\lambda \mu} \frac{d^2}{1 - n} i, \]  \hspace{1cm} (1)

where \( v \) is the seepage velocity of water (cm/s); \( i \) is the hydraulic gradient of soil; \( n \) is the porosity of soil; \( \gamma_w \) is the bulk weight of water (g/cm\(^3\)); \( \mu \) is the dynamic viscosity coefficient of water (g/s/cm\(^3\)); \( \beta \) is a coefficient, depending on the influence of adjacent particles; \( \beta = \pi/6 \) for a ball; and \( d \) is the diameter of soil particles (cm).

According to Equation (1), the relationship between permeability coefficient \( k \) and physical parameters of soil and fluid is as follows:

\[ k = \frac{\beta d^2 \gamma_w}{\lambda \mu} n^2 \left(1 - n\right). \]  \hspace{1cm} (2)

According to the survey data, the initial permeability coefficient \( k_0 \) and initial porosity \( n_0 \) can be obtained. Substituting them into Equation (2), it can be rewritten as

\[ k_0 \frac{n_0}{n^2} = \frac{\beta d^2 \gamma_w}{\lambda \mu}. \]  \hspace{1cm} (3)

Substituting Equation (3) into Equation (2), the relationship between permeability coefficient \( k(t) \) at time \( t \) and initial void ratio \( e_0 \) and porosity \( n(t) \) at time \( t \) is obtained as follows:

\[ k(t) = k_0 \frac{1 - n_0}{n_0} \frac{n \left(1 - n(t)\right)^2}{1 - n(t)}. \]  \hspace{1cm} (4)

Using this formula, other micromechanical parameters of soil can be replaced with the initial permeability coefficient and porosity of soil, which simplifies the process of calculation. It is assumed that the total volume \( V \) of soil in the process of muddy water seepage remains invariant, and the muddy water seepage only leads to the change of porosity in the area affected by piping. Thus, the porosity \( n(t) \) can be obtained as follows:

\[ n(t) = \frac{Q_s(t) + V_0}{V} = \frac{Q_s(t)}{V} + n_0, \]  \hspace{1cm} (5)

where \( n(t) \) is the porosity of area within the influence of piping at time \( t \), \( V \) is the total volume of area within the influence of piping, \( V_0 \) is the total volume of area within the influence of piping at the initial time \( t_0 \), and \( Q_s(t) \) is the sediment volume in water seepage from the piping outlet at time \( t \).
The total seepage flow per unit area after time for muddy water seepage is as follows:

\[ Q(t) = \int_0^T k(t) \, idt. \]  
\[ (6) \]

Assuming that the volume content percentage of sediment taken away by muddy water seepage is \( \alpha \), the sediment volume \( Q_s(t) \) is obtained as follows:

\[ Q_s(t) = \alpha Q(t). \]  
\[ (7) \]

Substituting Equation (7) into Equation (5) and then substituting it into Equation (6), the equation of muddy water seepage is obtained as follows:

\[ Q(t) = \int_0^T k(t) \, idt. \]  
\[ k_0 \left( 1 - n_0 \right) \left[ aQ(t)/V + n_0 \right]^2 \frac{1}{1 - n_0 - aQ(t)/V} idt. \]
\[ (8) \]

The integral equation is not easy to be solved, and its differential form is derived. Assuming that the muddy water seepage problem can be regarded as a stable seepage problem in a tiny time period, the seepage flow per unit area in this tiny time period is as follows:

\[ Q(t + \Delta t) - Q(t) = k(t) i \Delta t. \]  
\[ (9) \]

Both sides of the equation are divided by \( \Delta t \); Equation (9) can be rewritten as follows:

\[ \frac{Q(t + \Delta t) - Q(t)}{\Delta t} = k(t) i. \]  
\[ (10) \]

Taking the limit of \( \Delta t \) for the both sides of Equation (10), the equation of muddy water seepage in differential form is obtained as follows:

\[ Q'(t) = k(t) i = k_0 \frac{1 - n_0 \left[ aQ(t)/V + n_0 \right]^2}{n_0^2} \frac{1}{1 - n_0 - aQ(t)/V} \frac{1}{i}. \]  
\[ (11) \]

This differential equation can be solved numerically by the difference method. The difference format is as follows:

\[ Q(t + \Delta t) = k_0 \frac{1 - n_0 \left[ aQ(t)/V + n_0 \right]^2}{n_0^2} \frac{1}{1 - n_0 - aQ(t)/V} \frac{1}{i} \Delta t + Q(t) \Delta t. \]  
\[ (12) \]
The form of Equation (12) is simple. As long as the stable seepage flow $Q(t_0)$ of area within the influence of piping at $t_0$ time is known, the seepage flow at the next moment can be calculated. Then, the porosity $n(t)$ and permeability coefficient $k(t)$ of area within the influence of piping can be obtained. However, the difficulty lies in how to determine the stable seepage flow $Q(t_0)$ of area within the influence of piping at $t_0$ time.

3. Application for the New Control Equation of Piping Model

It is assumed that the generation mechanism of piping is that the sand particles at the front end of the piping channel are unstable under the action of seepage force, then are taken away by seepage flow, and gradually develop into the piping channel until centralized seepage channels are formed. Therefore, only when the hydraulic gradient reaches the critical hydraulic gradient of soil, the soil can produce piping phenomenon. The formula proposed by Terzaghi [22] $i_{cr} = (G_s - 1)(1 - n)$ is used to estimate the critical hydraulic gradient of soil in this manuscript.

3.1. Solution Process of Equation. According to the physical and mechanical parameters of soil given in reference [23], the stable seepage flow or the critical seepage flow $Q(t_0) = k_0 \cdot i_{cr} \cdot A_0$ (where $A_0$ is the cross-sectional area of the piping affected, which is mainly influenced by the porosity or permeability coefficient and critical hydraulic gradient of soil; the values of $A_0$ were assumed to be 50, 75, and 100 cm$^2$, respectively, in this manuscript; and the influences of $A_0$ on the generation and development of piping were discussed in this manuscript). That is to say, as long as the initial permeability coefficient $k_0$ and critical hydraulic gradient $i_{cr}$ of soil are determined, the critical seepage flow $Q(t_0)$ can be determined. And the initial permeability coefficient $k_0$ and critical hydraulic gradient $i_{cr}$ of soil mass can be obtained from survey data or laboratory tests. The seepage flow $Q(t)$ at $t$ time can be obtained by substituting them into Equation (12). The sediment volume $Q_s(t)$ can be obtained by substituting $Q(t)$ into Equation (7). Then, the porosity $n(t)$ of the area within the influence of piping at $t$ time can be calculated by substituting $Q_s(t)$ into Equation (5). Finally, the permeability coefficient $k(t)$ of area within the influence of piping at $t$ time can be calculated by substituting $n(t)$ into Equation (4). The specific process with solving the control equation of piping model based on the theory of muddy water seepage is shown in Figure 1 as follows:

3.2. Calculation Results and Analysis. According to the new control equation of piping model, combined with the physical parameters of two kinds of soil (as shown in Table 1), the generation mechanism and development process of piping for white fine sand and sand gravel are explored. It should be noted that when the new control equation of piping model is solved, the attention should be paid to the time step $\Delta t = t_i - t_{i-1}$ during the process of cyclic calculation, which should be as small as possible. The specific research results are as follows:

Figures 2 and 3 are the variation curves between porosity, permeability coefficient and time at the free surface of soil

![Figure 2: Variation curve between porosity and time at the free surface of soil with different $A_0$.](image1)

![Figure 3: Variation curve between porosity and time at the free surface of soil with different $A_0$.](image2)

### Table 1: Physical and mechanical parameters of materials [23].

<table>
<thead>
<tr>
<th>Materials</th>
<th>Relative density $G_s$</th>
<th>Initial porosity $n_0$</th>
<th>Initial permeability coefficient $k_0$ (cm/s)</th>
<th>Critical hydraulic gradient $i_{cr}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>White fine sand</td>
<td>2.68</td>
<td>0.48</td>
<td>$7.2 \times 10^{-4}$</td>
<td>0.87</td>
</tr>
<tr>
<td>Sand gravel</td>
<td>2.65</td>
<td>0.21</td>
<td>$8.7 \times 10^{-3}$</td>
<td>1.30</td>
</tr>
</tbody>
</table>
with different $A_0$, respectively. The porosity and permeability coefficient at the free surface of soil increase exponentially with time. The results calculated by the new control equation of piping model based on the theory of muddy water seepage are basically consistent with the previous research results and variation trends [23]. It is shown that the new control equation of piping model established by this manuscript is reasonable and has a certain reference value. According to Figure 2, the porosity at the free surface of soil changes little (only accounting for 1.24%) during the initial stage of piping (0 s $\sim$ 3000 s for white fine sand and 0 s $\sim$ 250 s for sand gravel). With the development of piping, the porosity gradually increases. The growth gradient of porosity becomes larger and larger. The centralized channel of piping is generated. The porosity tends to be stable, which will remain unchanged finally. According to Figure 3, the permeability coefficient at the free surface of soil changes little during the initial stage of piping. But with the increase of porosity, the permeability coefficient gradually increases, increases rapidly during the middle and later stages of piping, and will eventually tend to be stable. This is because a few soil particles are taken away from the connected pore channel during the initial stage of piping. With the continuous development of piping, the porosity increases. The permeability coefficient increases. The seepage velocity increases. The soil particle number taken away increases, and the soil particle size also increases. Thus, the pore channel diameter increases rapidly. In this cycle, it finally becomes stable during a certain period of time. If the corresponding treatments are not carried out in time, the seepage failure will occur. The seepage failure will occur. Compared with white fine sand, the porosity and permeability coefficient of sand gravel increase faster. When the porosity is the same, the permeability coefficient of sand
gravel is larger. The smaller the area affected by piping, the faster the increase of porosity and permeability coefficient at the free surface of soil.

Figures 4 and 5 are the variation curves between porosity, permeability coefficient and time at the free surface of soil with different \( k_0 \) and \( n_0 \), respectively. According to Figures 4 and 5, the porosity and permeability coefficient at the free surface of soil change little during the initial stage of piping. With the development of piping, the porosity and permeability coefficient gradually increase. Whether it is white fine sand or sand gravel, the smaller the initial porosity and permeability coefficient, the later the generation and development of piping.

Figures 6 and 7 are the variation curves between seepage flow, sediment volume and time at the free surface of soil with different \( A_0 \), respectively. The seepage flow and sediment volume at the free surface of soil increase exponentially with time. According to Figure 6, the seepage flow at the free surface of soil increases linearly with time during the initial stage of piping. It indirectly reflects the changes of porosity and permeability change are very small (it approximately belongs to stable seepage). With the continuous development of piping, the seepage flow at the free surface of soil increases with time from linear to nonlinear, and the gradient of curve is more and more steep. It indirectly reflects the porosity and permeability coefficient are larger and larger (it
belongs to unstable seepage). According to Figure 7, the sediment volume at the free surface of soil changes little during the initial stage of piping (it approximately belongs to clear water seepage), which shows again that the porosity and permeability coefficient are almost unchanged. With the continuous development of piping, the sediment volume at the free surface of soil increases sharply, which shows again that the porosity and permeability coefficient are larger and larger (it belongs to muddy water seepage). Compared with white fine sand, the seepage flow and sediment volume of sand gravel increase faster. When the porosity is the same, the seepage flow and sediment volume of sand gravel are larger. The smaller the area affected by piping, the faster the increase of seepage flow and sediment volume at the free surface of soil, and the smaller the seepage flow and sediment volume when the porosity and permeability coefficient are same.

Overall, the best time to repair piping failure is the early stage of development for piping. Once this time is missed, it will be more and more difficult to repair.

4. Advantages for the New Control Equation of Piping Model

The typical control equation of piping [24] did not consider the erosion process of piping. Although many scholars later introduced the erosion constitutive equation [23, 25], they did not consider the change for the porosity or permeability coefficient of soil during piping. Based on fully considering the generation, development, and stability of piping, the new control equation of piping model based on the theory of muddy water seepage is established. The equation is not only simpler in form but also easier to solve, which can
reveal the characteristics for the generation, development, and stability process of piping. The new control equation of piping model has obvious advantages. The specific advantages are discussed as follows.

4.1. Considering the Time Effect and Introducing the n(t) and k(t). Piping actually belongs to muddy water seepage. With the phenomenon of sediment taken away by seepage, the centralized seepage channel will be caused directly by piping. Because the small amount of sediment is taken away by seepage during the early stage of piping, it is still reasonable and desirable to use the theory of clear water seepage to calculate at this time. However, with the continuous development of piping, the seepage speed increases. Thus, the sediment volume taken away by seepage will increase sharply. If the theory of clear water seepage is used again to calculate at this time, it will have a great impact on the calculation results. In fact, the development process of piping is the function of time. The longer the generation of piping, the larger the porosity of soil, and the larger the permeability coefficient and seepage velocity. The increase of seepage velocity will enhance the ability to erode the wall of piping channel. Then, the sediment volume taken away by seepage will increase. This is the complex process of mutual correlation and mutual promotion. Therefore, the porosity and permeability coefficient of soil in the area affected by piping change with time, so it is more reasonable to introduce n(t) and k(t).

4.2. Introducing the Volume Content Percentage of Sediment α. The sediment taken away by muddy water seepage causes the increase for the porosity and permeability coefficient of soil in the area affected by piping. Then, the seepage velocity increases. This cycle accelerates the formation of piping concentration channel. When the sediment in the area affected by piping is taken away by seepage water, a weak area will occur in the soil, which cannot be considered in the traditional theory of clear water seepage. But the theory of muddy water seepage considers the influence for the change of sediment volume during the development process of piping. It is mainly reflected on the changes for the porosity and permeability coefficient of soil. According to the sediment dynamics and reference [26], the sediment volume taken away by seepage is greatly related to the seepage velocity, which is generally proportional to the power of 1.0–3.0 of the seepage velocity. When α is 1.0, it means that it is in the porous medium with laminar flow. When α is 3.0, it means that it is in a pipe or crack without filler. These are two limits. Once piping generates, α gradually increases from 1.0 to 3.0 with the increase of permeability coefficient. Based on the comprehensive consideration of permeability coefficient and sediment content and combined with the theory of muddy water seepage, it is considered that the sediment volume taken away by seepage is proportional to the quadratic power of seepage velocity. That is $\alpha = \eta \cdot \left(\frac{Q(t)}{A_0}\right)^2$, where $\eta$ is a coefficient (in this manuscript, $\eta$ is assumed to be 1.0).

4.3. Wide Application for the New Control Equation of Piping Model. Many control equations of piping can be only applied to a single homogeneous soil. This limits the applicability scope for the control equations of these piping models. The new control equation of piping model established by this manuscript can overcome the disadvantages. It is not only suitable for single homogeneous soil but also can be extended to different soil layers via equivalent permeability coefficient method. In addition, the new control equation of piping model is applicable not only to the two-dimensional model but also to the three-dimensional model. The difference between the former and the latter is as follows: $V = A_0$ in the case of the two-dimensional model, $V = A_0 \cdot L$ in the case of the three-dimensional model. In addition, the old control equation of piping model cannot consider the changes for the porosity and permeability coefficient of soil in the area affected by piping. The n(t), k(t), and α are introduced by the new control equation of piping model. The new control equation of piping model is more consistent with the actual development process of piping, and the calculation results are more accurate.

5. Conclusions

Based on the theory of muddy water seepage, the new control equation of piping model is derived. The generation mechanism and development process of piping for white fine sand and sand gravel are explored. The advantages for the new control equation of piping model are discussed. The main conclusions are as follows:

1. Based on the theory of muddy water seepage, the porosity n(t), permeability coefficient k(t), and volume content percentage of sediment taken away by muddy water seepage α are selected to replace the erosion equation. The new control equation of piping model is established via combining with mathematical methods. The equation is not only simpler in form but also easier to solve, which can reveal the characteristics for the generation, development and stability process characteristics of piping.

2. The porosity, permeability coefficient, and sediment volume at the free surface of soil are almost unchanged at the initial stage of piping, and the seepage flow increases linearly (it belongs to clear water seepage). With the continuous development of piping, the porosity, permeability coefficient, seepage flow, and sediment volume increase sharply (it belongs to muddy water seepage) and will eventually tend to be stable. The calculation results for the new control equation of piping model are consistent with the development law of piping.

3. Compared with white fine sand, the porosity, permeability coefficient, seepage flow, and sediment of sand gravel increase faster with time. When the porosity is the same, the permeability coefficient, seepage flow, and sediment volume of sand gravel are larger. The smaller the area affected by piping, the faster the increase of porosity, permeability coefficient, seepage flow, and sediment volume. When the porosity and permeability coefficient are same,
the smaller the seepage flow and sediment volume. The smaller the initial porosity and permeability coefficient, the later the generation and development of piping.

**Data Availability**

The data used to support the findings of this study are included within the article.

**Conflicts of Interest**

The authors declare that no conflict of interest exists in this manuscript.

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