

## Research Article

# A Binary Medium Model for Frozen Silty Sand Simplified by Breakage Parameter

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In order to investigate the strength-deformation characteristics of frozen silty sand, the triaxial compressive strength tests of saturated frozen silty sand under different fine particle contents were carried out, and the binary medium theory was introduced to interpret the stress-strain relationship. Due to the characteristics of the existing binary medium model with many parameters and complicated determination method, a simplified binary medium model based on breakage parameter is proposed. The derived model was verified by the triaxial tests of frozen silty sand. The results show that the stress-strain relationship can be divided into three stages with the increase of axial strain, namely, linear elastic deformation stage, plastic deformation stage, and strain softening stage. All three stages can be well explained by the transformation theory of bonded element and frictional element with the binary medium model. In the linear elastic deformation stage, the external stress is mainly borne by the bonded element. In the plastic deformation stage, the stress sharing ratio of the bonded element decreases and that of the frictional element increases. In the strain softening stage, the stress sharing ratio of the bonded element decreases rapidly, while that of the frictional element increases rapidly. Under the same confining pressure, both deviator stress and the maximum values of bulk expansion decrease, while the shear strength decreases linearly with the increase of fine particle content. By comparing the measured deviator stress in triaxial test with the calculated values of binary medium constitutive model simplified by breakage parameter, the proposed model can better simulate the stress-strain relationship of frozen silty sand. The results of the study can provide some theoretical reference for the constitutive model of seasonal frozen soil.

## 1. Introduction

Frozen soil is a four-phase system composed of mineral particles, pore ice, unfrozen water, and air. Due to the restriction of cemented ice, the mechanical behavior of frozen soil becomes more complex after being stressed [1]. Among them, the strength of frozen soil is one of the important mechanical properties of frozen soil, and it is the ability of frozen soil to resist external damage. In the construction of hydraulic engineering, traffic engineering, and industrial and civil buildings in permafrost areas, it is often necessary to check the strength of permafrost and calculate its defor-

mation [2]. The constitutive model of frozen soil is the key to analyze the stress-strain relationship of frozen soil, and it is also an important basis for determining whether the various links of frozen soil engineering construction in cold regions are reasonable [3]. At present, the constitutive models of frozen soil mainly include nonlinear elastic constitutive model based on generalized Hooke's law [4–6] and elastic-plastic constitutive model based on incremental elastic-plastic theory [7–9].

The shear strength of geotechnical materials consists of cohesion and internal friction, but these two parts of shear strength do not play a role at the same time. Cohesion can

reach its peak when deformation is small, while friction can play its full role only when deformation is sufficient. Thus, cohesion has brittle property and friction has plastic property [10, 11]. Because plastic mechanics, fracture mechanics, and damage mechanics cannot reasonably describe the brittle damage of geotechnical materials, Shen [10, 11] put forward the theory of geotechnical damage mechanics based on quasi-continuous medium hypothesis, brittle elastic damage hypothesis, and shared hypothesis. In this theory, structural rock and soil are abstracted as a binary medium composed of strongly cemented structural body and weakly cemented weak zone. During the deformation process, the structural body gradually transforms into weak zone, and they share the external load [12]. At present, many scholars have applied the binary medium model to a variety of geotechnical materials. Liu et al. [13] established a binary medium model of rock materials and compared it with the triaxial test of sandstone samples. In order to study the structure and deformation characteristics of fractured loess, Fan et al. [14] regarded it as a binary medium composed of cemented block and weak zone and obtained the single parameter stress-strain relationship of fractured loess. Lu et al. [15] established a two-parameter binary medium model of fractured loess and compared it with the results of triaxial test. Li et al. [16] simulated the development process of shear zone in structured soil under plane strain compression by using two-parameter binary medium model and numerical analysis method. Li et al. [17] established the constitutive model of saturated loess by combining the unified strength theory, binary medium model, and equivalent strain principle. Liu and Shen [18] simulated the stress-strain relationship of rock samples during unloading by using the binary medium model and analyzed the damage development process and displacement state of geotechnical materials under different lateral stress states through numerical calculation. Zhou et al. [19], based on the binary medium model and the concept of series-parallel connection in physics, derived the series-parallel stress model of soil-rock mixture considering the influence of particle fragmentation. Wu et al. [20] established a generalized Hardin model based on the binary medium model, which is used to evaluate the small strain shear modulus of sandy soil under different effective confining pressure, void ratio, and fine particle content. Based on the triaxial compression tests, the disturbed state concept, and homogenization theory, Liu et al. [21] proposed a binary medium constitutive model for artificially structured soils. When it comes to laboratory study of frozen soils, some scholars have made a lot of research in this field. Under different confining pressures and temperatures, Luo et al. [22] carried out a series of cryogenic triaxial compression tests to study the strength properties of frozen moraine soils. In order to describe the effect of particle breakage of frozen sandy soil on the stress-stain relationships, He et al. [23] proposed an elastoplastic constitutive model for frozen sandy soil with a series of triaxial compression tests. Zhu et al. [24] carried out the triaxial creep tests on the frozen sand under different conditions to study the temperature, dry density, and particle size distribution on the creep properties. Li et al. [25] carried out strain-

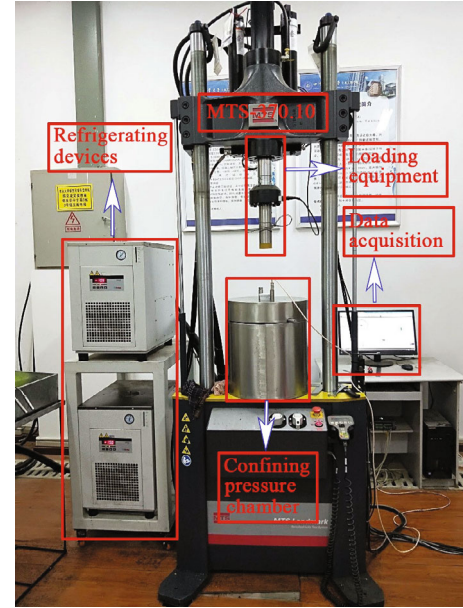


FIGURE 1: MTS-Landmark 370.10 triaxial testing instrument.

TABLE 1: Basic physical parameters of natural silty sand.

Properties	Unit	Value
Uniformity coefficient	—	1.90
Curvature coefficient	—	0.95
Specific gravity	$\text{g/cm}^3$	2.64
Permeability coefficient	$\text{cm/s}$	$6.98\text{e-}3$
Freezing temperature	$^{\circ}\text{C}$	-0.09
10 mm liquid limit	%	22.14
Plastic limit	%	10.61
Plasticity index	—	11.53

controlled monotonic triaxial tests and stress-controlled cyclic triaxial tests to study the influence of deviator stress history on the cyclic behavior of compacted frozen soils.

At present, few articles have made relevant analysis on the research of binary medium model and damage mechanism of frozen soil. Zhang et al. [26–28] took frozen silt at  $-6^{\circ}\text{C}$  as an example, introduced binary medium model to discuss the stress-strain relationship of frozen silt, and verified and predicted the proposed model combined with triaxial compression test of frozen soil. Based on test results of creep deformation of frozen soils, Wang et al. [29] discussed the creep breakage mechanisms of frozen soils by the concept of binary medium model and put forward a new binary medium creep constitutive model for frozen soils. Based on the meso-mechanics and homogenization theory, Liu et al. [30] proposed a constitutive model for tailing soils under freeze-thaw cycles. Combined with the rock failure characteristics and the binary medium concept, Li et al. [31] studied the load sharing of frozen red sandstone. Frozen soil is a special kind of soil with stiffness between soft soil and rock. Frozen soil skeleton is used as bonded element and thawed

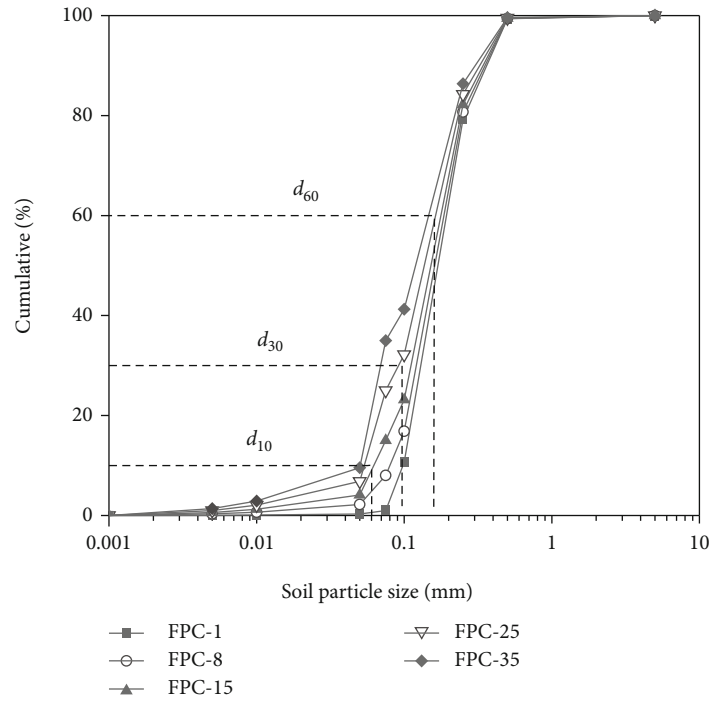


FIGURE 2: Particle sized distribution curves of silty sand with different silty particle content.

TABLE 2: Uniformity coefficient and curvature coefficient of 5 kinds silty sand.

Properties	FPC-1	FPC-8	FPC-15	FPC-25	FPC-35
Uniformity coefficient	1.90	2.21	2.56	2.70	2.83
Curvature coefficient	0.95	0.99	1.06	0.93	0.67



(a) Before demoulding



(b) After demoulding

FIGURE 3: Shear failure mode of soil sample.

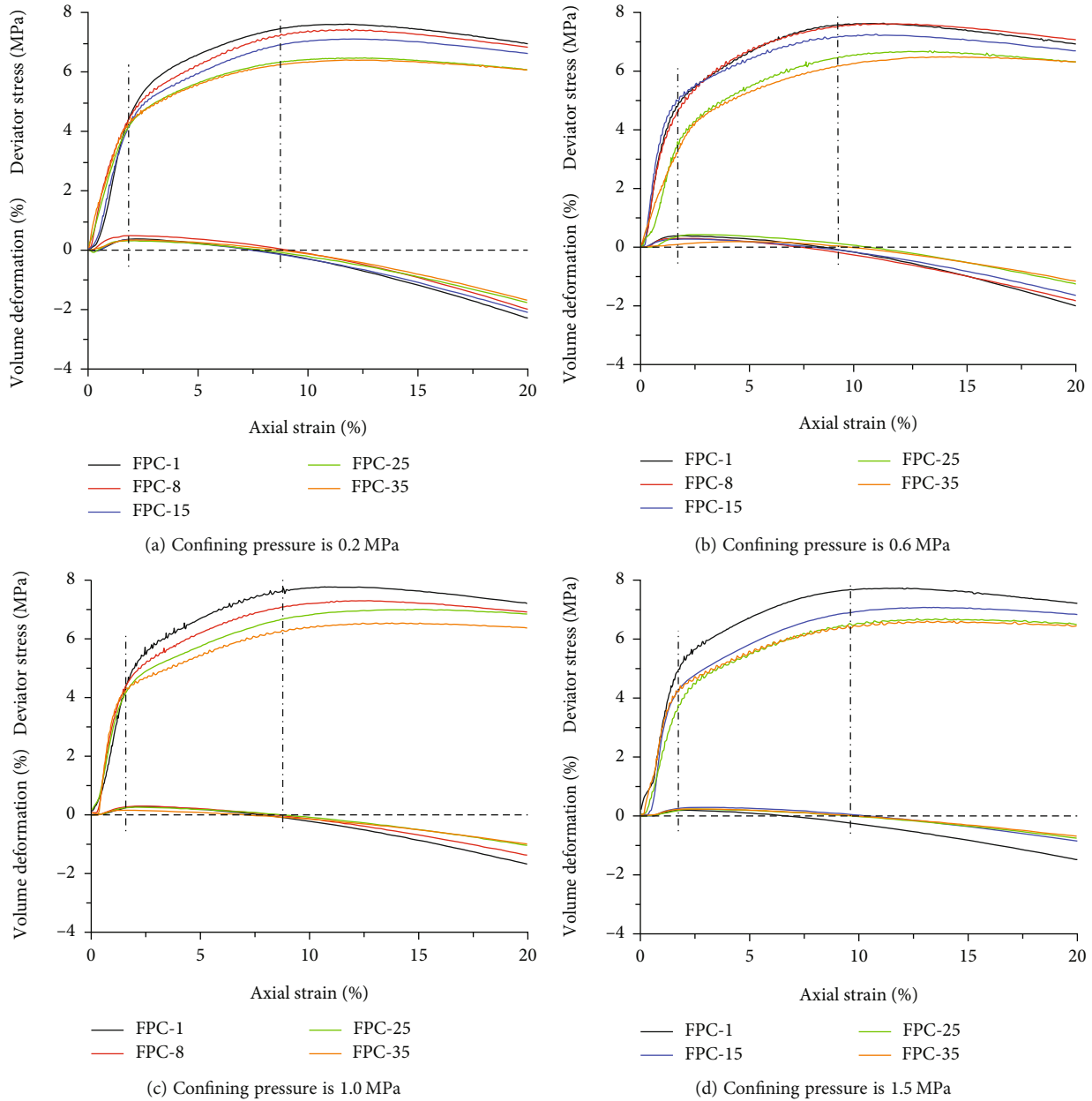


FIGURE 4: Stress-strain curves and volume deformation curves of frozen silty sand under different confining pressures.

soil skeleton as frictional element. Binary medium model can be used to describe the damage mechanism and stress sharing of frozen soil. Compared with existing constitutive models, the binary medium model can be divided into the bonded element and the frictional element, in which the non-uniform distribution of stress and strain can be considered, and the strain softening and dilatancy for geological materials can also be easily modelled by the binary medium model when the bonding elements transfer to frictional elements [30, 31]. As an empirical model, the binary medium model has a good simulation effect, but it involves many parameters. It is sometimes too subjective to determine the parameters by the method of “assuming first and then verifying” [32]. In addition, many scholars have studied and analyzed the stress-strain relationship and shear strength of

frozen soil under different variables such as confining pressure, temperature, ice content, or stress path, but there are few reports on the sensitive variable of fine particle content change in frozen soil triaxial tests. Based on the triaxial compression test of frozen saturated silty fine sand under low confining pressure, the freezing strength and deformation characteristics of frozen silty fine sand with different fine particle contents are studied in this paper, and the simplified binary medium model based on breakage parameter is verified and analyzed.

## 2. Triaxial Test and Result Analysis

**2.1. Soil Samples.** The MTS-Landmark 370.10 dynamic and static triaxial instrument for frozen soil is used in the triaxial

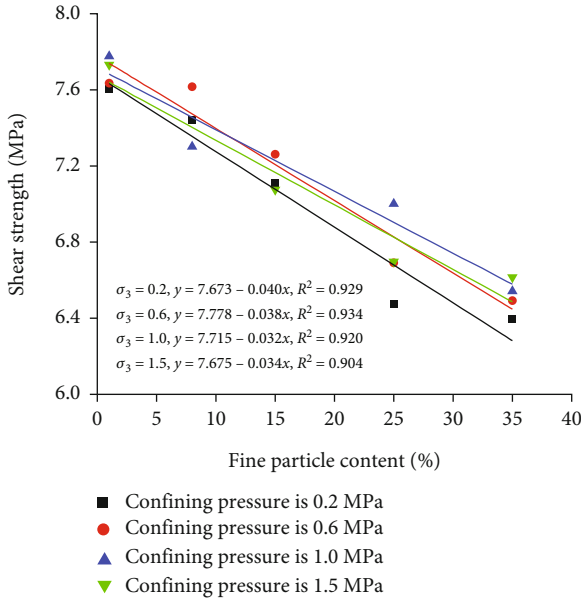


FIGURE 5: Variation curves of shear strength with fine particle content.

test of frozen soil (see Figure 1). The instrument adopts strain rate control mode and is mainly composed of loading frame system, triaxial pressure chamber, volume servo controller, circulating refrigeration system, and acquisition system. The temperature control range is  $-30\sim 80^\circ\text{C}$ , the temperature control accuracy is  $0.1^\circ\text{C}$ , the maximum confining pressure is 20 MPa, and the maximum axial pressure is 100 kN.

The soil used in the test is taken from the subgrade of Gongga Mountain Tunnel entrance in Lhasa-Linzhi section of Sichuan-Tibet Railway. The soil property is silty sand, and the fine particle content (particle size less than 0.075 mm) is 1.03%. The basic physical characteristics are shown in Table 1.

In order to explore the influence of different fine particle content on the strength and deformation characteristics of frozen soil, 5 types of soils were divided by screening method: natural soil (FPC-1), soil with fine particle content accounting for 8% (FPC-8), 15% (FPC-15), 25% (FPC-25), and 35% (FPC-35). The particle distribution curves of 5 kinds of sand are shown in Figure 2, and the uniformity coefficient and curvature coefficient are shown in Table 2.

## 2.2. Test Scheme

**2.2.1. Soil Saturation.** Take the dry soil through 5 mm sieve (remove large stones and sundries), spray and mix the soil according to the optimal water content (12%), and then seal it with plastic wrap for 12 hours to make its water vapor uniform. The dry density was controlled to  $1.5\text{ g/cm}^3$ , and the soil samples were compacted into a three-lobe mold with an inner diameter of 61.8 mm and a height of 125 mm in three layers, and the density was controlled to be medium density. Then place the soil and the three-lobe mold together

into a vacuum saturation cylinder for 12 hours to ensure that the soil sample is fully saturated.

**2.2.2. Temperature Preset and Consolidation.** The saturated soil samples were frozen in freezing environment for 24 h, and the size and weight of the samples were measured after demoulding. Combined with the quality and relative density of the soil, the water content, void ratio, and saturation of frozen soil column can be calculated. After being encapsulated with thin latex sleeve, the labels were pasted and put into a constant temperature test chamber with the same design temperature for 24 hours. The temperature of aviation hydraulic oil (for applying confining pressure and ambient temperature) in the triaxial pressure chamber is set to the design temperature, and then the soil sample is put into the triaxial pressure chamber. When the temperature of the soil sample is the same as that of confining pressure liquid, the design confining pressure is applied to consolidate the soil sample for 40 min.

**2.2.3. Shear Test.** After consolidation, the shear test was started. The shear rate was  $1.25\text{ mm}\cdot\text{min}^{-1}$  under constant strain loading, and the test was terminated when the strain of the specimen reached 20%. Triaxial tests were carried out on 5 kinds of silty sands at temperature of  $-8^\circ\text{C}$  and confining pressures of 0.2, 0.6, 1, and 1.5 MPa. The whole process was collected and recorded by electronic data acquisition instrument in real time. After triaxial test, the frozen soil specimen shows swelling deformation in the middle and lower part, and generally presents drum failure mode (see Figure 3).

**2.3. Analysis of Test Result.** Figure 4 shows the stress-strain curves and volume deformation curves of frozen silty sand with 5 kinds of fine particle contents under confining pressures of 0.2, 0.6, 1.0, and 1.5 MPa, and the freezing temperature is  $-8^\circ\text{C}$ . Correlation analyses are described as follows:

- (1) With the increase of axial strain, the deviator stress shows three stages: linear elastic deformation stage, plastic deformation stage, and strain softening (deviator stress decreasing) stage [31]. In the linear elastic growth stage, the bonded element composed of pore ice and soil particles plays a major role. When the strain is small, there is no crushing and melting phenomenon of pore ice and no cracks in the structure, so the deformation modulus of frozen soil is large enough and the deviator stress increases linearly at this time. In the plastic deformation stage, because the strain is more than 2%, micro-cracks begin to appear in the pore ice, which makes the bonded element structure gradually destroy and transform into frictional element. Because the deformation modulus of the frictional element is smaller than that of the bonded element, the deformation modulus of the frozen soil structure decreases, and the deviator stress increases slowly with axial strain [26]. In the strain softening stage, pore ice breaks and forms local shear bands, which leads to a sharp decrease

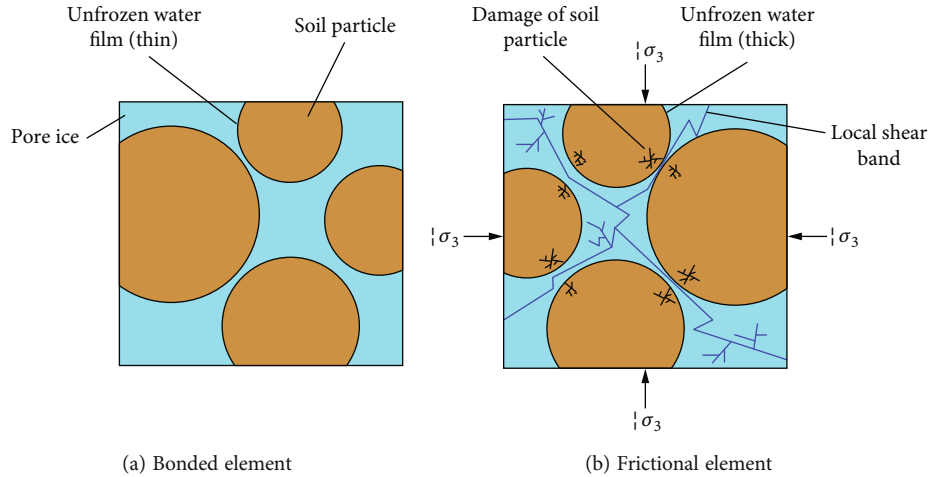


FIGURE 6: Structural model of saturated frozen soil.

in the number of bonded elements and an increasing number of frictional elements, resulting in the overall destruction of frozen soil structure, and the soil sample presents a strain softening state

- (2) The deviator stress decreases with the increase of fine particle content under the same confining pressure. This is because with the increase of the content of fine particles, the increased fine particles occupy part of the position of pore ice, which makes the bearing structure of frozen soil change from coarse particles and pore ice to coarse particles, fine particles, and pore ice. On the one hand, under the action of external force, thin pore ice is more likely to produce crack germination and expansion, which makes the thickness of pore ice decrease. On the other hand, fine particles and unfrozen water film between coarse particles show lubrication, which increases the rolling friction and sliding friction between particles and reduces the shear strength of frozen soil structure [33].
- (3) The turning points of the three stages of the development of the deviator stress coincide with the two points of the extreme point of the volume shrinkage and the turning point of the volume shrinkage and expansion in the volume deformation, which indicates that the volume deformation has a significant effect on the development of the deviator stress
- (4) With the increase of axial strain, the volume deformation of frozen silty sand first appears volume shrinkage and then develops to volume expansion. In the range of axial strain, under the same confining pressure, the larger the content of fine particles, the smaller the maximum volume expansion. Combined with deviator stress and volume deformation, it can be seen that the greater the maximum volume expansion, the greater the extreme value of deviator

stress corresponding to soil samples. This is because for dense frozen silty sand, the volume expansion of the sample needs to do positive work, which makes the shear stress increase and the shear strength increase accordingly. With the increase of fine particle content, the soil properties develop towards cohesive soil, and the dilatancy performance decreases, so the peak strength decreases. From the curve of shear strength with fine particle content in Figure 5, it can be seen that the linear goodness of fit under different confining pressures is greater than 0.9, and there is a strong linear correlation between shear strength and fine particle content

### 3. Binary Medium Model of Frozen Soil

**3.1. Frozen Soil Structure Model.** The strength of frozen soil is the ability of resisting external damage, and it is one of the important mechanical properties of frozen soil. Unsaturated frozen soil is a discontinuous four-phase system, including soil skeleton, unfrozen water, ice, and air, while saturated frozen soil is a three-phase system, including soil skeleton, unfrozen water, and pore ice. Under the action of temperature and pressure, pore water in frozen soil is transformed into pore ice, which changes the connection mode of internal structure of soil. This leads to the change of the interaction mode between soil particles and pore ice, thus changing the strength characteristics of soil. The interaction between soil particles and pore ice mainly includes molecular bonded force, structural bonded force, and ice cementation bonded force. In addition, there is aggregation between mineral particles and pore ice, and viscosity between ice and film water, which makes the action mode of elements in frozen soil obviously different from that in unfrozen soil.

The structural model of saturated frozen soil can be seen in Figure 6. According to the concept of binary medium model, the stable ice-soil skeleton mainly composed of soil particles and pore ice can be used as bonded element, and the bonded element has good integrity and large deformation modulus. Under the action of external force or external

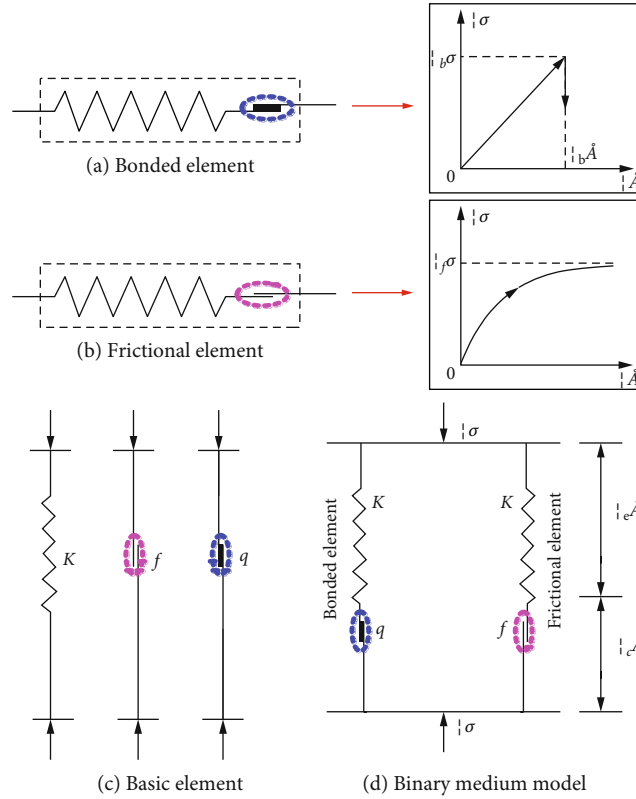


FIGURE 7: Simplified description of binary medium model.

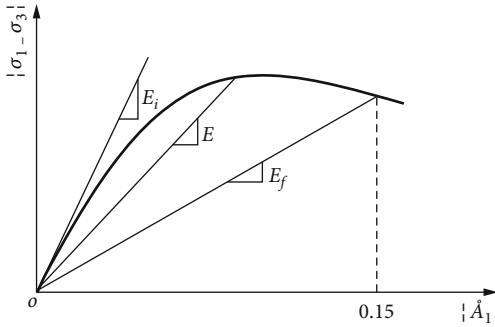


FIGURE 8: Illustration for determination of damage parameters.

temperature, the internal damage to the bonded element occurs, such as crushing and melting of pore ice or breaking of soil particles, and local shear bands are gradually formed under the action of concentrated force, which leads to the transformation of the bonded element into frictional element. In reality, there are numerous bonded elements and frictional elements coexisting in the frozen soil system, and they are transformed with each other under the action of the outside world, so the strength characteristics of frozen soil are directly related to the quantitative comparison between bonded elements and frictional elements. Triaxial test of frozen soil is a process from bonded element to frictional element. The bonded element plays a major role in the linear elastic deformation stage, while the bonded element is continuously transformed into frictional element

in the plastic deformation stage, and the role of frictional element is continuously increased in the strain softening stage.

In the linear elastic deformation stage of triaxial test of frozen soil, the bonded element plays a major role. As a material without damage, the bonded element can be regarded as an ideal elastic material, and its elastic modulus can be determined by the initial slope of stress-strain curve [12, 15]. The stress-strain characteristics of bonded element are shown in Figure 7.  $\epsilon_b$  is the fracture limit strains of bonded element, and the values of each bonded element are not uniform. In the interval of 0 to  $\epsilon_b$ , the stress of the bonded element increases linearly and then disappears to zero quickly after reaching the limit strain. Because the friction element has elastic-plastic characteristics, its stress-strain curve shows hyperbolic change (Figure 7). Hyperbolic models such as Duncan-Chang model, modified Duncan-Chang model, or Shen Zhujiang three-parameter model can be used for friction element [12]. Here, Duncan-Chang model is used for the next analysis, and its expression can be expressed by equation (1):

$$\sigma_f = \frac{\epsilon_f}{a + b\epsilon_f} \tag{1}$$

In the equation,  $\sigma_f$  is the stress borne by the frictional element;  $\epsilon_f$  is the strain of frictional element;  $a$  and  $b$  are the test parameters.

Elasticity, plasticity, and brittleness are three basic properties of solids, which can be represented by spring ( $K$ ),

TABLE 3: Parameters of binary medium simplified model of frozen silty sand.

Number	Fine particle content	Confining pressure (MPa)	$E_i$ (MPa)	$E_f$ (MPa)	$(\varepsilon_1)_m$ (%)	$(\sigma_1 - \sigma_3)_m$ (MPa)	$a$	$b$	$\alpha$
1	FPC-1	0.2	300.750	49.546	11.498	7.607	0.078	0.131	0.22
2	FPC-1	0.6	379.290	49.208	11.199	7.635	0.052	0.133	0.32
3	FPC-1	1.0	322.020	50.831	10.700	7.776	0.079	0.127	0.22
4	FPC-1	1.5	542.916	50.775	12.000	7.732	0.072	0.128	0.23
5	FPC-8	0.2	254.367	48.409	11.833	7.440	0.102	0.133	0.18
6	FPC-8	0.6	398.981	49.824	10.266	7.616	0.075	0.130	0.22
7	FPC-8	1.0	370.303	48.163	12.799	7.302	0.114	0.132	0.16
8	FPC-15	0.2	378.973	46.744	8.135	7.111	0.118	0.137	0.16
9	FPC-15	0.6	338.530	47.043	10.800	7.261	0.052	0.140	0.32
10	FPC-15	1.5	624.892	46.976	13.300	7.075	0.162	0.132	0.12
11	FPC-25	0.2	252.937	42.608	12.565	6.472	0.101	0.152	0.20
12	FPC-25	0.6	312.925	44.037	13.301	6.691	0.172	0.142	0.12
13	FPC-25	1.0	387.492	46.676	14.999	7.001	0.174	0.133	0.11
14	FPC-25	1.5	349.952	44.389	13.934	6.697	0.187	0.139	0.11
15	FPC-35	0.2	263.966	42.209	8.833	6.397	0.107	0.152	0.19
16	FPC-35	0.6	210.315	43.165	14.399	6.492	0.214	0.142	0.10
17	FPC-35	1.0	471.014	43.422	13.066	6.542	0.164	0.144	0.13
18	FPC-35	1.5	527.216	43.713	14.332	6.615	0.157	0.143	0.13

plastic slider ( $f$ ), and brittle bond ( $q$ ), respectively [18], as shown in Figure 7. Elastic-plastic elements can be composed of spring and plastic slider, and elastic-brittle elements can be composed of spring and brittle bond. Without considering the plastic deformation caused by structural plane slip, the binary medium model can be represented by the lower right figure of Figure 7.  $\varepsilon^e$  represents the elastic deformation of structural body and structural plane, and  $\varepsilon^c$  is the deformation caused by structural body damage. When the strain is small, the bonded element plays a bearing role. After the cemented rod breaks, the bonded element changes into a frictional element and plays a bearing role together with other frictional elements.

**3.2. Simplified Model of Binary Medium.** Because of the uneven distribution of cementing force in space, structural rock and soil can be regarded as heterogeneous materials. According to the homogenization theory of heterogeneous composite materials, the average stress and strain of bonded element and frictional element in representative elements can be expressed by equations (2) and (3):

$$\bar{\sigma}_i = \frac{1}{V_i} \int \sigma_{loc} dV_i; \bar{\sigma}_f = \frac{1}{V_f} \int \sigma_{loc} dV_f, \quad (2)$$

$$\bar{\varepsilon}_i = \frac{1}{V_i} \int \varepsilon_{loc} dV_i; \bar{\varepsilon}_f = \frac{1}{V_f} \int \varepsilon_{loc} dV_f. \quad (3)$$

The volume fraction of the bonded elements is defined as  $\lambda = V_f/V$  [29], and the macroscopic average stress can be

fitted by equation (4):

$$\begin{aligned} \bar{\sigma} &= \frac{1}{V} \int \sigma_{loc} dV = \frac{1}{V} \int \sigma_{loc} d(V_i + V_f) \\ &= \frac{1}{V} \int \sigma_{loc} dV_i + \frac{1}{V} \int \sigma_{loc} dV_f = \frac{V_i}{V} \cdot \left[ \frac{1}{V_i} \int \sigma_{loc} dV_i \right] + \frac{V_f}{V} \\ &\quad \cdot \left[ \frac{1}{V_f} \int \sigma_{loc} dV_f \right] = \frac{V_i}{V} \bar{\sigma}_i + \frac{V_f}{V} \bar{\sigma}_f = (1 - \lambda) \bar{\sigma}_i + \lambda \bar{\sigma}_f. \end{aligned} \quad (4)$$

Likewise, the macroscopic average strain can be expressed by equation (5):

$$\bar{\varepsilon} = (1 - \lambda) \bar{\varepsilon}_i + \lambda \bar{\varepsilon}_f. \quad (5)$$

In the equation,  $V_i$  is the volume of the bonded elements for the representative volume element (RVE);  $V_f$  is the volume of the frictional elements for the RVE;  $\bar{\sigma}$  is the macroscopic average stress;  $\bar{\varepsilon}$  is the macroscopic average strain;  $\bar{\sigma}_i$  and  $\bar{\sigma}_f$  are the average stresses of bonded element and frictional element, respectively;  $\bar{\varepsilon}_i$  and  $\bar{\varepsilon}_f$  are the average strains of bonded element and frictional element, respectively;  $\sigma_{loc}$  is the local stress for the RVE;  $\varepsilon_{loc}$  is the local strain for the RVE.

From the structural analysis of frozen soil model, it is known that bonded element can be regarded as ideal elastic material as a material without damage, its elastic modulus can be determined by the initial slope of stress-strain curve in triaxial test, and friction element has elastic-plastic characteristics, and its stress-strain curve changes in hyperbola. Then, equation (4) can be expressed by equation (6) under



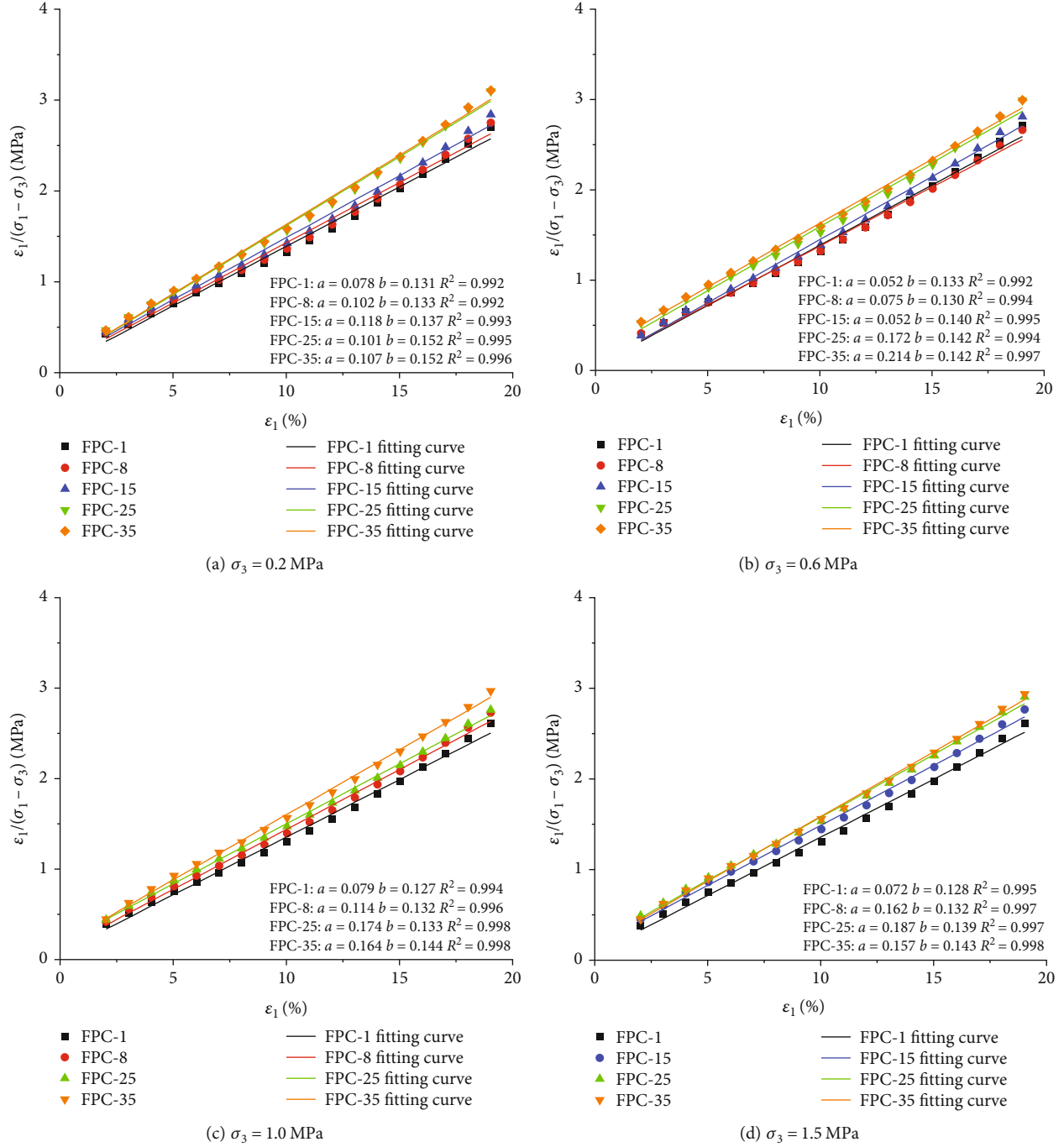


FIGURE 9: Relation curves  $\epsilon_1/(\sigma_1 - \sigma_3) \sim \epsilon_1$  of frozen silty sand under different confining pressure.

triaxial shear test:

$$\sigma_1 - \sigma_3 = (1 - \lambda)E_i\epsilon_1 + \lambda \frac{\epsilon_f}{a + b\epsilon_f}. \quad (6)$$

Considering that the local strain coefficient is scalar, the expression of the local strain coefficient is:

$$c = \frac{\epsilon_f}{\epsilon}. \quad (7)$$

According to equation (5) and (7), there is:

$$\epsilon_i = \frac{1 - c\lambda}{1 - \lambda} \epsilon. \quad (8)$$

In the equation,  $\sigma_1$  is the axial stress;  $\sigma_3$  is the consolidation confining pressure.

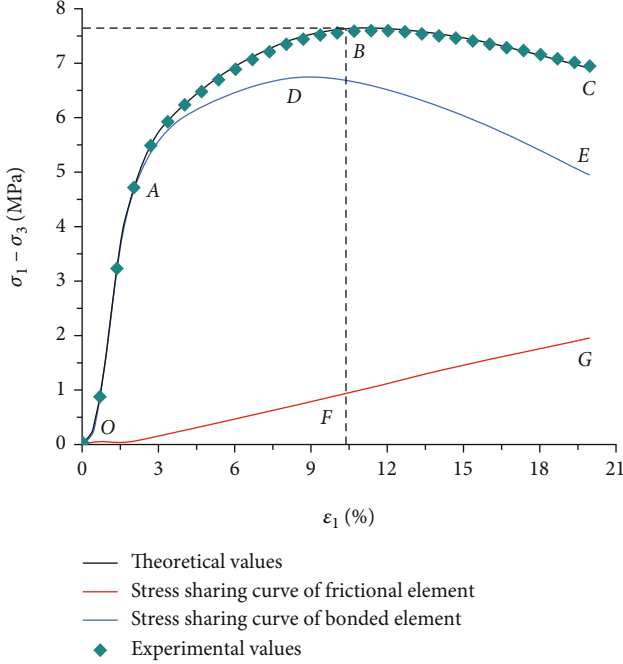


FIGURE 10: Comparison between theoretical values and the experimental values (FPC-1 under 0.2 MPa confining pressure).

Substituting equations (7) and (8) into equation (6), equation (9) can be expressed as follows [26, 27]:

$$\sigma_1 - \sigma_3 = (1 - c\lambda)E_i\varepsilon + c\lambda \frac{\varepsilon}{a + bc\varepsilon}. \quad (9)$$

Breakage parameter ( $\delta = c\lambda$ ) is an internal variable related to volume breakage rate and strain coefficient, which can also be called friction stress sharing rate. The above formula can be abbreviated as:

$$\sigma_1 - \sigma_3 = (1 - \delta)E_i\varepsilon + \frac{\delta\varepsilon}{a + bc\varepsilon}. \quad (10)$$

Combined with the triaxial stress-strain relationship of frozen soil, the breakage parameter can be determined by equation (11):

$$\delta = \frac{E_i - E}{E_i - \alpha E_f}. \quad (11)$$

In the equation,  $E_i$  is the modulus of bonded element;  $E_f$  is the nominal frictional element modulus;  $E$  is secant modulus in the change process of stress-strain curve;  $\alpha$  is the modulus correction coefficient of nominal friction element.

Combined with the stress-strain curve of triaxial shear test of frozen soil, it is assumed that the modulus at the beginning of shear is the bonded element modulus, and the secant modulus at the deformation of 15% is the frictional element modulus. The breakage parameter increases with the axial strain, which matches the transformation process from bonded element to frictional element in binary medium model [11, 34]. With the increase of axial strain,

the elastic-brittle bonded element is gradually damaged and transformed into elastic-plastic frictional element, and the ratio of frictional element is gradually increased. The illustration for determination of damage parameters can be seen in Figure 8.

The simplified model formula of frozen soil binary medium obtained by formulas (10) and (11) is as follows:

$$\sigma_1 - \sigma_3 = \left(1 - \frac{E_i - E}{E_i - \alpha E_f}\right)E_i\varepsilon_1 + \frac{E_i - E}{E_i - \alpha E_f} \cdot \frac{\varepsilon_1}{a + bc\varepsilon_1}. \quad (12)$$

**3.3. Determination of Parameters.** The initial modulus of elasticity is derived from the slope of stress-strain curve when the strain is 0.8%, that is, the modulus of bonded element ( $E_i$ ). The local strain coefficient ( $c$ ) is determined by assuming first and then verifying [12, 21, 35]. By comparing the stress-strain curves, the local strain coefficient of silty sand with different fine particle content in this paper can be fixed at 0.6. Under the condition that the local strain coefficient is determined, the modulus correction coefficient of nominal frictional element ( $\alpha$ ) is selected by comparing the theoretical curve with the measured curve.  $a$  and  $b$  can be determined by the calculation method of Duncan-Chang model test constants in triaxial tests. The calculation equation is shown in equation (13):

$$\frac{\varepsilon_1}{\sigma_1 - \sigma_3} = a + b\varepsilon_1. \quad (13)$$

The test data are arranged and plotted according to  $\varepsilon_1/(\sigma_1 - \sigma_3) \sim \varepsilon_1$ , and the slope is  $b$  value and the intercept is  $a$  value [19, 36–38]. The fitting calculation values of each parameter obtained by the above parameter determination method can be seen in Table 3. Figure 9 shows the  $\varepsilon_1/(\sigma_1 - \sigma_3) \sim \varepsilon_1$  curves of 5 kinds of frozen silty sand with different fine particle content under confining pressures of 0.2, 0.6, 1.0, and 1.5 MPa. It can be seen from Figure 9 and Table 3 that the  $b$  value increases with the increase of fine particle content under the same confining pressure; and the influence of confining pressure on the  $b$  value is not obvious under the condition of the same fine particle content.

**3.4. Model Validation.** In order to verify the calculation accuracy of the model, the calculated value of the simplified model of frozen soil binary medium proposed in this paper is compared with the measured value of frozen soil triaxial test. The stress sharing curves of bonded element and frictional element are drawn by formula (15), which can be seen in Figure 10.

Figure 10 is the stress-strain curve of FPC-1 under confining pressure of 0.2 MPa. It can be seen from the figure that the calculated values of the model are very close to the measured values and can reach a good matching degree in linear elastic deformation stage, plastic deformation stage, and strain softening stage, which shows that the simplified model can better simulate the stress-strain relationship of silty sand under low-temperature triaxial compression test. The total stress curve (OABC) is composed of the stress sharing curve of bonded element (OADE) and the stress

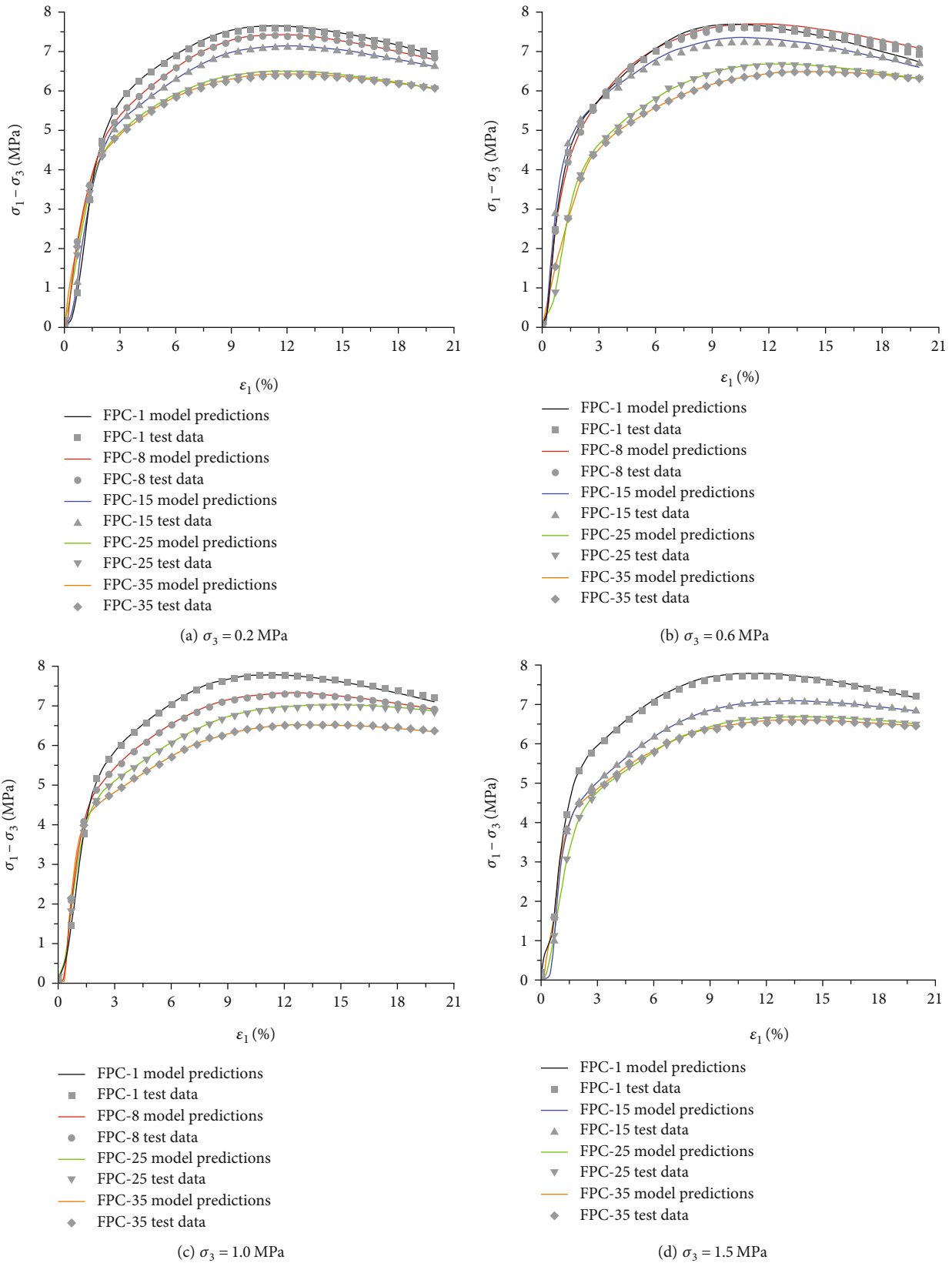


FIGURE 11: Comparison between model predictions and the test data under different confining pressure.

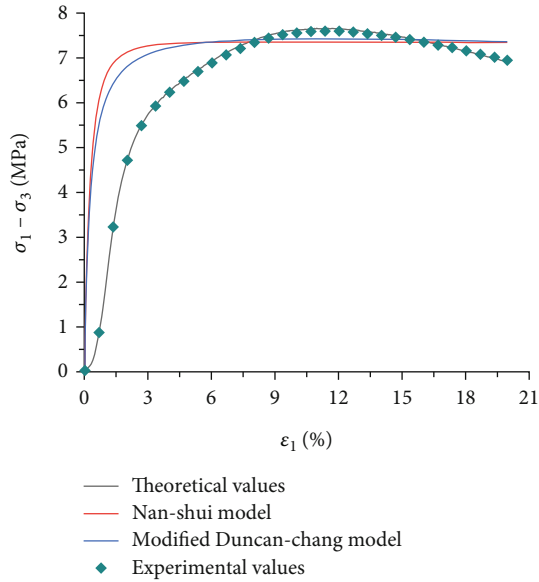


FIGURE 12: Comparison of theoretical model, Nan-Shui model, and Duncan-Chang model (FPC-1 under 0.2 MPa confining pressure).

sharing curve of frictional element (OFG). In the linear elastic deformation stage, the stress sharing curve of bonded element is consistent with the total stress curve, while the value of stress sharing curve of frictional element is small, which reflects that the integrity of cemented element is better in this stage, and the bonded element mainly plays a bearing role under the action of external force. In the plastic deformation stage, the stress sharing curve of frictional element begins to increase, while that of bonded element begins to increase slowly, and the difference between bonded element and total stress curve increases after point A. The stress sharing curve of bonded element begins to decrease after reaching the maximum at point D, and the total stress curve reaches the peak at point B, and point D appears earlier than point B in axial strain direction. The total stress peak shows that the sum of stress sharing curves of bonded element and frictional element reaches the maximum. According to the change trend of stress curves, it can be seen that the internal damage to bonded element begins to appear in the plastic deformation stage, and the bonded element changes to the frictional element continuously. During the strain softening stage, the stress sharing curve of bonded element decreases further and the stress sharing curve of frictional element increases rapidly. This shows that the pore ice in the bonded element crushes and melts and forms a local shear zone under the action of concentrated force, and the soil sample presents a failure mode. In addition, it can be seen from the figure that the stress sharing curve of cemented element is always larger than that of friction element within the range of 20% axial strain.

Figure 11 is a comparative analysis of the theoretical calculation values of the model and the measured values of triaxial tests for silty sand with different fine particle contents under confining pressures of 0.2, 0.6, 1.0, and 1.5 MPa, respectively. As can be seen from the figure, the theoretical values of the model are very close to the measured values

under different confining pressures and different fine particle contents.

Figure 12 is a comparative analysis of the theoretical model, Nan-Shui model, and modified Duncan-Chang model of FPC-1 under 0.2 MPa confining pressure. It can be seen from the figure that in the process of stress-strain curve change, the theoretical model is in good agreement with the experimental values, while the Nan-Shui model and the modified Duncan-Chang model are quite different from the measured values. In the linear elastic deformation stage, plastic deformation stage, and strain softening stage, the simplified model data has a high degree of matching with the measured data, which further verifies that the model can better simulate the stress-strain relationship curve of frozen silty sand.

## 4. Conclusions

In this paper, triaxial tests of silty sand under 5 kinds of fine particle content and 4 kinds of confining pressures are carried out, and a simplified binary medium model based on breakage parameter is derived for frozen soil triaxial tests, and then the reliability of the derived model is studied and analyzed in combination with triaxial tests. According to the analysis above, some key conclusions are drawn below:

- (1) Under the same confining pressure, the shear strength decreases linearly with the increase of fine particle content, and the maximum volume expansion decreases with the increase of fine particle content
- (2) Silty fine sand with 5 fine particle contents shows strain softening characteristics under 4 confining pressures. With the increase of axial strain, the volume deformation of frozen silty fine sand first shows the form of volume shrinkage and then develops to the form of volume expansion
- (3) Based on binary medium model, all three stages can be well explained by the transformation theory of bonded element and frictional element. In the linear elastic deformation stage, the external stress is mainly borne by the bonded element. In the plastic deformation stage, the stress sharing ratio of the bonded element decreases and that of the frictional element increases. In the strain softening stage, the stress sharing ratio of the bonded element decreases rapidly and that of the frictional element increases rapidly
- (4) By comparing the measured deviator stress in triaxial test with the calculated values of binary medium constitutive model based on simplified breakage parameter, the proposed model can better simulate the stress-strain relationship of frozen silty sand. In addition, it is worth noting that the applicability of the constitutive model in this paper under complex stress paths (cyclic loading, the rotation of the principal axial stress) needs further study

## Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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