

Research Article

Natural Frequency of Coal: Mathematical Model, Test, and Analysis on Influencing Factors

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The difficulty in enhancing the low permeability of deep coal seams is the key problem restricting gas extraction. The technology of coal rock resonance and permeability enhancement excited by vibration wave is hailed as a new technology to enhance coal seam permeability. In particular, the effect of resonance and permeability enhancement is remarkable when the excitation frequency is exactly the same as the natural frequency of coal. In order to promote the application of the technology, the first step is to explore the variation characteristics of coal natural frequency and its influencing factors. In this study, two mathematical models of coal natural frequency were established, and the variations and influencing factors of coal natural frequency were discussed through an experiment on the natural frequency of coal. The results show that coal vibration has multiorder natural frequency which grows with the increase of the order. In addition, the natural frequency of coal is closely related to its elastic modulus, density, size, mass, stiffness, and other physical and mechanical parameters. The larger the coal size and mass are, the lower the natural frequency would be. The natural frequency parallel to the bedding plane is higher than that perpendicular to the bedding plane. For the saturated coal sample, moisture changes its density and reduces its elastic modulus. Consequently, its natural frequency is lower than that of the dried coal sample. The difference of organic matter and mineral content coal of different rank affects the physical and mechanical properties of coal, which leads to the difference in natural frequency of different-rank coals. The natural frequencies of different-rank coal show bituminous > anthracite > lignite. The natural frequencies of coal samples under different influencing factors are all tens of Hz. Thus, the vibration excitation of coal under the low-frequency condition is the focus of future research. The study can provide a theoretical basis for the technology of coal resonance and permeability enhancement excited by vibration wave.

1. Introduction

The ground stress and gas pressure in coal seams increase as coal mining continues to deepen, which brings difficulty to gas extraction. Coal, as an organic matter containing unconventional natural gas resources, forms different types of pores during coalification [1]. Pore is the main place to store coalbed methane and fracture system is the main way of gas migration in coal seam. The abundance of micro-nano pore determines the permeability and gas adsorption capacity of coal seam, which directly affects the extraction of coalbed methane [2]. In addition, the

permeability of coal is also affected by effective stress, matrix shrinkage and expansion, and gas slippage [3]. This shows that the mechanical properties of coal also affect the production of coalbed methane. Therefore, aiming at low-permeability coal seam, seeking a scientific new technology to improve the permeability of coal seam is the key to the success of coalbed methane exploitation.

Yuan [4] proposed that it is necessary to carry out research on the mechanism of modified anti-reflection of deep low-permeability coal seam to realize the large-scale anti-reflection of coal seam. The technology of permeability enhancement excited by mechanical wave vibration can not

only disturb the coal seam and increase pores and cracks in coal but also modify coal seam gas adsorption/desorption to promote gas desorption and seepage. The technology has become a hot spot of the research on the permeability enhancement of low-permeability coal seams by means of an external physical field.

In the prevention and control of coal mine gas disaster, permeability enhancement mechanisms such as vibration-induced coal and gas outburst [5, 6], deep-hole blasting [7–9], CO₂ presplitting [10, 11], hydraulic fracturing [12–15], and acoustic or ultrasonic vibration [16, 17] are related to the theories of coal rock excitation by vibration wave. As a new technology, vibration wave excitation boasts great potential in improving the permeability of coal rock. Scholars have carried out extensive researches on gas desorption and coal rock crushing excited by vibration wave.

In terms of gas desorption excited by vibration wave, Chen et al. [18, 19] studied the influence of vibration on gas desorption diffusion in granular coal and drew the following conclusions: (1) With the increase of vibration frequency, gas desorption of coal samples first increased and then decreased. (2) Mechanical vibration could generate shear force in adsorbed gas to promote its desorption. (3) A larger vibration amplitude is conducive to gas desorption. Jiang et al. [20] proved that sound field can improve the desorption volume of methane, and the desorption volume increases with the increase of sound intensity. According to the above studies, vibration can promote gas desorption in coal, but the gas desorption effect varies greatly under different vibration frequencies.

In terms of coal rock crushing excited by vibration wave, Li et al. [21] proved that with the increase of excitation frequency, the drilling speed of rock increased and reached the maximum value at the natural frequency of rock. Yin et al. [22] found that the natural frequency of high-density hard rock ranged from 20 kHz to 38 kHz. Li et al. [23] adopted a large-scale vibration test device, and it is concluded that the coal body accelerates rupture when resonance occurs. Li et al. [24] put forward the theory of rock modal analysis and concluded that the cracks inside the rock would lead to the decrease of its natural frequency. The theory and practice [25, 26] show that the permeability enhancement is the best when the frequency of vibration wave is the same as the natural frequency of coal and rock. The study confirmed that the natural frequency of coal rock was related to the mass and stiffness of the system. The natural frequency of soft rock and coal, which generally ranged from tens of Hz to hundreds of Hz, belonged to low-frequency vibration. At the same time, the natural frequency of coal rock decreases with the fracturing of coal rock.

The technology of coal rock resonance and permeability enhancement excited by vibration wave (hereafter referred to as the RPEEVW technology) works in the following way. When the frequency of vibration wave is close to the natural frequency of coal, the amplitude of coal rises after forced resonance, so that pores and cracks in coal expand significantly. Obviously, the realization of this technology is closely related to the natural frequency and vibration characteristics of coal. However, compared with rock, coal has a

more complex and its natural frequency is affected by more factors. Therefore, the study on the variations of coal natural frequency and its influencing factors is the premise and basis for the implementation of the RPEEVW technology.

In this study, considering the working requirements for permeability enhancement excited by vibration wave in low-permeability coal, the mathematical models of natural frequency of coal were established based on theoretical analysis on the natural frequency of coal. Moreover, an experiment was performed on the variations of the natural frequency of coal. In this way, the influences of factors such as coal bedding, size, coal rank, and moisture on the natural frequency of coal were explored. The research results can provide a theoretical basis for the RPEEVW technology.

2. Models of Natural Frequency of Coal

Natural frequency is a kind of inherent property of structural specimen, coal, and rock materials that have their own natural frequency. In this paper, it is assumed that coal is isotropic and homogeneous. On the premise of ignoring the influences of pressure and temperature on coal vibration, the vibration system of coal body is simplified according to the relevant vibration theories [27]. Based on the physical model of single-degree-of-freedom coal, the entire mass of coal is concentrated on the particle m , and all of its elasticity is concentrated in the equivalent spring. In this case, the stiffness k of the coal is the equivalent stiffness of the equivalent spring. All the damping existing in the coal is equivalent to a damper represented by c (Figure 1).

When the coal is in a static equilibrium position, the generated velocity \dot{x} and acceleration \ddot{x} are both functions of time t . According to Newton's Second Law, the free vibration differential equation of single-degree-of-freedom coal is:

$$m\ddot{x} + c\dot{x} + kx = 0, \quad (1)$$

where m is the mass of coal, kg; c is the damping of coal, dimensionless; k is the stiffness of coal, N/m; x is the displacement of coal relative to the equilibrium position, m.

Eq. (1) is a linear homogeneous second-order differential equation with constant coefficients. Let $\omega_n = \sqrt{k/m}$, $\xi = (c)/(2\sqrt{km})$. The initial conditions are:

$x(0) = x_0$ and $\dot{x}(0) = \dot{x}_0$. Laplace transformation was conducted on both sides of Eq. (1) to obtain the solution of the differential equation of vibration frequency of single-degree-of-freedom coal:

$$x(t) = e^{-\xi\omega_n t} B \sin(\omega_d t + \varphi), \quad (2)$$

where $B = \sqrt{x_0^2 + (\dot{x}_0 + \xi\omega_n x_0/\omega_d)^2}$, $\xi = c/2\sqrt{km}$, $\omega_d = (1 - \xi^2/2\pi)\sqrt{k/m}$, $\varphi = t g^{-1}(x_0\omega_d/\dot{x}_0 + \xi\omega_n x_0)$;

B is the amplitude, m/s²; ξ is damping ratio, dimensionless; ω_d is the natural frequency of coal, Hz; φ is the phase angle, rad. Assuming that coal is an elastic medium, then its stiffness can be obtained from the stiffness equation of

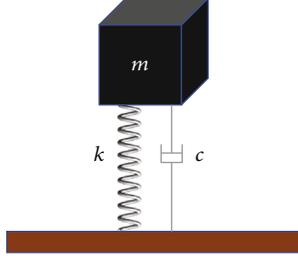


FIGURE 1: Vibration model of single-degree-of-freedom coal.

elastic material [28, 29]:

$$k = \frac{EA}{L}, \quad (3)$$

where E is the elastic modulus of coal, N/m; A is the cross-sectional area of coal, m^2 ; L is the length of coal, m.

By substituting Eq. (3) into ω_d in Eq. (2), the calculation equation of coal natural frequency can be obtained:

$$\omega_d = \frac{1 - \xi^2}{2\pi} \sqrt{\frac{EA}{mL}}. \quad (4)$$

When the coal is a cube or cylinder, its natural frequency can be expressed as:

$$\omega_d = \frac{1 - \xi^2}{2\pi} \sqrt{\frac{E}{\rho L^2}}, \quad (5)$$

where ρ is the density of coal, kg/m^3 .

It is concluded that the natural frequency of macroscopic coal is not only related to the damping ratio and size of coal but also closely related to the physical and mechanical parameters such as density and elastic modulus of coal. Among them, the damping ratio of coal and rock is generally much less than 10%, so the influence of damping ratio on the natural frequency of coal can be ignored.

In fact, as a special continuous elastic body, the internal structure of coal is quite complex. Coal contains not only pores and fractures but also cleft bedding structure. The interaction between adjacent cleats and beddings can be assumed as free vibration between homogeneous blocks. Therefore, when the damping is ignored, the interaction between coal cleats and beddings can be represented by the physical model of the multi-degree-of-freedom system (Figure 2).

According to D'Alembert's principle, the differential equation of free vibration of multi-degree-of-freedom coal is:

$$M\ddot{X} + KX = 0. \quad (6)$$

In Eq. (6), M is the mass matrix of adjacent bedding coal; K is the stiffness matrix of adjacent bedding coal; X is the displacement vector of coal.

The coal itself has n degrees of freedom due to the free vibration between coal matrix blocks, and its vibration natu-

ral frequency can be expressed by the n -dimensional matrix or the column vector:

$$\begin{cases} (K - \omega^2 M)\phi = 0 \\ X(t) = \phi \sin(\omega t + \theta) \end{cases}, \quad (7)$$

where ϕ is the n -dimensional column vector; ω is the natural frequency of multi-degree-of-freedom coal, Hz; θ is the phase angle, rad.

Conditions for the non-zero solution of Eq. (7) are as follows:

$$|K - \omega^2 M| = 0. \quad (8)$$

The solution of Eq. (8) yields n values which can be arranged in an ascending order:

$$0 < \omega_1 \leq \omega_2 \leq \dots \leq \omega_n, \quad (9)$$

where $\omega_i (i = 1, 2, 3, \dots, n)$ is the i -th natural frequency of coal vibration.

In the multi-degree-of-freedom coal vibration model, the natural frequency of coal is closely related to the distribution, mass, and stiffness of bedding, and the vibration of coal has multiorder natural frequency, which increases with the increase of the order.

Based on the above established natural frequency vibration models on single-degree-of-freedom and multi-degree-of-freedom coals, it can be seen that coal vibration has multiorder natural frequency which grows with the increase of the order. In addition, the natural frequency of coal is directly determined by the physical and mechanical parameters such as elastic modulus, density, size, bedding, mass, and stiffness.

Taking coal as a whole specimen, the single-degree-of-freedom vibration model of coal is established. The single-degree-of-freedom vibration model of coal is used to illustrate the macroscopic influence factors of natural frequency. However, any continuous structure can be regarded as composed of infinite number of micro-rigid bodies. From the microscopic point of view, the interaction between adjacent cleft bedding inside coal can be considered the free vibration between homogeneous blocks. The multi-degree-of-freedom vibration model of coal body closely links the stiffness matrix and mass matrix with the macroscopic physical parameters such as elastic modulus, density, and size of the specimen. It lays a foundation for the industrial analysis of coal samples, the test of physical and mechanical parameters, and the analysis of natural frequency experimental results.

3. Experiment on Coal Natural Frequency

In order to further study the change characteristics of coal natural frequency, typical experimental coal samples with different rank of high, middle, and low coal were collected and prepared. Furthermore, an experiment was performed on the variations of coal natural frequency with the aid of

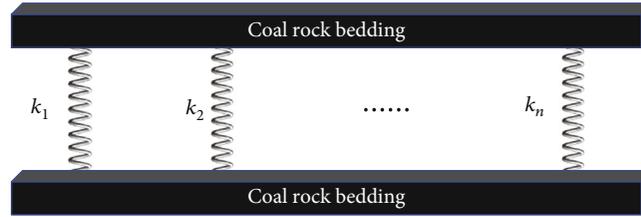
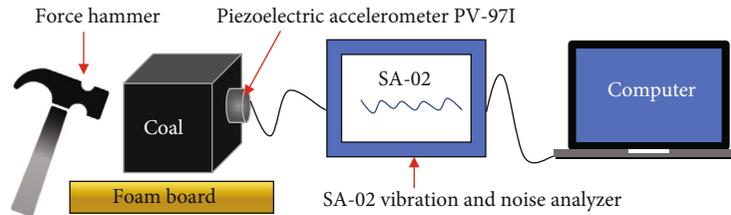
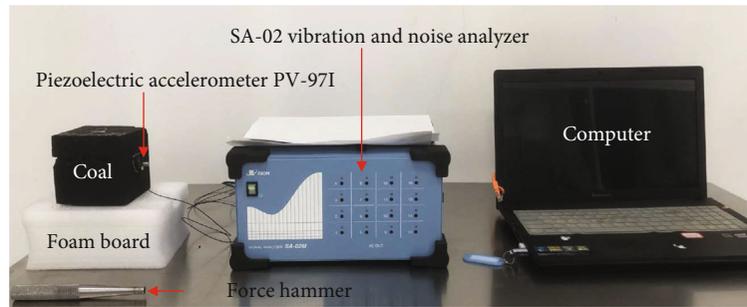


FIGURE 2: Vibration model of multi-degree-of-freedom coal.



(a) Schematic diagram



(b) Physical map

FIGURE 3: Testing device of coal natural frequency.

the experimental system setup using the relevant equipment of the State Key Laboratory Cultivation Base Forgas Geology and Gas Control relying on Henan Polytechnic University.

3.1. Test Principle and Device

(1) Test principle

Natural frequency is the inherent property of coal and rock material. When the coal body is subjected to external excitation, it will vibrate freely at a specific frequency, which is called the natural frequency of the coal body [23]. In this paper, instantaneous hammering method is used to test the natural frequency of coal. The specific operation is as follows: firstly, the acceleration sensor is installed on the surface of the cube coal sample to be tested by means of vaseline adhesive; secondly, using force hammer for instantaneous percussion hammer of coal samples, coal sample under a vibration shock pulse, the specimen will be its own inherent low order natural frequency of free vibration; finally, the frequency domain curve of the coal sample was obtained by applying fast Fourier transform (FFT) to the collected vibration signal, and the frequency corresponding to the acceleration peak, namely, the natural frequency of coal, was found on the curve.

(2) Test device

The test device of coal natural frequency comprises a force hammer, a foam board, the experimental coal sample, a vibration sensor, a vibration parameter detector, and a computer. The schematic diagram and photo of the test device are shown in Figure 3.

The vibration sensor used in the experiment is a three-direction piezoelectric accelerometer PV-97I sensor produced by RION, Japan. The sensor has a frequency measurement range of 1-10,000 Hz and a maximum test acceleration of 500 g. Since the natural frequency of coal is below 100 Hz, the sampling frequency in this experiment is set to 1-200 Hz, and the sampling number is 1,024. In addition, in order to avoid the influence of external medium on the vibration signal acquisition of acceleration sensor to the greatest extent, the foam board was used to support the coal sample for testing.

The vibration parameter detector is a SA-02 vibration noise analyzer produced by RION, Japan. The analyzer boasts the function of multichannel vibration signal acquisition and analysis. The analysis software that matches the vibration noise analyzer, i.e., the exclusive SA-02BASE vibration noise signal analysis system produced by RION, Japan, can conduct FFT analysis on the vibration signal data obtained and calculate the natural frequency of coal vibration.

3.2. Vibration Signal Processing

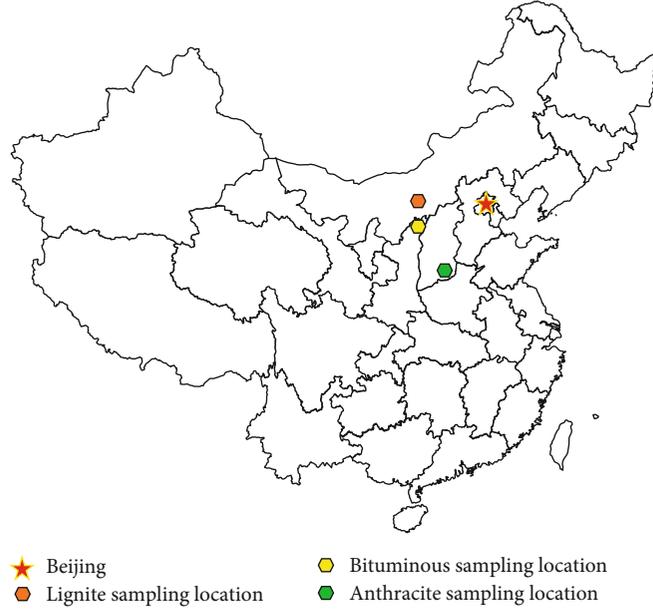


FIGURE 4: Sampling locations of experimental coal samples.

TABLE 1: Proximate analysis results of coal samples.

Type of coal	Proximate (wt%)			
	M_{ad}	A_{ad}	V_{ad}	FC_{ad}
Lignite	5.82	9.56	35.88	48.74
Bituminous	3.71	11.30	27.17	57.82
Anthracite	1.72	8.06	15.73	74.49

Note: M_{ad} : moisture content in the experimental coal sample; A_{ad} : ash content on an air-dried basis; V_{ad} : volatile matter content on an air-dried basis; FC_{ad} : fixed carbon content on an air-dried basis.

TABLE 2: Mechanical parameters of dried coal samples.

Type of coal	Elasticity modulus (GPa)	Poisson's ratio	Uniaxial compressive strength (MPa)
Lignite	1.11	0.32	9.36
Bituminous	3.16	0.29	18.69
Anthracite	2.67	0.30	14.73

(1) FFT transform

The original signal obtained by the signal acquisition system is the time-domain signal that changes with time. The vibration noise analysis software SA-02 BASE can be used to collect the time-domain signal for fast Fourier transform (FFT) to get the frequency domain signal, in the frequency domain diagram to determine the natural frequency of the sample. The formula of FFT transform is as follows [27]:

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft} dt, \quad (10)$$

where $X(f)$ is the frequency domain representation of the signal; $x(t)$ is the time-domain representation of the signal.

(2) Signal interception

Because the time-domain signal we collected is of finite time length, so it is necessary to intercept the signal before FFT transformation. The time-domain weighting function used for intercepting signals is called window function. The selection of window function depends on the purpose of analysis and the type of vibration signal, which is generally selected according to the following principles: (1) If the truncated signal is a periodic signal, there is no leakage and no window is needed. (2) If the signal is a random signal or there are multiple frequency components in the signal, and the test focuses on the frequency point rather than the energy size, Hanning window should be selected. (3) For calibration, accurate amplitude is required and flat-top window can be selected.

Combined with the selection principle of the above window function, the natural frequency of the coal sample is the focus of the analysis when the "Instantaneous Hammering method" is used to test the natural frequency of the coal body. Therefore, Hanning window is selected for signal interception in this paper.

(3) Frequency domain average

In order to improve the reliability of the test results, after collecting the time-domain data of the specified test times, the vibration and noise analysis software SA-02 BASE is used. Firstly, the time-domain signal data collected each time are transformed by FFT to obtain the corresponding frequency domain data. Then, the acceleration amplitudes of the same frequency points in the frequency domain signal are automatically summed and averaged, and the final spectrum diagram with dominant frequency is obtained.

3.3. *Experimental Coal Samples*. Experimental lignite coal with a low metamorphic degree from Dongsheng Coal Mine

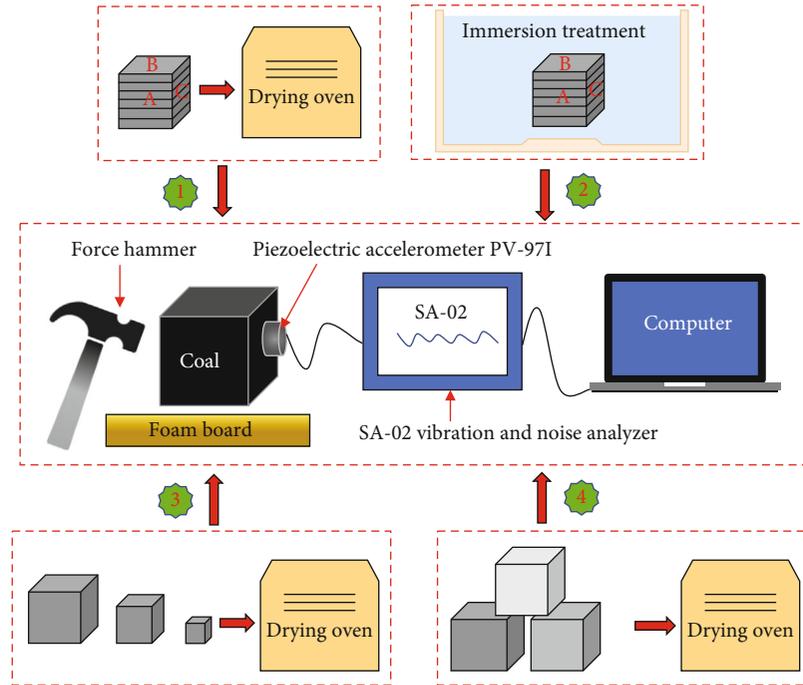


FIGURE 5: Experimental schemes and flowchart.

in Inner Mongolia, bituminous coal with a medium metamorphic degree from Daliuta Coal Mine in Shenmu City, Shaanxi Province, and anthracite coal with a high metamorphic degree in Chengzhuang Coal Mine in Shanxi Province, China, were selected as experimental coal samples. The sampling locations are displayed in Figure 4.

Proximate analysis was conducted on the three experimental coal samples to determine four indexes including moisture, ash, volatile matter, and fixed carbon. The determination results are given in Table 1, and the mechanical parameters of dried coal samples are exhibited in Table 2.

The lignite with the original structure was processed into cubes with sizes of $30\text{ mm} \times 30\text{ mm} \times 30\text{ mm}$, $50\text{ mm} \times 50\text{ mm} \times 50\text{ mm}$, and $100\text{ mm} \times 100\text{ mm} \times 100\text{ mm}$ based on the experimental needs. The bituminous coal and anthracite coal were processed into $100\text{ mm} \times 100\text{ mm} \times 100\text{ mm}$ cubes. Then, the experimental coal samples were put into an incubator at 55°C to be dried for 24 h. After the coal samples cooled to room temperature, they were put into a dryer for sealing and preservation for later use.

3.4. Test Schemes. In the hope of investigating the influences of bedding, moisture, size, and coal rank on the natural frequency of coal, four experimental schemes were developed. The experimental schemes and flowchart are exhibited in Figure 5.

- (1) The influence of bedding on the natural frequency of coal: Among three adjacent faces of the $100\text{ mm} \times 100\text{ mm} \times 100\text{ mm}$ cubic dried lignite coal sample with an obvious bedding structure, the face parallel to the bedding was marked as Face B, and the two faces perpendicular to the bedding were

labeled as Faces A and C; the faces corresponding to these three faces were marked as Faces a, b and c, respectively. Twenty natural frequency tests were carried out on each side, and the frequency domain signals of the six planes were obtained by the aforementioned “frequency domain average” method, and then the influence law of bedding on the natural frequency of coal was analyzed

- (2) The influence of moisture on the natural frequency of coal: The $100\text{ mm} \times 100\text{ mm} \times 100\text{ mm}$ cubic lignite coal sample in Scheme (1) was soaked in water to obtain the saturated coal sample. The natural frequency of the saturated coal sample was tested and analyzed for the Faces A, B, and C, respectively. The results were compared with those measured in Scheme (1).
- (3) The influence of size on the natural frequency of coal: The $30\text{ mm} \times 30\text{ mm} \times 30\text{ mm}$, $50\text{ mm} \times 50\text{ mm} \times 50\text{ mm}$, and $100\text{ mm} \times 100\text{ mm} \times 100\text{ mm}$ cubic dried lignite coal samples, ignoring the influence of bedding, were selected for obtaining frequency domain signals of different-size coal samples for the Face A according to the method in Scheme (1). Next, the values of natural frequencies of the coal samples for the Face A were used for analyzing the influence of size on natural frequency
- (4) The influence of coal rank on the natural frequency of coal: Cubic dried lignite, bituminous coal, and anthracite coal with the size of $100\text{ mm} \times 100\text{ mm} \times 100\text{ mm}$, ignoring the influence of bedding, were selected for obtaining frequency

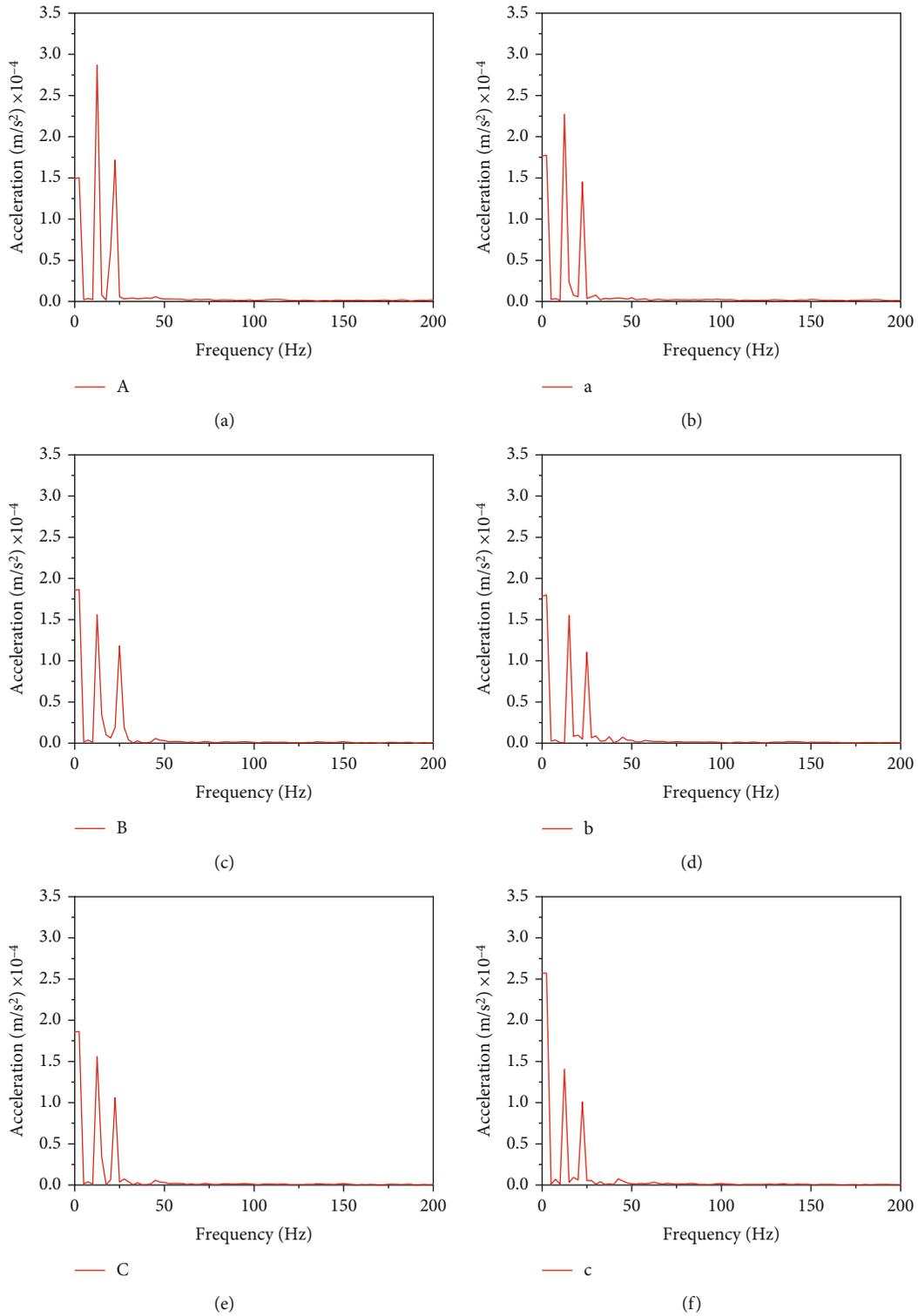


FIGURE 6: Frequency domains for the six faces of the dried lignite coal sample.

domain signals for the Face A of different-rank coal samples according to the method in Scheme (1). Next, the values of natural frequencies of the coal samples for the Face A were used for analyzing the influence of coal rank on natural frequency

4. Analysis and Discussion on the Experimental Results

4.1. *Influence of Bedding.* To analyze the influence of bedding on the natural frequency of coal, the frequency domains for

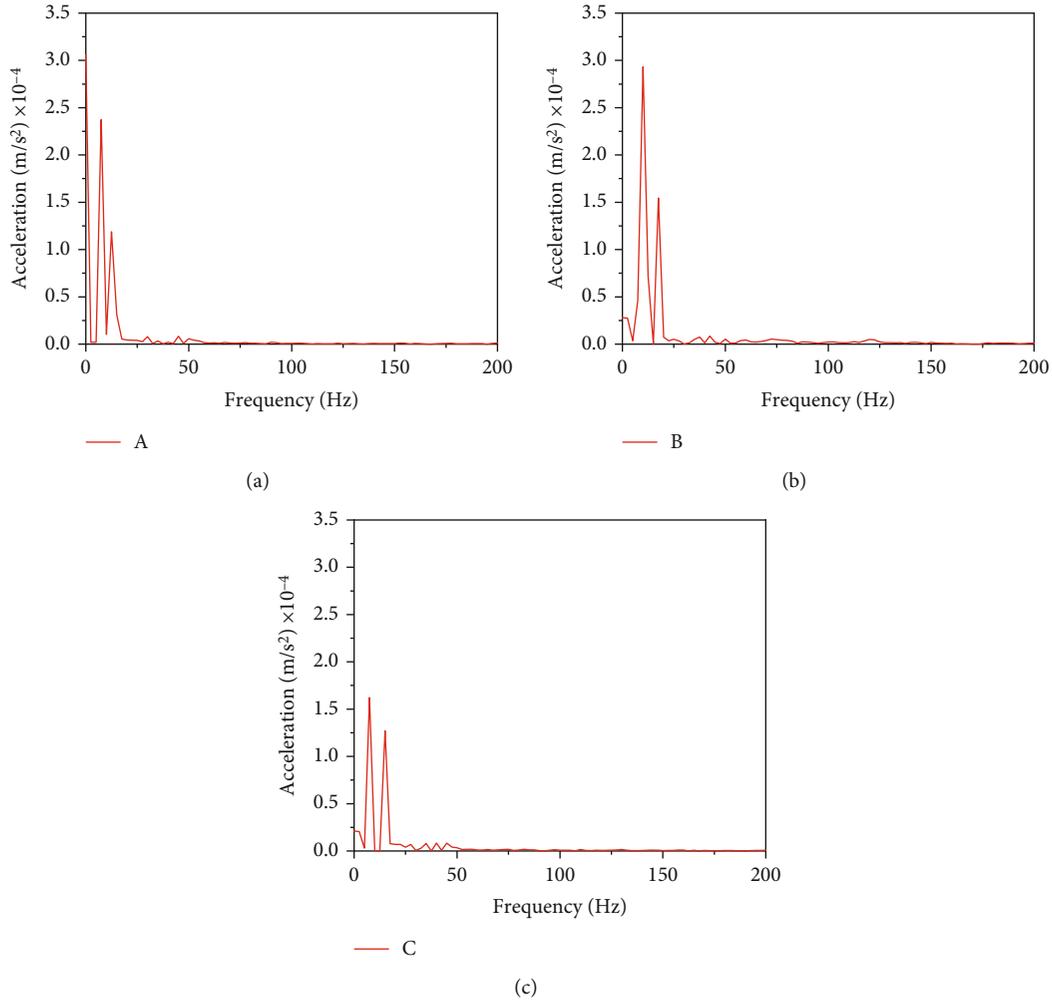


FIGURE 7: Frequency domains for the three adjacent faces of the saturated lignite coal sample.

TABLE 3: Mechanical parameters of dried and saturated lignite coal samples.

State of coal sample	Elasticity modulus (GPa)	Poisson's ratio	Uniaxial compressive strength (MPa)
Dried	1.11	0.30	9.36
Saturated	0.95	0.27	7.69

the six faces of the 100 mm × 100 mm × 100 mm cubic dried lignite coal sample were obtained according to Scheme (1) in Section 3.3. The test results are displayed in Figure 6.

The following findings are obtained from the frequency domain diagrams for the six faces of the dried lignite coal sample:

(1) The amplitudes of acceleration signals for different faces are not the same. This is because when the natural frequency of the coal sample is measured by the instantaneous hammering method, it was difficult to ensure that the intensity of each percussion is exactly the same. The amplitude of acceleration signals represents the magnitude of the percussion force to a

certain extent, but this does not affect the test and analysis on the natural frequency of coal.

- (2) Obvious natural frequencies of the first two orders, whose values lie in the range of 0-30 Hz, are excited for all the faces of the coal sample. Among them, the first-order and second-order natural frequencies for Faces A and a are both 12.5 Hz and 22.5 Hz; those for Faces B and b are both 15 Hz and 25 Hz; those for Faces C and c are both 12.5 Hz and 22.5 Hz. This indicates that the measured results of natural frequencies of coal samples conform to the reciprocity theorem. In other words, in the case of single excitation, the response remains unchanged when the positions of the excitation port and the response port are exchanged.
- (3) For all the faces of the coal sample, the second-order natural frequency is higher than the first-order natural frequency, which agrees with the conclusion that the natural frequency of the coal sample increases with the rise of the order described in the multi-degree-of-freedom theoretical model.

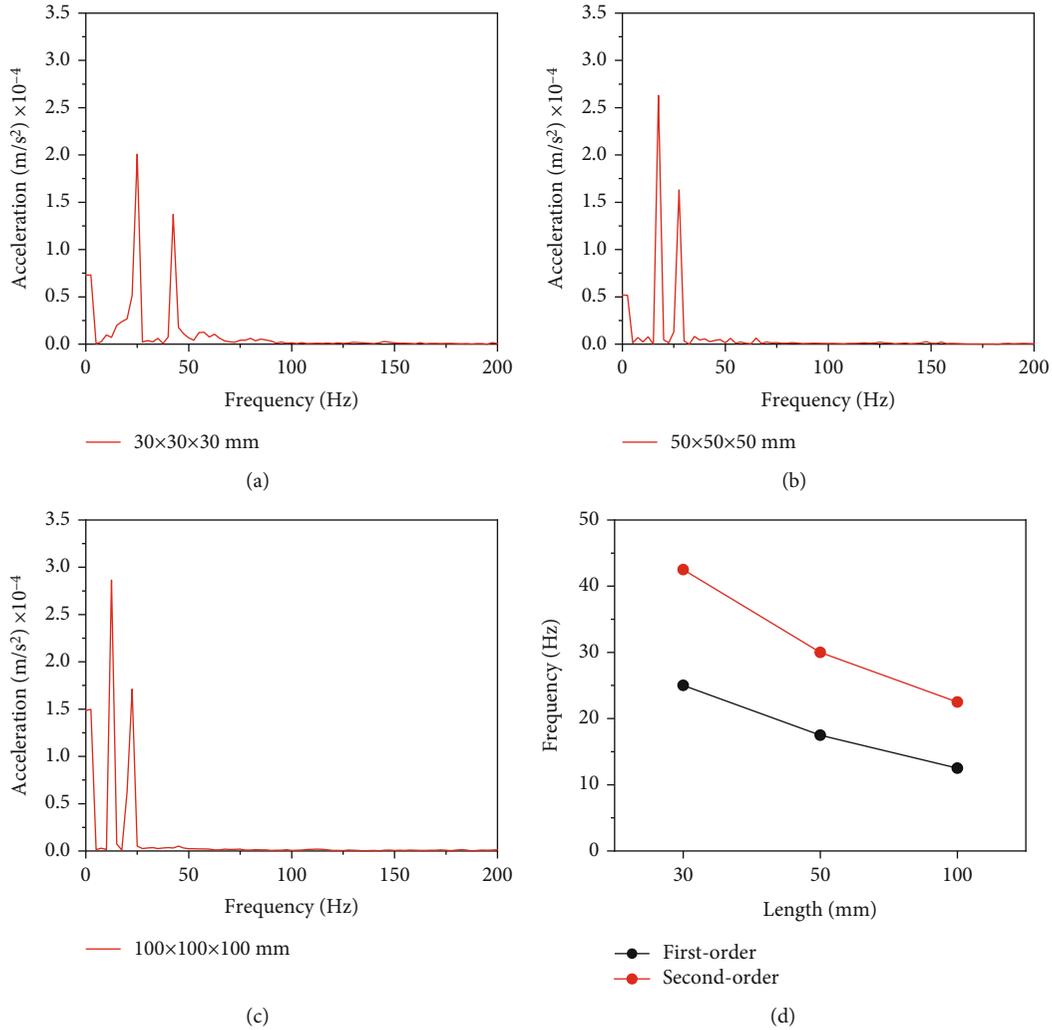


FIGURE 8: Frequency domain diagrams and natural frequency variation pattern diagram of different-size dried lignite coal samples.

(4) The first-order and second-order natural frequencies for Faces B and b are higher than those for Faces A, a, C, and c, mainly because the complexity of the internal structure of the coal sample results in the discrepancy of the direction of its internal mechanical properties.

In the literature [30], the influence of anisotropy on the mechanical response characteristics of coal was expounded in detail. The uniaxial compressive strength and peak strength parallel to the Faces of bedding are higher than those perpendicular to the Faces of bedding. The directional distribution characteristics of microstructures in the coal sample cause regular changes in mechanical properties. Therefore, the natural frequencies measured for faces parallel to the bedding are higher than those measured perpendicular to the bedding.

4.2. Influence of Moisture. Microscopically, coal has a porous structure of pores and cracks, and its molecular structure is rich in oxygen-containing functional groups, showing strong hydrophilicity [31]. The change of coal natural frequency is

an external expression of its internal structural parameters. The influence of moisture erosion on coal natural frequency cannot be ignored.

According to Scheme (2) in Section 3.3, the natural frequencies for the six faces of the saturated coal sample were measured. The natural frequency of coal satisfied the reciprocity theorem based on the analysis in Scheme (2) in Section 4.1. Hence, the frequency domain diagrams for the three adjacent Faces A, B, and C of the saturated coal sample were plotted (Figure 7).

From the frequency domain diagrams for the three adjacent faces of the saturated lignite coal sample, it can be concluded that:

- (1) When it comes to the saturated lignite coal sample, the first-order and second-order natural frequencies for Face A are 7.5 Hz and 15 Hz; those for Face B are 10 Hz and 17.5 Hz; those for Face C are 7.5 Hz and 15 Hz, respectively. Compared with the test results of the Faces A, B, and C of dried lignite samples in Section 4.1, the natural frequencies of saturated water coal sample are significantly reduced

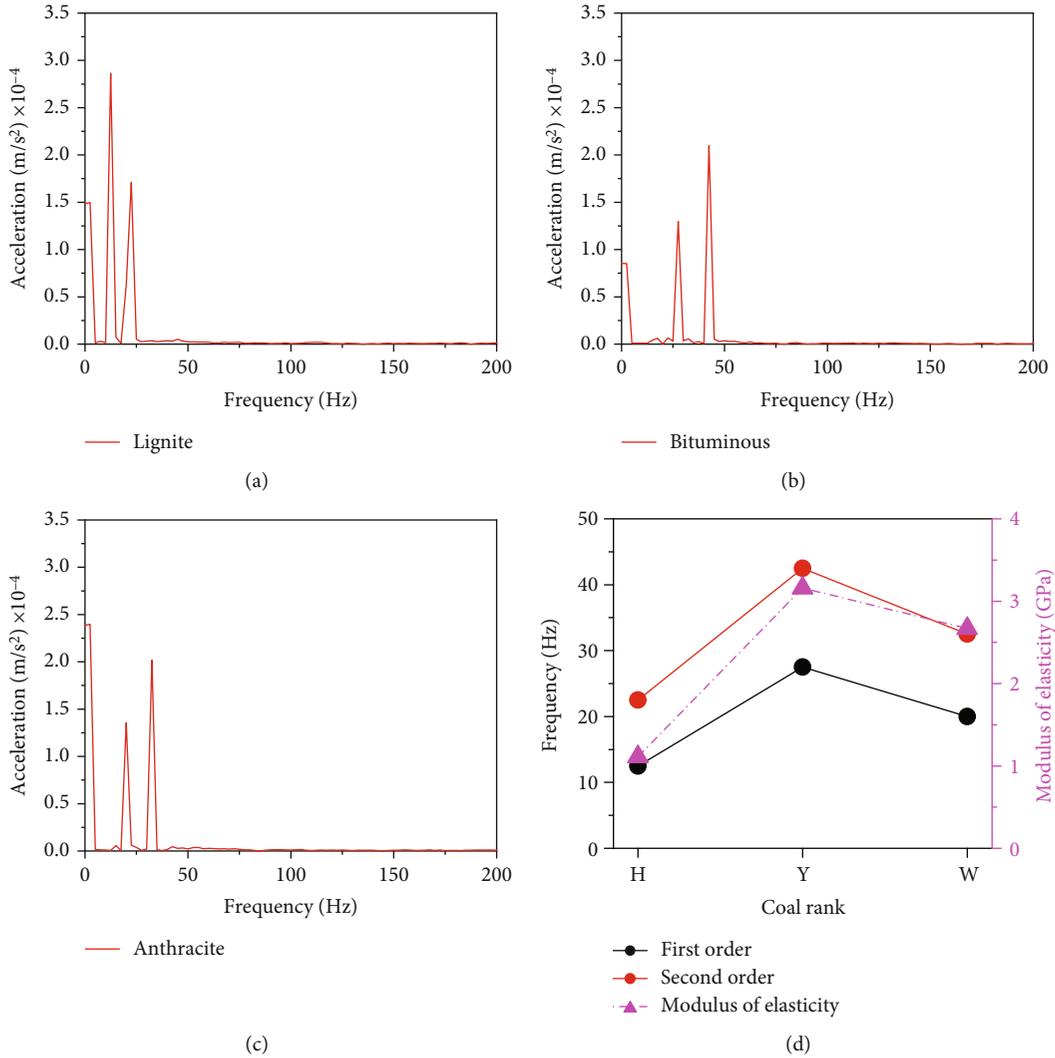


FIGURE 9: Frequency domain diagrams and natural frequency variation pattern diagram of different-rank dried lignite coal samples.

- (2) Under water erosion, the natural frequencies for Face B are still higher than those for Faces A and C even of the same coal sample, which further verifies the influence of bedding structure on the natural frequency of coal
- (3) The density of the coal sample increases after saturation treatment. According to Eq. (5) of the mathematical model of natural frequency of coal in section 2, the increase of coal sample density is bound to induce the reduction of natural frequency
- (4) The moisture content of coal also exerts a significant effect on its mechanical parameters. The elastic modulus and uniaxial compressive strength of coal samples after saturated treatment are significantly reduced, which leads to the decrease of natural frequency of coal samples. The comparative test results of some mechanical parameters of dried and saturated lignite coal samples are listed in Table 3

4.3. Influence of Size. Due to the limitations of coal damping and vibration wave propagation energy, the size of coal also considerably influences on its natural frequency in the range of vibration wave transmission. A relevant study discloses that the strength of rock decreases monotonously and tends to approach its minimum strength value with the increase of its size [32]. In order to reveal the influence of size on the natural frequency of coal, frequency domain diagrams for Face A of dried lignite coal samples with the sizes of 30 mm \times 30 mm \times 30 mm, 50 mm \times 50 mm \times 50 mm, and 100 mm \times 100 mm \times 100 mm were obtained according to Scheme (3) in Section 3.3 (Figure 8).

From the frequency domain diagrams and natural frequency variation pattern diagram of different-size dried lignite coal samples, the following conclusions can be drawn:

- (1) The first-order and second-order natural frequencies for Face A of the 30 mm \times 30 mm \times 30 mm dried lignite coal sample are 25 Hz and 42.5 Hz; those for Face A of the 50 mm \times 50 mm \times 50 mm dried lignite

coal sample are 17.5 Hz and 30 Hz; those for Face A of the 100 mm × 100 mm × 100 mm dried lignite coal sample are 12.5 Hz and 22.5 Hz, respectively. Under the same condition, the natural frequency decreases with the increase of coal sample size

- (2) When the elastic modulus, density, moisture, bedding, and other conditions of coal remain constant, the natural frequency of coal is inversely proportional to its size. That is, the larger the coal size is, the lower the natural frequency will be. Through formula (5), the experimental results agree well with the theoretical model

Hence, the size effect of coal should be considered when using vibration wave for permeability enhancement excitation. Low-frequency vibration excitation should be applied to large-size coal for the purpose of achieving resonance and permeability enhancement effect.

4.4. Influence of Coal Rank. For different-rank coals, pores and cracks are distributed in different ways and the contents of organic matters and minerals also differ notably, which directly determines and affects the mechanical properties [33, 34]. In order to explore the influence of coal rank on the natural frequency of coal, frequency domain diagrams for Face A of the 100 mm × 100 mm × 100 mm typical dried coal samples of high, middle, and low rank were obtained according to Scheme (4) in Section 3.3 (Figure 9).

The following conclusions can be drawn from the frequency domain diagrams and natural frequency variation pattern diagram of different-rank dried lignite coal samples:

- (1) Natural frequencies of different-rank coal samples are significantly different. The first-order and second-order natural frequencies of lignite are 12.5 Hz and 22.5 Hz; those of bituminous coal are 27.5 Hz and 42.5 Hz; those of anthracite coal are 20 Hz and 32.5 Hz. Under the conditions of identical bedding, size, and moisture, the first-order and second-order natural frequencies of different-rank coals follow the order: bituminous coal > anthracite coal > lignite
- (2) The main frequency of signal refers to the frequency with the greatest response energy under the same impulse excitation. The main frequencies of signals generated by different-rank coal samples also differ. The main frequency of signals generated by lignite coal is located at the first-order natural frequency, while those of bituminous coal and anthracite coal are at the second-order natural frequency. Such a result suggests that the coal samples have multiple orders of natural frequencies, but the response degrees of different frequencies differ under the same impulse excitation
- (3) A comparison of the relevant parameters of different-rank coal samples in Tables 1 and 2 demonstrates that the ash contents (one of the main

parameters of mineral content) in the three coal samples follow the order: bituminous > anthracite > lignite. Besides, minerals fill the pores and cracks in coal, strengthen the compressive strength of the matrix skeleton, and improve the mechanical strength and elastic modulus. In the corresponding dry coal samples of the same size, the elastic modulus and uniaxial compressive strength of bituminous coal are the largest, followed by anthracite and lignite. According to the single-degree-of-freedom theoretical model, the larger the elastic modulus of the material is, the larger the natural frequency is. The physical and mechanical parameters of coal directly determine the natural frequency of coal samples of different rank

5. Conclusions

In this study, the natural frequency of coal was theoretically analyzed first. Based on the theoretical analysis, mathematical models of coal natural frequency were established, and an experiment was performed on the variations of coal natural frequency. Furthermore, the main factors influencing coal natural frequency were explored. The following conclusions were drawn:

- (1) Based on the established natural frequency vibration models of coal, it is concluded that coal vibration has multiorder natural frequency which grows with the increase of the order. In addition, the natural frequency of coal is directly determined by the physical and mechanical parameters such as size, elastic modulus, density, bedding, mass, and stiffness
- (2) The bedding structure characteristics of the experimental coal sample lead to regular variations of its mechanical properties, so that the natural frequency parallel to the bedding plane is higher than that perpendicular to the bedding plane. Besides, the size of the coal sample primarily exerts an influence on its mass. The larger the coal size and mass are, the lower the natural frequency would be
- (3) For the saturated coal sample, moisture changes its density and reduces its elastic modulus. Resultantly, its natural frequency is lower than that of the dried coal sample. Moreover, the physical and mechanical properties of different-rank dried coal samples are affected by the content of organic matter and minerals, which results in the difference of natural frequencies of different-rank dried coal samples. Thus, the natural frequencies of different-rank coal samples follow the order: bituminous > anthracite > lignite
- (4) The experiment results indicate that the natural frequencies of coal samples under different influencing factors are all tens of Hz. Thus, in the application of the RPEEVW technology, attention should be paid to the vibration excitation under the low-frequency condition

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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