

Research Article

Time-Dependent Creep Constitutive Model of Roadway Surrounding Rock Based on Creep Parameters

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In order to better study the creep deformation characteristics of rock under different stresses, the generalized Burgers model was used as the basic model after analyzing the creep characteristics of rock and its relationship with stress and strain, and its application scope was verified. Based on the damage mechanics theory, a viscoplastic body considering aging damage was established, which was connected with the generalized Burgers model in series. A new nonlinear creep constitutive model was obtained, which was extended to three dimensions. In addition, the relationship between model parameters and damage variables was established by introducing damage variables that considered damage effects. The unsteady creep constitutive model of rock is obtained, and the correctness and rationality of the model are verified by test data. The results show that this model not only accurately reflects the creep characteristics of attenuation and steady creep stage but also overcomes the defect that the generalized Burgers model is difficult to describe the accelerated creep. Considering the deterioration of creep parameters with time, the creep damage process of rock under different stress states can be better described, which provides a new idea for establishing unsteady creep model and determining model parameters.

1. Introduction

With the rapid development of underground and tunnel engineering, the geotechnical mechanics involved in complex geological environment becomes more and more complicated. This also poses great challenges for rock engineering. Due to the interaction of multiple factors such as gravity, tectonic stress, and underground flowing water, the deep rock mass often shows obvious creep characteristics [1]. Since the creep characteristics of rock are closely related to the long-term stability of rock mass [2], such as tunnel surrounding rock affected by in situ stress and seepage force often showed obvious internal crushing, as well as the rheological characteristics and strong weathering properties, with deep underground rock tunnel under the external load deformation of surrounding rock, lining under the interac-

tion of supporting structure and surrounding rock mechanics engineering practical problems such as deformation time effect. Therefore, enough attention must be paid to creep characteristics of rock in engineering. The study on creep characteristics of roadway surrounding rock considering its timeliness can not only provide a reasonable solution for the failure of roadway surrounding rock but also provide a certain theoretical basis for the excavation and support technology of tunnel engineering.

The constitutive model is the main form to reflect the characteristics of rock and soil mechanics and deformation. The current constitutive model of roadway surrounding rock is mainly formed by free combination of components which can reflect different properties. The established model is flexible and simple, but it is still difficult to describe the mechanical characteristics of rock, especially the accelerated

creep characteristics of rock. As is known to all, creep deformation of rocks accumulates gradually under external stress conditions. Changes in external stress conditions affect the development of creep deformation. When a certain stress level is reached, rocks exhibit accelerated creep behavior. Therefore, a scientific and reasonable creep constitutive model of rock must be able to accurately simulate the accelerated creep characteristics of rock. In this respect, some scholars have carried out pioneering work. In order to reflect the viscoelastic-plastic characteristics of soft rock under different stress levels, Chen et al. [3] proposed the plastic body of fracture and nonlinear creep and constructed a composite rheological mechanics model, which can well reflect the three-stage creep characteristics and crack closure effect. Yang and Xu [4] used the Kachanov damage definition to deduce the damage evolution equation of different creep stages and established a nonlinear damage rheological model of rock based on the view of effective stress, which can better describe the three-stage creep characteristics. Zhi-lei et al. [5] established a fractional-order creep model by replacing Newtonian bodies in the Westland model with soft components. Jiawen et al. [6] obtained a nonlinear creep model that could describe the accelerated creep characteristics of greenschist by improving the generalized Bingham model. Yajing et al. [7] proposed a nonlinear stick-pot element with strain-triggering and presented its constitutive relation. A new constitutive equation reflecting the whole process of rock creep was established by connecting the element with the traditional western element model in series. Fengnian and Kuiying [8] found that most of the traditional viscoelastic models could not describe the whole process of rock creep effectively through analysis, and then discussed the nonlinear viscoelastic constitutive model from the elastic model, the relationship between strength and strain velocity, and the influence of the change of creep experimental parameters on the curve. [9] proposed a new element combining the classical Mohr coulomb criterion and then connected it in series with the traditional Burgers model. The new creep model obtained can make up for the defect that the Burgers model cannot reflect the accelerated creep stage of rock. Tested the long-term creep mechanical behavior of salt rock under low stress (7.5~15 MPa) and proposed a nonlinear creep constitutive model of salt rock considering damage. Mansouri and Ajalloeian [10] researched the mechanical characteristics of salt rock using uniaxial compression tests and creep tests in a salt diapir located in the south of Iran. Based on the creep experiment of triaxial salt rock, Fei et al. [11] extended the traditional fractional derivative model from one-dimensional case to three-dimensional case. A new creep model with varying order fractional derivative is proposed to describe the total creep process of salt rock. Hou et al. [12] studied the influence of initial damage on rock aging behavior by analyzing the creep test results of sandstone with different initial damage degrees and proposed a coupled creep damage constitutive model that can be used to predict the time-varying behavior of rock under different initial damage states. Yonggang and Xiue [13] established the unsteady creep model and deduced its creep compliance based on the unsteady sticky pot, which can better reflect

the nonlinear characteristics of creep. Sterpi and Gioda [14] established a constitutive model that can better describe the creep behavior of tuff by establishing the relationship between creep strain rate and rheological parameters. Junzhen [15] explored the relationship between brittle creep failure time of hard rock and postfailure stress deformation in view of the brittle creep failure phenomenon of hard rock. Haifeng et al. [16] introduced fractional calculus to describe the viscoelastic and viscoplastic strain of creep of soft rock and connected it with damaged elastomer to obtain a new creep damage model, which proved that it could better reflect the creep deformation characteristics of soft rock. Xuecheng [17] established a nonlinear creep model for the three-stage creep process based on the creep aging and damage characteristics of marble. Yongjun et al. [18] established a nonlinear Burgers model with FC component combination, which can describe the nonlinear creep stage of rock well. Yang et al. [19] proposed a nonlinear sticky pot element to improve the Burgers model and extended the creep equation of the unsteady Burgers model to three-dimensional stress state. Kaiyun et al. [20] established an unsteady nonlinear model by introducing a nonlinear viscoplastic body, considering the weakening law of elastic modulus with time. Junbao et al. [21] found an empirical model that could describe the whole process of creep and made the model curve have a good fitting degree with the test data. Based on the fractional order theory, Zhi-lei et al. [5] established a nonlinear model that could overcome the shortcoming of traditional Westland model that was difficult to describe accelerated creep on the basis of considering the unsteady characteristics of parameters. Zhao et al. [22], based on Kachanov damage theory, combined Burgers model and nonlinear M-C plastic element in series to form the BNMC creep damage model, which can reasonably simulate the viscoelastic-plastic and creep damage of rock. Wang et al. [23] established damage variable expression based on the difference between initial pressure and unloading pressure and introduced Burgers model to establish nonlinear creep model, which can describe the whole process of rock creep better.

The above pioneering work provides a good theoretical basis for the establishment of constitutive models considering the accelerated creep characteristics of rock. However, most models cannot describe the accelerated creep behavior of soft rock and hard rock at the same time. Based on this, in this paper, the generalized Burgers creep model is used as the basic model, the damage mechanics theory is introduced, the nonlinear viscoelastic-plastic creep damage model of rock is established, and the model is extended to the three-dimensional stress state. In addition, this paper also gives a method to calculate the model parameters and verifies the rationality and feasibility of the model by comparing with the experimental results.

2. Nonlinear Creep Characteristics of Rock

After rock is subjected to constant load, it will produce a transient deformation at first, and then enter the stage of creep deformation, which is generally divided into three stages: attenuation creep, stable creep and accelerated creep.

Not all rocks undergo creep stage after loading, which leads to failure. The creep deformation behaviour of rocks is affected by its own structural properties, stress level and loading time. According to the stress level, creep can be divided into steady creep and unsteady creep processes. [1, 24–26]. The details are as follows:

- (1) When the stress level is less than the long-term strength of the rock, the rock exhibits the property of attenuation creep. The creep curve presents an upward convex shape, and the deformation value approaches to a constant value. And with the increase of time, the rock will not occur creep damage
- (2) When the stress level is greater than the long-term strength of the rock, it exhibits the property of nonattenuation creep. Under the long-term loading, the deformation of rock increases with time, and the creep curve presents an inverse “s” type, which gradually goes through the stages of attenuation creep, stable creep, and accelerated creep and finally reaches the ultimate strain and occurs instability failure

It should be noted that not all rocks exhibit three-stage creep when subjected to horizontal stress loads. Because of the difference of rock mechanical properties and loading methods, the duration of each creep stage of rock is different. At this point, the cracks and damage inside the rock caused by the load are also different. When the stress is greater than the long-term strength of the rock, the creep rate of the second creep stage is low and lasts for a long time at a low stress level, and the third creep stage is difficult to observe, mainly because the internal damage of the rock will not continue to develop at a low stress level. At a high stress level, the creep rate increases sharply and soon enters the third creep stage. The second creep stage lasts for a short time, and it is even difficult to observe the phenomenon of stable creep. This is mainly due to the rapid development of internal damage of rock at a high stress level, resulting in the nonobvious stable creep stage. A complete three-stage creep curve can be observed only under moderate stress conditions.

When the stress level is lower than point A, there is no creep phenomenon and only transient deformation. When the stress level is higher than point A and lower than point B, the creep property is only attenuated. At the same time, the creep stops when it intersects the creep termination trajectory, and the rock specimen is in a stable state. The final creep deformation will fall at the intersection of AE line and strain parallel line. The strain corresponding to point B is defined as the strain corresponding to the long-term strength of rock, and the AE curve is defined as the termination track line of creep under low stress level. When the stress level is greater than point B, the rock will exhibit three-stage creep and nonattenuation creep and eventually will fail over time. Moreover, the higher the stress level is, the shorter the creep failure time is, and the smaller the final creep strain value is. As can be seen from the Figure 1, when the external load is C point, the final creep reaches point D failure. When the external load is greater than point C, the final creep fails before point B, and the creep duration of

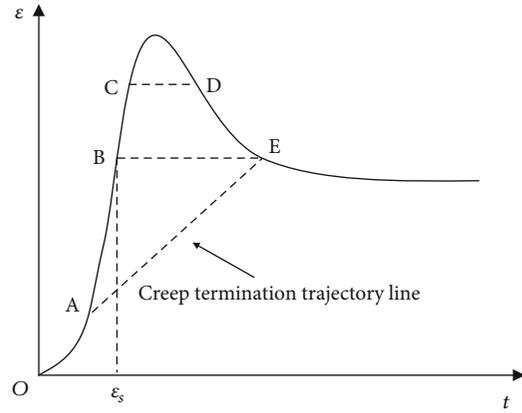


FIGURE 1: Creep and stress-strain relationship.

the former is longer than that of the latter and the final creep strain value is larger.

According to the above analysis of creep characteristics, when the external load stress level is lower than the long-term strength, the rock will not occur creep failure. When the external load stress level is greater than the long-term strength, rock failure will occur with the development of time. It is therefore important to determine the long-term strength or strain of the rock at this time. It is very important for the stability of rock mass engineering to control the external load (strain) below the long-term strength (strain corresponding to long-term strength). When the external load cannot be controlled, effective engineering support should be carried out before accelerating creep to enhance the mechanical properties of rock mass and the ability to resist deformation and failure, so as to avoid the occurrence and development of engineering accidents.

3. Establishment of the Nonlinear Creep Model

3.1. Base Model Selection. As a common complex nonlinear geological material, there are some basic mechanical characteristics in rock body, such as elasticity, viscosity, viscoelasticity, and viscoplasticity. There are three kinds of elements commonly used in rock constitutive models: elastic element, viscous element, and plastic element. Since a single element can only describe a certain fixed property of rock body, the model of rock composite element can be constructed by combining different elements. The generalized Burgers model is developed from the classical Burgers model, which can be regarded as a generalized Burgers creep model with wider application scope by connecting a Kelvin body in series on the basis of the traditional Burgers model. The schematic diagram of the generalized Burgers creep model is shown in Figure 2.

where E_1 and η_1 are the elastic modulus and viscosity coefficient of Maxwell body, E_2 and η_2 are the elastic modulus and viscosity coefficient of Kelvin-I body, and E_3 and η_3 are the elastic modulus and viscosity coefficient of Kelvin-II body, respectively.

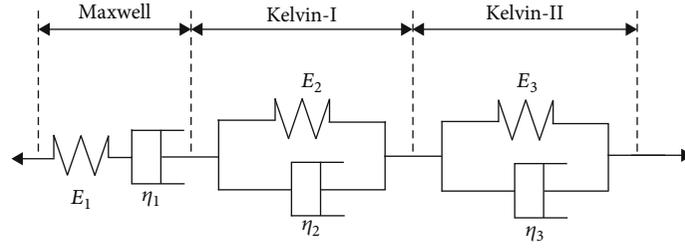


FIGURE 2: Generalized Burgers creep model.

According to the rheological model theory, the generalized Burgers model satisfies the following relations in one-dimensional state:

$$\begin{cases} \sigma = \sigma_1 + \sigma_2 + \sigma_3, \\ \varepsilon = \varepsilon_1 + \varepsilon_2 + \varepsilon_3, \end{cases} \quad (1)$$

where σ is the stress, ε is the creep deformation of rock, and subscripts 1, 2, and 3 are state variables of Maxwell body, Kelvin-I body, and Kelvin-II body, respectively.

According to the mechanical characteristics of the component itself, the one-dimensional creep equation of the generalized Burgers model can be obtained:

$$\varepsilon = \frac{\sigma}{E_1} + \frac{\sigma}{\eta_1} t + \frac{\sigma}{E_2} \left[1 - \exp\left(-\frac{E_2}{\eta_2} t\right) \right] + \frac{\sigma}{E_3} \left[1 - \exp\left(-\frac{E_3}{\eta_3} t\right) \right]. \quad (2)$$

3.2. Improved Generalized Burgers Model with Fixed Parameters. The creep curves of intact rock specimens are mainly composed of initial creep stage, stable creep stage, and accelerated creep stage. The creep curve of generalized Burgers model with fixed parameters is shown in Figure 3. For the generalized Burgers model, attenuation creep and stable creep deformation can be well described. However, the accelerated creep deformation cannot be well described because the components used in the generalized Burgers model are ideal linear components. In order to enhance its describing ability, the generalized Burgers model is improved to reflect the creep characteristic curves under different stresses.

At the microscopic level, the creep deformation of rock is the accumulation of cracks and damage inside the rock. When the cracks develop to a certain extent, the rock will be destroyed. Considering that the deformation of rock is mainly plastic deformation after the accelerated creep, it is more reasonable to select a strain parameter as the threshold value of rock entering the accelerated creep. In this paper, a nonlinear viscous pot with strain triggering proposed by Yajing et al. [7] was used to describe the deformation of rock in the accelerated creep stage. The improved generalized Burgers model was obtained by connecting this model in series

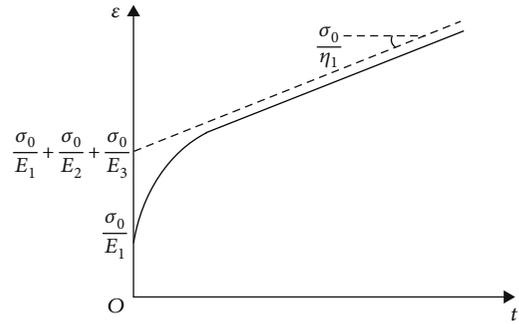


FIGURE 3: Generalized Burgers creep curve.

to the generalized Burgers model, as shown in Figure 2. The constitutive model of the nonlinear sticky pot is

$$\begin{cases} \sigma = \eta_n \ddot{\varepsilon}_n & (\varepsilon \geq \varepsilon_s), \\ \varepsilon_n = 0 & (\varepsilon < \varepsilon_s), \end{cases} \quad (3)$$

where ε_n is the deformation caused by accelerated creep, and η_n is the viscosity coefficient of nonlinear sticky pot.

As shown in Figure 4, when $\varepsilon < \varepsilon_s$, the deformation produced by nonlinear sticky pot is zero. Therefore, the model can be reduced to a generalized Burgers model, as shown in Equation (2).

However, when $\varepsilon \geq \varepsilon_s$, the rock enters an accelerated creep phase. At this point, the nonlinear sticky pot comes into play. According to the series relationship of model components, the total strain of the improved generalized Burgers model can be expressed as

$$\varepsilon = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_n. \quad (4)$$

Combining Equations (2)–(4) with Laplace transform, one can be obtained as

$$\tilde{\varepsilon}_1(s) + \tilde{\varepsilon}_2(s) + \tilde{\varepsilon}_3(s) = \frac{\sigma}{E_1 s} + \frac{\sigma}{\eta_1 s^2} + \frac{\sigma}{(E_2 + \eta_2 s)s} + \frac{\sigma}{(E_3 + \eta_3 s)s}, \quad (5)$$

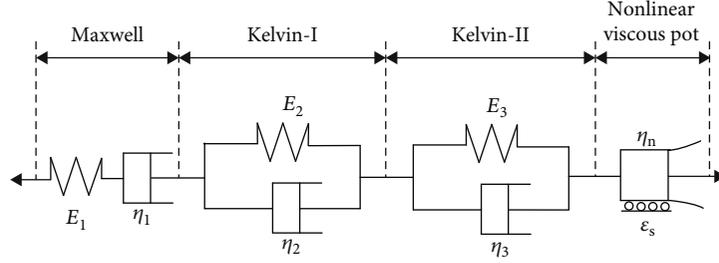


FIGURE 4: Improved generalized Burgers creep model.

$$\tilde{\varepsilon}_n(s) = \frac{\sigma}{\eta_n s^3}, \quad (6)$$

$$\tilde{\varepsilon}(s) = \tilde{\varepsilon}_1(s) + \tilde{\varepsilon}_2(s) + \tilde{\varepsilon}_3(s). \quad (7)$$

where $\tilde{\varepsilon}$ is the Laplace transform of ε , and s is the complex variable of the Laplace transform space.

The combined Equations (5)-(7), $\tilde{\varepsilon}(s)$, can be expressed as

$$\tilde{\varepsilon}(s) = \frac{\sigma}{E_1 s} + \frac{\sigma}{\eta_1 s^2} + \frac{\sigma}{(E_2 + \eta_2 s)s} + \frac{\sigma}{(E_3 + \eta_3 s)s} + \frac{\sigma}{\eta_n s^3}. \quad (8)$$

The inverse Laplace transform is applied to Equation (8). When rock deformation is greater than ε_s , the stress-strain

relationship of the improved generalized Burgers model can be expressed as

$$\varepsilon = \frac{\sigma}{E_1} + \frac{\sigma}{\eta_1} t + \frac{\sigma}{E_2} \left[1 - \exp\left(-\frac{E_2}{\eta_2} t\right) \right] + \frac{\sigma}{E_3} \left[1 - \exp\left(-\frac{E_3}{\eta_3} t\right) \right] + \frac{\sigma}{2\eta_n} \xi^2, \quad (9)$$

where $\xi = t - t_s$, t_s , is the time when the rock just enters accelerated creep.

Combining Equations (2), (3), and (9), a complete improved generalized Burgers one-dimensional creep model can be obtained as follows:

$$\begin{cases} \varepsilon = \frac{\sigma}{E_1} + \frac{\sigma}{\eta_1} t + \frac{\sigma}{E_2} \left[1 - \exp\left(-\frac{E_2}{\eta_2} t\right) \right] + \frac{\sigma}{E_3} \left[1 - \exp\left(-\frac{E_3}{\eta_3} t\right) \right] & (\varepsilon < \varepsilon_s), \\ \varepsilon = \frac{\sigma}{E_1} + \frac{\sigma}{\eta_1} t + \frac{\sigma}{E_2} \left[1 - \exp\left(-\frac{E_2}{\eta_2} t\right) \right] + \frac{\sigma}{E_3} \left[1 - \exp\left(-\frac{E_3}{\eta_3} t\right) \right] + \frac{\sigma}{2\eta_n} \xi^2 & (\varepsilon \geq \varepsilon_s). \end{cases} \quad (10)$$

3.3. Improved Constant Three Dimensional Generalized Burgers Model. Under three-dimensional stress conditions, the total strain of the improved generalized Burgers model can be expressed by a tensor:

$$\varepsilon_{ij} = \varepsilon_{ij}^1 + \varepsilon_{ij}^2 + \varepsilon_{ij}^3 + \varepsilon_{ij}^n, \quad (11)$$

where ε_{ij} is the total strain of the improved generalized Burgers model under three-dimensional stress state, and ε_{ij}^1 , ε_{ij}^2 , ε_{ij}^3 , and ε_{ij}^n are the strains of Maxwell body, Kelvin-I body, Kelvin-II body, and nonlinear pot, respectively.

According to the generalized Hooke's law, the three-dimensional constitutive relation of elastomers is

$$\begin{cases} e_{ij} = \frac{1}{2G_0} s_{ij}, \\ \varepsilon_{ii} = \frac{1}{3K} \sigma_{ii}, \end{cases} \quad (12)$$

where s_{ij} and e_{ij} represent partial stress and partial strain tensors, respectively; accordingly, σ_{ii} and ε_{ii} represent volumetric stress and volumetric strain, respectively, and G_0 and K represent shear modulus and volumetric modulus, respectively.

The stress tensor σ_{ij} can be further decomposed into

$$\sigma_{ij} = s_{ij} + \delta_{ij} \sigma_{mm}. \quad (13)$$

It is generally considered that the bulk strain of rock can be ignored in the whole creep stage of rock; so, the creep characteristics of rock are mainly reflected in shear deformation. Therefore, an improved generalized Burgers three-dimensional creep equation is established without considering volumetric creep. Thus, Equation (10) is extended to three-dimensional stress state, and the following equation can be obtained:

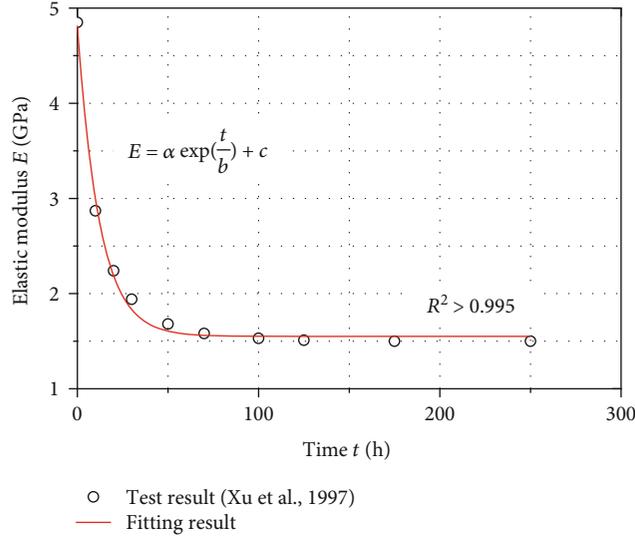


FIGURE 5: Relation between elastic modulus and time [29].

$$\begin{cases} \varepsilon_{ij} = \frac{s_{ij}}{2G_1} + \frac{\sigma_{mm}}{3K} \delta_{ij} + \frac{s_{ij}}{2H_1} t + \frac{s_{ij}}{2G_2} \left[1 - \exp\left(-\frac{G_2}{H_2} t\right) \right] +, \\ \frac{s_{ij}}{2G_3} \left[1 - \exp\left(-\frac{G_3}{H_3} t\right) \right] (\varepsilon_{11} < \varepsilon_s), \\ \varepsilon_{ij} = \frac{s_{ij}}{2G_1} + \frac{\sigma_{mm}}{3K} \delta_{ij} + \frac{s_{ij}}{2H_1} t + \frac{s_{ij}}{2G_2} \left[1 - \exp\left(-\frac{G_2}{H_2} t\right) \right] +, \\ \frac{s_{ij}}{2G_3} \left[1 - \exp\left(-\frac{G_3}{H_3} t\right) \right] + \frac{\xi^2}{4H_n} s_{ij} (\varepsilon_{11} \geq \varepsilon_s), \end{cases} \quad (14)$$

where G_1 , G_2 , and G_3 correspond to the shear moduli of elastic moduli E_1 , E_2 , and E_3 , respectively, and H_1 , H_2 , H_3 ,

and H_n , respectively, represent the viscosity coefficients under three-dimensional stress state at each stage.

The conventional triaxial stress path satisfies the following relationship:

$$\begin{cases} \sigma_2 = \sigma_3 < \sigma_1, \\ \sigma_{mm} = (2\sigma_3 + \sigma_1)/3, \\ s_{11} = 2(\sigma_1 - \sigma_3)/3. \end{cases} \quad (15)$$

By substituting Equation (15) into Equation (14), the improved generalized Burgers model under three dimensional stress can be expressed as

$$\begin{cases} \varepsilon_{11}(t) = \frac{\sigma_1 - \sigma_3}{3G_1} + \frac{\sigma_1 + 2\sigma_3}{9K} \delta_{ij} + \frac{\sigma_1 - \sigma_3}{3H_1} t + \frac{\sigma_1 - \sigma_3}{3G_2} \left[1 - \exp\left(-\frac{G_2}{H_2} t\right) \right] +, \\ \frac{\sigma_1 - \sigma_3}{3G_3} \left[1 - \exp\left(-\frac{G_3}{H_3} t\right) \right] (\varepsilon_{11} < \varepsilon_s), \\ \varepsilon_{11}(t) = \frac{\sigma_1 - \sigma_3}{3G_1} + \frac{\sigma_1 + 2\sigma_3}{9K} \delta_{ij} + \frac{\sigma_1 - \sigma_3}{3H_1} t + \frac{\sigma_1 - \sigma_3}{3G_2} \left[1 - \exp\left(-\frac{G_2}{H_2} t\right) \right] +, \\ \frac{\sigma_1 - \sigma_3}{3G_3} \left[1 - \exp\left(-\frac{G_3}{H_3} t\right) \right] + \frac{(\sigma_1 - \sigma_3)\xi^2}{6H_n} (\varepsilon_{11} \geq \varepsilon_s). \end{cases} \quad (16)$$

4. Unsteady Model Parameters

In complex and deep geological environment, the mechanical properties of surrounding rock cannot be explained by conventional mechanical theory, and the mechanical properties and creep characteristics of surrounding rock have obvious nonlinear characteristics. Therefore, creep param-

eters can no longer be taken as fixed values to describe the nonlinear creep deformation characteristics. The reason why the generalized Burgers model cannot accurately describe the deformation characteristics of rock in various creep stages is that rock is regarded as an ideal fluid, and shear modulus and viscosity coefficient are considered to be constant parameters in the creep process. In fact, in the

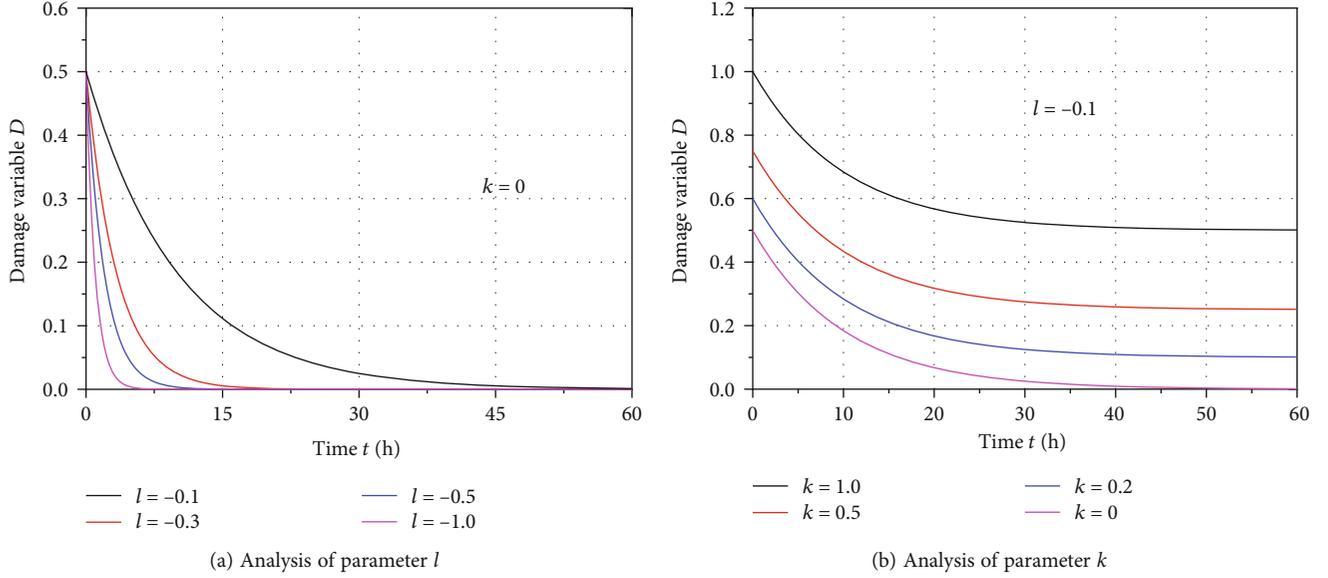


FIGURE 6: Influence of fitting parameters on damage variable.



FIGURE 7: Basic condition of roadway.

creep process of rock, the primary fractures are gradually pressed and closed, and the new fractures initiation, expansion, and connection and the mechanical properties of rock gradually deteriorate. Therefore, it is very important to clarify the damage mechanism and its quantitative expression for the establishment of a complete creep model of rock. In damage mechanics, there are mainly two ways to define damage factor D : one is to define damage variable according to the change of effective loading area in the process of material damage; the other is to define the damage variable according to the change of elastic modulus during material loading. The first definition method requires the use of analysis equipment to constantly monitor the occurrence of damage phenomenon in the loading process of specimens to define the effective bearing area; the second method requires processing of cyclic loading and unloading data to obtain isochronous stress-strain curves, so as to obtain the elastic modulus of specimens at different times. In contrast, the second method is more convenient to define the damage variables. In this paper, the basic damage variable is introduced according to the strain equivalence hypothesis, and the damage variable is regarded as a function only related

to deviational stress and time [27, 28]; following his ideas, this paper proposed the damage variables as follows:

$$D(t) = 1 - \frac{E(t)}{E_0(t)}, \quad (17)$$

where E_0 represents the elastic modulus of rock in the initial state, and E represents the current t elastic modulus of rock at a certain moment.

Hongfa [29] carried out uniaxial creep compression test of soft rock and obtained the quantitative relationship between strength and elastic modulus of rock mass, as shown in Figure 4. It can be seen from the figure that the elastic modulus decreases gradually with the increase of time, which also indirectly indicates that the creep damage of rock has a significant impact on the mechanical parameters of rock. In order to further clarify its quantitative relationship, it can be seen from Figure 5 that when $t = 0$, the rock is in the initial state and no damage has occurred to the rock. When $t \rightarrow \infty$, it is assumed that the damage to the rock has been stabilized and $D = E_0 - E_\infty/E_0$; so, Equation (17) can be further written as

$$D(t) = \frac{E_0 - E_\infty}{E_0} [\exp (lt) + k], \quad (18)$$

where k and l are fitting parameters. In order to further explore the influence of the above fitting parameters on the value of the damage variable, as $E_0 - E_\infty/E_0$ in the expression of the damage variable can be determined by experiment, the following analysis is focused on the influence of fitting parameters k and l on the damage variable. For simplicity, it is assumed that $E_0 - E_\infty/E_0 = 0.5$, and the effects of different fitting parameters are shown in Figure 6. It can be seen from the figure that the smaller l is, the faster the

TABLE 1: Results of model parameters.

Deviatoric stress (MPa)	G_2 (MPa)	G_3 (MPa)	H_1 (GPa·h)	H_2 (GPa·h)	H_3 (GPa·h)	H_n (GPa·h)	t_s (h)
3	17.21	6.46	2617.5	0.96	404.57	—	—
4	19.42	12.35	2138.4	0.10	91.38	—	—
5	18.48	16.63	1585.6	0.71	54.96	—	—
6	25.61	16.07	853.2	0.48	32.57	—	—
7	29.89	17.38	283.7	0.36	3.69	365.9	365.3

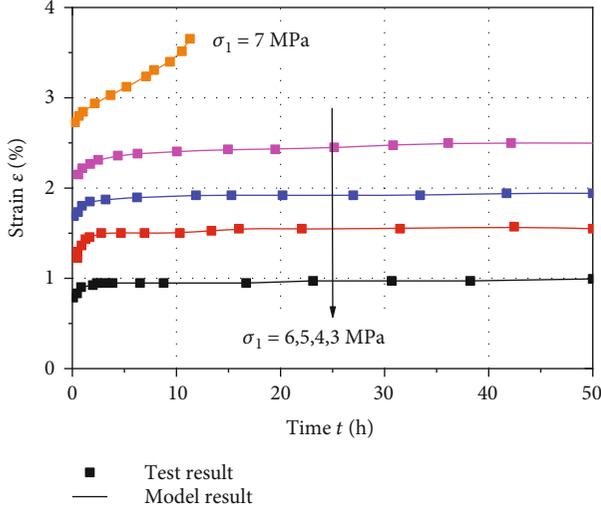


FIGURE 8: Comparison between experimental results and model calculation results.

speed of the damage variable tends to zero, and the damage speed is faster at this time, indicating that the main influence of l is the description of the damage process by the damage variable D . The smaller k is, the smaller the initial value of the damage variable is, and the initial damage degree is more significant at this time, which indicates that the main influence of l is the description of the initial state of the damage by the damage variable D . Therefore, both parameters k and l have significant influence on damage parameter D .

It is assumed that the material is isotropic and the damage law of each parameter is the same. Then, the degradation change of any rheological parameter $x(t)$ with time can be expressed as

$$\begin{aligned} x(t) &= x_t = x_0(1 - D) \\ &= x_0 \frac{-E_0[\exp(lt) + (k - 1)] + E_{\infty}[\exp(lt) + k]}{E_0}, \end{aligned} \quad (19)$$

where x_0 represents the initial rheological parameter.

It can be seen from Equation (19) that, in the same way, both elastic modulus and viscosity coefficient can be calculated by the following equation:

$$\begin{aligned} E(t) &= E_t = E_0(1 - D) \\ &= E_0 \frac{-E_0[\exp(lt) + (k - 1)] + E_{\infty}[\exp(lt) + k]}{E_0}, \end{aligned} \quad (20)$$

$$\begin{aligned} \eta(t) &= \eta_t = \eta_0(1 - D) \\ &= \eta_0 \frac{-E_0[\exp(lt) + (k - 1)] + E_{\infty}[\exp(lt) + k]}{E_0}, \end{aligned} \quad (21)$$

where η_0 represents the initial viscosity coefficient. By substituting the above parameters into Equation (16), an improved generalized Burgers model with creep parameters can be obtained.

5. Model Parameter Determination

The rheological model established in this paper can be divided into steady-state creep (rock creep has only the first three stages) and unsteady creep according to the condition with or without accelerated creep stage. When the model is steady-state creep, the least square method is used for parameter fitting, and the parameter fitting effect is usually better. When the creep type of rock is unsteady creep, the results of parameter fitting by the least square method cannot meet the requirements. Through the analysis of the test curve, this paper proposed the following parameter determination method:

- (1) According to the test curve, the creep curve of rock can be divided into steady state and unsteady state; thus, the time t_s for entering the accelerated creep stage can be determined, and the creep curve of rock can be divided into two stages $(0, t_s)$ and (t_s, t)
- (2) For steady-state creep stage, creep parameters ($G_1, G_2, G_3, H_1, H_2, H_3, K$) can be obtained by fitting the steady-state creep calculation method shown in Equation (16) with the test data
- (3) Based on the rheological test data of rock in steady creep stage, a state variable T is assumed. For the initial state, the state variable T can be regarded as a function of the creep parameters ($G_1, G_2, G_3, H_1, H_2, H_3, K$) and is calculated iteratively based on these values
- (4) By putting the state variable T into the unsteady creep stage, the theoretical strain value $\hat{\epsilon}_i$ is obtained. Based on this, the objective function $f(T)$ of rock creep function is established. Taking the sum of squares of the difference between the objective function and the actual experimental results of strain values as the objective function, the following equation can be obtained:

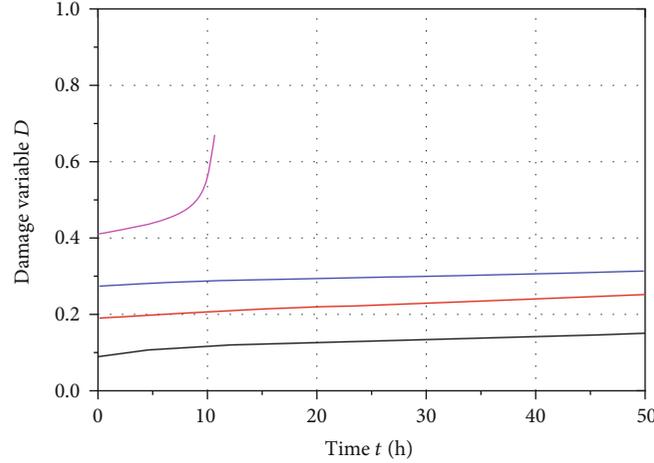


FIGURE 9: Variation of damage parameter D with time.

$$f(T) = \sum_{i=1}^n (\hat{\varepsilon}_i - \varepsilon_i)^2. \quad (22)$$

- (5) Determine whether the objective function $f(T)$ has reached a minimum. If so, the set design variable T is the final result, otherwise the design variable is constantly modified until the objective function reaches the minimum value

For entering the accelerated creep stage (t_s, t), the same nonlinear least square method as above is applied to the second formula in Equation (16). Since the creep parameters ($G_1, G_2, G_3, H_1, H_2, H_3, K$) have been determined, the rheological parameter η_n can be obtained by fitting the accelerated creep curve with the least square method. At this point, all parameters in the model have been calculated.

6. Model Validation and Parameter Analysis

The roadway surrounding rock of a coal mine in Shandong is adopted, and the roadway is shown in Figure 7. The surrounding rock was made into a cylindrical specimen with a height of 100 mm and a diameter of 50 mm, and the triaxial creep test was carried out in laboratory by the method of gradual loading of single specimen. The test plan is as follows: firstly, the confining pressure is loaded to a predetermined value of 2 MPa, and the stress level is 3, 4, 5, 6, and 7 MPa. After the confining pressure is stabilized, the axial pressure is applied. The loading rate is set to 50 N/s, and the confining pressure must be kept within a controllable range when the axial stress is applied. When the creep deformation at this stress level enters stable creep, the next load is applied. In this way, cyclic loading is repeated until the rock sample is destroyed. Results of model parameters obtained by fitting are shown in Table 1, where $G_1 = 5.78$, $K = 9.05$.

As can be seen from Figure 8 and Table 1, the calculated results are in good agreement with the test data under different stress levels. This model not only accurately reflects the creep characteristics of attenuation and steady creep stage but also overcomes the defect that the generalized Burgers model is difficult to describe the accelerated creep. Considering the deterioration of creep parameters with time, the creep damage process of rock under different stress states can be better described, which provides a new idea for establishing unsteady creep model and determining model parameters.

Figure 8 shows the variation of damage variables over time in the above tests. It can be seen from Figure 8 that damage variable D varies with stress level at different times. Under the first-stage load, the damage variable of rock is not zero at time zero, indicating that rock itself has damage nature, and rock damage expands instantaneously during initial loading. Under the action of the first three levels of load, the damage amount of rock is in a relatively stable state, and the damage variable increases slightly, indicating that the strain deformation amount of rock is small at this time, the overall creep deformation is in the attenuation and stable creep stage, and the microcracks and micropores and other defects in the rock are slowly developing. When the next level of load is applied each time, there is a jump increase in the damage variables, which indicates that the internal defects of rock expand greatly at the moment of loading. Under the action of the last stage load, the damage variable of rock is relatively gentle in the initial stage, and this damage trend will be intensified with the cumulative effect of the continuous development of microcracks and fissures. The microcracks converge in the weak region, and the damage rate increases further. Then, the local damage began to expand and connect with each other to form a macroscopic fracture surface and suddenly linearly increased at about 11 h, followed by the failure, resulting in a sudden increase of the damage variable to 1. Under different confining pressures, the variation trend of damage variables under low stress is basically the same, but when the last stage rock is destroyed, the damage variables D keep a slow growth in the early stage and suddenly increase when the failure is near.

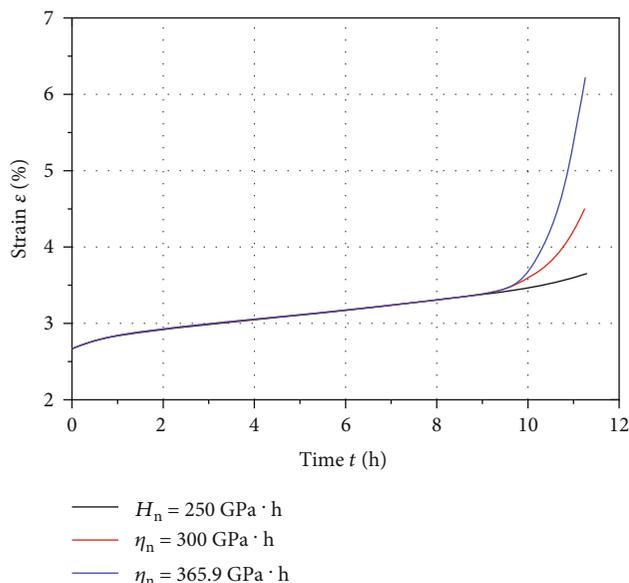


FIGURE 10: Variation of viscosity coefficient H_n with time in the accelerated creep stage.

The influence of elastic modulus on rock creep is mentioned in the above study, which will not be described here. Figure 9 shows the influence law of nonlinear viscosity coefficient on the curve of the whole process of rock creep. Only H_n sensitivity analysis is carried out for nonlinear viscosity coefficient, and the values listed in Table 1 are used for other model parameters. It can be seen from Figure 10 that, with the decrease of nonlinear viscosity coefficient H_n , the faster the axial strain increases after the rock enters the accelerated creep stage, the shorter the failure time will be, indicating that the improved generalized Burgers model has a good adaptability to the deformation process after the rock enters the accelerated creep stage and has the value of popularization and application.

7. Conclusions

Based on the generalized Burgers model, a new unsteady creep model considering accelerated creep was proposed, which reasonably introduced damage variables to describe the aging damage characteristics of rock and used the rheological model to identify the whole process curve of shale creep test. The main conclusions can be drawn as follows:

- (1) In this paper, an improved generalized Burgers model is proposed by connecting a nonlinear viscose pot with strain-triggered on the generalized Burgers model, and the creep constitutive equations of the new model under one and three dimensional stress states are derived. In order to facilitate the model parameter identification of creep test results under conventional triaxial compression, the axial creep equation of rock specimens under conventional triaxial compression was given

- (2) Compared with the experimental curves, it is shown that the model can describe the creep law of rock well in the accelerated creep stage. Accelerated creep is the key stage of tunnel collapse prediction. The successful description of accelerated creep stage shows that the model is of great significance to the prediction of tunnel surrounding rock collapse. The improved generalized Burgers model only introduces a new pot element, and the model parameters are relatively few. In addition, the derivation process of creep equation is simple and easy; so, it has certain application value
- (3) In order to consider the characteristics of rock aging damage, a new definition of rock damage variable is proposed and introduced into the proposed model to further improve the model, and the parameters of the unsteady model can more reasonably simulate the creep characteristics of rock
- (4) The sensitivity analysis of the nonlinear viscosity coefficient introduced by the new model shows that with the decrease of the nonlinear viscosity coefficient H_n , the faster the axial strain increases when the rock enters the accelerated creep stage, the shorter the time tending to failure

Data Availability

The original contributions presented in this study are all included in this manuscript, and further inquiries can be directed to the corresponding authors.

Conflicts of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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