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Research Article

Mechanical Constitutive and Seepage Theoretical Model of Water Storage Media Based on Fractional Derivative

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Using an abandoned underground goaf in coal mines as water reservoirs has been successfully applied for protecting mine water resources in western China. The water storage media are composed of broken rock masses and the voids between the rock masses. It is critical for reservoir capacity calculation that the deformation characteristics and seepage evolution of the water storage media under triaxial stress are theoretically described. In this study, broken rock masses and the space among the rock masses are simplified as two springs in a series. The mechanical behavior of the broken rock mass is described by Hooke's law of linear elasticity, and the deformation characteristics of the space among the rock masses are represented by a nonlinear elastic constitutive model. The nonlinear stress-strain constitutive model of the water storage media is established by combining Hooke's law and the fractional derivative stress-strain model. Similarly, a non-Darcy seepage model of the water storage media is obtained. The nonlinear stress-strain model is verified by mechanical experiments, physical simulation tests, and field measured data, and parameter sensitivity analysis is performed. The non-Darcy seepage equation is fitted and analyzed by using the seepage experimental data of the broken rock mass under triaxial compression conditions. The fractional non-Darcy model rather than the Forchheimer equation can more accurately describe the nonlinear seepage process in water storage media.

1. Introduction

Western China has large reserves of coal resources, with shallowly buried coal seams and relatively simple geological and hydrological conditions. After more than 30 years of applying coal mining technology in China's western mining area, a modern fully mechanized coal mining technology characterized by safety, efficiency, and high recovery rate has evolved, and this technology provides an important guarantee for China's coal supply. Geographically, this region is located in an arid and semiarid zone with reduced atmospheric precipitation, resulting in problems such as lack of water resources, serious soil erosion, and land desertification. Due to the shortage of groundwater resources and the fragile ecological environment, coal mining has led to surficial ecological damage and a large amount of mine water drainage, intensifying the problem of developing coal resources while protecting groundwater resources and the ecology of the surface [1–3]. To address the difficult problem of groundwater protection during coal mining in western China, China has successfully developed the technology of mine water protection by underground reservoirs in coal mines (URCM) shown in Figure 1. This technology uses the rock mass gap in the goaf formed by coal mining to connect the discontinuous coal pillars with an artificial dam to form reservoir dams; this forms a relatively closed water storage space. Meanwhile, mine water injection facilities and water intake facilities have also been constructed to

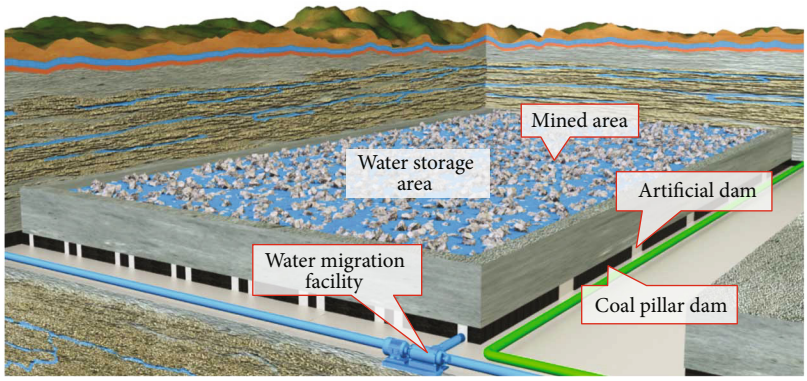


Figure 1: Technical idea sketch of the underground reservoir of a coal mine.

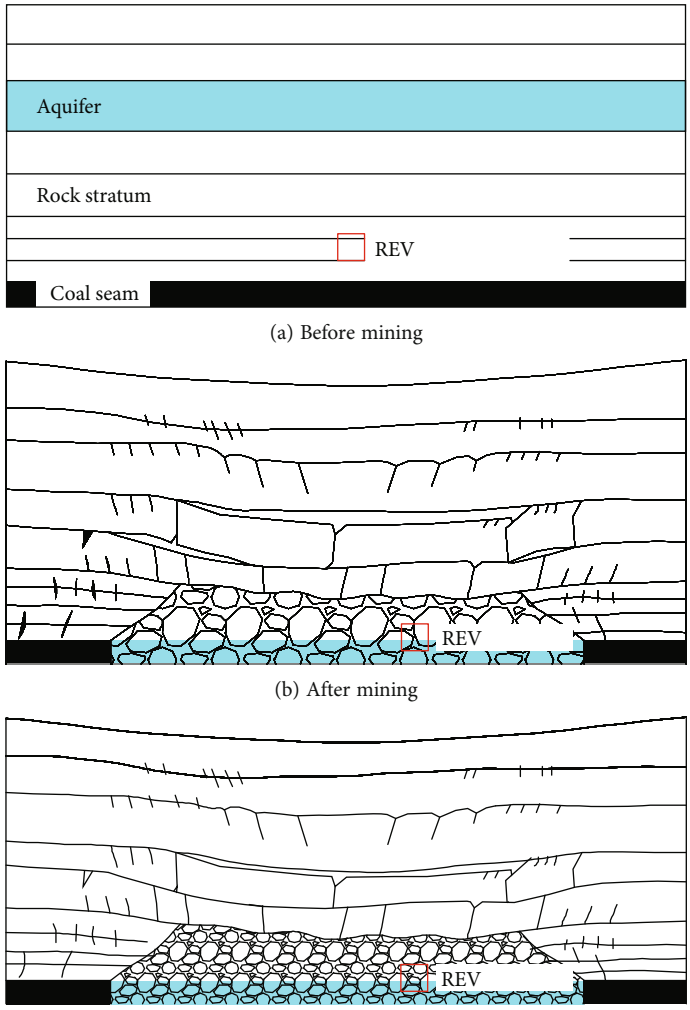
make full use of the natural purification effect of the goaf rock mass on mine water to achieve mine water storage and utilization [4, 5].

After the coal seam is mined out, the overlying strata bend, break and collapse to the goaf. The collapse zone, fracture zone and bending subsidence zone are divided according to the vertical movement of the stratum. The water storage space of URCM is composed of the collapse zone and fractured zones. As the water storage media of URCM, the deformation characteristics of the broken rock mass are affected by the rock mass's strength and rock particle size. Under the same stress, the higher the strength of the rock mass, the smaller the deformation, and the smaller the strain growth rate of the rock with larger particle sizes at the initial stage of loading. With the increase in stress, the strain growth rate of the rock sample with a large particle size becomes less than that of the rock sample with a small particle size [6]; also, the stress acting on the broken rock mass is affected by the structure of the overlying strata. The water storing rock mass in the underground reservoir in a coal mine is compressed and deformed under the action of mining stress, and its stress-strain curve presents nonlinear characteristics. The conventional constitutive equation cannot accurately reflect its variation.

The broken rock mass in a goaf is not only the main structure for water storage in the underground reservoirs of coal mines but also the main channel of mine water seepage. The presence of the goaf makes it difficult to carry out in situ test research of the broken rock mass. Existing studies have mainly indirectly studied the pressure distribution in the goaf and the stress-strain characteristics of the broken rock mass through theoretical assumptions or mechanical experiments. It is difficult to describe the stress distribution of a goaf's rock mass by a unified method due to the change in the original rock stress field and the presence of various broken rock mass structures [7, 8]. A tensile stress concentration area is usually formed around the boundary of the goaf [7]. From the boundary to the center of the goaf, the stress distribution can be divided into stress concentrated area, stress relaxed area, and stable stress area [8]. The broken rock mass can be assumed to be a large number of rock mass accumulations of different scales. Experiments have shown that stress-strain of broken rock mass curve indicates strong nonlinearity, and stress-strain equation can be fitted into a fourth-order polynomial function [9], the Salamon filling equation [10], and the hyperbolic function [11]. The compaction characteristics of broken rock mass change due to different lithologies, block sizes, and water immersion conditions [12–14]. For example, soft rock goes through the progressive compaction stage of crushing→compaction→recrushing→recompaction [12]. The compaction characteristics of the roof sandstone and sandy mudstone show obvious segmentation under saturated water conditions in Xin'an Coal Mine, Yima city in China. Further, the coefficient of dilatancy increases with the magnification in the size of the rock mass [13] and follows the negative exponential change with stress [14]. Based on the stress-strain characteristics of the broken rock mass in a goaf, an empirical formula for the correlation between stress recovery and land subsidence can be established, and this basically meets the exponential function relationship [15–17]. The stress-strain relationship of broken rock mass has mainly focused on the qualitative analysis of the compaction law and the fitting equation obtained from the experimental data. However, theoretical research on the stress-strain constitutive model of a broken rock mass is urgently needed.

As an unconsolidated material without cohesion, the permeability of broken rock mass is obviously different from that of porous media containing pores and fissures [18]. Some scholars have performed a large number of detailed studies on the water seepage characteristics of coal, coal gangue, mudstone, limestone, sandstone, shale, and other broken rock masses by developing a broken rock mass seepage device matched with MTS testing machine [18–22]. The results show that lithology, particle size, and axial stress affect the change in permeability. The increase in the axial stress makes the void ratio decrease, which leads to a decrease in the number of seepage channels and the permeability of the fractured rock mass. The larger the particle size, the easier it is to be compacted, and the greater the permeability and porosity change with the same axial compression. When the fluid pressure is large, the rock mass seepage presents non-Darcy-Forchheimer flow.

The mine water is stored in the space among broken rock masses in underground reservoir [23]. The stress-



(a) Before mining

(b) After mining

(c) Deformation of the rock mass for water storage

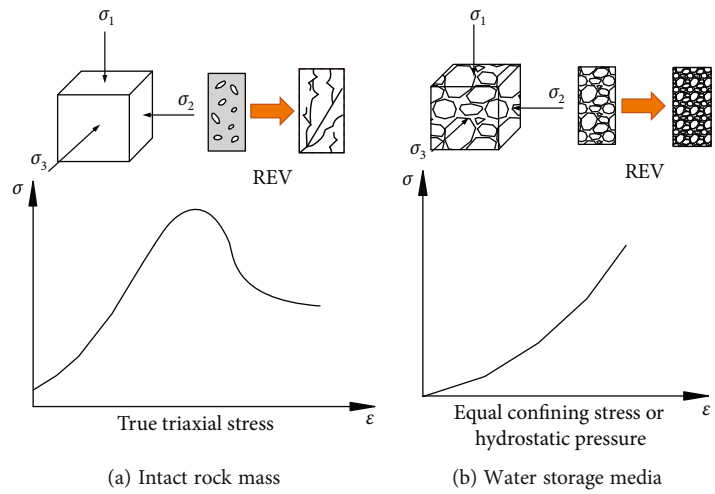
Figure 2: Evolution of the water storage media in the underground reservoir of a coal mine.

strain constitutive model has become the basis whether it is the evolution of the storage capacity or the flow characteristics of mine water in the water storage media. Although many scholars have carried out a large number of mechanical experiments on broken rock mass, the stress-strain constitutive model has not been put forward theoretically. In addition, Forchheimer equation is mostly used for the water flow in broken rock mass, and its applicability to the seepage characteristics of water storage media in URCM still needs to be studied. In this paper, the water storage media in URCM is taken as a whole and simplified into two series springs. Based on the fractional calculus theory, a nonlinear stress-strain constitutive model and a fractional non-Darcy seepage equation are proposed, which are verified by experimental data.

2. Morphological Evolution of the Water Storage Media

Coal mining changes the stress field of the coal-bearing strata, resulting in varying degrees of disturbance to the rock mass within the mining range [24]. After the coal is mined, a large goaf area is formed, and the fracture and collapse of the overlying strata occur in the goaf. Further, combined with the protective coal pillars and artificial dams, all these constitute the underground reservoir in coal mines [4]. The water-storing rock mass is in an intact state before the coal is mined out (as shown in Figure 2(a)). After destruction of intact rock, it collapses in the goaf and presents a loose state (as shown in Figure 2(b)). With the compaction of the overburden rock mass under the action of gravity, the water storage space is gradually compressed (as shown in Figure 2(c)). The occurrence form of the overlying strata before and after coal mining is shown in Figure 2.

The morphological evolution of the underground water storage media in a coal mine can be described using the stress state and the existing form of the rock mass. In Figures 2(a) and 2(b), when the rock mass is not disturbed by coal mining, its initial stress is in a three-dimensional unequal pressure state. Due to the mining disturbance of the coal face, the rock mass goes through elastic state and plastic state until failure under the mining stress path, and



(a) Intact rock mass (b) Water storage media

Figure 3: Stress-strain curves before and after the formation of the water storage media.

its stress-strain curve is shown in Figure 3(a) [25]. The rock mass gradually evolves from its initial intact state to a fractured rock mass with apparent cracks (see Figure 3(a)), thereby becoming a part of the water storage media in the underground reservoir of the coal mine. In Figures 2(b) and 2(c), the water storage media of the underground reservoir is composed of broken rock masses and the voids between the rock masses. The water storage media is in a state of loose accumulation in the early stage of formation. When the overlying rock mass sinks under the action of gravity, the water storage media is gradually compressed and deformed. The confining pressure depends on the stress distribution state, and the stress-strain curve of the water storage media is shown in Figure 3(b) [8, 18]. The water storage media itself has discontinuity and incompleteness, and its deformation state mainly depends on the voids and loads between the rock masses, which makes it outside the research scope of conventional rock mechanics.

According to the rock mechanics theory, the stress-strain process for rock starts from the "zero point" of the stress and strain and ends at the postpeak failure stage. Based on the morphological evolution of the broken rock mass for underground reservoir in coal mines (URCM), the definition of the stress-strain curve of the rock mass can be expanded as the entire life cycle of the overlying rock mass before, during, and after mining coal seams. The stress-strain relation for a rock mass in the entire life cycle can be defined as the process starting from the in situ stress state of the intact rock mass and ending with the recompaction of the broken rock mass to the original stress state of the rock. The stress-strain characteristics of the water storage media in URCM correspond to the recompaction stage of the broken rock mass, as shown in Figures 2(c) and 3(b).

3. Nonlinear Stress-Strain Model

3.1. Fractional Stress-Strain Constitutive Equation. The water storage media in URCM is formed by the accumulation of broken rock mass. There is no cohesive contact between each rock mass, and a large number of voids are formed. The deformation characteristics of broken rock mass are obviously different from those of porous and fractured rock masses. When Liu et al. [26] studied the elastic deformation characteristics of a porous (fractured) rock mass under stress, they proposed a series spring model to characterize the nonlinear elastic stress-strain relationship of the porous (fractured) rock mass [26]. In this perspective, the underground water storage media is regarded as a special rock mass with a high porosity and fracture rate, in which the pores and fractures are voids, and the rock mass contact is a physical contact type without cohesion. Therefore, the deformation characteristics of the underground water storage media in a coal mine are simplified and some assumptions are made to put forward a stress-strain constitutive model of the water storage media.

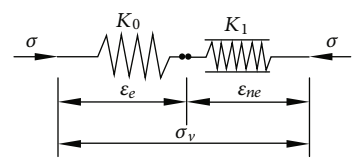


Figure 4: Stress-strain model of broken rock.

Assuming that the water storage media is elastically deformed under the action of three-direction stresses, the deformation consists of linear elastic deformation of the broken rock mass and nonlinear elastic deformation of the gaps between the rock masses. The two media are simplified as two discontinuous contact series springs, and the model can only stand compression but not tension, as shown in Figure 4.

The total strain of the water storage media in URCM can be expressed as

$$\varepsilon_v = \varepsilon_e + \varepsilon_{ne}, \tag{1}$$

Table 1: Model parameters of water storage media.

References	Sample collection sites	Sample type	E_0 (MPa)	E_1 (MPa)	α	R^2
Ma et al. [13]	Xin'an Coal Mine	Mudstone	16110	499.85	2.131	0.9420
		Sandy mudstone	34620	2893.6	2.788	0.9140
		Sandstone	47330	15699	3.436	0.9128
Wang et al. [30]	Shendong Mining Area	Dry sandstone	5582	53.04	2.185	0.9657
		Saturated sandstone	6531	130.20	2.88	0.9763
Wang [16]	Taiping Coal Mine	Mudstone and sandstone	257800	39.48	1.311	0.9978

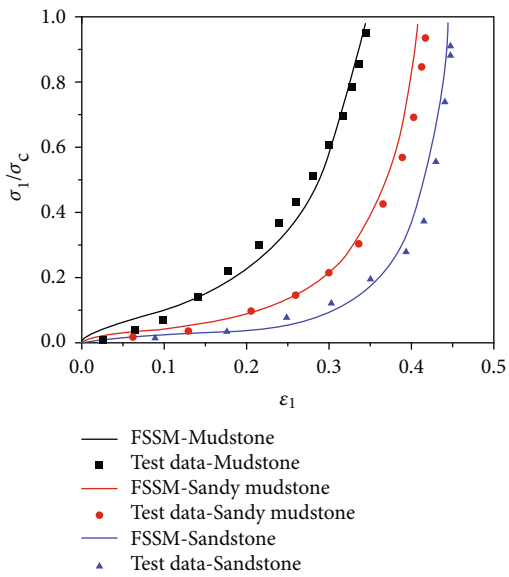


Figure 5: Stress-strain curves of the broken rock with different lithologies.

where ε_v is the volumetric strain of the water storage media, ε_e is the volumetric strain of the broken rock mass in the underground reservoir, and ε_{ne} is the volumetric strain between the voids of the broken rock mass.

Hooke's law is used to express the stress-strain relation in the linear elastic constitutive model as

$$\sigma = K_0 \varepsilon_e, \tag{2}$$

where σ is the volume stress and K_0 is the bulk modulus of the broken rock mass.

Based on the differential form of the linear elastic Hooke's law, the stress-strain constitutive relation of the voids between the fractured rock mass can be extended to a nonlinear Hooke's law varying with gradient by a nonlocal derivative [27]:

$$K_1 = \frac{d^\alpha \sigma}{d\varepsilon_{ne}^\alpha}, \tag{3}$$

where K_1 denotes the void bulk modulus, α is the nonlinear elastic influence coefficient of the water storage media, and $d^\alpha/d\varepsilon_{ne}^\alpha$ is the Caputo fractional differential operator, which is defined as [27]

$$\frac{d^\alpha f(t)}{dt^\alpha} = I^{n-\alpha} f^{(n)}(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-s)^{n-\alpha-1} f^{(n)}(s) ds. \tag{4}$$

For a given function $f(t)$ and $\alpha > 0$, n is the smallest integer greater than α. I^n is a Riemann-Liouville fractional integral operator, which is defined as [28]

$$I^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} f^{(n)}(s) ds, \tag{5}$$

where $\Gamma(\cdot)$ is the gamma function and $\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t} dt$. In addition, when $\alpha = 0$, $d^\alpha/d\varepsilon^\alpha$ represents the differential operator of the integer order.

Taking Equation (5) into Equation (3) and considering the initial conditions $\sigma(0) = 0$, the explicit expression of Equation (3) is obtained:

$$\sigma = K_1 \frac{\varepsilon_{ne}^\alpha}{\Gamma(1+\alpha)}. \tag{6}$$

By substituting Equations (2) and (6) into Equation (1), the stress-strain constitutive equation of the water storage media can be obtained:

$$\varepsilon_v = \frac{\sigma}{K_0} + \left[\frac{\sigma}{K_1} \Gamma(1+\alpha)\right]^{1/\alpha}. \tag{7}$$

The elastic constitutive relation of coal and rock masses can usually adopt the same form of the constitutive equation [29]. To verify the accuracy of the above constitutive equation through experimental data, the expressions of the axial compression deformation and the axial stress of the water storage media were obtained according to Equation (7):

$$\varepsilon_1 = \frac{\sigma_1}{E_0} + \left[\frac{\sigma_1}{E_1} \Gamma(1+\alpha)\right]^{1/\alpha}, \tag{8}$$

where ε_1 is the axial compressive strain of the water storage media, σ_1 is the axial stress of the water storage media, E_0 is the elastic modulus of the broken rock mass, and E_1 is the elastic modulus of the voids.

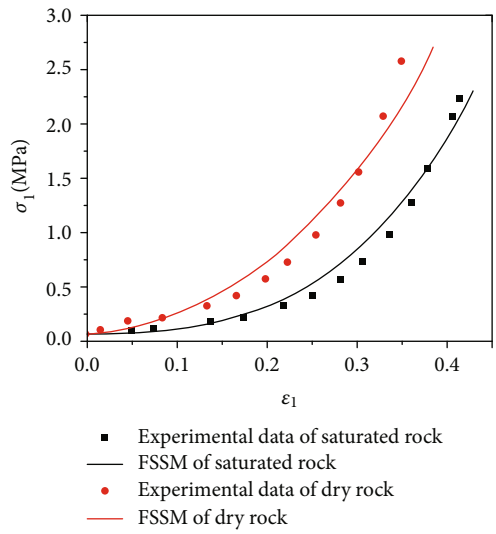


Figure 6: Stress-strain curves of the water storage media.

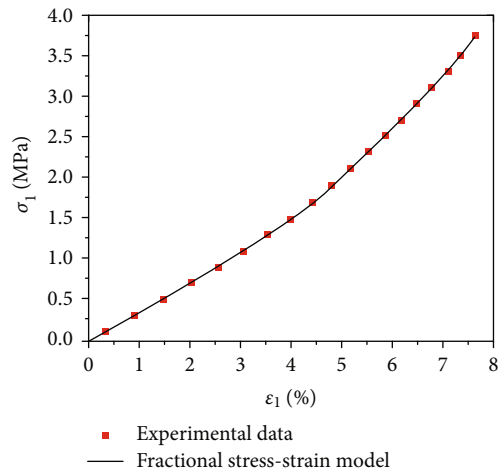


Figure 7: Stress-strain curves of the broken rock in the goaf.

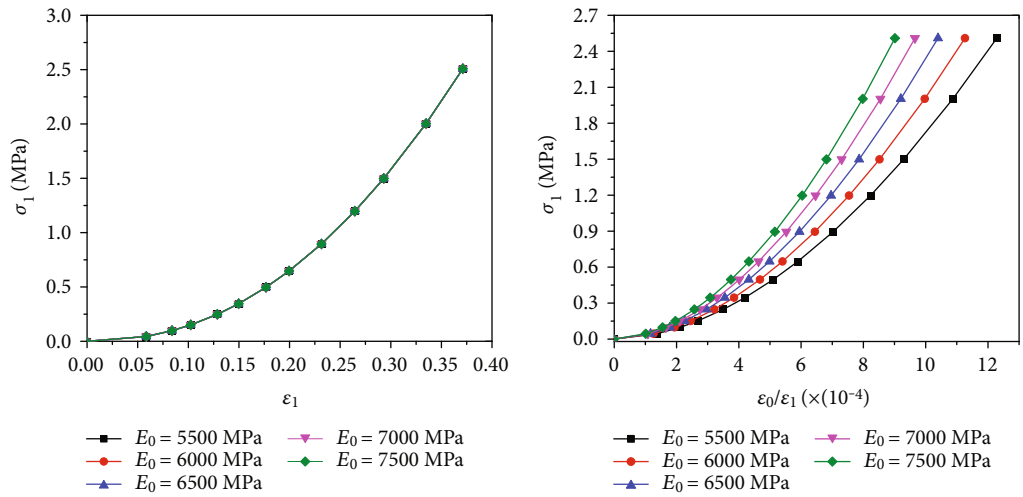
3.2. Modelling Verification. The deformation characteristics of the underground water storage media in a coal mine can be reflected by studying the compaction characteristics of the broken rock mass in the goaf of the coal mine. The commonly used experiments include mechanical tests of laboratory small-scale samples, physical simulation tests of similar materials, and on-site monitoring of the compaction deformation of the collapsed rock mass in the goaf.

3.2.1. Laboratory Experiment. To study the influence of the compaction characteristics of a broken rock mass on the damage degree of the water-resisting layer and the surface subsidence after coal mining, Su et al.[13] took the typical rock mass in Xin'an Coal Mine as the object and carried out compaction deformation experiments of mudstone, sandy mudstone, and sandstone with different rock block sizes. The experimental data of the crushed rock samples with a size of 10~15 mm were selected for the data fitting analysis of Equation (8). The fitting parameters are shown in Table 1, and the fitting curve is shown in Figure 5. The ordinate value is expressed as the ratio of the compressive stress to the uniaxial compressive strength of the intact rock sample by the normalization method.

According to the curve in Figure 5, the deformation characteristics of the three kinds of broken rocks during compaction are basically similar and can be divided into three stages: rapid deformation stage, transition stage, and slow deformation stage. At the initial loading stage of broken rock samples, each rock block is in an unstable accumulation state and has a high degree of freedom; the rock sample can be rotated and translated easily under a small stress gradient, resulting in large deformation. As the stress increases, the contact area of the rock blocks increases and the degree of freedom decreases, extrusion deformation and damage occur between the rock blocks, and the convex areas of the rock blocks are eroded and damaged each other, which leads to a reduction in the deformation rate of the broken rock samples. As the stress exerted on the sample increases further, the contact area of the rock blocks increases gradually, and the degree of freedom is further limited. At this time, the deformation of the broken rock mass is mainly determined by the deformation of the rock block itself. According to the experimental results, the greater the compressive strength of the intact rock samples, the greater the deformation of the broken samples, the greater the coefficient of the crushing expansion after the failure of the intact rock masses, and the larger the water storage space.

3.2.2. Physical Simulation Experiment. To study the evolution of rock's crushing expansion coefficient with stress in a goaf by physical simulation tests, Wang et al. [30] remanufactured crushed prototype rock samples according to the size of the overlying strata and carried out compaction experiments on the rock samples in a dry and saturated state, respectively. During the experiment, the variation of the compression amount with the stress of the broken samples was tested. Based on this, in this study, the variation curve of the compression strain (axial strain) with stress was calculated and drawn, as shown in Figure 6. Under the dry and saturated conditions, the stress-strain relationship of the broken rock mass is nonlinear, and the compressive strain increases rapidly in the initial stage of stress application, and the strain rate shows a decreasing trend. The compression deformation of the saturated samples is larger than that of the natural samples under the same stress state. The nonlinear stress-strain constitutive Equation (8) and linear elastic constitutive Equation (1) of the water storage media proposed in this paper were used to fit the above test data, and the results are shown in Figure 6. The relevant model parameters are shown in Table 1.

The fitting results in Figure 6 show that the nonlinear constitutive model of the underground water storage media proposed in this paper can well describe the stress-strain relationship of the broken rock masses and that the linear elastic Hooke's law cannot accurately reflect the nonlinear deformation characteristics of the broken rock masses under three-dimensional pressure. Although the compaction deformation characteristics of a saturated rock sample were



(a) Stress-strain relationship

(b) Ratio of block strain to the total strain

Figure 8: Sensitivity of the stress-strain curves to the elastic modulus of broken rock.

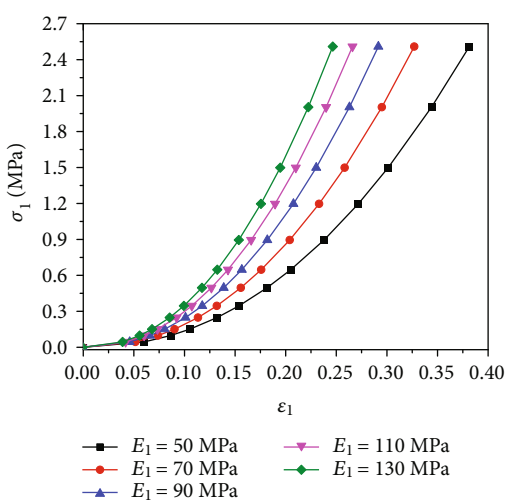


Figure 9: Sensitivity of the stress-strain curves to the elastic modulus of the voids between the broken rocks.

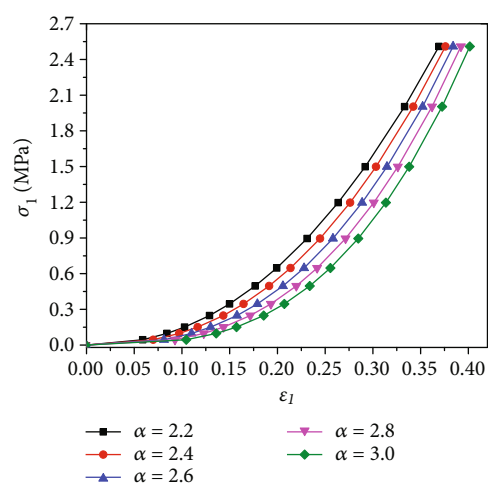


Figure 10: Sensitivity of stress-strain curves to the α in the case.

tested in the experiment, they are still different from the compaction deformation characteristics of a fractured rock mass immersed in water for a long time in a coal mine's underground reservoir. The above analysis only reflects the stress-strain law of a broken rock mass in the natural dry state and under the condition of water saturation.

3.2.3. Field Measurement. Because of the hidden structure of a coal mine's underground reservoir, it is difficult to measure the deformation of the water storage media under the action of mining pressure. Based on the measured data of surface subsidence, Wenxue et al. [16] theoretically obtained the relationship between the deformation of a caved rock mass and the stress recovery, as shown in Figure 7. Equation (8) is used to fit the deformation data of the caving rock mass; the stress-strain curve of the caving rock mass in the goaf was obtained (Figure 7), and the related parameters are shown in Table 1. According to the fitting data in Table 1, Equation (8) can better express the evolution of the deformation of the collapsed rock mass along with the stress in the goaf.

3.3. Parameter Sensitivity Analysis

3.3.1. Influence of the Elastic Modulus of the Rock Mass. According to the fitting results of the constitutive model of the underground water storage media of the coal mine and referring to the experimental data of dry sandstone samples in the literature [30], the parameter $E_1 = 53.04$ MPa, $\alpha = 2.185$ is substituted into the stress-strain constitutive Equation (8). By changing the elastic modulus values of the rock blocks in the water storage media (5500, 6000, 6500, 7000, and 7500 MPa, respectively), a set of stress-strain curves under different elastic modulus conditions were obtained (Figure 8). It can be seen that the volume modulus of the

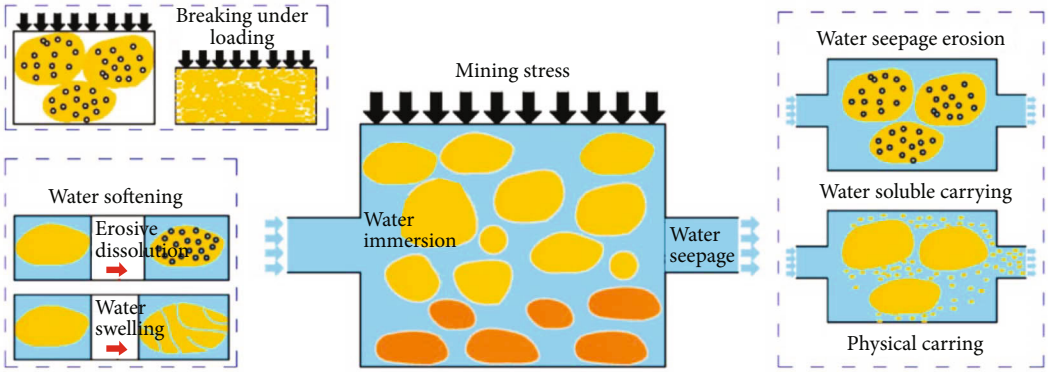


Figure 11: Mine water seepage in broken rock mass [23].

broken rock blocks in the underground reservoir has little influence on the shape of the water storage media's stress-strain curve (Figure 8(a)). To clearly reflect the influence of the rock block's elastic modulus on the stress and strain of the water storage media, the ratio of the rock block's strain to the water storage media's strain was taken as the horizontal axis to draw the stress-strain curve (Figure 8(b)). The results show that the larger the elastic modulus of the rock mass, the smaller the strain of the rock mass. The difference between the rock mass's strain and the overall strain is 4 orders of magnitude, which indicates that the strain of the rock mass can be ignored in the deformation of the water storage media.

3.3.2. Influence of Void Elastic Modulus on the Water Storage Media. According to the parameter fitting results of the stress-strain constitutive equation of the natural samples in Table 1, under the condition of keeping the parameter $E_0 = 5582$ MPa, $\alpha = 2.185$ unchanged, and the void elastic modulus of the water storage media was changed to 50, 70, 90, 110, and 130 MPa, respectively; then, the stress-strain curves under different void elastic modulus were obtained (Figure 9). It can be seen that the void elastic modulus is an important influencing factor on the stress-strain evolution of the water storage media. The larger the void elastic modulus, the smaller the compressive strain value under the same stress state, and the more stable the packing shape of the water storage media.

3.3.3. Influence of Parameter α. Keeping the elastic modulus of the rock block and the void elastic modulus unchanged ($E_0 = 5582$ MPa and $E_1 = 53.04$ MPa) and only changing the nonlinear elastic influence coefficient of the water storage media to 2.2, 2.4, 2.6, 2.8, and 3.0, respectively, the variation of the stress-strain curve under different nonlinear elastic coefficients of the water storage media was obtained (Figure 10). With the increase in the nonlinear elastic influence coefficient, the compression strain and the nonlinear range of the stress-strain curve also increase, indicating that the nonlinear elastic influence coefficient does not change the monotonicity of the stress and strain but affects the curvature of the stress-strain curve.

It can be seen that the main parameters affecting the stress-strain curve of the underground reservoir's water storage media are the rock block elastic modulus, the void elastic modulus, and the nonlinear elastic influence coefficient. The elastic modulus of the rock block has the least influence on the stress-strain curve of the water storage media and is the basic parameter of the constitutive model. The void elastic modulus directly determines the strain level of the water storage media and the change rate of the strain with stress. The nonlinear elastic influence coefficient mainly determines the range of the nonlinear stage in the stress-strain curve of the water storage media. The void elastic modulus is affected by many factors, such as the rock block size, shape, accumulation state, and contact area. The nonlinear elastic influence coefficient mainly reflects the expansion characteristics of the broken rock, which is described quantitatively by the fragmentation expansion coefficient [15]. The constitutive equation comprising the above three basic parameters can accurately describe the stress-strain relationship of the coal mine's underground water storage media.

4. Non-Darcy Seepage Model

4.1. Non-Darcy Seepage Equation with Fractional Derivative. The water storage media in URCM is mainly formed by the accumulation of collapsed broken rock masses. Mine water flows in the gaps in the rock masses as shown in Figure 11 [23]. Compared with the viscous force, the fluid inertia force is dominant, resulting in high-speed non-Darcy seepage in the water storage media.

The differential form of Darcy's law is

$$\frac{dq}{di} = k, \tag{9}$$

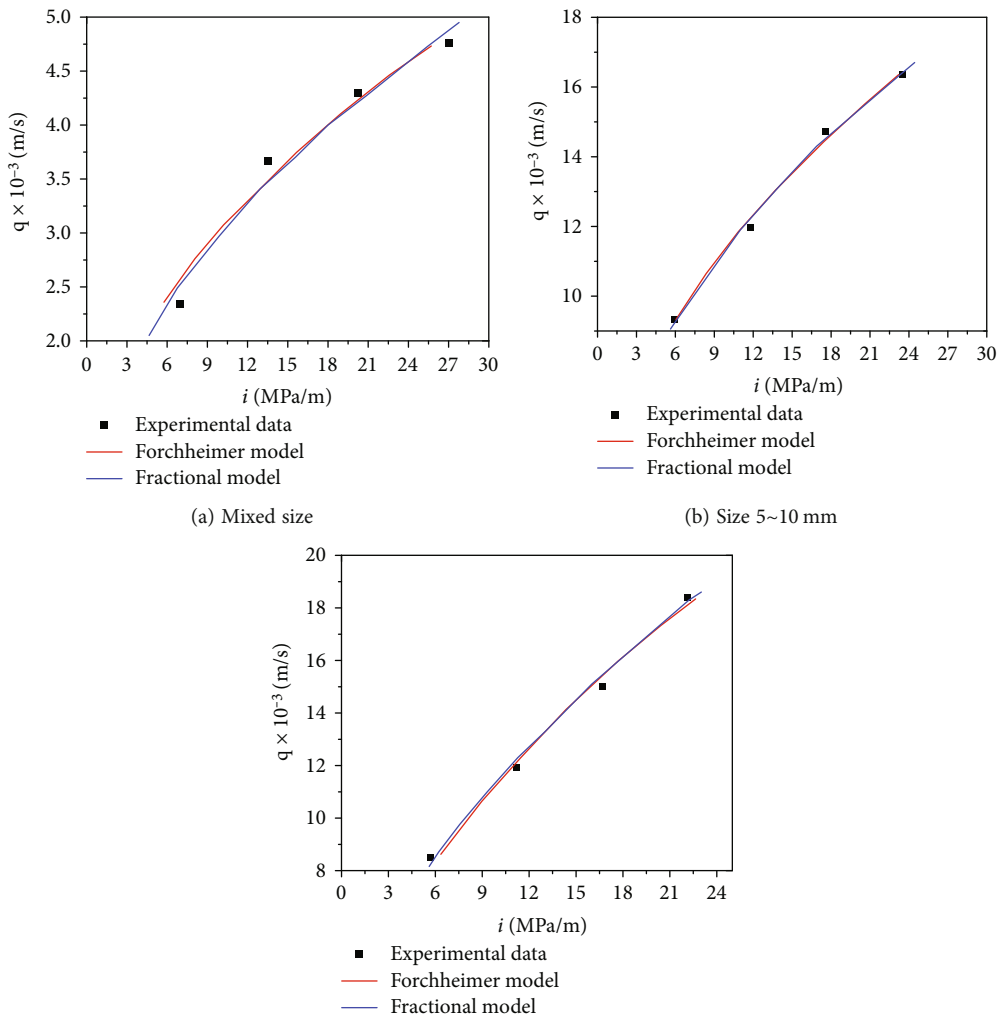
where q is the seepage velocity of the mine water in the water storage media, i is the hydraulic gradient, and k is the permeability coefficient of the water storage media.

Similar to Equation (3), Equation (9) is extended to the non-Darcy seepage equation based on a fractional derivative:

$$\frac{d^\gamma q}{di^\gamma} = k, 0 \le \gamma \le 1 \tag{10}$$

Table 2: Seepage model parameters of water storage media in underground reservoir of coal mine.

Size (mm)	Fractional model			Forchheimer model	
	$k \times 10^{-3}$ (m/s)	γ	R^2	Fitting equation	R^2
Mixed size	0.8605	0.4893	0.9943	$i = 1.269 \times 10^{12} q^2 + 6.136 \times 10^8 q$	0.9781
5~10	3.864	0.4199	0.9992	$i = 1.121 \times 10^{11} q^2 + 4.106 \times 10^8 q$	0.9945
10~15	2.653	0.5828	0.9980	$i = 5.086 \times 10^{10} q^2 + 3.036 \times 10^8 q$	0.9918



(a) Mixed size

(b) Size 5~10 mm

(c) Size 10~15 mm

Figure 12: Fitting curves given by fractional non-Darcian model.

where γ is the nonlinear Darcy seepage influence factor. Using the solution process of Equation (6), the analytical solution of non-Darcy seepage is obtained:

$$q = k \frac{i^{\gamma}}{\Gamma(1+\gamma)}. \tag{11}$$

When $\gamma = 1$, Equation (11) degenerates into the classical Darcy equation, indicating that Darcy's law is a special form of the non-Darcy seepage model.

4.2. Parameter Determination of the Non-Darcy Seepage Model. Zang et al. [22] carried out seepage experiments on broken sandstone with different particle sizes to address the water inrush problem on the roof; they obtained the evolution of the seepage characteristics of the broken sandstone under triaxial stress and found that the seepage of water in broken sandstone does not comply with Darcy's law. In this study, seepage experimental data of broken sandstone with mixed particle sizes (5~10 mm and 10~15 mm) from the literature were selected, and the nonlinear least square fitting method was implemented based on the fractional non-

Darcy seepage model as Equation (11). The model parameters obtained are shown in Table 2. The fitting curve is shown in Figure 12. The fitting analysis results in Table 2 and Figure 12 show that the fractional non-Darcy model matches well with the experimental data, which verifies that the fractional non-Darcy model rather than the Forchheimer equation can more accurately describe the nonlinear seepage process.

5. Conclusions

(1) The morphological evolution of water storage media in URCM was analyzed, and the stress-strain relationship of the entire life cycle of the rock was obtained. The stress-strain evolution during the rock mass's entire life cycle starts from the original stress state of the intact rock mass and ends at the moment when the broken rock mass is recompacted to the original rock stress

(2) The water storage media in URCM is composed of broken rock masses and the voids among the rock masses; this media was condensed into a mechanical model composed of two series springs. The nonlinear elastic stress-strain constitutive equation was further derived based on Hooke's equation

(3) The rationality of the nonlinear elastic constitutive model was verified by laboratory experiments, physical simulations, and field monitoring data of the water storage media in URCM. It is concluded that the void elastic modulus and the nonlinear elastic coefficient have a great influence on the stress-strain curve

(4) It is clear that the flow of mine water in underground reservoirs belongs to high-speed non-Darcy seepage. Based on Darcy's law and fractional calculus, the non-Darcy seepage model of the water storage media was constructed. The experimental data show that the model can accurately describe the flow state of mine water in water storage media

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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