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Research Article

Pumping-Induced Non-Darcian Flow in a Finite Confined Aquifer

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An analytic model for depicting non-Darcian flow caused by pumping in a finite confined aquifer with an outer barrier boundary is established. The model considers the wellbore storage and skin effect. And the semi-analytical solution is derived with the aid of the Izbash's law-based non-Darcian flow and the method for linearization procedure combined with the Laplace transformation. The new presented solution can reduce to some available solutions for the confined aquifer of unlimited extension. The drawdowns in abstraction well and aquifer are explored. The results suggest that the presence of the no-flux outer boundary makes no difference to the early-time drawdowns, the late-time drawdowns in the unlimited confined aquifer are smaller than those in the finite confined aquifer with a zero-flux outer boundary, and the flow in this finite confined aquifer cannot approach quasi-steady state, especially to a smaller finite outer boundary distance radius. The change of power index n in the non-Darcian flow equation cannot affect the early-time drawdowns in abstraction well, while the drawdowns in aquifer are underestimated for Darcian flow case at early pumping time. The late-time drawdowns in the pumping well and aquifer are significantly overestimated under the assumption of Darcian flow. The early-time drawdowns in abstraction well and aquifer are significantly affected by wellbore storage, and a larger wellbore storage coefficient leads to a smaller drawdown. The skin factor can impact the intermediate-time drawdowns in abstraction well, while the early and intermediate-time drawdowns in aquifer are influenced by skin effect.

1. Introduction

For groundwater flow to a large-dimeter production well, there are many available analytic solutions that can be used to explore the drawdown distribution in confined aquifers [1–3], unconfined aquifers [4–6], and leaky confined aquifers [7–9]. These works illustrated that the influence of wellbore storage on the drawdown response needs to be taken into consideration, especially at the early stage of pumping. In addition, the effect of wellbore skin on the drawdown in the large-dimeter pumping well is also significant. The wellbore skin is usually caused by well development or well clogging [10]. Skin factor or finite thickness skin can often be used to depict the skin effect in the existing literature [4, 9–16]. The less parameter is needed for an infinitesimal skin in comparison with a finite thickness skin; thus, the nondimensional skin factor is presented to describe the skin effect in this study, which is also available for the previous research [2, 4, 12, 13, 16]. A large number of these analytic solutions considering wellbore storage effect or well-bore skin were developed on the assumption that the groundwater flow follows Darcy law. However, the non-

Darcy flow occurs under the circumstances that the flow velocity is high or the well discharge is relatively large [10, 15].

For non-Darcy flow to a pumping well, many studies are available in well hydraulic research on the basis of the two frequently used groundwater flow with Forchheimer's law [17] and Izbash's law [18]. For example, by means of the Forchheimer's law of groundwater flow, Sen [19] took the Boltzmann transformation and developed the drawdown solution in an infinite confined aquifer with a large-diameter pumping well. Mathias et al. [20] proposed an approximate drawdown solution in an infinite confined aquifer under the condition of well full penetration. With the aid of the Izbash's law of groundwater flow, Sen [21] also applied the Boltzmann transform and obtained a drawdown solution for unsteady radial Izbash flow to an infinitesimal-radius pumping well in a confined aquifer. Wen et al. [22] introduced a linearization method and developed a solution for Izbash flow induced by pumping in a confined aquifer of infinite extent. Remarkably, many studies demonstrated that the Izbash's law of groundwater flow is a better choice than the Forchheimer's law of groundwater flow in describing non-Darcian flow in coarse porous medias [10, 23–25]. In recent years, Wen et al. [26, 27] investigated the drawdown distribution for non-Darcian flow to a partially penetrated pumping well neglecting or taking the effect of wellbore storage into consideration. Feng and Wen [15] considered the impact of the finite skin zone and explored the Izbash flow behavior due to pumping in a confined aquifer. Zhu and Wen [28] included the influence of wellbore storage and the nonuniformly distributed flux boundary along the wellbore and investigated the drawdown response for non-Darcian flow in a semiconfined aquifer.

The majority of the aforesaid analytical solutions for flow to a pumping well in different types of aquifer systems are developed on the assumption that the aquifers are treated as an infinite extension in the lateral direction. However, in reality, the aquifer may come across a physical obstruction (e.g., natural or artificial impervious media) assumed as a barrier boundary (zero-flux condition) [10, 29–33]. Under these circumstances, the aquifer has a limited extent in horizontal orientation, and such assumption for the aquifer of infinite extent is not valid; the impact of the outer barrier boundary on drawdown must be addressed. Although it may be convenient for using the image well theory or superposition principle to obtain the drawdown solution in finite aquifers, the analytical solution cannot be developed without the existing solutions of infinite aquifers. Accordingly, there are some studies focusing on flow in such aquifer system. For example, Kuiper [34] provided an analytic solution of drawdown in a finite confined aquifer limited by a barrier boundary at outer distance from a full penetration abstraction well. Mishra and Chachadi [35] further included the influence of wellbore storage with the use of the solution of Kuiper [34]. Butler and Tsou [36] determined the leakage in a laterally bounded leaky aquifer using image well theory with the aid of the available solution for laterally unlimited aquifers. Zhou et al. [37] obtained the solution of pressure change in a finite leaky confined aquifer limited by no-flux outer boundary. Recently, Lin et al. [38] derived drawdown solutions for groundwater flow in a finite two-zone-confined aquifer restricted by the Robin outer boundary. Feng and Zhan [16] developed semi-analytical solutions for flow to a constant-head pumping well in an aquitard-aquifer system with taking account of three remote lateral boundaries. Feng et al. [39] gave semi-analytical solutions for flow to an abstraction well of constant pumping rate in a leaky confined aquifer limited by zero drawdown and zero-flux condition at the outer boundaries.

After a careful literature review, most of well-hydraulic literature for pumping-induced non-Darcian flow are developed with the statement that the aquifers are of unlimited extension in horizontal orientation. And the available researches are rarely concerned with pumping-induced non-Darcian flow in a closed confined aquifer, and the calculation equations have not been presented so far. Thus, in this work, a solution for non-Darcian flow toward an abstraction well in a finite aquifer restricted by a zero-flux condition at the outer boundary is derived directly from the governing equation, and a straightforward semi-analytical solution including wellbore storage and skin effect is provided with the help of the Izbash equation and the methods of linearization approximation and the Laplace transform. The developed solution can be employed to explore drawdown responses in pumping well and aquifer under the impact of non-Darcian, wellbore storage, skin effect, and no-flow outer boundary; describe the groundwater flow or predict pressure distribution in a closed reservoir; help the design problem of dewatering in geological engineering; and provide a theoretical base for hydraulic parameter estimation of a confined aquifer of finite barrier boundary.

2. Methods

2.1. Mathematical Statement. Figure 1 demonstrates a centrally located fully penetrated well of finite radius pumping at steady discharge Q, in a confined aquifer of a finite radial extension. Additionally, this study also assumes that (1) the aquifer of constant thickness B is homogeneous and isotropic; (2) the radial zero-flux boundary is imposed at r_0 from the well center; and (3) the flow in the horizontal direction follows Izbash non-Darcian equation. It is obvious that the most appropriate location for pumping groundwater is the central zone of the finite aquifer.

Accordingly, the governing flow equation for the model follows that [21, 22, 40]

$$\frac{\partial q(r,t)}{\partial r} + \frac{q(r,t)}{r} = S_s \frac{\partial s(r,t)}{\partial t}, r_w \le r \le r_0, \tag{1}$$

where s is the drawdown; r refers to the radial distance; t is the time; r_w designates the radius of the well screen; q implies the specific discharge; and S_s indicates the specific storage.

The initial condition makes

$$s(r,0) = 0. \tag{2}$$

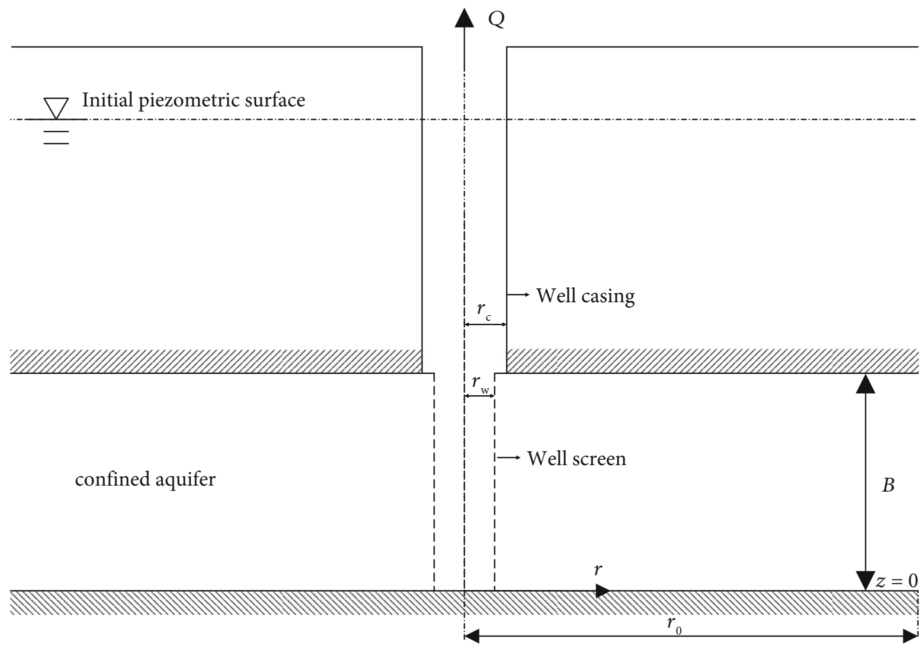

Figure 1: Schematic showing of a finite confined aquifer system.

The outer barrier boundary condition at $r = r_0$ yields

$$\frac{\partial s(r_0, t)}{\partial r} = 0. \tag{3}$$

Assuming the uniform flux along the well screen [41–45], and taking the wellbore storage and skin effect into account, the well-face boundary condition is [4, 41]:

$$Q = -2\pi r_w B q(r_w, t) + \pi r_c^2 \frac{\partial H}{\partial t}, r = r_w, \tag{4}$$

where

$$H = s(r, t) - r_w S_k \frac{\partial s(r, t)}{\partial r}, r = r_w, \tag{5}$$

in which r_c shown in Figure 1 represents the well casing radius, H refers to the drawdown in pumping well, and $S_k = K_r d_s/(K_s r_w)$ is skin factor that influences skin conductance [2, 41], where d_s designates well skin thickness [L]. In a similar way, K_s can also be regarded as the apparent hydraulic conductivity of skin. It should be mentioned that the well skin storage is ignored for a thin skin in this study.

2.2. Solutions. The Izbash non-Darcian flow equation takes the form [22]:

$$-|q_r|^n = -(-q_r)^n = K \frac{\partial s(r, t)}{\partial r}, \tag{6}$$

where n represents a power index and its value is between one and two for describing non-Darcian flow caused by pumping due to the occurred large flow rates in a coarse aquifer media [15, 19, 40]. K refers to non-Darcy empirical constant, and one can regard it as the apparent hydraulic conductivity [22]. Notably, one can obviously see that Eq. (6) is the same as the Darcy equation if $n = 1$ and K is the hydraulic conductivity.

Substituting Eqs. (6) into Eq. (1) leads to

$$\frac{K}{n(-q)^{n-1}} \frac{\partial^2 s(r, t)}{\partial r^2} + \frac{K}{r(-q)^{n-1}} \frac{\partial s(r, t)}{\partial r} = S_s \frac{\partial s(r, t)}{\partial t}, r_w \le r \le r_0. \tag{7}$$

Eq. (7) is a nonlinear equation due to the existence of $(-q)^{n-1}$ terms. By the use of the linearization method proposed by Wen et al. [22, 40, 46], one can apply the following equation (Eq. (8)) to solve it analytically:

$$(-q)^{n-1} \approx \left(\frac{Q}{2\pi r B}\right)^{n-1}, \tag{8}$$

such that Eq. (7) is rewritten as

$$\frac{\partial^2 s(r, t)}{\partial r^2} + \frac{n}{r} \frac{\partial s(r, t)}{\partial r} = \frac{n}{K} \left(\frac{Q}{2\pi r B}\right)^{n-1} S_s \frac{\partial s(r, t)}{\partial t}, \quad r_w \le r \le r_0. \tag{9}$$

The wellbore boundary condition can be expressed as

$$\Delta \frac{\partial s(r_w, t)}{\partial r} - \pi r_c^2 \frac{\partial H}{\partial t} = -Q, \tag{10}$$

where $\Delta = 2\pi K r_w B [Q/(2\pi r_w B)]^{1-n}$.

With the help of Laplace transform, one can obtain the solution in the aquifer, and one can find the derivation process in the Appendix. The drawdown solution takes the form

$$\bar{H} = \frac{Q}{p(\Phi_2 - \Phi_1)}\left[\varphi_1 K_{\frac{2}{3-n}}\left(\frac{2}{3-n} r_0^{\frac{3-n}{2}}\sqrt{\delta}\right) + \varphi_2 I_{\frac{-2}{3-n}}\left(\frac{2}{3-n} r_0^{\frac{3-n}{2}}\sqrt{\delta}\right)\right], \tag{11}$$

and the Laplace domain drawdown solution in the aquifer is

$$\begin{aligned}\bar{s}(r,p) = {} & \frac{Q}{p(\Phi_2 - \Phi_1)} r^{\frac{1-n}{2}}\left[K_{\frac{2}{3-n}}\left(\frac{2}{3-n} r_0^{\frac{3-n}{2}}\sqrt{\delta}\right) I_{\frac{1-n}{3-n}}\right. \\ & \cdot\left(\frac{2}{3-n} r^{\frac{3-n}{2}}\sqrt{\delta}\right) + I_{\frac{-2}{3-n}}\left(\frac{2}{3-n} r_0^{\frac{3-n}{2}}\sqrt{\delta}\right) K_{\frac{1-n}{3-n}} \\ & \left.\cdot\left(\frac{2}{3-n} r^{\frac{3-n}{2}}\sqrt{\delta}\right)\right],\end{aligned} \tag{12}$$

where $\delta = (S_s np/K)(Q/2\pi B)^{n-1}$, p denotes the Laplace transform parameter, overbar refers to terms in Laplace space. $I_m(\cdot)$ and $K_m(\cdot)$ are, respectively, the m-th order modified

Eq. (14) is the accordance with the solution of Wen et al. [22] describing the pumping-induced non-Darcian flow in an unlimited confined aquifer under the condition of a full penetration well with wellbore storage.

When flow is Darcian ($n = 1$) flow, Eq. (14) becomes the Laplace domain solution of Papadopulos and Cooper [1].

3. Results and Discussion

In what follows, the drawdown responses to the impact of the power index n, wellbore storage, skin factor, and no-flux boundary are mainly discussed. All the used default parameter values include the following: $Q = 50\,\mathrm{m^3/h}$, $B = 20$ m, $K_r = 0.1(\mathrm{m/h})^{1/n}$, $n = 1.5$, $S_s = 0.0001\,\mathrm{m^{-1}}$, $r_0 = 80$ m, $r_w = 0.3$ m, $r_c = 0.3$ m, and $r = 5$ m.

3.1. Comparison with Previous Solutions. Figure 2 demonstrates the drawdown behavior in the pumping well determined by the present solution and other available solutions, such as the solution of Wen et al. [22] for non-Darcian flow in an infinite confined aquifer with taking into account wellbore storage and the Darcy's law-based solution of Papadopulos and Cooper [1] for flow in an unlimited confined aquifer with the consideration of wellbore storage

Bessel function of the first and second kinds. The Appendix lists the complete expression of φ_1, φ_2, Φ_1, and Φ_2. The time-domain drawdown solutions in the pumping well and the aquifer can be available by carrying out the Laplace transform inversion numerically with the often used Stehfest method [47].

2.3. Special Solutions. If $r_0 \longrightarrow \infty$, the new developed solution reduces to

$$\bar{s}(r,p) = \frac{Q r^{1-n/2} K_{1-n/3-n}\left((2/3-n) r^{3-n/2}\sqrt{\delta}\right)}{p\left[\Delta r_w^{1-n}\sqrt{\delta} K_{2/3-n}\left((2/3-n) r_w^{3-n/2}\sqrt{\delta}\right) + \pi r_c^2 p \varphi_2\right]}. \tag{13}$$

Eq. (13) is the solution including wellbore storage and skin effect for pumping-induced non-Darcian flow in a confined aquifer of unlimited extension.

Neglecting the skin effect, Eq. (13) becomes

time, the drawdowns for the case of non-Darcian flow are smaller than those of Darcian flow, and the drawdowns in the unlimited confined aquifer are smaller than those in the closed confined aquifer, which is caused by the no-flux outer boundary without providing groundwater. In addition, one also can see that the intermediate-drawdown response to the impact of skin is striking and the behavior will be detailly explored below.

Figure 3 illustrates the drawdown behavior in aquifer ($r = 5\,m$) with the use of the proposed analytical model and the other analytical model that have been used in Figure 2. One can see that the early-time drawdowns for the case of non-Darcian flow are larger than those of Darcian flow, while the late-time drawdown behavior agrees with that shown in Figure 2. One may also find that the outer boundary remarkably affects the late-time drawdowns, and the drawdowns in the closed aquifer are also larger than those in the infinite aquifer. Moreover, the early and intermediate-time drawdowns in aquifer are influenced by skin effect, as shown in Figure 3.

3.2. Effect of Outer Boundary. Figure 4 displays the drawdown response (a) in the pumping well and (b) in the aquifer ($r = 5\,m$) with different finite outer radius r_0. Note that

$$\bar{s}(r,p) = \frac{Q r^{1-n/2} K_{1-n/3-n}\left((2/3-n) r^{3-n/2}\sqrt{\delta}\right)}{p\left[\Delta r_w^{1-n}\sqrt{\delta} K_{2/3-n}\left((2/3-n) r_w^{3-n/2}\sqrt{\delta}\right) + \pi r_c^2 p r_w^{1-n/2} K_{1-n/3-n}\left((2/3-n) r_w^{3-n/2}\sqrt{\delta}\right)\right]}. \tag{14}$$

effect. It can be seen from Figure 2 that the drawdowns in abstraction well for all cases are almost the same at early time; this is because that at this stage, the pumping groundwater originates from wellbore storage. At late pumping

the commonly used case for the aquifer of infinite extent ($r_0 \to \infty$) is contained in this figure for comparison. Figures 4(a) and 4(b) show that the behaviors of drawdown in pumping well are similar with those in aquifer under

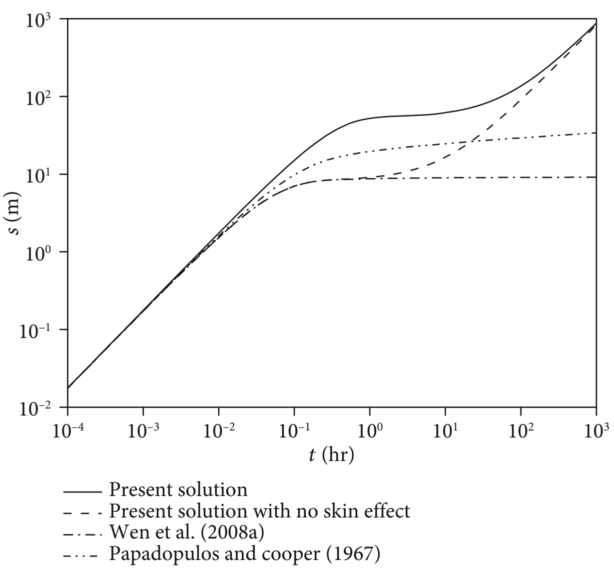

FIGURE 2: Comparison of the drawdown behavior in pumping well in the present study and the available studies.

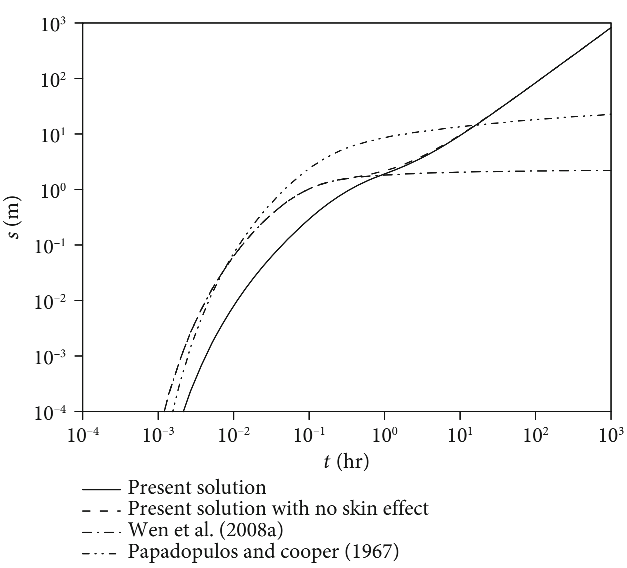

FIGURE 3: The drawdown behavior in aquifer determined using the present study and the available studies.

various r_0. The change of r_0 has no effect on the early-time drawdown, as expected. A larger r_0 results in a smaller late-time drawdown. It is because a bigger r_0 indicating more groundwater is accumulated in aquifer. In addition, Figure 4 also illustrates that the influence of the outer no-flux boundary on the drawdown is much faster for a smaller r_0. Additionally, the late-time drawdown behavior shows that the flow in the closed confined aquifer cannot approach a quasi-steady state, especially to a smaller r_0. However, the flow in an infinite confined aquifer can approach a quasi-steady state, as reflected in Figure 4. In general, both the drawdowns in pumping well and aquifer are affected by the outer no-flow boundary, especially for the late pumping time, and the drawdown is significantly be underestimated under the assumption of the aquifer of infinite area extent.

3.3. Effect of Non-Darcian Flow (n). The drawdown responses in pumping well and aquifer to the influence of non-Darcian flow with different power index n are demonstrated in Figure 5. It should be noted that $n = 1$ indicates the flow is Darcian and considered in this figure. As illustrated in Figure 5(a), no matter what the value of power index n is, the drawdowns in the pumping well for all cases have the same values during the early pumping stage because of taking the wellbore storage effect into account; while a bigger n leads to a bigger aquifer drawdown, a bigger n indicates that the degree of flow turbulence is much greater and the stored water in the aquifer might release much faster; therefore, one can observe a larger drawdown at early pumping time, as shown in Figure 5(b). As for the drawdown behavior in the pumping well and in the aquifer at late pumping time, one can see the similar impact of non-Darcian flow that a greater n can cause to a smaller drawdown, a bigger n also implies that the flow resistance is much greater, and the drawdown may drop much faster; thus, a smaller drawdown can be observed in this figure. Additionally, Figure 5 displays that the late-time drawdowns in the pumping well and in the aquifer are significantly overestimated under the assumption of Darcian flow.

3.4. Effect of Wellbore Storage. Figure 6 shows the type curves of drawdown vs. time in (a) pumping well and (b) aquifer ($r = 5\,m$) with different values of C_w. It should be attention that $Cw = 0$ designates the case neglecting wellbore storage. The type curves for the early-time drawdown in abstraction well on log-log scale is linear with various values of C_w, as shown in Figure 6(a). However, in general, the type curves of drawdown in the aquifer are nonlinear and may nearly linear if C_w is large enough, as demonstrated in Figure 6(b), and accordingly, one can easily to distinguish the degree of the impact of wellbore storage on drawdown. Moreover, one can observe that a larger C_w leads to a longer impact of wellbore storage on drawdown and indicates that more groundwater is stored in the pumping well; thus, a smaller drawdown can be found.

3.5. Effect of Skin Factor. Figure 7 shows the drawdown in pumping well and aquifer with distinct value of skin factor (S_k). Remarkably, $Sk = 0$ represents the no skin case. Figure 7(a) shows that the drawdowns in pumping well at early time are mainly affected by wellbore storage, as expected. Furthermore, the change of skin factor impacts the intermediate-drawdown in pumping well. However, Figure 7(b) shows that the early and intermediate-time drawdowns in aquifer are influenced by the effect of skin. More importantly, one can see that as the constant skin factor increases, the drawdown in abstraction well also increases, while the drawdown in aquifer decreases. A smaller skin factor represents a much more powerful skin conductance, indicating that groundwater flow is much easier to enter the pumping well and aquifer response to the pumping is much faster. Therefore, one can a larger

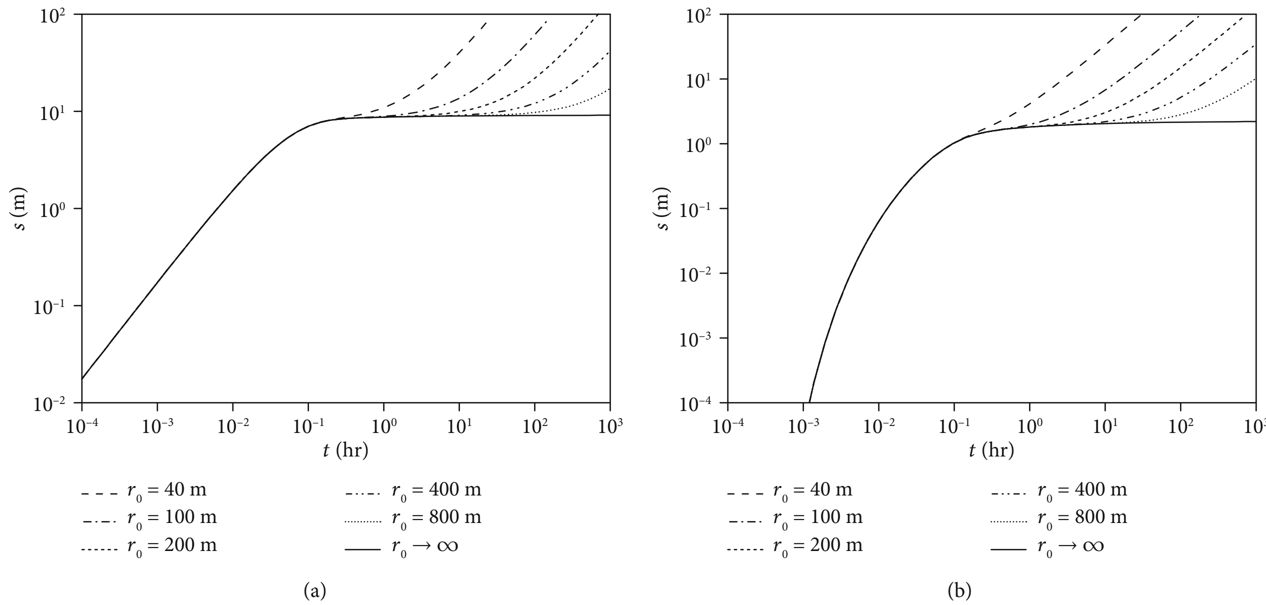

FIGURE 4: Impact of outer boundary on the drawdown in pumping well (a) and aquifer (b).

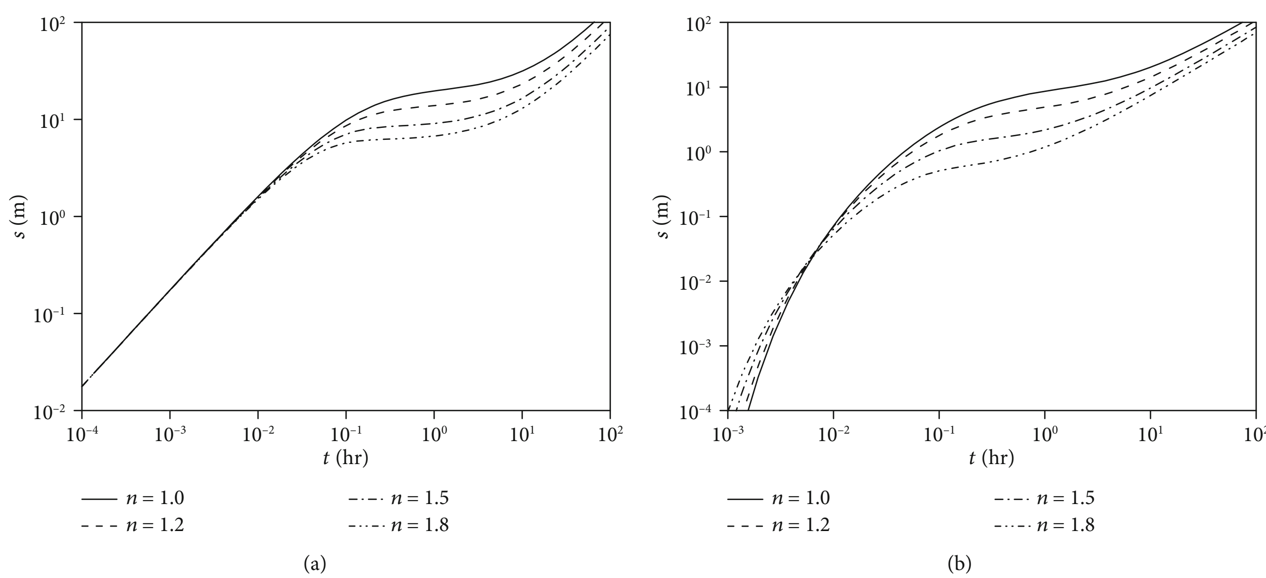

FIGURE 5: Impact of non-Darcy (n) on the drawdown in pumping well (a) and aquifer (b).

drawdown in pumping well be given by Figure 7(a), and a smaller aquifer drawdown shown in Figure 7(b).

3.6. Limitations of the Proposed Solution. One can find from the above analysis that the newly proposed solution can be directly applied for exploring the drawdown characteristics in confined aquifers of finite or infinite extension and investigating the effects of non-Darcy flow, wellbore storage, skin effect, and the outer no-flux boundary on the drawdown. Apart from that, the developed solution can be used to determine the hydraulic parameters of a finite aquifer with the collected data of drawdown in field, and more importantly, it could be very useful in practice, especially to the field of petroleum engineering and dewatering engineering in construction.

However, some limitations could also be addressed for better utilization of the obtained solution. First, the radial non-Darcy flow is depicted by the Izbash equation in this study, but one can use other equations to describe non-Darcy flow law (e.g., the Forchheimer equation) if necessary. Second, the non-Darcy flow is assumed to be occurred in the entire aquifer system, but the flow at the place far away from the pumping well may abide by Darcy law; thus, it is necessary to develop a two-region flow model for describing

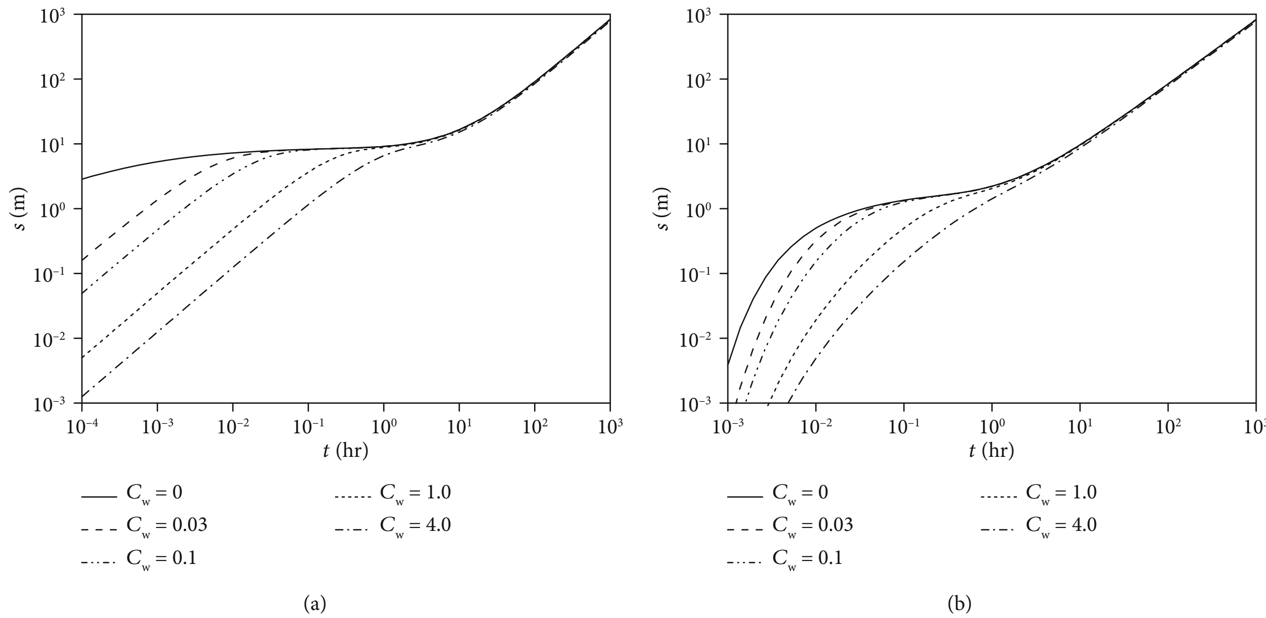

Figure 6: Impact of wellbore storage on the drawdown in pumping well (a) and aquifer (b).

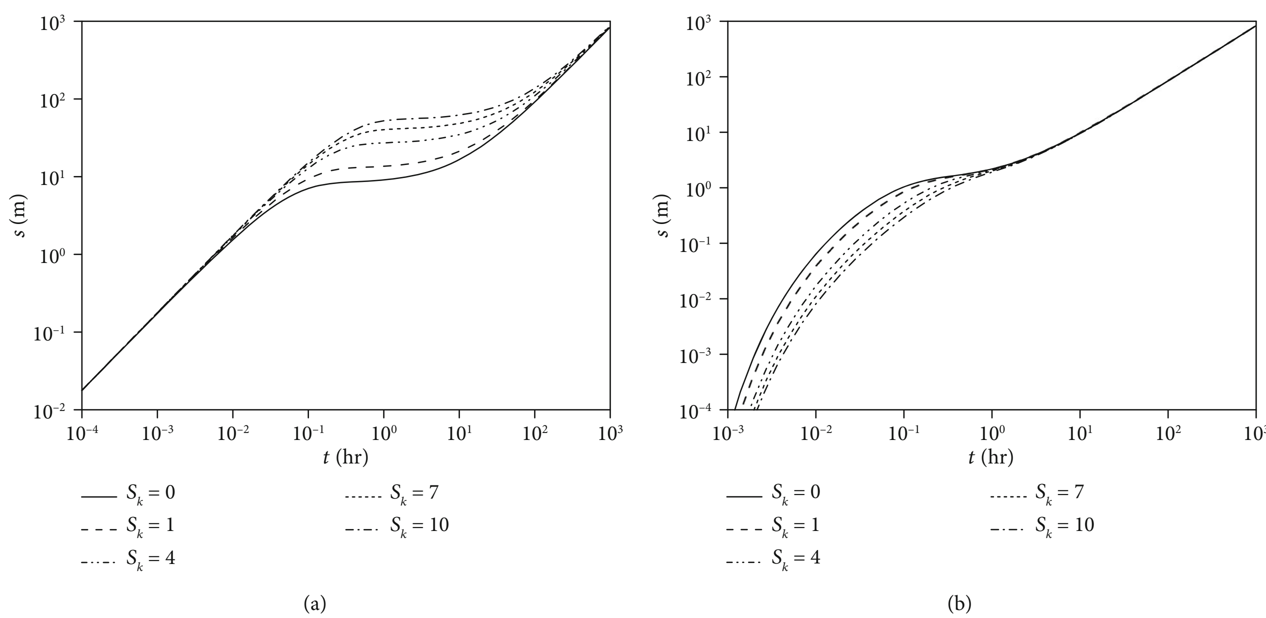

Figure 7: Impact of skin factor on the drawdown in pumping well (a) and aquifer (b).

non-Darcian and Darcian flows in a finite aquifer system in the future. Third, the collected data in the field or physical model tests can be further verified the application of the solution. Fourth, the influences of leakage, well partial penetration, finite thickness skin with storage, and other used type of pumping test (e.g., constant-head pumping test) cannot be investigated by using the proposed solution. Finally, the flow problem induced by an eccentric pumping well is also not addressed in this study. In summary, one can further investigate the above listed points based on this study in the near future.

4. Conclusions

A novel analytical model for pumping-induced non-Darcian flow in a bounded confined aquifer with a constant rate abstraction well has been proposed. The radial non-Darcian flow is depicted in Izbash equation. The drawdown solution is developed with the help of linearization approximation and the transformation of Laplace, and the solution is different from available solutions in taking account of the united impacts of non-Darcian, wellbore storage, skin factor, and outer boundary of no-flow. With the proposed solution, the drawdowns in pumping well and aquifer are explored. The main conclusions can be made as follows:

(1) The early-time drawdown is not influenced by the outer no-flux boundary, the late-time drawdowns in the infinite confined aquifer are smaller than those in the closed confined aquifer, and the flow in the closed confined aquifer cannot approach a quasi-steady state for a smaller finite outer radius r_0

(2) The change of power index n in the non-Darcian flow equation cannot affect the early-time drawdowns in abstraction well, while the drawdowns in aquifer is underestimated for Darcian flow case at early pumping time. The late-time drawdowns in pumping well and aquifer are significantly overestimated under the assumption of Darcian flow

(3) The early-time drawdowns in abstraction well and aquifer are significantly affected by wellbore storage. The early-time drawdowns in pumping well and aquifer are significantly affected by wellbore storage, and a larger wellbore storage coefficient C_w leads to a smaller drawdown

(4) The skin factor can impact the intermediate-time drawdowns in abstraction well, while the early and intermediate-time drawdowns in aquifer are influenced by the skin effect

Appendix

Applying Laplace transformation and taking the initial condition Eq.(3) into consideration, one finds

$$\frac{\partial^2 \bar{s}(r,p)}{\partial r^2} + \frac{n}{r}\frac{\partial \bar{s}(r,p)}{\partial r} = \delta r^{1-n}\bar{s}(r,p), \tag{A.1}$$

in which $\delta = (S_s np/K)(Q/2\pi B)^{n-1}$.

The well-face boundary condition of Eq. (4) in Laplace domain becomes

$$\Delta \frac{\partial \bar{s}(r_w,p)}{\partial r} - \pi r_c^2 p \bar{H}(p) = -\frac{Q}{p}, \tag{A.2}$$

where

$$\bar{H}(p) = \bar{s} - r_w S_k \frac{\partial \bar{s}}{\partial r}, r = r_w. \tag{A.3}$$

The outer no-flux boundary condition can be rewritten as

$$\frac{\partial \bar{s}(r_0,p)}{\partial r} = 0. \tag{A.4}$$

The general solution of Eq.(A.1) yields

$$\bar{s}(r,p) = r^{\frac{1-n}{2}}\left[C_1 I_{\frac{1-n}{3-n}}\left(\frac{2}{3-n} r^{\frac{3-n}{2}}\sqrt{\delta}\right) + C_2 K_{\frac{1-n}{3-n}}\left(\frac{2}{3-n} r^{\frac{3-n}{2}}\sqrt{\delta}\right)\right], \tag{A.5}$$

where C_1 and C_2 refer to constants of integration and can be determined using boundary conditions.

Substituting Eq. (A.5) into Eqs. (A.2) and (A.4), respectively, one makes

$$\begin{aligned} &C_1\left[\Delta r_w^{1-n}\sqrt{\delta} I_{\frac{-2}{3-n}}\left(\frac{2}{3-n} r_w^{\frac{3-n}{2}}\sqrt{\delta}\right) - \pi r_c^2 p \varphi_1\right] \\ &\quad - C_2\left[\Delta r_w^{1-n}\sqrt{\delta} K_{\frac{2}{3-n}}\left(\frac{2}{3-n} r_w^{\frac{3-n}{2}}\sqrt{\delta}\right) + \pi r_c^2 p \varphi_2\right] = -\frac{Q}{p}, \end{aligned} \tag{A.6}$$

$$C_1 r_0^{1-n}\sqrt{\delta} I_{\frac{-2}{3-n}}\left(\frac{2}{3-n} r_0^{\frac{3-n}{2}}\sqrt{\delta}\right) - C_2 r_0^{1-n}\sqrt{\delta} K_{\frac{2}{3-n}}\left(\frac{2}{3-n} r_0^{\frac{3-n}{2}}\sqrt{\delta}\right) = 0, \tag{A.7}$$

where

$$\begin{aligned} \varphi_1 &= r_w^{\frac{1-n}{2}} I_{\frac{1-n}{3-n}}\left(\frac{2}{3-n} r_w^{\frac{3-n}{2}}\sqrt{\delta}\right) - r_w S_k r_w^{1-n}\sqrt{\delta} I_{\frac{-2}{3-n}}\left(\frac{2}{3-n} r_w^{\frac{3-n}{2}}\sqrt{\delta}\right), \\ \varphi_2 &= r_w^{\frac{1-n}{2}} K_{\frac{1-n}{3-n}}\left(\frac{2}{3-n} r_w^{\frac{3-n}{2}}\sqrt{\delta}\right) + r_w S_k r_w^{1-n}\sqrt{\delta} K_{\frac{2}{3-n}}\left(\frac{2}{3-n} r_w^{\frac{3-n}{2}}\sqrt{\delta}\right), \end{aligned} \tag{A.8}$$

Using Eqs. (A.6) and (A.7), one computes

$$\begin{aligned} C_1 &= \frac{Q}{p(\Phi_2 - \Phi_1)} K_{\frac{2}{3-n}}\left(\frac{2}{3-n} r_0^{\frac{3-n}{2}}\sqrt{\delta}\right), \\ C_2 &= \frac{Q}{p(\Phi_2 - \Phi_1)} I_{\frac{-2}{3-n}}\left(\frac{2}{3-n} r_0^{\frac{3-n}{2}}\sqrt{\delta}\right), \end{aligned} \tag{A.9}$$

where

$$\begin{aligned} \Phi_1 &= K_{\frac{2}{3-n}}\left(\frac{2}{3-n} r_0^{\frac{3-n}{2}}\sqrt{\delta}\right)\left[\Delta r_w^{1-n}\sqrt{\delta} I_{\frac{-2}{3-n}}\left(\frac{2}{3-n} r_w^{\frac{3-n}{2}}\sqrt{\delta}\right) - \pi r_c^2 p \varphi_1\right], \\ \Phi_2 &= I_{\frac{-2}{3-n}}\left(\frac{2}{3-n} r_0^{\frac{3-n}{2}}\sqrt{\delta}\right)\left[\Delta r_w^{1-n}\sqrt{\delta} K_{\frac{2}{3-n}}\left(\frac{2}{3-n} r_w^{\frac{3-n}{2}}\sqrt{\delta}\right) + \pi r_c^2 p \varphi_2\right]. \end{aligned} \tag{A.10}$$

Data Availability

All data included in this study are available upon request by contact with the corresponding author.

Conflicts of Interest

The authors state that there is no conflict of interest.

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