

Research Article

Stability Evaluation of Sliding-Type Perilous Rock in Huangzangsi Hydrojunction Project Based on Natural Vibration Frequency

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The instability of perilous rock is mostly manifested as sudden collapse and failure without obvious displacement characteristics. Therefore, it is difficult to achieve the purpose of monitoring and early warning by conventional displacement monitoring. But the existing stability monitoring indicators are mostly deformation, stress, and strain. There is a problem that the stability evaluation parameters are inconsistent with the monitoring parameters. Taking sliding-type perilous rock as the research object, the structural plane is assumed to be homogeneous and isotropic, and linear elastic deformation in the amplitude range. Based on the dynamic theory and limit equilibrium model, the quantitative relationship model can be established involving safety factors, natural vibration frequency, structural surface bonding area, elastic modulus, and mass. The remote laser vibrometer is used to monitor the natural vibration frequency of the sliding-type perilous rock on the slope of the Huangzangsi Hydrojunction, and the stability evaluation of the perilous rock is achieved based on the quantitative relationship model between the safety factor can be monitored. The results are basically the same with the safety factor calculated by the limit equilibrium method, indicating that the method is correct and feasible. The research has high theoretical significance and practical value for the safety monitoring and advanced warning of sliding perilous rock.

1. Introduction

There are a large number of rock slopes in China. The perilous rock on the slopes are unstable due to natural or human factors such as rainfall, earthquakes, and vibrations which bring great threats to the safety of engineering and people's lives and properties, the social impact is great. At present, the theoretical analysis of perilous rock is usually carried out qualitatively or quantitatively through the traditional stress-strain model, and little attention is paid to the structural damage of perilous rock after external disturbance. Recent research results show that the dynamic characteristic parameters of perilous rock will change regularly with the weakening of the strength of the structural plane, and the change of dynamic characteristic parameters reflects the change of the safety factor of perilous rock. Therefore, how to introduce new parameters to reflect the degree of structural damage after the disturbance of perilous rock, and carry out the stability analysis and early warning monitoring of perilous rock based on the change of this parameter, is one of the problems that need to be solved urgently. According to the theory of structural dynamics, the dynamic characteristic parameters are related to the physical properties of the structure. The damage of the structure will inevitably lead to the change of the physical parameters of the structure, which provides a new technical idea for the safety monitoring and damage evaluation of perilous rock. The damage degree of the dangerous rock structural plane can be qualitatively or quantitatively judged by the change of parameters such as natural vibration frequency. This method provides basic data support for the safety evaluation of perilous rock, and also it provides scientific and reasonable technical indicators for engineering monitoring. In recent years, with the improvement of laser vibration measurement accuracy and the improvement of performance such as measurement distance, the acquisition of natural vibration frequency has become more convenient and accurate. Therefore, the stability evaluation method of perilous rock based on natural vibration frequency has the advantages of economy, high efficiency, dynamic and accuracy. This technical is playing an important role in effective monitoring, forecasting, analysis and risk assessment of major disasters, and has strong research value and practical significance.

At present, the research on the stability of perilous rock mainly focuses on five methods: limit equilibrium method, limit analysis method, numerical simulation method, mathematical model method [1], and stability evaluation method based on dynamic characteristic parameters. The limit equilibrium method is simple and has clear physical parameters and is widely used. However, rock and soil mass is an extremely complex elastic-plastic body [2-5]. Chen introduced the limit analysis method of plastic mechanics to the field of rock and soil stability evaluation and formed the famous upper and lower limit theory [6]. Pan questioned the hypothesis put forward by Hoek and Bray, and proposed the "Maximum and Minimum Theory" based on the upper and lower limit theory. He believes that if a landslide can slide along many sliding surfaces, it will fail along the sliding surface with the least resistance when it becomes unstable [7]. According to the question raised by Pan Jiazheng, Chen et al. obtained the generalized solution of the safety factor of perilous rock by using the principle of virtual work and the upper and lower bound theorems of plastic mechanics [8]. Although the limit equilibrium method and limit analysis method are simple and effective to realize the stability evaluation of rock and soil mass, it is not enough to realize the stability evaluation of rock and soil mass with complex boundary conditions, discontinuity, and anisotropy. With the development of computer technology, the emergence of numerical methods is a good solution to the evaluation of rock and soil stability with complex boundary conditions. And numerical methods mainly include finite element, discrete element, finite difference, and the discontinuous medium mechanics numerical analysis method (DDA method). Zhang used the finite element method to analyze the stability of perilous rock [9]. Zheng et al. analyzed the stability of perilous rock downstream of the left abutment of Baihetan Hydropower Station based on discrete element and Lagrangian finite difference method [10]. The DDA method proposed by Shi reflects the discontinuity and large deformation characteristics of rock mass deformation [11]. The numerical simulation method simplifies the mechanical parameters and boundary conditions of rock and soil, and at the same time, it is inevitably affected by objective and sub-

jective factors in the simulation process. Therefore, many scholars have doubts about the reliability and credibility of numerical simulation. With the development of mathematical methods, some scholars have introduced mathematical methods such as grey theory, fuzzy mathematics, and reliability into the field of rock and soil stability evaluation. Wang et al. proposed a fuzzy point estimation method for perilous rock [12]. Sun et al. used the inverse algorithm to obtain the shear strength value of the sliding surface [13]. Tang et al. used norm to quantify the factors affecting the stability of perilous rock [14]. Xie and Xia realized the stability analysis of perilous rock based on grey clustering in grey system theory [15]. The mathematical methods in the above methods are difficult to quantitatively evaluate the stability of rock and soil. The limit equilibrium method and limit analysis method can well achieve the stability evaluation of perilous rock [16, 17]. However, the key parameters affecting the stability of perilous rock, cohesion and internal friction angle, are inconsistent with the monitoring indicators, making it impossible to efficiently and dynamically obtain the safety factor of perilous rock.

Since the perilous rock must be damaged before it is destroyed, the damage identification of the perilous rock is an effective method to realize the stability evaluation of the perilous rock. At present, the research on structural damage identification mainly focuses on high-rise buildings, bridges, machinery, and other fields. Damage identification methods mainly include modal natural frequency modal [18], vibration modal [19], modal damping [20], modal flexibility matrix [21], modal mass [22], modal stiffness [23], modal curvature [24], and modal strain [25]. Although the above methods are widely used in the research and application of large-scale structural damage identification, their application in the evaluation of perilous rock stability is still in its infancy. Saito et al. found that the stability of perilous rock is closely related to dynamic characteristics [26]. Ma et al. found that the bonding area was positively correlated with the natural vibration frequency [27]. Fukata et al. found that the burial depth of surface perilous rock is closely related to the quality, Young's modulus, natural vibration frequency, and Poisson's ratio of the perilous rock [28]. Xie et al. found that a ground-penetrating radar (GPR) survey can be used to evaluate the slope stability [29]. Du et al. and Jia et al. further conducted experimental research on dumping perilous rock and found that the natural vibration frequency is closely related to the stability [30-33]. However, the above research has just started and is in the experimental stage, and the theory has not yet been perfected [34, 35]. And the above test found that the dynamic characteristic parameters of the perilous rock are closely related to the stability [36]. Therefore, the natural vibration frequency can be used to identify and monitor the damage of the perilous rock, and then judge its stability.

2. Project Overview

Huangzangsi Hydrojunction is located on the main stream of Heihe River, about 11 km downstream of Huangzangsi Village, Qinghai Province. It is a roller compaction concrete



FIGURE 1: A typical slip-type perilous rock model.



FIGURE 2: Simple harmonic vibration.



FIGURE 3: Vibration model of perilous rock.

gravity dam. The dam catchment area is 7648 km², the total storage capacity of the reservoir is 403 million m³, the installation capacity of the power station is 490,000 kilowatts, and

the investment is 2,782.09 million yuan. The valley is narrow and in the shape of a "V". The elevation of the bottom of the valley is 2520.85–2533.50 m, the elevation of the left bank top is 3121–3130 m, and the maximum elevation of the right bank top is 3254 m. The bank slopes on both sides of the dam axis are steep, where the left bank is more than 2576 m, the slope is generally $60^{\circ} \sim 80^{\circ}$, and the right bank slope is generally $45^{\circ} \sim 60^{\circ}$. The engineering site is located in the Liupanshan-Qilianshan seismic zone. According to the seismic risk analysis results, the peak ground acceleration in the site is 0.19 g, and the corresponding basic earth-

quake intensity is VII degree. The stratigraphic lithology in the dam site area is mainly Cambrian Middle System metamorphic rock and Quaternary loose accumulation layer, and the overall lithology is mainly chlorite, muscovite, and quartz schist, which is medium hard rock, and contains soft rocks such as chlorite schist or mica in some parts. The dam site area is located in the North Qilian Fold Belts. Affected by multiphase tectonic movements, faults and other structural planes are developed and the left bank slope is stable, but there are different scales of collapse. The right abutment has near rivertrending faults and moderately steep dip angle structural planes, and the integrity of the rock mass is poor. Under the action of gravity, vibration load and fissure water pressure, when the shear strength of the structural surface is not enough to resist the sliding force, the sliding damage along the main control structure surface will seriously threaten the safety of life and property of the construction personnel and the construction of the project. A typical slip-type perilous rock model is shown in Figure 1.

3. Stability Evaluation Model of Sliding Perilous Rock Based on Natural Vibration Frequency

3.1. Natural Vibration Frequency. Vibration is an inherent property of objects. From nature to industrial fields, vibration phenomena are ubiquitous. Perilous rock, landslides, various machinery, power devices, instruments, buildings, and bridges, etc., each object is in different freedoms. When the structure of the object is damaged, the natural vibration frequency of the object will change in different degrees of freedom. At present, the principle of natural vibration frequency is widely used in the structural damage monitoring of bridge structures, building structures, aviation instruments, ships, etc., and the structural health status or damage degree of the monitored object is analyzed through monitoring data. Therefore, it is considered to introduce the theory of natural vibration frequency into the structural damage identification of perilous rock to realize its stability evaluation.

Fixed frequencies are also called natural frequencies. When an object vibrates freely, the object performs periodic reciprocating motion with time, and the period of the reciprocating cycle is fixed. The most typical vibration is simple harmonic vibration, and the formula for calculating the natural vibration frequency of simple harmonic vibration is Equation (1). The simple harmonic vibration is shown in



FIGURE 4: Sliding-slip perilous rock on both banks of the dam site.



FIGURE 5: RSV-150 remote laser vibrometer.

Figure 2.

$$f = \frac{1}{2\pi} \sqrt{\frac{K}{M}},\tag{1}$$

where f is the undamped natural vibration frequency (Hz); K is the equivalent stiffness coefficient (N/m); and M is the mass (kg).

3.2. Construction of Dynamic Characteristic Model. Assume that the perilous rock is homogeneous and isotropic, the main control structure surface is a single plane, and the system damping ratio is less than 1 and the deformation in the amplitude range is linear elastic deformation, and then the vibration model of the perilous rock can be simplified to a simple harmonic vibration model, as shown in Figure 3.

The base can be regarded as an infinite block relative to the perilous rock. The overall stiffness of the base and the perilous rock is relatively large, and the structural plane strength is relatively weak compared to the perilous rock and the base. When disturbed by external forces, the block can be regarded as a proton, the structural surface can be regarded as a spring, the base transmits power to the perilous rock through the structural surface, and the perilous rock vibrates up and down along the structural surface. Since the base is regarded as an infinite block, so the base has no effect on the vibration of the system. Therefore, the dynamic characteristic model of sliding perilous rock can be regarded as simple harmonic vibration.

K in Formula (1) is used to describe the basic physical quantity of the elastic deformation form of the material under the action of external force, more generally speaking, the force required to generate unit displacement. The expression is Equation (2).

$$K = \text{ES}/D.$$
 (2)

Putting Equation (2) into Equation (1), the vibration frequency formula of the perilous rock is

$$f = \frac{1}{2\pi} \sqrt{\frac{\text{ES}}{\text{MD}}}.$$
 (3)

The system damping is not considered in the above theoretical derivation. When the system group damping is equal to 1 (critical damping) or greater than 1 (over damping), the vibration of the system is rapidly attenuated, and there is no vibration characteristic, and the vibration frequency of the perilous rock does not exist. Therefore, the model is not suitable for critically damped and overdamped perilous rock systems; when the damping ratio of the perilous rock is less than 1 (weak damping system), the vibration frequency of the damping system and the theoretically derived vibration frequency have the following relationship.

$$f_{\rm d} = f \sqrt{1 - \xi^2},\tag{4}$$

where ξ is the damping ratio of the system, dimensionless and f_d is the damped vibration frequency of the system (Hz).

By substituting Equation (4) into Equation (3), the natural vibration frequency relationship of the perilous rock under the condition of weak damping can be obtained as

$$f_d = \frac{\sqrt{1-\xi^2}}{2\pi} \sqrt{\frac{\text{ES}}{\text{MD}}}.$$
 (5)

3.3. Stability Evaluation Model Based on Natural Vibration Frequency. According to the limit equilibrium method, the safety factor of sliding-type perilous rock is Equation (6).

$$K_{\rm f} = \frac{\left(W\cos\alpha - \varepsilon W\sin\alpha - V\right)\tan\phi + cS}{W\sin\alpha + \varepsilon W\cos\alpha},\qquad(6)$$

Geofluids

Parameter Parameter ranges Parameter Parameter ranges Laser Wave length 1550 nm Velocity resolution $<0.5 \,\mu m/s/\sqrt{Hz}$ Output speed 0.4 mm/s-100 mm/s Bandwidth 0 Hz~25 kHz Resolution 7.5 mm@100 m Maximum distance >300 m



FIGURE 6: Results of the first monitoring.

where V is the fissure water pressure (kN/m); ε is the seismic horizontal coefficient, unitless; *c* is the cohesion force (kPa); φ is the friction angle (°); α is the inclination angle of the slip surface; *W* is the dead weight of the perilous rock; and *S* is the bonding area between the perilous rock and the base (m²).

Substitute Equation (5) into Equation (6) to obtain Equation (7).

$$K_{\rm f} = \frac{(W \cos \alpha - \varepsilon W \sin \alpha - V) \tan \phi}{W \sin \alpha + \varepsilon W \cos \alpha} + \frac{4\pi^2 f_d^2 {\rm MD} c}{(W \sin \alpha + \varepsilon W \cos \alpha) E \left(1 - \xi^2\right)}.$$
(7)

According to Equation (7), when the mass, dip angle, seismic horizontal coefficient, cohesion, internal friction angle, elastic modulus, and structural surface damping of sliding-type perilous rock are known, the safety factor of perilous rock can be calculated by monitoring the natural vibration frequency.

4. Stability Evaluation of Sliding-Type Perilous Rock on the Slope of Huangzangsi Dam Site

4.1. Selection of Perilous Rock. After excavation and support, the overall stability of the left-and right-side slopes of the dam site area is good, and there are empty or relatively loose rock and soil masses in some unsupported areas. Under the influence of construction or rainfall, the perilous rock in the above-mentioned areas may become unstable. The main unstable areas are the broken surface of the slope above No. 2 Road on the right bank of the dam site and the loose slope on the downstream side of the excavation area on the left bank. Select a typical sliding-type perilous rock in each of these two areas, use a remote laser vibrometer as a tool to monitor the natural vibration frequencies of these two perilous rocks, and then calculate the safety factor based on the theoretical model proposed in this paper, and then compare it with the current situation. Some limit equilibrium methods are used to calculate the safety factor of perilous rock, and the typical sliding-type perilous rock is shown in Figure 4.

4.2. Instrument Test. The instrument test is the RSV-150 remote laser vibrometer produced by Polytec. The RSV-150 laser vibrometer uses a dual-frequency laser vibrometer, which is different from the single-frequency laser vibrometer with strong resistance ability to interference and high signal-to-noise ratio. The RSV-150 remote laser vibrometer is shown in Figure 5, and the technical parameters are shown in Table 1.

4.3. Perilous Rocks Monitoring

4.3.1. Monitoring Process. Field engineering data collection: It mainly includes dip angle of main structural plane, size of perilous rocks, unit weight, thickness of structural plane, elastic modulus, cohesion, and internal friction angle.

Data acquisition: Remote laser vibrometer is arranged on site. Collect the dynamic characteristic data of each perilous rock, and measure each perilous rock at least three times.

Data analysis: The collected data shall be screened and the monitoring data with large amplitude shall be selected, so that the data signal-to-noise ratio is high. Fast Fourier

TABLE 1: Technical parameters of RSV-150 remote laser vibrometer.



FIGURE 7: Results of the second monitoring.



FIGURE 8: Results of the third monitoring.

transform is used to analyze the vibration frequency of Perilous rocks, and the average value of three monitoring results is taken. Based on the stability evaluation model of dangerous rock constructed in this paper, the safety factor of perilous rocks is calculated.

4.3.2. Monitoring Results. Place the remote laser vibrometer within 300 meters away from perilous rock. The computer connected to the remote laser vibrometer collects the time domain diagram of vibration. The frequency domain diagram is obtained by fast Fourier transform. From May 2019 to October 2021, natural vibration frequency was collected for three times at two monitoring points. The average damping ratio of perilous rock A is 0.0249 and B is 0.0588, so it is suitable for the dynamic characteristic model of perilous rock established in this paper. The results of the first monitoring are shown in Figure 6. The results of the second monitoring are shown in Figure 8.

It can be seen from Figure 6 that the natural vibration frequency of the first monitoring of perilous rock A is

8.22 Hz, and the natural vibration frequency of perilous rock B is 7.53 Hz.

It can be seen from Figure 7 that the natural vibration frequency of the second monitoring of perilous rock A is 8.20 Hz, and the natural vibration frequency of the second monitoring of perilous rock B is 7.51 Hz.

It can be seen from Figure 8 that the natural vibration frequency of the third monitoring of perilous rock A is 8.19 Hz, and the natural vibration frequency of the third monitoring of perilous rock B is 7.50 Hz. The natural vibration frequencies of perilous rock A and perilous rock B show a decreasing trend. The natural vibration frequencies of perilous rock A and perilous rock A and perilous rock Hz.

4.4. Stability Calculation. The perilous rock was scanned by a three-dimensional laser scanner, and parameters such as the size of the perilous rock, the thickness of the structural plane, the dip angle, and other parameters were analyzed according to the scanning results (Figure 9). Figure 10 is the field work diagram of the three-dimensional laser scanner.



FIGURE 9: 3D laser scanner scanning results.



FIGURE 10: 3D laser scanner working diagram.

By 3D laser scanner scanning, the perilous rock A is $6.08 \text{ m} \log 5.91 \text{ m}$ wide, 1.98 m thick, 71.15 m^3 in volume, 47.3° in inclination of the structural plane, 0.015 m thick in the structural plane, and 179290.30 kg in mass; perilous rock B is $4.88 \text{ m} \log$, the width is 8.02 m, the thickness is 3.54 m, the volume is 138.55 m^3 , the inclination angle of the structural plane and 179290.30 kg models.

ture surface is 38.5°, the thickness of the structure surface is 0.015 m, and the mass is 349138.70 kg. The peak acceleration of ground motion with a probability of exceeding 10% in 50 years in the dam site area is 0.19 g, and the corresponding basic earthquake intensity is VIII degree, with the seismic horizontal coefficient of 0.19. Referring to the survey and design data, the parameters of the perilous rock are shown in Table 2.

Substitute the natural vibration frequency and structural plane strength parameters into Equation (7) to calculate the safety factors of the two perilous rocks, as shown in Table 3.

From the Table 3, it can be seen that the natural vibration frequency is monotonically positively correlated with the safety factor of perilous rock. The natural vibration frequency of perilous rock A has reduced from 8.22 HZ to 8.19 HZ, and the safety factor from 2.43 to 2.41. The natural vibration frequency of perilous rock B has reduced from 7.53 HZ to 7.50 HZ, and the safety factor from 2.08 to 2.06. The calculation results show that under the influence of external environment such as rainfall, vibration, and ice melting, with the gradual deterioration of the structural plane of the perilous rock A and the perilous rock B, the strength decreases, the natural vibration frequency decreases, and the safety factor decreases. The safety factor is always greater than 2.0 and is in a steady state. According to the design data, using the limit equilibrium method, the safety factor of perilous rock A is 2.52, and the safety factor of perilous rock B is 2.12, which is close to the safety factor of perilous rock calculated in this paper, indicating that the method in this paper is correct and feasible. By monitoring the natural vibration frequency of perilous rock and calculating the safety factor in real time, the stability evaluation of perilous rock can be

TABLE 2: Parameter values.

| Perilous rock | Cohesion/kPa | Internal friction/(°) | Elastic modulus/GPa | Structural surface thickness/m | Sliding surface inclination/(°) |
|---------------|--------------|-----------------------|---------------------|--------------------------------|---------------------------------|
| А | 800 | 32.1 | 2.5 | 0.05 | 46.1 |
| В | 500 | 31.5 | 2.3 | 0.05 | 39.6 |

TABLE 3: Measurement results of natural vibration frequency of perilous rock.

| Perilous rock | 1st safety factor | 2nd safety factor | 3rd safety factor |
|------------------|-------------------|-------------------|-------------------|
| А | 2.43 | 2.42 | 2.41 |
| В | 2.08 | 2.07 | 2.06 |

realized quickly in real time. The research has high theoretical significance and application value for the stability evaluation and safety monitoring and early warning of sliding perilous rock.

5. Conclusion

Based on the dynamic theory, a quantitative relationship model between the natural vibration frequency, bonding area, elastic modulus, and mass of the sliding-type perilous rock was established. And the limit equilibrium model was introduced to perform the stability evaluation of the sliding-type perilous rock based on the natural vibration frequency. The problem of inconsistency between the stability evaluation index of perilous rock and the monitoring index of perilous rock has been solved. This model can calculate the safety factor in real-time by monitoring the natural vibration frequency of perilous rock. Furthermore, the stability evaluation of perilous rock in real time can be determined quickly. This technical has strong research value and publicical significance.

The remote laser vibrometer is used to monitor the natural vibration frequency of the sliding-type perilous rock on the slope of the Huangzangsi Hydrojunction, and the stability evaluation of the perilous rock is achieved based on the quantitative relationship model between the safety factor and the natural vibration frequency. Compared with the safety factor calculated by the limit equilibrium method, the results are basically the same, indicating that the method is correct and feasible. And the related research results can provide significant guidance for the safety monitoring and advanced warning of sliding perilous rock.

Data Availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that the publication of this paper has no conflicts of interest.

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