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Research Article

Research on Nonroutine Problems: A Hybrid Didactical Design for Overcoming Student Learning Obstacles

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The multiplication of fractions was generally considered easy, but it became challenging when presented as a nonroutine problem. One solution to these challenges was integrating technology into learning. Therefore, the study is aimed at proposing a hybrid module as a solution. The design in this research was a didactical design research. The study involved 56 participants, aged 13–18 years, mainly female students from the Sasak tribe in an Indonesian junior high school. Researchers employed fraction comprehension tests, in-depth interviews, and a hybrid hypothesis module as primary instruments. For learning outcomes, NVivo-12-assisted thematic analysis was used, and for learning obstacles, retrospective-based qualitative analysis was used. The study findings revealed several factors that caused learning obstacles, including conceptual ontogenic, epistemological factors, and students' restricted Internet access. The hybrid hypothesis module proved effective in assisting students by providing additional problem contexts and scaffolding, addressing previous learning obstacles. After implementation, there was a redesign of the hybrid module in the form of additional scaffolding to help students construct the concepts studied. This research concluded that the application of technology in the form of hybrid modules was able to minimize barriers to student learning.

1. Introduction

Several previous studies [1–3] reveal that fractions play a crucial role in mathematics, as well as in other scientific disciplines [4, 5], and everyday life [6]. However, fractions often pose challenges for students, such as students' limited concepts in understanding fractions, students' low ability to operate fractions [1–3], and teachers that tend not to integrate technology and nonroutine problems in learning [6], including at the junior high school level [7]. While significant research exists on fractions in elementary schools [1, 2], limited attention has been given to understanding the factors that contribute to learning obstacles in solving nonroutine problems involving fraction multiplication and the potential of hybrid modules as alternative solutions, particularly in junior high schools and in the context of didactical design research (DDR) post-COVID-19 pandemic.

Research conducted in Jakarta, Indonesia [8], explores elementary school students' knowledge and constraints when learning multiplication of fractions. Using a qualitative approach, the study reveals that students have a limited understanding of fractions, which affects their interpretation of the context in fraction multiplication problems. Another study in Bali, Indonesia [9], examines elementary school teachers' understanding of the fraction concept. Through qualitative case studies, it is found that teachers have a limited comprehension of fractions as parts of a whole. Similarly, a study in the USA [10] investigates teachers' understanding of fractions using a survey, and the findings indicate a limited grasp of arithmetic involving fractions among math teachers.

In contrast to previous research, the present study is aimed at identifying the factors that contribute to students' learning obstacles and developing hybrid modules as an alternative solution to mitigate these obstacles. The research design employs DDR, which aligns with the interpretive paradigm [11] to examine the factors causing learning obstacles and the critical paradigm [12] to propose hypothetical hybrid modules as alternative solutions. To achieve the research objectives, several research questions are formulated, including the following:

(a) What are the factors that contribute to students' learning obstacles in solving nonroutine problems related to fraction multiplication?

(b) How can a hypothetical hybrid module be described to minimize learning obstacles for students solving nonroutine problems involving fraction multiplication?

(c) What is the implementation of the hypothetical hybrid module, and how does it minimize learning obstacles for students when solving nonroutine problems related to fraction multiplication?

(d) How can the revised hypothetical hybrid module be described in minimizing learning obstacles for students when solving nonroutine problems related to fraction multiplication?

2. Materials and Methods

2.1. Literature Review. Mathematics learning facilitated the formation of ways of thinking and understanding [11–13]. Ideally, mathematics learning should have presented problems as situations that helped students construct mathematical concepts or procedures [14]. However, mathematics learning in schools often fell short. Students sometimes faced learning obstacles [15, 16], which hindered the optimal development of their competencies. Based on external factors causing learning obstacles, these obstacles could be categorized into three types [16, 17]. Firstly, didactic obstacles [7, 11] were learning obstacles that students experienced due to teachers' limited understanding in knowledge transposition. Secondly, epistemological obstacles [12, 18] occurred due to limited context or scaffolding used by mathematics teachers during learning. Thirdly, ontogenetic obstacles [14, 16] were learning obstacles that arose when students were unprepared to engage in the learning process. There were three types of ontogenetic obstacles: conceptual, psychological, and instrumental [11, 16]. Conceptual ontogenetic obstacles occurred when students had limited understanding of prerequisite concepts or lacked relevant experiences. Instrumental ontogenetic obstacles were learning obstacles that arose from technical or operational unpreparedness during learning. Psychological ontogenetic obstacles referred to learning obstacles caused by uninspiring mathematics instruction that failed to capture students' interest, motivation, or enthusiasm for learning [11, 12, 16].

Several approaches could be used to minimize learning obstacles, one of which was designing learning experiences that were tailored to the factors causing students to experience learning obstacles [19]. Such designs were often referred to as didactic designs [11, 12]. In this research, didactic design referred to the use of learning modules. These learning modules should have been flexible [20, 21], capable of adapting to various school situations. Furthermore, these modules could have been transformed into hybrid modules [22, 23], which could have been implemented in different learning modes, such as online learning and home visits [23, 24]. In this study, the hybrid modules followed the flow of didactic situations [12, 25], which consisted of four stages: action, formulation, validation, and institutionalization [14, 25]. The action-formulation situation occurred when students responded to a problem by mentally engaging with it. The final condition of the action-formulation situation was reached when students found a solution to the problem. The validation situation was when students connected the previous problem-solving process or solution to underlying mathematical concepts or procedures. The outcome of this situation was the discovery of axiomatic-formal mathematical concepts or procedures. The institutionalization situation was when students used the discovered mathematical concepts or procedures to solve problems in different contexts [14, 16].

2.2. Research Design. This research adopted a qualitative approach with DDR [14]. DDR was used for two reasons. Firstly, the interpretive paradigm [11] within DDR addressed the research questions related to the factors causing students to experience learning obstacles in the post-COVID-19 pandemic context. Secondly, the critical paradigm [12] within DDR was used to answer research questions related to the implementation of hybrid mathematics learning modules during the post-COVID-19 pandemic period. The research procedure followed the steps of DDR, including prospective analysis, metapedadidactic analysis, and retrospective analysis [14]. In the prospective analysis, the research is aimed at investigating the factors causing students to experience learning obstacles in mathematics learning during the post-COVID-19 pandemic period. The outcome of this analysis was the hypothetical hybrid module. The metapedadidactic analysis involved implementing the hypothetical hybrid module in mathematics learning in schools during the post-COVID-19 pandemic. Finally, the retrospective analysis is aimed at reflecting on the implementation results to improve the hypothetical hybrid module. The hypothetical hybrid module that had gone through this analysis was referred to as the empirical hybrid module [11, 12]. For more details regarding research procedures, you could have referred to Figure 1.

2.3. Participants. The participants in this study were 56 students from a junior high school in Indonesia. The school was chosen as it was considered exemplary but faced challenges in teaching fractions. Meanwhile, we selected the 56 students from a group of students who came from classes with an average mathematics score below the minimum completeness criteria. If this research yielded significant results for the participants, it could be inferred that it would also have a significant impact on other students at the school. Therefore, the 56 students were considered representative of the school as participants in this research. The participants were

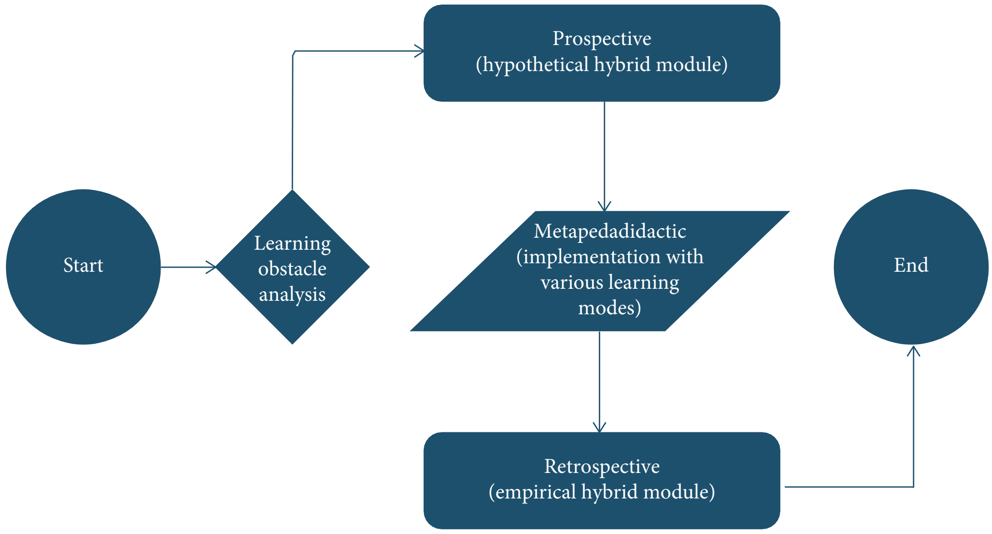


Figure 1: Research procedure.

divided into two groups. The first group consisted of 29 students aged 14 to 18, predominantly female, with parents working as entrepreneurs, and belonging to the Sasak tribe. This group was chosen because they had studied fractions. The second group included 27 students aged 13 to 15, predominantly female, with parents working as entrepreneurs, and also belonging to the Sasak tribe. This group was chosen because they will study fractions. Another participant in this research was a 26-year-old mathematics teacher with approximately 8 years of teaching experience.

2.4. Instrument and Data Collection Procedure. The main instruments used in this research were the researcher [26, 27], along with additional instruments such as fraction comprehension tests, in-depth interview guidelines [28, 29], and the hypothetical hybrid module. The fraction comprehension tests and in-depth interview guidelines were used to collect data related to the factors causing students to experience learning obstacles. The hypothetical hybrid module was used to collect data regarding the implementation of learning during the post-COVID-19 pandemic. These instruments underwent content validity testing by seven experts. The CVR test results [30] yielded a CVR value of 1, indicating the essentiality of the hypothetical hybrid module for learning.

2.5. Data Analysis. Thematic analysis [31, 32] supported by NVivo-12 [33] was employed to analyze data related to the factors causing students to experience learning obstacles. The analysis involved several stages, including familiarizing with the data (reading the data multiple times), initial coding, identifying themes based on similar characteristics of initial codes, reviewing themes, and naming themes [32, 34]. For example, we grouped students who could not perform multiplication or division of whole numbers correctly (initial code) into one theme, such as prerequisite abilities. These two operations were classified as basic mathematical operations that served as prerequisite material for almost all mathematical concepts in school. Qualitative data analysis [35] combined with retrospective data analysis [11] was used to analyze data related to the implementation of the hypothetical hybrid module. The qualitative data analysis consisted of three stages: data reduction, presentation of the reduced data in various forms, and drawing conclusions regarding the implementation results of the hypothetical hybrid module [35]. Triangulation of sources and data collection methods was conducted to strengthen the findings of this research [36]. The research adhered to ethical guidelines by obtaining informed consent from the parents of the students before conducting interviews and maintaining anonymity and confidentiality in reporting the data [29, 37].

Table 1: Description of learning barrier themes.

Theme code	Description
T-1	Students cannot get the prerequisite material.
T-2	Students cannot make mathematical models.
T-3	Students do not know the procedure or formula for multiplying fractions.

3. Results and Discussion

3.1. Results

3.1.1. What Are the Factors That Contribute to Students' Learning Obstacles in Solving Nonroutine Problems Related to Fraction Multiplication? Based on the results of the thematic analysis, information was obtained that two themes were formed when students solved nonroutine problems involving the multiplication of fractions. A description of the three themes can be seen in Table 1. The prerequisite material in T-1 was related to the multiplication of integers. In other words, students' limitations in performing integer operations were prerequisite materials that caused students to experience learning obstacles.

Table 2: Snippet of the answers from teacher interviews.

Researcher's questions	Teacher's answers
Do you ever start learning by giving non-routine problems to students?	*Yes, sir, but the average student gets confused. They do not understand how. So, let me first explain how, but not all students understand. There are also some students who, maybe because of story questions, immediately say they do not understand when given story questions. In fact, they need to first figure out what they know and what they are being asked.*
In your opinion, does the learning you do by telling students the formula directly, without asking them to construct it themselves, have an impact on the students?	*Yes, sir, it does. Students tend to quickly forget when formulas are directly given. They usually do not remember the formulas taught in the previous class.*

Table 3: Alternative solutions to overcome learning obstacles.

Learning obstacles	Alternative solutions
The low prerequisite skills of students (integer operations).	Designing *Let us Guess* LCM and GCD value activities to strengthen students' ability to operate integers. Only a few multiples of a number are written down, so students are expected to find the value of other multiples of that number themselves. This activity is expected to strengthen students' ability to operate whole numbers. For students who forget how to find LCM and GCD, scaffolding is provided in the form of multiples of numbers and factor trees of a number so that students can easily find the LCM and GCD values.
Limitations of the problem context given by the teacher.	Designing the *Let us Find* activity consisting of nonroutine problems to facilitate students in discovering the procedure for multiplying fractions. This activity is expected to be able to make students remember the multiplication procedure for a long time.
Limited Internet quota that students have.	Providing Internet quota assistance to students.

After an in-depth analysis of T-2, information was obtained during the interview that students did not understand the meaning of the questions, so they were confused when making appropriate mathematical models. From the results of these interviews, information was also obtained that students rarely encountered word problems during learning. Students revealed that math teachers tended to directly provide formulas, sample questions, and practice questions to students. The sample questions and practice questions given also tended to be the routine problems. This result was also confirmed by the teacher during the interview with the teacher. Snippets of the interview results with teachers can be seen in Table 2.

In relation to T-3, the students also mentioned that they had forgotten the formula for multiplying fractions. The students really did not remember the formula. This result could be attributed to the fact that the lessons given by the teacher tended not to be memorable, causing the students to quickly forget. These results were in line with the answers provided by the teacher during the interview, as shown in Table 2. Based on the interview results, it was also discovered that the students had experienced learning obstacles during the post-COVID-19 period due to the limited Internet quota they had. Based on the previous description, it can be concluded that there were three factors that caused the students to experience learning obstacles in solving nonroutine problems involving fractions during the post-COVID-19 pandemic. These factors were the low prerequisite abilities of the students (integer operations), the limited problem contexts provided by the teacher in learning fractions, and the limited Internet quota that the students had.

3.1.2. How Can a Hypothetical Hybrid Module Be Described to Minimize Learning Obstacles for Students Solving Nonroutine Problems Involving Fraction Multiplication? Based on the learning obstacles students experienced before, several solutions had to be included in the hybrid module. The full description could be seen in Table 3. Based on Table 3, not all alternative solutions could be included in the hybrid module, such as obstacles related to Internet quota. This solution was not included in the hybrid module because it is related to technical matters that supported the implementation of the learning process during the post-COVID-19 pandemic. Alternative solutions related to prerequisite material and problem context could be seen in Figures 2 and 3, respectively. The complete description regarding the hybrid module could be accessed on the following page: https://shorturl.at/kRW37.

3.1.3. What Is the Implementation of the Hypothetical Hybrid Module, and How Does It Minimize Learning Obstacles for Students when Solving Nonroutine Problems Related to Fraction Multiplication?

(1) Online Learning Mode (Zoom Meeting). In the *Let us Guess* activity, students were able to correctly guess the LCM values of 4 and 6. The students revealed that the LCM values of the two numbers were 12. Similarly, for the

Activity 1 30 Minutes

Prerequisite Material 15 Minutes

Instruction:
Don't forget, prepare a notebook or worksheet and other writing tools, such as colored pencils, erasers, rulers and scissors. Do all activities carefully, manage your time well, and check your answers again when finished. Write your answers in your notebook or worksheet!

Example answer: the LCM value of 4 and 6 is 12.

Let's Guess LCM

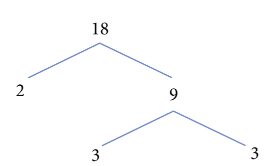
Look at several rows of multiples of numbers below.
Multiples of 4 are : 4, 8, ...
Multiples of 6 are : 6, 12, ...
Multiples of 12 are : 12, 24, ...

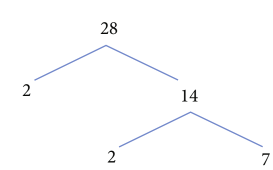
Answer the questions below by filling in the columns provided.

What is the LCM of 4 and 6?	
What is the LCM of 4, 6, and 12?	

Let's Guess GCD

Look at the factor tree below.





Answer the questions below by filling in the columns provided.

What is the FPB value of 18 and 28?	

Figure 2: Let us Guess activity.

LCM values of 4, 6, and 12, the students were able to answer correctly that the LCM values of the three numbers were 12. However, when guessing the GCD values of 18 and 28, students seemed to experience obstacles. Some students revealed that the GCD values of the two numbers were 14, while others answered 2. After the researcher gave several trigger questions, such as "are there the same factors between 18 and 28 in the factor tree?" and "what is the value of this factor?", finally, students found that the correct GCD value for 18 and 28 was 2. When working with the

Activity 2 30 Minutes

Action Situation 15 Minutes

Instruction :
Don't forget, preparea notebook or worksheet and other writing tools, such as colored pencils, erasers, rulers and scissors. Do all activities carefully, manage your time well, and check your answers again when finished. Write your answers in your notebook or worksheet!

Example answer : The food that Dad usually brings is *martabak*.

Problem

Who is one example of the most instrumental person in your life? Yes, the answer is Dad. Father is the head of the household and is the best leader in life. My father is the best father. Every time he comes home from work, Dad always brings food.

Now, look for the food in your house that your father usually brings. If there isn't any, then take whatever food is in your house. Don't forget to ask your parents' permission. After that, divide the food into several parts and eat one portion. Next, pay attention to the remaining parts that have not been eaten and ask other members of your family to eat some parts. What fraction represents the portion that your family members eat, compared to the original portion?

Figure 3: Let us Find activity.

illustration model, students were also able to pair the illustration model with the appropriate fraction. In fact, the students were able to provide correct explanations.

After ensuring that the students understood the prerequisite material well, the learning activities continued with *Let us Read*. In this activity, the students were able to read motivational stories well and express the ideas conveyed from these stories. The students revealed that the concept of fractions teaches a person to share with others according to their ability, not necessarily with money. For example, when a natural disaster occurs, someone who does not have

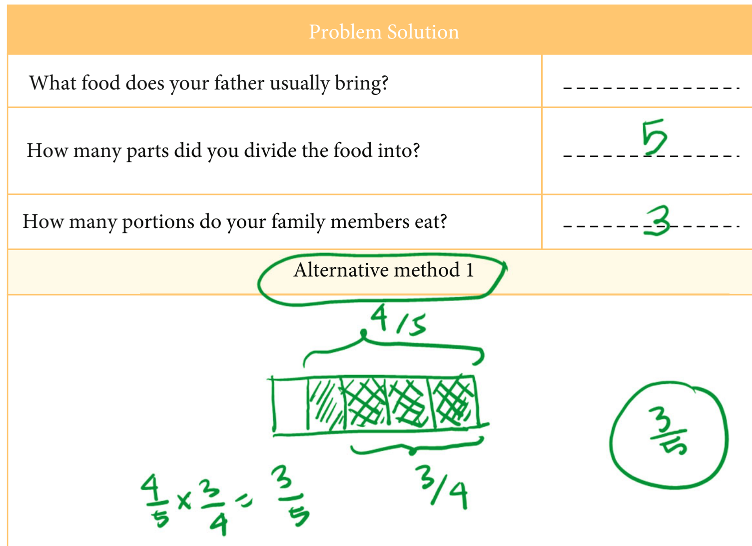


FIGURE 4: Snippet of group 2's answers to the *Let us Find* activity.

money can help by building a public kitchen. The next activity was *Let us Find*, where the students entered the breakout room (BOR) divided into two BORs. The students also had fruitful discussions in the BOR. After the allotted time was over, the learning activities continued with *Let us Tell Stories*. In this activity, group 2 presented their work first, while group 1 did not present their work as they had not finished solving the problem. A snippet of group 2's answers can be seen in Figure 4. Figure 4 provides information that group 2 divided the food into 5 parts and gave 3 parts to their family members. Therefore, group 2 concluded that the number of portions eaten by family members compared to the total portion was 3/5.

The next activity was *Let us Summarize*. In this activity, students were not able to conclude the multiplication fraction formula correctly. The researcher then gave a number of trigger questions so that students could find that the general formula for multiplying fractions was $(a/b) \times (c/d) = (a \times c)/(b \times d)$, where a, b, c, and d were integers and b and d were not equal to 0. After a while, the learning activities continued with *Let us Practice*. In this activity, students were able to solve problems together with the teacher. Snippets of the student answers can be seen in Figure 5. Figure 4 provides information that the surface area of a fish pond with a length of 45/4 m and a width of 16/3 m was not 70 m^2, but 60 m^2.

In the *Reflection* activity, students seemed to be able to perform all the activities correctly. They expressed their enjoyment of learning during the session and showed commitment to giving alms and assisting their mothers with homework. The complete story for this implementation can be accessed on the following page: https://www.youtube.com/watch?v=d2upy2x3yWQ.

(2) Home Visit Learning Mode. Three individuals participated in the learning activities using this mode because they did not have smartphones. In the *Let us Guess* activity, students were able to correctly guess the LCM score, although at times they appeared doubtful. The researcher provided reinforcement that the LCM score was related to the smallest multiple. Similarly, when determining the value of the GCD, students made mistakes initially. With some reinforcement from the researcher, the students were finally able to determine that the GCD value of 18 and 28 was 2. In the subsequent activity, students were able to match the illustration model with the correct fraction form.

The next activity was *Let us Read*. In this activity, students read carefully but struggled to express the essence of the motivational story. The researcher provided reinforcement that fractions teach about the principle of sharing according to one's abilities or profession. After completing the activity, the learning continued with *Let us Find* activity. Students were unable to solve the problems independently, so the researchers provided scaffolding to help them find solutions, specifically 2/4. The *Let us Tell Stories* activity was not conducted separately as it overlapped with the *Let us Find* activity. In the *Let us Summarize* activity, students, along with the researcher, concluded the general formula for multiplying fractions. In the *Let us Practice* activity, students were able to solve both problems correctly, as well as in the *Reflection* activities. It appeared that students were able to perform all the activities correctly.

3.1.4. How Can the Revised Hypothetical Hybrid Module Be Described in Minimizing Learning Obstacles for Students when Solving Nonroutine Problems Related to Fraction Multiplication? Based on the previous description, students did not encounter significant obstacles during the implementation of the hybrid module, both for students who used online learning modes and home visits. However, there was one significant obstacle, which was the *Let us Summarize* activity. In this activity, students were unable to find the multiplication fraction formula correctly. The revisions related to these activities can be seen in Table 4.

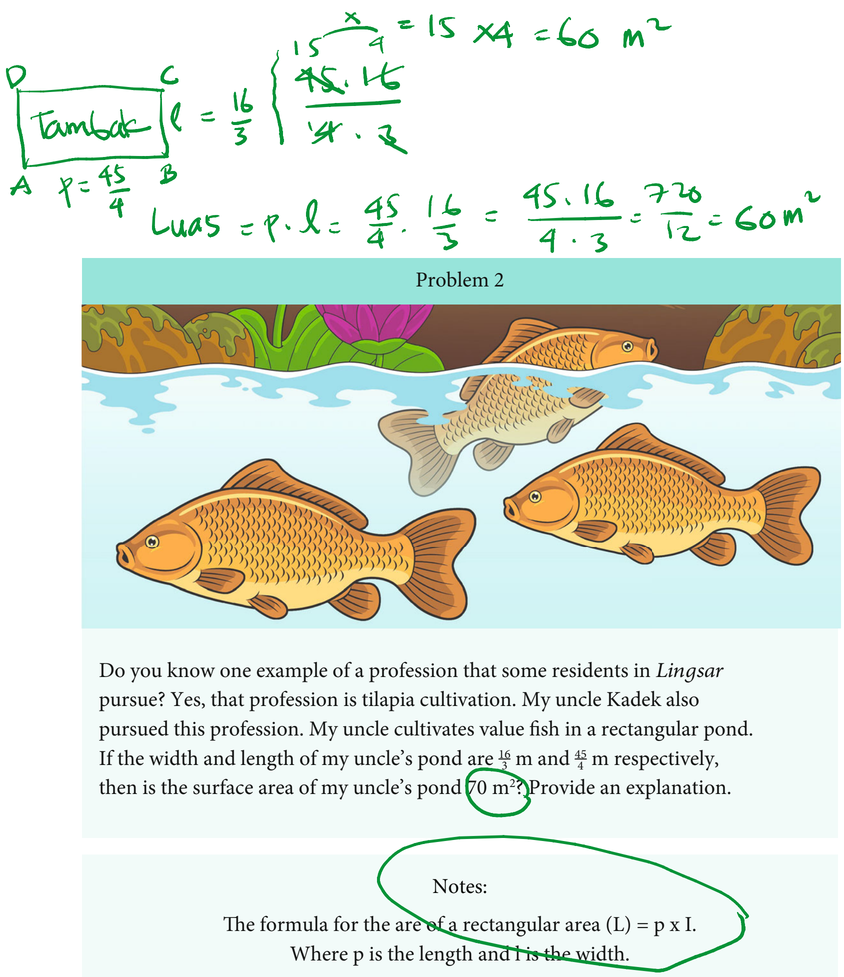


Figure 5: Snippets of the student answers in *Let us Practice* activities.

3.2. Discussion

3.2.1. What Are the Factors That Contribute to Students' Learning Obstacles in Solving Nonroutine Problems Related to Fraction Multiplication? When associated with the theory of learning obstacles [16, 25], we can conclude that students are indicated to experience two types of learning obstacles. Firstly, there are conceptual ontogenic obstacles caused by the lack of prerequisite skills (operating on integers) among students. A conceptual ontogenic obstacle is a learning obstacle caused by students' unpreparedness in terms of the lack of prerequisite concepts they possess and the need to learn a new concept [16]. Secondly, there are epistemological obstacles arising from the limited context used by mathematics teachers in their instruction. The results of this study align with several previous theories and studies [38–40] that highlight the low ability to operate integers as one of the factors contributing to students' difficulties in learning mathematics. Similarly, the limited context provided by teachers is supported by various theories and prior research [7, 16], confirming that restricted movement or limited student activity optimization, such as implementing a conventional approach, contributes to students facing challenges in mathematics learning.

In terms of limited Internet quota, this study concludes that it falls under the category of instrumental ontogenic obstacles. This is because it affects the implementation process of learning, especially during the post-COVID-19 pandemic [11, 16]. The findings of this study are consistent with several previous studies [41–43] that reveal the obstacle students face in accessing online learning due to limited Internet quota, both during and after the COVID-19 pandemic. However, the solution to this obstacle is not included in the hybrid module as it pertains to the operational implementation of learning rather than the content or didactic aspects of learning.

3.2.2. How Can a Hypothetical Hybrid Module Be Described to Minimize Learning Obstacles for Students Solving Nonroutine Problems Involving Fraction Multiplication? There are at least three types of factors causing students to experience learning obstacles: low prerequisite skills in operating integers, limited contexts used by teachers in learning, and limited Internet quota for students. The best solution to a problem is one that is tailored to the specific factors causing the problem. Therefore, the hybrid module offers alternative solutions, as shown in Table 3. To address the issue of low ability in operating integers, the hybrid module

Table 4: Revision of the hybrid module for the *Let us Summarize* activity.

Design revision	
Based on the previous activity, the multiplication of the two fractions above is equal to the fraction that represents the part your family members eat (compared to the number of original parts). So, you have $\frac{\cdots}{\cdots} \times \frac{\cdots}{\cdots} = \frac{\cdots}{\cdots}$	To fill in the blanks in the fraction below, follow these conditions: the quantifier for the denominator should be the same as the multiplication of the quantifier of the two fractions, and the denominator of the fraction should be the product of the denominators of the two fractions above. This way, you will get $\frac{\cdots}{\cdots} \times \frac{\cdots}{\cdots} = \frac{\cdots\times\cdots}{\cdots\times\cdots} = \frac{\cdots}{\cdots}$
Is the end result of the two multiplication fractions above the same?	Yes/no
Therefore, you can conclude that $\frac{a}{b} \times \frac{c}{d} = \frac{a\times\cdots}{\cdots\times d}$ where a, b, c, and d are integers and b and d are not equal to 0.	

provides early learning activities where students determine the LCM and GCD values of several integers. This helps train their ability to perform multiplication and division operations on integers [44].

Additionally, the hybrid module incorporates problem-based learning. In this approach, the problem presented in the design serves as a situation that facilitates students in discovering concepts and formulas related to the multiplication of fractions. This solution is supported by various theories and previous studies [45–47], indicating that problem-based learning can optimize students' mathematical abilities and reduce learning obstacles, particularly those related to epistemological obstacles.

The hybrid module is organized into three main activities: introduction, core activities, and closing. This structure is based on the theory that advocates for learning design to be arranged as a cohesive unit, comprising preparatory, lecture, and evaluation stages [48, 49]. The preparatory activities include *Let us Guess* and *Let us Read*. The lecture activities involve *Let us Find*, *Let us Tell Stories*, and *Let us Summarize*. The evaluation activities encompass *Let us Practice* and *My Reflection*. *Let us Guess* ensures students have the prerequisite abilities [38, 50]. *Let us Read* is aimed at stimulating students' interest and motivation in learning mathematics through motivational stories related to fractions [45, 46]. *Let us Find* facilitates students in discovering the concept or formula of multiplying fractions through problem-solving activities [47, 51, 52]. Apart from that, students are also facilitated with various illustrative models, such as area models, number lines, and collections of objects, to make it easier to solve problems. *Let us Tell Stories* helps students develop an understanding of the problem-solving process [53, 54]. *Let us Summarize* guides students in summarizing the concepts or procedures for multiplying fractions they have discovered [54, 55]. *Let us Practice* reinforces the concept or procedure by engaging students in solving nonroutine problems in different contexts [14]. *My Reflections* support students in conducting self-assessment, reflecting on the concept or procedure for multiplying fractions, their learning experience, and the character developed throughout the activities [56, 57].

The organization of the hybrid modules follows the theory of didactical situations [25]. It incorporates the flow of action-formulation situations, validation, and institutionalization [14]. The action-formulation situation corresponds to the *Let us Find* activity. Action-formulation situations are situations where students carry out mental actions and hands-on activities to find and implement strategies for solving nonroutine problems. Therefore, this situation is termed a *Let us Find* activity. The validation situation encompasses the *Let us Tell Stories* and *Let us Summarize* activities. A validation situation is a situation where the process and solutions that students obtain are presented or told in front of the class and then given responses by other students. The final result of this activity is that students agree to conclude the concept or formula found in the previous activity. The institutionalization situation relates to the *Let us Practice* activities [11, 12].

3.2.3. What Is the Implementation of the Hypothetical Hybrid Module, and How Does It Minimize Learning Obstacles for Students when Solving Nonroutine Problems Related to Fraction Multiplication? As previously described, hybrid modules are used because hybrid-based learning develops students' motivation, interest, and independence in learning [23, 58]. During the implementation, there are no significant obstacles that students experience, both for students who use the online learning mode and home visits. This is because the prerequisite skills and students' ability to operate ICT are quite good. In fact, students can solve problems in the *Let us Find* activity without using help or scaffolding included in the hybrid module. The results of this study are then aligned with the theory and several previous studies [59, 60], which reveal that good prerequisite skills and digital literacy have a fairly positive impact on student activities during learning.

Regarding the constraints that students experience in the *Let us Summarize* activity, the results of this study are consistent with several previous theories and studies [16, 61], which reveal that one of the obstacles students face in student-centered learning is difficulty in making connections between the learning process and problem-solving with the concepts or procedures they find. Usually, teachers' use of light questions or inappropriate scaffolding is to blame for this. The final result of this research is that the implementation of the hybrid module is able to minimize barriers to student learning. This is because the hybrid module integrates technology [22, 23] to facilitate students in discovering concepts through various didactic situations [12, 25]. These results are also in line with several previous studies [11, 12], which revealed that didactic design was able to minimize barriers to student learning.

3.2.4. How Can the Revised Hypothetical Hybrid Module Be Described in Minimizing Learning Obstacles for Students when Solving Nonroutine Problems Related to Fraction Multiplication? There are no significant revisions made to the hybrid module because the design is essentially developed based on the factors that cause students to experience learning obstacles. The only significant revision is found in the *Let us Summarize* activity. The revision is limited to replacing scaffolding or lighter questions with more relevant ones. This revision is based on previous theories and research that reveal how relevant scaffolding can facilitate students in understanding mathematical concepts or procedures. Additionally, this revision is also in line with several previous theories and studies [11, 57] that emphasize the importance of continuous improvement in learning design as a way to apply the principles of assessment in learning. It is hoped that this revision contributes to helping teachers teach multiplication of fractions by not only using one illustration model but various forms of illustration models, such as area models, number lines, and collections of objects.

4. Conclusion

Some factors causing students to experience learning obstacles in learning multiplication fractions are the low ability of students to perform multiplication and division of integer operations (conceptual ontogenic obstacle), limited context problems used by teachers in learning (epistemological obstacle), and limited Internet quota available to students (instrumental ontogenic obstacle). A solution offered to minimize learning obstacles is a hypothetical hybrid module. The design consists of activities that aim to strengthen students' abilities regarding multiplication and division operations on integers by determining the value of the LCM and GCD. Additionally, the hybrid module utilizes problem-based situations to facilitate students in finding multiplication fraction formulas. However, regarding quota limitations, the solution to these obstacles is not incorporated in the design; instead, researchers provide a solution by purchasing Internet quota for the students.

The hypothetical hybrid module comprises three activities: activity 1 includes *Let us Guess* and *Let us Read*; activity 2 includes *Let us Find*, *Let us Tell Stories*, and *Let us Summarize*; and activity 3 consists of *Let us Practice* and *My Reflection*. The hybrid module is based on action-formulation situations (*Let us Find*), validation (*Let us Tell Stories* and *Let us Summarize*), and institutionalization (*Let us Practice*). Based on the implementation results, no significant obstacles were encountered during the implementation of the hypothetical hybrid module. The researcher only identified one obstacle, which is the *Let us Summarize* activity. The scaffolding or trigger questions included in the design were found to be inappropriate. Following the implementation, the hypothetical hybrid module was revised by adding several triggering questions and referred to as the empirical hybrid module. The empirical hybrid module was also referred to as an epistemic learning pattern, because it facilitates students in constructing knowledge (mathematical concepts or formulas) through physical and mental activities during learning. Finally, this research concludes that hybrid modules that integrate technology are able to minimize barriers to student learning at school.

The empirical hybrid module is expected to serve as an alternative for learning multiplication fractions, particularly in the post-COVID-19 pandemic era. Mathematics teachers are encouraged to utilize various didactic situations [25] based on problems [47] as an alternative to minimize student learning obstacles, especially in the context of learning multiplication fractions. This research has limitations in that there is no inferential statistical evidence regarding the influence of the hybrid module on students' mathematical abilities. Therefore, this research recommends that future studies quantitatively examine the effectiveness of the empirical hybrid module on students' mathematical competence, particularly their mathematical digital literacy abilities.

Data Availability

The data can be accessed from the following link: https://shorturl.at/kRW37https://www.youtube.com/watch?v=d2upy2x3yWQ.

Conflicts of Interest

The authors declare no potential conflicts of interest regarding the research, authorship, or publication of this article.

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