

Research Article

A Robust Sidelobe Cancellation Algorithm Based on Beamforming Vector Norm Constraint

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Sidelobe cancellation (SLC) is a well-established beamforming technique for mitigating interference, particularly in the context of satellite communication (SATCOM). However, traditional SLC suffers from the issue of partially canceling the desired signal at high signal-to-noise ratio (SNR), primarily due to unconstrained beamforming processing. Extensive research has been conducted to address this problem; however, existing algorithms have limitations such as dependence on knowledge of signal array vectors or number of interferers and involve high computational complexity. In this paper, we propose a robust SLC algorithm based on beamforming vector norm constraint. Our proposal offers a practical solution by only requiring knowledge of the earth station antenna gain and maximum auxiliary array gain to the desired signal, both of which are fully known. Furthermore, compared to traditional SLC, our proposed method introduces additional computational complexity that only scales linearly with the size of the auxiliary array. Simulation results demonstrate comparable performance between our proposed method and existing techniques such as diagonal loading and spatial degrees-of-freedom control-based algorithms.

1. Introduction

Sidelobe cancellation (SLC) is widely employed to suppress interference in radio systems equipped with high-gain antennas [1, 2], particularly in the field of satellite communication (SATCOM) [3, 4], as well as radar [5, 6]. An SLC system typically consists of a high-gain antenna (the main antenna) for receiving the desired signal, and an auxiliary array with low-gain antenna elements for receiving interference signals. The beamforming algorithm is simple, i.e., minimizing the average output signal power by linearly weighting and combining the received signals of the auxiliary array and then subtracting from the received signal of the main antenna [7]. An intrinsic drawback of the traditional beamforming algorithm is that the desired signal is partially canceled when the received signal-to-noise ratio (SNR) is high. This problem is known as the desired signal cancellation problem [1, 8].

Multiple methodologies have been proposed in the existing literature to address the issue of desired signal cancellation. The first method involves using a blocking matrix to suppress the desired signal in the auxiliary array output before applying SLC beamforming [1]. However, obtaining

knowledge of the angle or array vector of the desired signal is challenging or even impossible when interference signals are present. Nonetheless, if the array vector of the desired signal is known, the desired signal received by the auxiliary array can be eliminated to prevent signal cancellation. The second method is diagonal loading (DL) methodology [9], i.e., adding a proper DL factor to the autocovariance matrix of the auxiliary array output signal. DL is a popular and effective method. Despite that the optimization of the DL factor remains a well-known challenge with no universally available simple solution [10–12]. These methods are typically limited to uniform one-/two-dimensional arrays in order to take advantage of the array geometry, e.g., [13]. The third method is to control the number of spatial degrees-of-freedom (DoF), specially, by reducing the auxiliary array signal dimension down to exactly the number of interferers. The dimension reduction procedure is typically based on principal component analysis (PCA) [8]. If the number of interferers is known exactly, DoF control can also achieve great performance. However, estimating the number of interferers is not an easy task especially for unknown signals [14]. Some algorithms, e.g., [7, 15], rely on Gram–Schmidt (GS) orthogonalization to control the DoF.

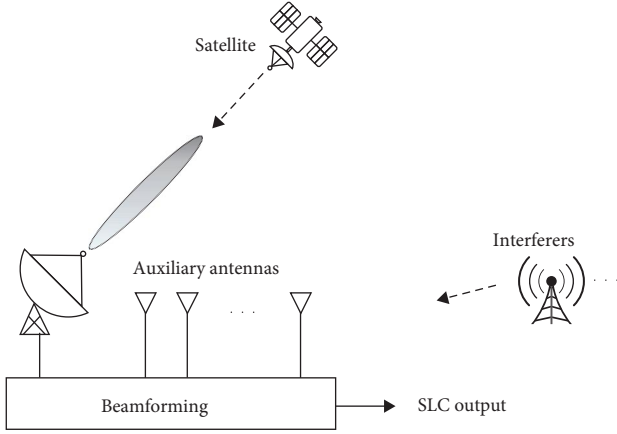


FIGURE 1: Illustration of SLC system.

However, this technique is limited to colocated uniform auxiliary array for which the fixed orthogonalization order does not impact performance. Another method is based on the concept of worst-case performance optimization, e.g., [16], which is also proposed for uniform arrays. However, it is associated with the drawback of high computational complexity due to involvement of convex/nonconvex optimization procedures.

In this study, we focus on an SLC system depicted in Figure 1. However, the auxiliary array geometry is not limited to any specific configuration. Our objective is to develop a beamforming algorithm that possesses the following properties: (1) not limited to uniform auxiliary arrays, (2) does not require channel knowledge or angles of either the signal-of-interest (SOI) or interference signals, and (3) has low computational complexity. To achieve this goal, we propose a robust SLC algorithm based on constraining the norm of the beamforming vector. Our proposal is motivated by the observation that in high SNR regimes, the norm of the beamforming vector primarily depends on and scales monotonically with SNR while being approximately independent of interference-to-noise ratio (INR). Therefore, our hypothesis is that constraining the norm of the beamforming vector may mitigate the signal cancellation effect without degrading interference cancellation performance.

The contributions of this paper are as follows:

- (1) We present mathematical analysis of the relationship between the norm of the beamforming vector and SNR, revealing that the norm of the beamforming vector exhibits an approximately linear behavior with respect to SNR when the SNR is large.
- (2) We demonstrate that imposing a constraint on the norm of the beamforming vector is tantamount to minimizing the degradation in SNR under the worst-case condition, thereby providing a physical interpretation for the norm constraint.
- (3) We propose a robust SLC algorithm that incorporates a beamforming vector norm constraint, i.e., the optimal beamforming vector is obtained by imposing restrictions on its vector norm. By transforming the beamforming algorithm into a quadratic

constraint convex optimization problem, we enable its solvability using well-established techniques such as Lagrange methodology. The key advantage of this algorithm lies in its reliance solely on the knowledge of earth station antenna gain and maximum auxiliary array gain to the desired signal, both of which are fully known. Consequently, this algorithm can be applied to arrays with arbitrary geometries.

- (4) To mitigate the computational complexity of the proposed algorithm, we introduce a Lagrange multiplier approximation that exhibits a linear complexity in relation to the number of antennas. Simulations demonstrate excellent performance in comparison with existing algorithms.

The paper is structured as follows. Section 2 presents the signal model. Section 3 elaborates on the proposed robust SLC algorithm. Section 4 presents discussions. Section 5 showcases simulation results, and conclusions are drawn in Section 6.

2. Signal Model

Consider an SLC system with a high-gain antenna and an auxiliary array with arbitrary geometry and low-gain elements, as illustrated in Figure 1.

The received signal of the high-gain antenna is as follows:

$$d(n) = g_s s(n) + \mathbf{g}_c \mathbf{c}(n) + n_d(n), \quad (1)$$

where $s(n)$ is the SOI, $\mathbf{c}(n) = [c_k]_{K \times 1} \in \mathbb{C}^{K \times 1}$ is the signal vector of the K interferers, $n_d(n)$ is the receiver noise of the earth station antenna; g_s is the gain of the earth station antenna to the SOI, and $\mathbf{g}_c = [g_{c,k}]_{1 \times K} \in \mathbb{C}^{1 \times K}$ is a row vector consisting of the gains to the K interferers.

The received signal vector of the auxiliary array is as follows:

$$\mathbf{x}(n) = \mathbf{a}_s s(n) + \mathbf{A}_c \mathbf{c}(n) + \mathbf{n}_x(n), \quad (2)$$

where $\mathbf{a}_s \in \mathbb{C}^{N \times 1}$ is the array vector of the SOI, $\mathbf{A}_c = [\mathbf{a}_{c,k}]_{N \times K} \in \mathbb{C}^{N \times K}$ is the matrix composed by the array vectors of the interferers, and $\mathbf{n}_x(n) = [n_{x,k}]_{N \times 1} \in \mathbb{C}^{N \times 1}$ is the noise vector of the auxiliary array, where N is the number auxiliary antennas.

The output signal of the beamformer is as follows:

$$\begin{aligned} e(n) &= d(n) - \mathbf{w}^H \mathbf{x}(n) \\ &= (g_s - \mathbf{w}^H \mathbf{a}_s) s(n) + (\mathbf{g}_c - \mathbf{w}^H \mathbf{A}_c) \mathbf{c}(n) + n_d(n) - \mathbf{w}^H \mathbf{n}_x(n), \end{aligned} \quad (3)$$

where $\mathbf{w} \in \mathbb{C}^{N \times 1}$ is the beamforming vector.

The beamforming output SINR is as follows:

$$\text{SINR}_{\text{out}} = \frac{|g_s - \mathbf{w}^H \mathbf{a}_s|^2 p_s}{(\mathbf{g}_c - \mathbf{w}^H \mathbf{A})^H \mathbf{P}_c (\mathbf{g}_c - \mathbf{w}^H \mathbf{A}) + (1 + \|\mathbf{w}\|^2) p_n}, \quad (4)$$

where $p_s = E(|s(n)|^2)$ is the power of the SOI, $\mathbf{P}_c = \text{diag}\{p_{c,1}, \dots, p_{c,K}\}$, and $p_{c,k} = E(|c_k(n)|^2)$ is the power of the k th interference signal, and p_n is the receiver noise power.

3. Proposed Robust Beamforming Algorithm

We begin by analyzing the relation between beamforming vector norm and SNR, followed by the relation between beamforming vector norm constraint and worst-case SNR loss constraint. We then propose a robust SLC algorithm, which is defined as an optimization problem, and then discuss its approximate solution.

3.1. Beamforming Vector Norm versus SNR. For traditional SLC, the beamforming vector is found by solving the unconstrained optimization problem:

$$\text{Minimize } J(\mathbf{w}) = E(|e(n)|^2). \quad (5)$$

The solution is given by the following equation:

$$\mathbf{w}_{\text{SLC}} = \mathbf{R}_{\text{xx}}^{-1} \mathbf{R}_{\text{xd}}, \quad (6)$$

where $\mathbf{R}_{\text{xx}} = E(\mathbf{x}(n)\mathbf{x}(n)^H) = \mathbf{a}_s \mathbf{a}_s^H p_s + \mathbf{A}_c \mathbf{P}_c \mathbf{A}_c^H + p_n \mathbf{I}_N$ and $\mathbf{R}_{\text{xd}} = E(\mathbf{x}(n)d(n)^*) = a_s g_s^* p_s + \mathbf{A}_c \mathbf{P}_c \mathbf{g}_c^H$.

When SNR is high, we can have the following approximation (see Appendix A, and let $\lambda = 0$):

$$\|\mathbf{w}_{\text{SLC}}\| \approx \frac{p_s}{p_n} |g_s| \|\mathbf{a}_s\|. \quad (7)$$

That is, the norm of the SLC beamforming vector is linearly related to the SNR but is independent with the interference signal power. This observation inspires us to consider constraining the norm of the beamforming vector to mitigate the signal cancellation effect.

3.2. Worst-Case SNR Loss Constraint. In this section, we will find that constraining the norm of the beamforming vector can be interpreted as limiting the loss in SNR in the worst-case scenario.

When interference signals are absent, the signal cancellation effect is the most pronounced, which we refer to as the worst case. Under this condition, the beamforming output SNR is as follows:

$$\text{SNR} = \frac{|g_s - \mathbf{w}^H \mathbf{a}_s|^2 p_s}{(1 + \|\mathbf{w}\|^2) p_n}. \quad (8)$$

We define SNR loss as follows:

$$\rho = \text{SNR}_0 / \text{SNR} = \frac{p_s |g_s|^2}{p_n} / \frac{p_s |g_s - \mathbf{w}^H \mathbf{a}_s|^2}{(1 + \|\mathbf{w}\|^2) p_n} = \frac{|g_s|^2 (1 + \|\mathbf{w}\|^2)}{|g_s - \mathbf{w}^H \mathbf{a}_s|^2}, \quad (9)$$

where SNR_0 is the received SNR of the earth station antenna. We would expect the SNR loss to be minimal. Therefore, we restrict $\rho \leq \rho_{\text{loss}}$, where ρ_{loss} is a control variable that denotes the maximum SNR loss. Based on our experience, it is a good practice to set ρ_{loss} within the range of 0.1–1 dB.

By applying Cauchy–Schwarz inequality to Equation (9), we get the following equation:

$$\rho \leq \frac{|g_s|^2 (1 + \|\mathbf{w}\|^2)}{|g_s|^2 - \|\mathbf{a}_s\|^2 \|\mathbf{w}\|^2}, \quad (10)$$

where $\|\mathbf{a}_s\|^2$ is the gain of the auxiliary array to the SOI.

Obtaining \mathbf{a}_s becomes challenging or impossible in the presence of interferences, especially when the SNR is low, such as in SATCOM. However, the maximum gain of the auxiliary array, denoted by ε^2 , is a priori knowledge. By replacing $\|\mathbf{a}_s\|^2$ with ε^2 in Equation (10), we can get the following equation:

$$\rho \leq \frac{|g_s|^2 (1 + \|\mathbf{w}\|^2)}{|g_s|^2 - \varepsilon^2 \|\mathbf{w}\|^2} \leq \rho_{\text{loss}}. \quad (11)$$

Equation (11) can be seen as a *worst-case SNR loss constraint*. In other words, in the worst-case scenario that the gain of the auxiliary array to the SOI achieves the maximum, SNR loss should be smaller than a predefined value ρ_{loss} .

From Equation (11), we can derive the following equation:

$$\|\mathbf{w}\|^2 \leq \eta = \frac{(\rho_{\text{loss}} - 1) |g_s|^2}{|g_s|^2 + \rho_{\text{loss}} \varepsilon^2}. \quad (12)$$

Hence, the worst-case SNR loss constraint can be reformulated as a constraint on the norm of the beamforming vector. It is worth noting that the right-hand side of Equation (12) can be determined offline, making it easily applicable in practice.

3.3. Robust SLC Beamforming Algorithm. Imposing the constraint of Equation (12) on the optimization problem defined in Equation (5), we can formulate the following new optimization problem:

$$\text{Minimize } J(\mathbf{w}) \text{ s.t. } \|\mathbf{w}\|^2 \leq \eta. \quad (13)$$

This optimization problem can be solved using Lagrange multiplier methodology [17]. Note that when the solution of Problem (5) satisfies the constraint Equation (12), it is unnecessary to solve the constraint problem defined by Equation (13); Otherwise, if the constraint is violated, the Problem (13) is equivalent to the following equation:

$$\text{Minimize } J(\mathbf{w}) \text{ s.t. } \|\mathbf{w}\|^2 = \eta. \quad (14)$$

To solve Equation (14), first, form the Lagrangian as follows:

$$L(\mathbf{w}, \lambda) = \mathbf{w}^H \mathbf{R}_{xx} \mathbf{w} - \mathbf{w}^H \mathbf{R}_{xd} - (\mathbf{w}^H \mathbf{R}_{xd})^H + \lambda(\|\mathbf{w}\|^2 - \eta). \quad (15)$$

Then, derive the optimality conditions as follows:

$$\begin{aligned} \nabla_{\mathbf{w}} L(\mathbf{w}, \lambda) &= (\mathbf{R}_{xx} + \lambda \mathbf{I}_M) \mathbf{w} - \mathbf{R}_{xd} = 0 \\ \nabla_{\lambda} L(\mathbf{w}, \lambda) &= \|\mathbf{w}\|^2 - \eta = 0. \end{aligned} \quad (16)$$

Hence:

$$\mathbf{w}_{\text{opt}} = (\mathbf{R}_{xx} + \lambda \mathbf{I}_M)^{-1} \mathbf{R}_{xd}, \quad (17)$$

where λ is found by solving the following equation:

$$\|\mathbf{w}_{\text{opt}}\|^2 - \eta = 0. \quad (18)$$

Substituting Equation (17) into Equation (18), we get the following equation:

$$f(\lambda) = \mathbf{R}_{xd}^H (\mathbf{R}_{xx} + \lambda \mathbf{I}_M)^{-2} \mathbf{R}_{xd} - \eta = 0. \quad (19)$$

Applying eigendecomposition to \mathbf{R}_{xx} , we get $\mathbf{R}_{xx} = \mathbf{Q} \mathbf{\Gamma} \mathbf{Q}^H$. Then, we obtain the following equation:

$$\mathbf{R}_{xx} + \lambda = \mathbf{Q} \mathbf{\Gamma} \mathbf{Q}^H + \lambda \mathbf{I}_M = \mathbf{Q} (\mathbf{\Gamma} + \lambda \mathbf{I}_M) \mathbf{Q}^H. \quad (20)$$

Therefore:

$$\begin{aligned} f(\lambda) &= \mathbf{R}_{xd}^H (\mathbf{R}_{xx} + \lambda)^{-2} \mathbf{R}_{xd} - \eta \\ &= \mathbf{R}_{xd}^H \mathbf{Q} (\mathbf{\Gamma} + \lambda \mathbf{I})^{-2} \mathbf{Q}^H \mathbf{R}_{xd} - \eta \\ &= (\mathbf{Q}^H \mathbf{R}_{xd})^H (\mathbf{\Gamma} + \lambda \mathbf{I})^{-2} (\mathbf{Q} \mathbf{R}_{xd}) - \eta. \end{aligned} \quad (21)$$

Let $\mathbf{c} = \mathbf{Q}^H \mathbf{R}_{xd}$, and substituting it into Equation (21), we get the following equation:

$$f(\lambda) = \mathbf{c}^H (\mathbf{\Gamma} + \lambda \mathbf{I})^{-2} \mathbf{c} - \eta = \sum_i \frac{|c_i|^2}{(\gamma_i + \lambda)^2} - \eta. \quad (22)$$

Since we have the following inequality:

$$\frac{\partial f(\lambda)}{\partial \lambda} = \sum_i -2|c_i|^2 (\gamma_i + \lambda)^{-3} < 0 \text{ for } \lambda \geq 0. \quad (23)$$

The solution of Equation (22) is unique. Equation (23) has to be solved using numerical methods like Newton's method. The search procedure typically converges in a few iterations [18].

The proposed robust SLC beamforming algorithm (Algorithm 1) is summarized below:

The main computational complexity comes from the eigendecomposition of \mathbf{R}_{xx} in Step 3, which is about $O(N^3)$ FLOPs. Therefore, the complexity is significant when N is large.

Step 1: Compute \mathbf{R}_{xx} and \mathbf{R}_{xd} .
 Step 2: Use Equation (5) to calculate \mathbf{w}_{opt} (unconstraint SLC). If $\|\mathbf{w}_{\text{opt}}\|^2 \leq \eta$, go to Step 5; otherwise, go to Step 3.
 Step 3: Compute the eigendecomposition of \mathbf{R}_{xx} .
 Step 4: Use Equation (22) to find optimal λ by numerical search.
 Step 5: Use Equation (17) to calculate \mathbf{w}_{opt} .
 Step 6: Use Equation (3) to calculate SLC output signal.

ALGORITHM 1: Robust SLC algorithm.

3.4. Lagrange Multiplier Approximation. To reduce the complexity involved in finding the optimal Lagrange multiplier, we propose a method to approximate it, by taking advantage of some properties of the SLC system. As a result, the complexity is reduced to $O(N)$ FLOPs.

We find that the following approximation holds for SLC (see Appendix A for the proof):

$$\|\mathbf{w}\|^2 \approx \|\mathbf{w}_s\|^2 \approx \frac{p_s^2 |g_s|^2 \|\mathbf{a}_s\|^2}{(p_n + \lambda)^2}. \quad (24)$$

Then, let $\|\mathbf{w}\|^2 = \eta$. The solution of λ is as follows:

$$\bar{\lambda} = \frac{p_s |g_s| \|\mathbf{a}_s\|}{\sqrt{\eta}} - p_n. \quad (25)$$

Unfortunately, the knowledge of p_s , p_n , g_s , and $\|\mathbf{a}_s\|^2$ is required for calculating $\bar{\lambda}$. Usually, $|g_s|$ and $\|\mathbf{a}_s\|$ are a priori information, and p_n can be easily measured. It is, however, hard to estimate p_s particularly when interferences are present.

We observe that it is possible to use Equation (24) to get a rough estimate of p_s , or $p_s |g_s| \|\mathbf{a}_s\|$ as a whole, when SNR is high. This is explained in the following. First, let $\lambda = 0$, i.e., without using the SNR loss constraint, then Equation (24) is an approximation of $\|\mathbf{w}_{\text{SLC}}\|^2$ calculated by Equation (5), i.e.:

$$\|\mathbf{w}_{\text{SLC}}\|^2 \approx \left(\frac{p_s}{p_n} |g_s| \|\mathbf{a}_s\| \right)^2. \quad (26)$$

Then, a rough estimate of p_s is then given by the following equation:

$$p_s \approx \frac{p_n \|\mathbf{w}_{\text{SLC}}\|}{|g_s| \|\mathbf{a}_s\|}. \quad (27)$$

Substituting Equation (26) or Equation (27) into Equation (25), we finally can get the following equation:

$$\hat{\lambda} = p_n \left(\frac{\|\mathbf{w}_{\text{SLC}}\|}{\sqrt{\eta}} - 1 \right). \quad (28)$$

Note that now we do not need the information of p_s , g_s , and $\|\mathbf{a}_s\|^2$ at all, but only p_n , to calculate the $\hat{\lambda}$. The computational complexity of Equation (28) is only $O(N)$.

Step 1: Compute \mathbf{R}_{xx} and \mathbf{R}_{xd} .
 Step 2: Use Equation (5) to calculate \mathbf{w}_{opt} (unconstraint SLC). If $\|\mathbf{w}_{\text{opt}}\|^2 \leq \eta$, go to Step 5; otherwise, go to Step 3.
 Step 3: Calculate λ using Equation (28) and (29).
 Step 4: Use Equation (17) to calculate \mathbf{w}_{opt} .
 Step 5: Use Equation (3) to calculate SLC output signal.

ALGORITHM 2: Robust SLC algorithm with approximated Lagrange multiplier.

Since λ should be non-negative, we define λ as the following equation:

$$\lambda = \begin{cases} \hat{\lambda}, & \hat{\lambda} \geq 0 \\ 0, & \hat{\lambda} < 0 \end{cases}. \quad (29)$$

Then, λ can then be substituted into Equation (17) to calculate \mathbf{w} . As a result of Equation (29), λ is set to zero when SOI power is low, i.e., the proposed algorithm degrades to traditional SLC.

The robust SLC algorithm using an approximated Lagrange multiplier is summarized in the following.

4. Discussion

The proposed algorithm can be implemented with either an iterative or noniterative structure. For the iterative implementation, the algorithm structure resembles that of stochastic gradient algorithms, such as least mean square (LMS). However, a key distinction lies in the weight updates being determined by $\mathbf{w}(n+1) = \mathbf{w}(n) + \mu \hat{\mathbf{x}}(n)e^*(n)$, where $\hat{\mathbf{x}}(n) = \mathbf{x}(n) + \sqrt{\lambda} \bar{\mathbf{n}}(n)$ and $\bar{\mathbf{n}}(n)$ represents i.i.d. Gaussian noise vector. This similarity arises from the observation that, $E(\hat{\mathbf{x}}(n)\hat{\mathbf{x}}^H(n)) = \mathbf{R}_{xx} + \lambda \mathbf{I}_N$, and Equation (17) resembles the Wiener filter. On the other hand, for noniterative implementation, Step 1 computes the covariance matrix \mathbf{R}_{xx} and cross-correlation vector \mathbf{R}_{xd} using received signal samples, without explicitly requiring instant channel knowledge or antenna array vectors. The remaining steps are straightforward.

The limitations of this study are as follows. First, we have primarily focused on Gaussian noises, similar to previous relevant works, as the main objective is to enhance robustness against desired signal cancellation effects. However, it is also crucial to improve robustness against non-Gaussian noises in practical scenarios. In this regard, incorporating ideas proposed in [19, 20], particularly regarding iterative implementation, can be considered. Second, if an iterative structure is employed for implementing the proposed algorithm, investigating its tracking performance would be intriguing. Specifically, tracking the power of the desired SOI becomes important when dealing with scenarios where SOI power frequently changes. The SOI power remains constant in SatCom system due to the line-of-sight channel, thus obviating the need for intensive tracking.

TABLE 1: Simulation parameters.

Interferer angles	$ \theta_{c,k} - \theta_s \geq 10$
Number of interferers, K	1–3
SNR, ρ_s	0–30 dB
INR, $\max_k \rho_{c,k}$	0–30 dB
Auxiliary array size, N	8

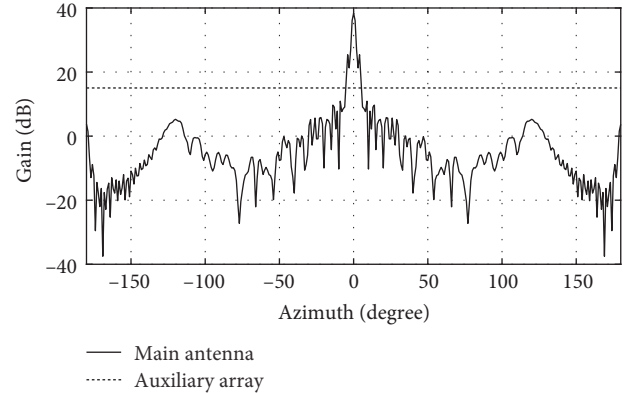


FIGURE 2: Comparison of earth station antenna gain and auxiliary array gain.

5. Simulation Results

In this section, simulations are conducted to compare the performance of the proposed Robust SLC algorithm with existing algorithms, including traditional SLC, PC method proposed in [8], diagonal loading method in [13], while the diagonal loading factor optimized by numerical search. Minimum power distortionless response (MPDR), assuming knowledge of the SOI array vector, is considered as a performance benchmark.

The main simulation parameters are listed in Table 1. The earth station antenna is a Ku band parabolic antenna of 0.8 m diameter, with a maximum gain of 38 dB; the auxiliary array is a uniform circular array with diameter of 1 m, surrounding the main antenna. The auxiliary antenna element is an isotropic antenna with a gain of 6 dB. A comparison of the earth station antenna pattern and the auxiliary array gain is shown in Figure 2. The interferers are located within the sidelobe angles of the earth station antenna. The size of the auxiliary array is chosen to ensure that its gain is much larger than the sidelobe gains toward the interferers. The ranges of SNR and INR are determined based on practical considerations [21], and relevant literature such as in [13, 22].

5.1. Validation of Lagrange Multiplier Approximation. The only difference between the Robust SLC algorithm (Algorithm 2) and the preliminary version (Algorithm 1) lies in the utilization of Lagrange multiplier approximation. In this subsection, we present numerical results to validate the approximation method of the Lagrange multiplier.

The simulation parameters are as follows: K ranges from 1 to 3, and the interferers are positioned at angles of 30° ,

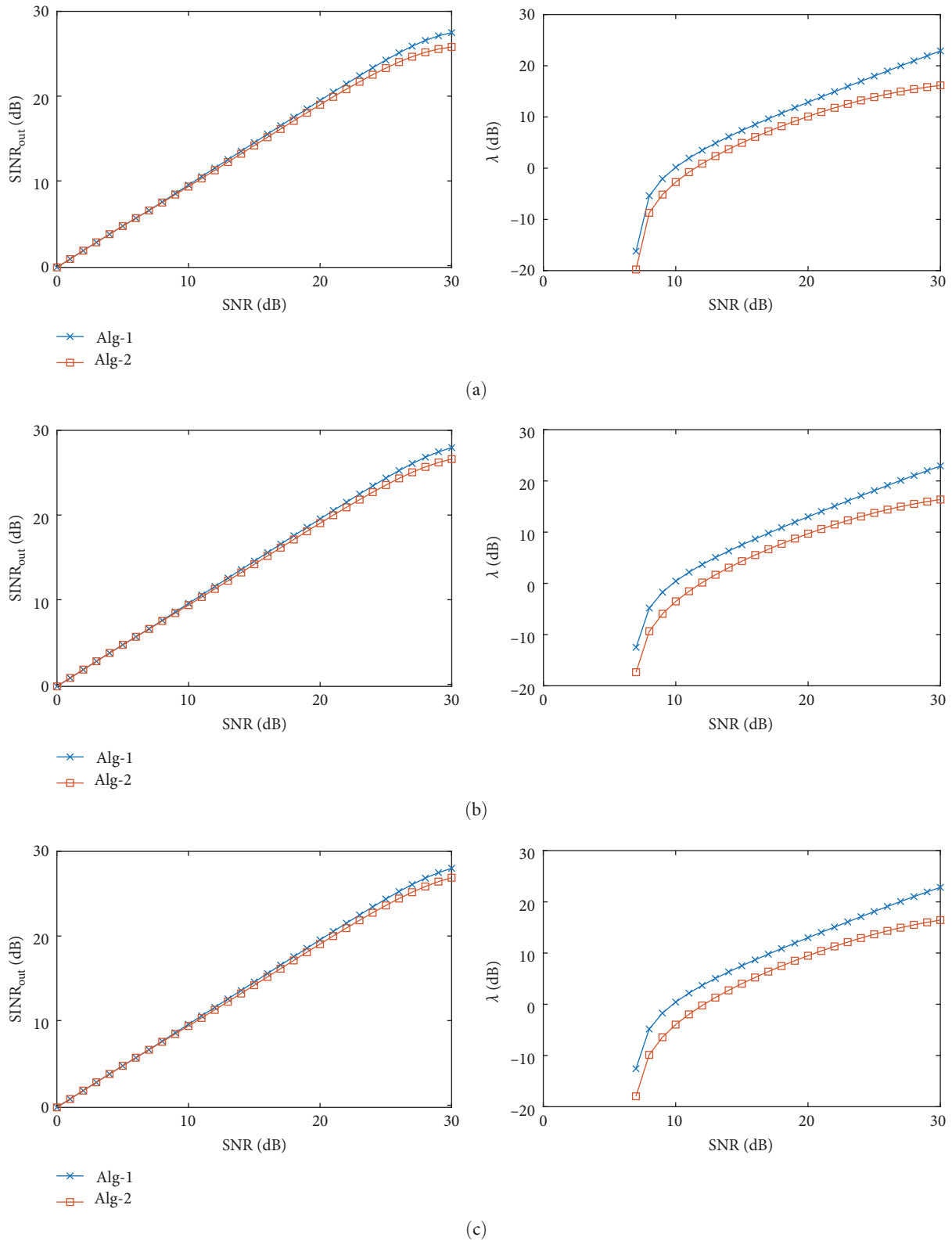


FIGURE 3: Example of output SINR and Lagrange multiplier using Algorithms 1 and 2: (a) $K=1$, (b) $K=2$, and (c) $K=3$.

-30° , and 60° , respectively; $\text{INR} = 30$ dB, and $\rho_{\text{loss}} = 0.5$ dB. Figure 3 presents an example of the beamforming output SINR and Lagrange multiplier. Note that the Lagrange multiplier is normalized with respect to the noise power.

The approximated Lagrange multiplier is generally observed to be smaller than the optimal one, resulting in a slight decrease in output SINR. This reduction primarily occurs at high SNR levels, particularly when SNR exceeds approximately 25 dB.

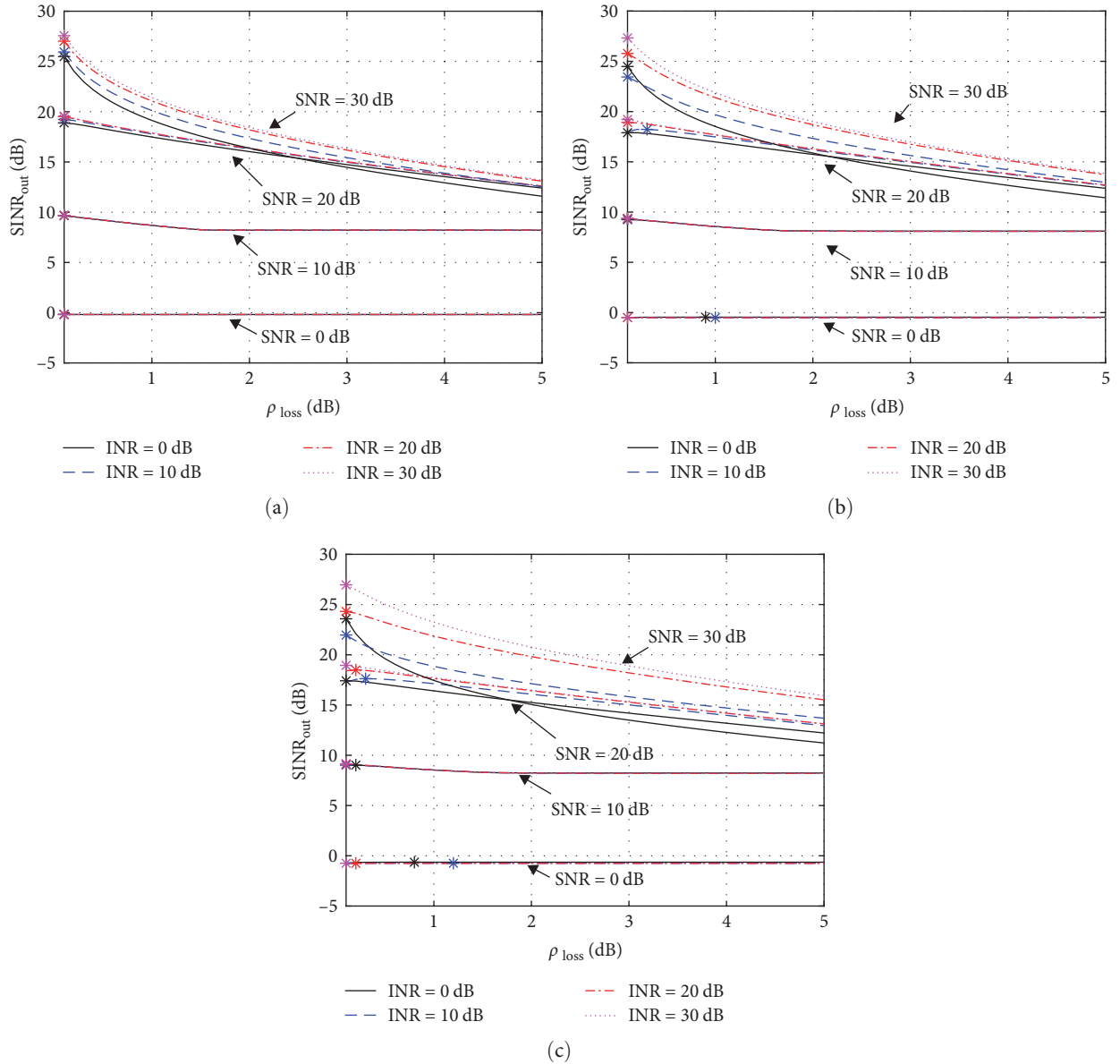


FIGURE 4: Output SINR versus ρ_{loss} : (a) single interferer, (b) two interferers, and (c) three interferers.

When the SNR is below 7 dB, both the optimal and approximated Lagrange multipliers are zero (thus not visible on the logarithmic scale). Subsequent sections will mainly focus on analyzing Algorithm 2 due to its practicality stemming from low computational complexity.

5.2. Choice of ρ_{loss} for Robust SLC Algorithm. The choice of ρ_{loss} is important for the performance of the proposed robust SLC algorithm. If it is too small, the constraint will be too restrictive, there may be no viable solution; if it is too large, the algorithm will revert to traditional SLC, hence no improvement is obtained. Since there is no theoretical method to obtain the optimal value of ρ_{loss} , we turn to numerical analysis to examine the effect of it.

Figure 4 presents results of average output SINR versus ρ_{loss} , for various SNRs, INRs, and number of interferers. We observe that:

- (1) For low SNRs (≤ 10 dB), output SINR is almost constant with respect to ρ_{loss} . Therefore, it is unnecessary to use the constraint.
- (2) For high SNRs (> 10 dB), the optimal SINR is achieved when ρ_{loss} is around 0.1–1 dB. INR has negligible effect on the optimal ρ_{loss} .
- (3) The optimal ρ_{loss} is slightly larger for a larger number of interferers.
- (4) The output SINR decreases slowly with ρ_{loss} , we would expect a rough setting of ρ_{loss} will not significantly degrade the performance.

Hence, in this paper, we consider ρ_{loss} to be in the range of 0.1–1 dB.

5.3. Beamforming Performance. Figure 5 presents the average beamforming output SINR versus receive SNR for different

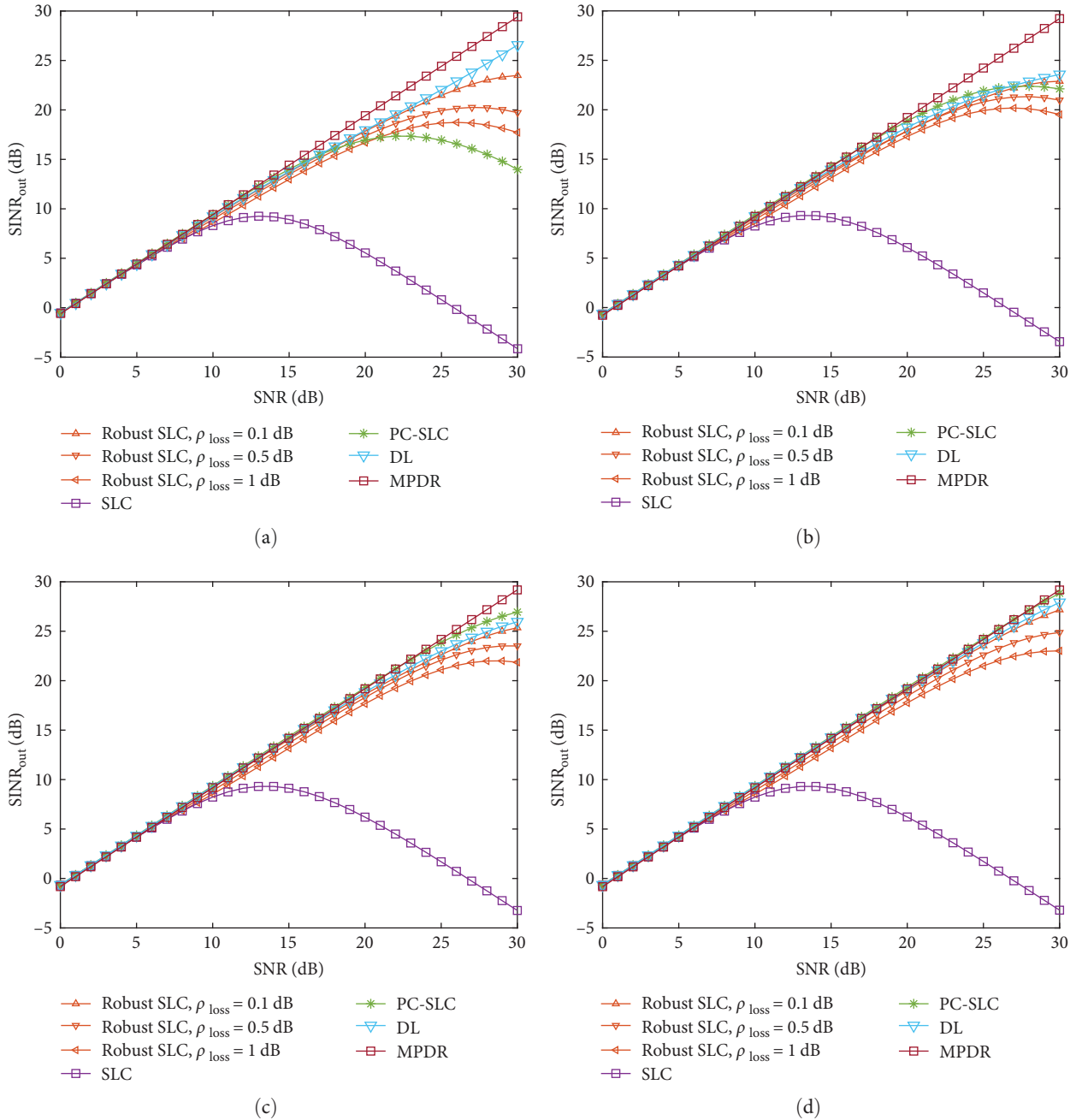


FIGURE 5: Output SINR versus receive SNR for different algorithms: (a) INR = 0 dB, (b) INR = 10 dB, (c) INR = 20 dB, and (d) INR = 30 dB.

INRs. The average is taken with respect to random interference directions. For the proposed Robust SLC algorithm, $\rho_{\text{loss}} = 0.1, 0.5,$ and 1 dB are considered, respectively. We observe that:

- (1) For traditional SLC, the output SINR starts to decrease when SNR is larger than about 10 dB and experiences a more significant degradation as the SNR increases.
- (2) The DL method achieves close-to-optimal performance as MPDR for low and moderate SNRs (≤ 20 dB) but performs poorly for high SNRs. The PC-based

method achieves a similar performance as DL but deteriorates when the SNR is high ($\text{SNR} \geq 20$ dB).

- (3) The robust SLC algorithm achieves approximately the same performance as the DL method when using a small ρ_{loss} , for example, 0.1 dB. This result is observed across various simulated SNRs and INRs, demonstrating the effectiveness of our proposal.
- (4) As the SNR constraint becomes relaxed, i.e., when ρ_{loss} increases, we observe that the performance curve of the Robust SLC algorithm approaches that of traditional SLC (still significantly superior). Therefore, it

is advantageous to employ a smaller ρ_{loss} for Robust SLC.

6. Conclusions

We propose a robust SLC algorithm based on beamforming vector norm constraint to address the signal cancellation problem in SLC systems. The algorithm is formulated as a quadratic optimization problem with a quadratic inequality constraint, which is then solved using Lagrange methodology. To reduce the computational complexity of finding the Lagrange multiplier, we propose an approximation method. This results in a linear computation complexity with respect to the auxiliary array size, contrasting with the cubic complexity of Lagrange methodology. Simulation results demonstrate that our proposed algorithm performs comparably to optimal diagonal loading and principal component-based algorithms.

Appendix

A. Proof of Equations (7) and (24).

First, rewrite Equation (17) as follows:

$$\begin{aligned} \mathbf{w} &= (\mathbf{R}_{xx} + \lambda \mathbf{I}_N)^{-1} \mathbf{R}_{xd} \\ &= (\mathbf{R}_{xx} + \lambda \mathbf{I}_N)^{-1} \left(\mathbf{a}_s g_s^* p_s + \sum_k \mathbf{a}_{c,k} g_{c,k}^* p_{c,k} \right) \\ &= \underbrace{(\mathbf{R}_{xx} + \lambda \mathbf{I}_N)^{-1} \mathbf{a}_s g_s^* p_s}_{\mathbf{w}_s} + \sum_k \underbrace{(\mathbf{R}_{xx} + \lambda \mathbf{I}_N)^{-1} \mathbf{a}_{c,k} g_{c,k}^* p_{c,k}}_{\mathbf{w}_{c,k}}, \end{aligned} \quad (\text{A.1})$$

where \mathbf{w}_s is the component contributed by SOI, and $\mathbf{w}_{c,k}$, $k = 1, \dots, K$ are the components contributed by the interference signals.

Next, we will show that $\|\mathbf{w}_s\|^2 \gg \|\mathbf{w}_c\|^2$, where $\mathbf{w}_c = \sum_k \mathbf{w}_{c,k}$, when SNR is high, thus we can have the approximation that $\|\mathbf{w}\|^2 \approx \|\mathbf{w}_s\|^2$.

By using the Woodbury equation, we get the following equation:

$$\begin{aligned} (\mathbf{R}_{xx} + \lambda \mathbf{I}_N)^{-1} &= (p_s \mathbf{a}_s \mathbf{a}_s^H + (\mathbf{R}_{in} + \lambda \mathbf{I}))^{-1} \\ &= (\mathbf{R}_{in} + \lambda \mathbf{I})^{-1} - \frac{p_s (\mathbf{R}_{in} + \lambda \mathbf{I})^{-1} \mathbf{a}_s \mathbf{a}_s^H (\mathbf{R}_{in} + \lambda \mathbf{I})^{-1}}{\mathbf{I} + p_s \mathbf{a}_s^H (\mathbf{R}_{in} + \lambda \mathbf{I})^{-1} \mathbf{a}_s}. \end{aligned} \quad (\text{A.2})$$

Therefore:

$$\begin{aligned} \mathbf{w}_s &= \left((\mathbf{R}_{in} + \lambda \mathbf{I})^{-1} - \frac{p_s (\mathbf{R}_{in} + \lambda \mathbf{I})^{-1} \mathbf{a}_s \mathbf{a}_s^H (\mathbf{R}_{in} + \lambda \mathbf{I})^{-1}}{1 + p_s \mathbf{a}_s^H (\mathbf{R}_{in} + \lambda \mathbf{I})^{-1} \mathbf{a}_s} \right) (\mathbf{a}_s g_s^* p_s) \\ &= (\mathbf{R}_{in} + \lambda \mathbf{I})^{-1} \mathbf{a}_s g_s^* p_s - \frac{p_s (\mathbf{R}_{in} + \lambda \mathbf{I})^{-1} \mathbf{a}_s \mathbf{a}_s^H (\mathbf{R}_{in} + \lambda \mathbf{I})^{-1} \mathbf{a}_s g_s^* p_s}{1 + p_s \mathbf{a}_s^H (\mathbf{R}_{in} + \lambda \mathbf{I})^{-1} \mathbf{a}_s} \\ &= p_s (\mathbf{R}_{in} + \lambda \mathbf{I})^{-1} \mathbf{a}_s g_s^* - \frac{p_s (\mathbf{R}_{in} + \lambda \mathbf{I})^{-1} \mathbf{a}_s g_s^* p_s \mathbf{a}_s^H (\mathbf{R}_{in} + \lambda \mathbf{I})^{-1} \mathbf{a}_s}{1 + p_s \mathbf{a}_s^H (\mathbf{R}_{in} + \lambda \mathbf{I})^{-1} \mathbf{a}_s} \end{aligned} \quad (\text{A.3})$$

Based on eigenvalue decomposition, we can get $\mathbf{R}_{in} = \sum_{i=1}^K \gamma_i \mathbf{u}_i \mathbf{u}_i^H + \sum_{i=K+1}^N p_n \mathbf{u}_i \mathbf{u}_i^H$, and then the following equation:

$$\begin{aligned} \mathbf{a}_s^H (\mathbf{R}_{in} + \lambda \mathbf{I})^{-1} \mathbf{a}_s &= \mathbf{a}_s^H \sum_{i=1}^N \frac{\mathbf{u}_i \mathbf{u}_i^H}{\gamma_i + \lambda} \mathbf{a}_s \\ &= \sum_{i=1}^K \frac{\mathbf{a}_s^H \mathbf{u}_i \mathbf{u}_i^H \mathbf{a}_s}{\gamma_i + \lambda} + \sum_{i=K+1}^N \frac{\mathbf{a}_s^H \mathbf{u}_i \mathbf{u}_i^H \mathbf{a}_s}{p_n + \lambda} \\ &= \frac{\|\mathbf{U}_I^H \mathbf{a}_s\|^2}{\bar{\gamma} + \lambda} + \frac{\|\mathbf{U}_N^H \mathbf{a}_s\|^2}{p_n + \lambda} \\ &\approx \frac{\|\mathbf{a}_s\|^2}{p_n + \lambda}. \end{aligned} \quad (\text{A.4})$$

The approximation here is due to the fact that the signal power of the SOI falls mainly into the noise subspace.

Similarly:

$$\mathbf{a}_s^H (\mathbf{R}_{in} + \lambda \mathbf{I})^{-2} \mathbf{a}_s \approx \frac{\|\mathbf{a}_s\|^2}{(p_n + \lambda)^2}. \quad (\text{A.5})$$

Substituting Equation A.4 into Equation A.3, we get the following equation:

$$\begin{aligned} \mathbf{w}_s &\approx p_s (\mathbf{R}_{in} + \lambda \mathbf{I})^{-1} \mathbf{a}_s g_s^* - \frac{p_s (\mathbf{R}_{in} + \lambda \mathbf{I})^{-1} \mathbf{a}_s g_s^* p_s \frac{\|\mathbf{a}_s\|^2}{p_n + \lambda}}{1 + p_s \frac{\|\mathbf{a}_s\|^2}{p_n + \lambda}} \\ &= p_s g_s^* (\mathbf{R}_{in} + \lambda \mathbf{I})^{-1} \mathbf{a}_s \left(1 - \frac{p_s \|\mathbf{a}_s\|^2}{p_n + \lambda + p_s \|\mathbf{a}_s\|^2} \right) \\ &= p_s g_s^* (\mathbf{R}_{in} + \lambda \mathbf{I})^{-1} \mathbf{a}_s \frac{p_n + \lambda}{p_n + \lambda + p_s \|\mathbf{a}_s\|^2} \\ &\approx p_s g_s^* (\mathbf{R}_{in} + \lambda \mathbf{I})^{-1} \mathbf{a}_s, \end{aligned} \quad (\text{A.6})$$

where it is assumed that $p_s \|\mathbf{a}_s\|^2 \ll p_n$, i.e., the SOI power received by the auxiliary array is much smaller than the receiver noise. Then, we can obtain the following equation:

$$\|\mathbf{w}_s\|^2 \approx p_s^2 |g_s|^2 \mathbf{a}_s^H (\mathbf{R}_{in} + \lambda \mathbf{I})^{-2} \mathbf{a}_s \approx \frac{p_s^2 |g_s|^2 \|\mathbf{a}_s\|^2}{(p_n + \lambda)^2}. \quad (\text{A.7})$$

Therefore, $\|\mathbf{w}_s\|^2$ increases quadratically with the SOI power p_s .

Like the derivation of $\|\mathbf{w}_s\|^2$, we can get the following equation:

$$\begin{aligned} \mathbf{w}_{c,k} &= (\mathbf{R}_{in} + \lambda \mathbf{I})^{-1} \mathbf{a}_{c,k} g_{c,k}^* p_{c,k} \\ &\quad - \frac{p_s (\mathbf{R}_{in} + \lambda \mathbf{I})^{-1} \mathbf{a}_s \mathbf{a}_s^H (\mathbf{R}_{in} + \lambda \mathbf{I})^{-1} \mathbf{a}_{c,k} g_{c,k}^* p_{c,k}}{1 + p_s \mathbf{a}_s^H (\mathbf{R}_{in} + \lambda \mathbf{I})^{-1} \mathbf{a}_s}. \end{aligned} \quad (\text{A.8})$$

Since \mathbf{a}_s is approximately orthogonal to the interference subspace, and $\mathbf{a}_{c,k}$ is orthogonal to the noise subspace [13], we can get the following approximation:

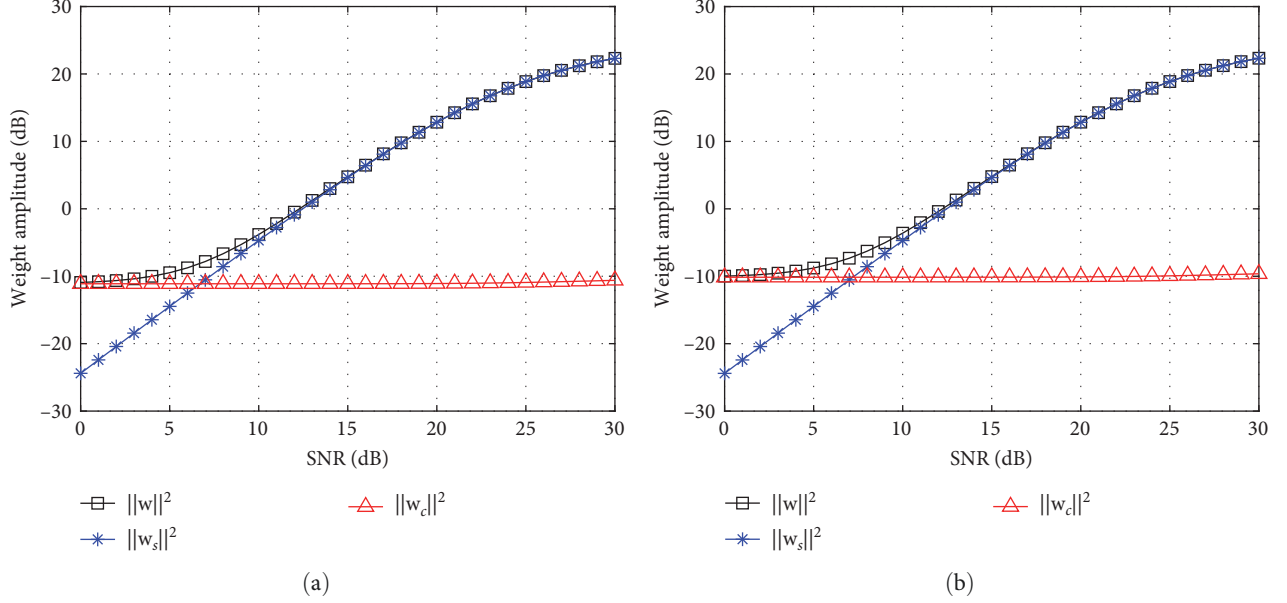


FIGURE 6: Weight norm versus SNR: (a) INR = 10 dB and (b) INR = 30 dB.

$$\begin{aligned}
 \mathbf{a}_s^H (\mathbf{R}_{in} + \lambda \mathbf{I})^{-1} \mathbf{a}_{c,k} &= \mathbf{a}_s^H \sum_{i=1}^N \frac{\mathbf{u}_i \mathbf{u}_i^H}{\gamma_i + \lambda} \mathbf{a}_{c,k} \\
 &= \sum_{i=1}^K \frac{\mathbf{a}_s^H \mathbf{u}_i \mathbf{u}_i^H \mathbf{a}_{c,k}}{\gamma_i + \lambda} + \sum_{i=K+1}^N \frac{\mathbf{a}_s^H \mathbf{u}_i \mathbf{u}_i^H \mathbf{a}_{c,k}}{p_n + \lambda} \approx 0.
 \end{aligned} \tag{A.9}$$

Based on the above equation, we have the following equation:

$$\mathbf{w}_{c,k} \approx (\mathbf{R}_{in} + \lambda \mathbf{I})^{-1} \mathbf{a}_{c,k} \mathcal{G}_{c,k}^* p_{c,k}. \tag{A.10}$$

Thus, $\mathbf{w}_{c,k}$ is then not related to SOI. Then, we have the following equation:

$$\|\mathbf{w}_c\|^2 = \mathbf{g}_c \mathbf{P}_c \mathbf{A}_c^H (\mathbf{A}_c \mathbf{P}_c \mathbf{A}_c^H + (p_n + \lambda) \mathbf{I}_N)^{-2} \mathbf{A}_c \mathbf{P}_c \mathbf{g}_c^H. \tag{A.11}$$

It is, however, not possible to further simplify the above equation. Therefore, we consider an assumption that the array vectors of the interferers are mutually orthogonal. This is approximately true when the interferers are separated larger than a beam width [13]. Since the auxiliary array has a large aperture, the beam width is small; therefore, the assumption can be employed safely. Under this assumption, we have the following equation:

$$\|\mathbf{w}_c\|^2 \approx \sum_k \frac{p_{c,k}^2 \|\mathbf{a}_{c,k}\|^2 |g_{c,k}|^2}{(p_{c,k} \|\mathbf{a}_{c,k}\| + p_n + \lambda)^2}. \tag{A.12}$$

We can see that $\|\mathbf{w}_c\|^2$ increases with the interference power. When the interference power is high, we have the following equation:

$$\|\mathbf{w}_c\|^2 \approx \sum_k \frac{|g_{c,k}|^2}{(1 + \lambda)^2}. \tag{A.13}$$

Therefore, $\|\mathbf{w}_c\|^2$ tends to level off when interference power increases, and finally becomes upper bounded.

By comparing Equations (A.7) and (A.13), we can conclude that $\|\mathbf{w}_s\|^2$ will be much larger than $\|\mathbf{w}_c\|^2$ when the SNR is large. Hence, $\|\mathbf{w}\|^2 \approx \|\mathbf{w}_s\|^2$. Note that $|g_s|^2 \gg |g_{c,k}|^2$ since the main antenna has low sidelobes, while usually $\|\mathbf{a}_s\|^2 \geq \max(|g_{c,k}|^2)$.

To illustrate the above findings, we present simulation results of the average of $\|\mathbf{w}\|^2$, $\|\mathbf{w}_s\|^2$, and $\|\mathbf{w}_c\|^2$ versus SNR in Figure 6. The number of the interferers is $K = 2$. The other parameters used in this section remain consistent with those in Section 5. We can see that when SNR is larger than around 10 dB, $\|\mathbf{w}\|^2 \approx \|\mathbf{w}_s\|^2$, and $\|\mathbf{w}_c\|^2$ is almost constant for different SNRs. In fact, when $\text{SNR} \geq 10$ dB, $\|\mathbf{w}_s\|^2$ start to dominates, then the output SINR starts to drop dramatically.

Based on the above approximations, we can finally get the following equation:

$$\|\mathbf{w}\|^2 \approx \|\mathbf{w}_s\|^2 \approx \frac{p_s^2 |g_s|^2 \|\mathbf{a}_s\|^2}{(p_n + \lambda)^2}. \tag{A.14}$$

Data Availability

The data supporting the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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