

Research Article

Adaptive Backstepping Attitude Control Law with L_2 -Gain Performance for Flexible Spacecraft

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In this paper, an observer-based adaptive backstepping attitude maneuver controller (briefly, OBABC) for flexible spacecraft is presented. First, an observer is constructed to estimate the flexible modal variables. Based on the proposed observer, a backstepping control law is presented for the case where the inertia matrix is known. Further, an adaptive law is developed to estimate the unknown parameters of the inertia matrix of the flexible spacecraft. By utilizing Lyapunov theory, the proposed OBABC law can guarantee the asymptotical convergence of the closed-loop system in the presence of the external disturbance, incorporating with the L_2 -gain performance criterion constraint. Simulation results show that the attitude maneuver can be achieved by the proposed observer-based adaptive backstepping attitude control law.

1. Introduction

A new generation of large spacecraft have a complex structure that can include flexible appendages such as solar panels, antennas, and space manipulators. These flexible appendages may effect the attitude control performance of spacecraft due to the strong coupling between the spacecraft main body and flexible appendages [1]. Therefore, the attitude controller design for a flexible spacecraft becomes a crucial topic [2]. Many researches have proposed various attitude control approaches for flexible spacecraft [3–10]. A PD control method was presented for an attitude tracking problem in [3], which analyzed three controller structures in detail including model-independent, model-dependent, and parameter adaptive control structures. In [4], an antidisturbance PD controller was proposed to deal with attitude control of a flexible spacecraft in the presence of multiple disturbances. Sliding mode controllers were developed in [5, 6] to solve the attitude tracking problem for flexible spacecraft in the presence of external disturbance and model uncertainties. A second-order sliding mode

controller was designed by using a supertwisting approach for flexible spacecraft attitude tracking issue in [7]. In [8], a delay-dependent disturbance observer was utilized to estimate the main disturbance caused by flexible appendages, and an H_∞ control scheme was developed to attenuate exogenous bounded disturbance. Moreover, some other control approaches have also been investigated for the attitude control problem of flexible spacecraft, such as the angular velocity feedback control without angular velocity measurement [9] and finite-time control technique [10].

The control techniques mentioned above are based on the assumption that the inertia matrix of spacecraft is known in advance. In fact, estimating the inertia matrix of the spacecraft effectively is a challenging task for spacecraft attitude control law designers [11]. Several solutions to the inertia matrix estimation problem have been presented. An adaptive control law was presented in [12] to compensate the unknown inertia matrix for a rigid spacecraft tracking issue. In [13], an adaptive variable structure tracking control law was presented for rigid spacecraft with inertia uncertainty. An adaptive sliding mode control strategy with a synthesized

hybrid sliding surface was designed in [14]. In [15], a new simple adaptive control law for the rotational maneuver of a flexible spacecraft was designed. A model reference adaptive control was utilized to design an output feedback variable structure adaptive controller in [16]. In recent years, a recursive controller design called adaptive backstepping has received much attention [17]. Adaptive backstepping, which makes use of online parameter estimation laws to deal with parametric uncertainties, is a recursive, Lyapunov-based, nonlinear design method. The backstepping technique is adequate for the nonlinear system that can be transformed into a lower-triangular form. The basic idea is to use some states to construct virtual control laws, which can control other states. Based on this technique, a direct adaptive fuzzy backstepping controller for a class of nonlinear systems was developed in [18]. In [19], an adaptive controller for attitude maneuver of flexible spacecraft with nonlinear characteristics was proposed. In [20], an adaptive control law was presented to estimate the unknown model parameters for a large angle rotational maneuver of flexible spacecraft. In [21], a nonlinear adaptive control law was designed for a roll-coupled aircraft. On the basis of previous research work, a robust adaptive controller was proposed with angular velocity bounded in [22]. This controller was designed for attitude maneuver and vibration reduction with external disturbance and inertia matrix uncertainty. In [23], an adaptive attitude control with active disturbance rejection was presented for rigid spacecraft. An adaptive gain parameter was used to compensate disturbance with known bound. In [24], a quaternion feedback attitude tracking control law was developed for rigid spacecraft with uncertain disturbance. A fault-tolerant controller was designed for distributed tracking of a group of flexible spacecraft in [25] under an undirected communication graph. The singularity and ambiguity can be avoided effectively with this controller. An adaptive fault-tolerant control for attitude tracking of flexible spacecraft was proposed in [26]. The modal variable was compensated by an observer.

A typical feature of the aforementioned attitude control techniques is that the flexible modal variables are assumed to be measurable. Unfortunately, this requirement is not always satisfied in reality due to the impossibility or impracticability of using appropriate sensors. Motivated by the above discussion, we design an observer-based adaptive backstepping attitude controller in this paper for a flexible spacecraft in the presence of external disturbance, unknown inertia matrix parameters, and flexible appendage vibrations. First, an observer is constructed to estimate the flexible modal variables. Then, by designing a novel adaptive law, an OBABC law is developed. Lyapunov stability analysis shows that the proposed control law guarantees asymptotical convergence of the attitude angle and angular velocity of the flexible spacecraft in the presence of bounded disturbance, incorporating the L_2 -gain performance criterion constraint. These results are illustrated through various numerical simulations. Compared with the designed control law in [27, 28], the proposed controller in this paper possesses better performance.

2. Model Description and Problem Statement

2.1. Model Description. By using the Modified Rodrigues Parameters (MRPs), the kinematic equation of a flexible spacecraft can be given by [18]

$$\dot{\sigma} = M\omega, \quad (1)$$

where $\sigma \in \mathbb{R}^3$ denotes the attitude MRPs, $\omega \in \mathbb{R}^3$ represents the angular velocity, and

$$M := \frac{1}{4} [(1 - \sigma^T \sigma)I_3 + 2\sigma^\times + 2\sigma\sigma^T], \quad (2)$$

where $I_3 \in \mathbb{R}^3$ denotes the identity matrix. In addition, for any $\sigma \in \mathbb{R}^3$, σ^\times represents the following cross matrix:

$$\sigma^\times := \begin{bmatrix} 0 & -\sigma_3 & \sigma_2 \\ \sigma_3 & 0 & -\sigma_1 \\ -\sigma_2 & \sigma_1 & 0 \end{bmatrix}. \quad (3)$$

Without loss of generality, we consider a flexible spacecraft with one flexible appendage. Suppose that the flexible appendage has small elastic deformations, then the attitude dynamic equation of a flexible spacecraft can be expressed as [22]

$$\begin{cases} J_0 \dot{\omega} = -\omega^\times (J_0 \omega + \delta^T \psi) + \delta^T (C\psi + K\eta - C\delta\omega) + \mathbf{u} + \mathbf{d}, \\ \dot{\eta} = \psi - \delta\omega, \\ \dot{\psi} = -(C\psi + K\eta - C\delta\omega), \end{cases} \quad (4)$$

where δ is the coupling matrix between the flexible appendage and the rigid body; J is the total inertia matrix of the flexible spacecraft, which is an unknown positive symmetric constant matrix, satisfying $J = J_0 + \delta^T \delta$, where J_0 is the inertia matrix of the main body; η is the modal coordinate vector of the flexible appendage relative to the main body; $\mathbf{u} \in \mathbb{R}^3$ is the external torque acting on the main body; and $\mathbf{d} \in \mathbb{R}^3$ represents the external disturbance torque. In addition, for flexible spacecraft dynamic equation (4), N elastic modes are taken into consideration, with ω_{ni} , $i = 1, \dots, N$, being the natural frequencies and ξ_i , $i = 1, \dots, N$, being the associated damping ratio; \mathbf{C} and \mathbf{K} are the damping matrix and stiffness matrix of the flexible spacecraft, respectively, and are defined as

$$\begin{aligned} \mathbf{C} &:= \text{diag} \{2\xi_i \omega_{ni}, i = 1, 2, \dots, N\}, \\ \mathbf{K} &:= \text{diag} \{\omega_{ni}^2, i = 1, 2, \dots, N\}. \end{aligned} \quad (5)$$

Combining (1) and (4), the attitude control system of the flexible spacecraft can be obtained as

$$\begin{cases} \dot{\sigma} = M\omega, \\ \begin{bmatrix} \dot{\eta} \\ \dot{\psi} \end{bmatrix} = A \begin{bmatrix} \eta \\ \psi \end{bmatrix} + B\delta\omega, \\ J_0\dot{\omega} = -\omega^\times (J_0\omega + \delta^T\psi) + \delta^T(C\psi + K\eta - C\delta\omega) + \mathbf{u} + \mathbf{d}, \end{cases} \quad (6)$$

with

$$\begin{aligned} A &:= \begin{bmatrix} 0 & I_4 \\ -K & -C \end{bmatrix}, \\ B &:= \begin{bmatrix} -I_5 \\ C \end{bmatrix}. \end{aligned} \quad (7)$$

2.2. Problem Statement. In this paper, a typical rest-to-rest attitude maneuver control problem is considered for the flexible spacecraft system (6) in the presence of unknown inertia matrix, unmeasurable flexible modal variables, and external disturbance. Our control goal is to propose an attitude maneuver controller for the flexible spacecraft system (6) combining the criteria of control performance given by L_2 -gain constraint. In such a framework, all the solutions of the designed attitude control system are uniformly bounded. Furthermore, it can be achieved that the attitude variables (σ, ω) and flexible modal variables

(η, ψ) can be rendered small while arbitrarily attenuating the effect of the disturbance \mathbf{d} . In particular, for all $T > 0$, the closed-loop system satisfies the following inequality:

$$\int_0^T \|\gamma(t)\|^2 dt \leq \gamma^2 \int_0^T \|\mathbf{d}\|^2 dt + \beta, \quad \mathbf{d} \in L_2[0, T), \quad (8)$$

where γ is a constant which prescribes the level of vibration suppression. Furthermore, if $\mathbf{d} = 0$, the attitude variables σ, ω will asymptotically converge to zero. In addition, the vibrations induced by attitude maneuver operations are also damped out passively, i.e.,

$$\begin{aligned} \lim_{t \rightarrow \infty} \eta &= 0, \\ \lim_{t \rightarrow \infty} \psi &= 0. \end{aligned} \quad (9)$$

3. Observer-Based Adaptive Backstepping Attitude Control Law

3.1. Backstepping Attitude Control Law Design. First, we assume that the inertia matrix and the upper bound of the external disturbance are known. Then, the following theorem is developed to construct a backstepping attitude control law based on the observer.

Theorem 1. Consider a flexible spacecraft described by (6) with known inertia matrix, unmeasurable flexible modal variables, and external disturbance. When the following observer based backstepping control law is applied to system (6),

$$\begin{cases} \begin{bmatrix} \dot{\hat{\eta}} \\ \dot{\hat{\psi}} \end{bmatrix} = A \begin{bmatrix} \hat{\eta} \\ \hat{\psi} \end{bmatrix} + B(I + \lambda C)\delta\omega, \\ \mathbf{u} = \alpha(\sigma, \hat{\eta}, \hat{\psi}) + \omega^\times J_0\omega - J_0k_1\dot{\mathbf{f}}_1(\sigma) - J_0k_2\dot{\mathbf{f}}_2(\hat{\eta}, \hat{\psi}) + \omega^\times \delta^T \tilde{\psi} - \delta^T C \tilde{\psi} - \delta^T K \tilde{\eta} + \delta^T C \delta\omega - \mathbf{d}, \end{cases} \quad (10)$$

where $\hat{\eta}$ and $\hat{\psi}$ are the estimates of η and ψ , respectively; λ is a nonnegative number; A and B are defined in (7), k_1 and k_2 are positive numbers; and $\alpha(\sigma, \hat{\eta}, \hat{\psi})$, $\mathbf{f}_1(\sigma)$, and $\mathbf{f}_2(\hat{\eta}, \hat{\psi})$ are in the following:

$$\alpha(\sigma, \hat{\eta}, \hat{\psi}) := -k_1\mathbf{f}_1(\sigma) - k_2\mathbf{f}_2(\hat{\eta}, \hat{\psi}), \quad (11)$$

$$\mathbf{f}_1(\sigma) := (I + \sigma^T\sigma)\sigma, \quad (12)$$

$$\mathbf{f}_2(\hat{\eta}, \hat{\psi}) := \delta^T(C\hat{\psi} - 2K\hat{\eta}). \quad (13)$$

The attitude variables (σ, ω) converge to zero.

Proof. The proof is first to choose a Lyapunov function to design a virtual control $\alpha(\sigma, \hat{\eta}, \hat{\psi})$, with which the asymp-

totic stability of the attitude variable σ can be guaranteed. Then, by choosing another Lyapunov function, combined with the preceding stability result and the developed virtual control, the asymptotic stability of the angular velocity ω is guaranteed. The following two steps are considered.

Step 1. Consider the following Lyapunov function candidate,

$$\begin{aligned} V_0 &:= 2k_1\sigma^T\sigma + \frac{k_2}{2} \begin{bmatrix} \tilde{\eta}^T & \tilde{\psi}^T \end{bmatrix} \mathbf{P} \begin{bmatrix} \tilde{\eta} \\ \tilde{\psi} \end{bmatrix} \\ &\quad + \frac{k_2}{2} \begin{bmatrix} \eta^T & \psi^T \end{bmatrix} \begin{bmatrix} 2K + C^2 & C \\ C & 2I \end{bmatrix} \begin{bmatrix} \eta \\ \psi \end{bmatrix}, \end{aligned} \quad (14)$$

where $\tilde{\eta} = \hat{\eta} - \eta$ and $\tilde{\psi} = \hat{\psi} - \psi$ are the observer errors of η and ψ , respectively; and P is a symmetric positive definite matrix in the following form:

$$P := \begin{bmatrix} P_1 & P_2 \\ P_2^T & P_3 \end{bmatrix}, \quad (15)$$

with P_1, P_2 , and P_3 satisfying

$$\begin{aligned} P_1 &:= \frac{1}{\lambda} (2KC^{-1} + C), \\ P_2 &:= \frac{1}{\lambda} I, \quad P_2^T = P_2, \\ P_3 &:= \frac{2}{\lambda} C^{-1}. \end{aligned} \quad (16)$$

Denote

$$V_1 := \frac{k_2}{2} [\tilde{\eta}^T \quad \tilde{\psi}^T] P \begin{bmatrix} \tilde{\eta} \\ \tilde{\psi} \end{bmatrix}. \quad (17)$$

By taking time derivative of V_1 , we have

$$\begin{aligned} \dot{V}_1 &= k_2 \tilde{\eta}^T [-(P_2 C - P_1) \tilde{\psi} - P_2 K \tilde{\eta} + \lambda (P_2 C - P_1) C \delta \omega] \\ &\quad + k_2 \tilde{\psi}^T [-(P_3 C - P_2) \tilde{\psi} - P_3 K \tilde{\eta} + \lambda (P_3 C - P_2) C \delta \omega]. \end{aligned} \quad (18)$$

It can be derived from (14) that

$$\begin{aligned} \dot{V}_0 &= \dot{V}_1 - k_2 \psi^T C \psi - k_2 \eta^T C K \eta + [k_1 \sigma^T (I + \sigma \sigma^T) \\ &\quad + k_2 (\psi^T C - 2\eta^T K) \delta] \omega = -k_2 \tilde{\eta}^T P_2 K \tilde{\eta} - k_2 \tilde{\psi}^T (P_3 C - P_2) \tilde{\psi} \\ &\quad + k_2 \tilde{\eta}^T (-P_2 C + P_1 - K P_3) \tilde{\psi} - k_2 \psi^T C \psi - k_2 \eta^T C K \eta \\ &\quad + [k_1 \sigma^T (I + \sigma \sigma^T) + k_2 (\psi^T C - 2\eta^T K) \delta] \omega \\ &\quad + k_2 [\tilde{\psi}^T \lambda (P_3 C - P_2) C \delta + \tilde{\eta}^T \lambda (P_2 C - P_1) C \delta] \omega. \end{aligned} \quad (19)$$

Substituting (16) into (19) yields

$$\begin{aligned} \dot{V}_0 &= -k_2 \psi^T C \psi - k_2 \eta^T C K \eta - k_2 \lambda^{-1} \tilde{\eta}^T K \tilde{\eta} - k_2 \lambda^{-1} \tilde{\psi}^T \tilde{\psi} \\ &\quad + [k_1 \sigma^T (I + \sigma \sigma^T) + k_2 (\hat{\psi}^T C - 2\hat{\eta}^T K) \delta] \omega. \end{aligned} \quad (20)$$

Let

$$\omega = \alpha(\sigma, \hat{\eta}, \hat{\psi}). \quad (21)$$

Then, substituting the virtual control (11) into (20), one has

$$\dot{V}_1 \leq 0. \quad (22)$$

It is obvious that the following relations hold for the flexible spacecraft (6) under the virtual control (11),

$$\begin{aligned} \lim_{t \rightarrow \infty} \sigma &= 0, \\ \lim_{t \rightarrow \infty} \eta &= 0, \\ \lim_{t \rightarrow \infty} \psi &= 0, \\ \lim_{t \rightarrow \infty} \tilde{\eta} &= 0, \\ \lim_{t \rightarrow \infty} \tilde{\psi} &= 0. \end{aligned} \quad (23)$$

Step 2. Define a new variable z as

$$z := \omega - \alpha(\sigma, \hat{\eta}, \hat{\psi}). \quad (24)$$

Consider the following Lyapunov function candidate:

$$V_2 = V_1 + \frac{1}{2} z^T J_0 z. \quad (25)$$

By taking time derivative of V_2 , we have

$$\begin{aligned} \dot{V}_2 &= \dot{V}_0 + z^T J_0 \dot{z} = \dot{V}_0 + z^T [-\omega^\times J_0 \omega - J_0 \dot{\alpha}(\sigma, \hat{\eta}, \hat{\psi}) \\ &\quad - \omega^\times \delta^T \psi + \delta^T C \psi + \delta^T K \eta - \delta^T C \delta \omega + u + d]. \end{aligned} \quad (26)$$

It can be derived from (11), (12), (13), and (26) that

$$\begin{aligned} \dot{V}_2 &= -k_2 \psi^T C \psi - k_2 \eta^T C K \eta - k_2 \lambda^{-1} \tilde{\eta}^T K \tilde{\eta} - k_2 \lambda^{-1} \tilde{\psi}^T \tilde{\psi} \\ &\quad - \alpha^T(\sigma, \hat{\eta}, \hat{\psi}) \alpha(\sigma, \hat{\eta}, \hat{\psi}) + z^T [-\alpha(\sigma, \hat{\eta}, \hat{\psi}) - \omega^\times J_0 \omega \\ &\quad + J_0 k_1 \dot{f}_1(\sigma) + J_0 k_2 \dot{f}_2(\hat{\eta}, \hat{\psi}) - \omega^\times \delta^T \psi + \delta^T C \psi + \delta^T K \eta \\ &\quad - \delta^T C \delta \omega + u + d]. \end{aligned} \quad (27)$$

Substituting the control law (10) into above relation yields $\dot{V}_2 \leq 0$. Thus, the globally asymptotic stability of the closed-loop flexible spacecraft system (6) can be guaranteed by adopting the controller (10), which means

$$\begin{aligned} \lim_{t \rightarrow \infty} \sigma &= 0, \\ \lim_{t \rightarrow \infty} \hat{\eta} &= 0, \\ \lim_{t \rightarrow \infty} \hat{\psi} &= 0, \\ \lim_{t \rightarrow \infty} \tilde{\eta} &= 0, \\ \lim_{t \rightarrow \infty} \tilde{\psi} &= 0, \\ \lim_{t \rightarrow \infty} \omega &= 0. \end{aligned} \quad (28)$$

Thus, this proof is completed.

3.2. Adaptive Backstepping Attitude Control Law Design. Obviously, the developed observer-based backstepping

control law (10) in Theorem 1 depends explicitly on the inertia matrix J_0 and the upper bound of the external disturbance \mathbf{d} . In fact, the inertia matrix J_0 of the flexible spacecraft (6) is unknown, and the external disturbance is unmeasurable. To this end, an adaptive law is further proposed in the next theorem to estimate the inertia matrix J_0 . In addition, L_2 -gain constrain is used to improve the robustness of the closed-loop system with

the external disturbance \mathbf{d} . Finally, OBABC law is presented.

Theorem 2. Consider a flexible spacecraft described by (6) with known inertia matrix, unmeasurable flexible modal variables, and external disturbance. The following OBABC law can ensure the global asymptotic stability of the system (6):

$$\begin{cases} \begin{bmatrix} \dot{\hat{\boldsymbol{\eta}}} \\ \dot{\hat{\boldsymbol{\psi}}} \end{bmatrix} = A \begin{bmatrix} \hat{\boldsymbol{\eta}} \\ \hat{\boldsymbol{\psi}} \end{bmatrix} + B(I + \lambda C)\delta\boldsymbol{\omega}, \\ \mathbf{u} = \boldsymbol{\alpha}(\boldsymbol{\sigma}, \hat{\boldsymbol{\eta}}, \hat{\boldsymbol{\psi}}) - F(\boldsymbol{\sigma}, \boldsymbol{\omega}, \hat{\boldsymbol{\eta}}, \hat{\boldsymbol{\psi}})\hat{\boldsymbol{\theta}} + \delta^T C \delta\boldsymbol{\omega} + \boldsymbol{\omega}^\times \delta^T \hat{\boldsymbol{\psi}} - \delta^T C \hat{\boldsymbol{\psi}} - \delta^T K \hat{\boldsymbol{\eta}} - \frac{1}{2\varepsilon_1} \boldsymbol{\omega}^\times \delta^T \delta(\boldsymbol{\omega}^\times)^T \mathbf{z}, \\ \dot{\hat{\boldsymbol{\theta}}} = k_4 \Gamma^{-1} F^T(\boldsymbol{\sigma}, \boldsymbol{\omega}, \hat{\boldsymbol{\eta}}, \hat{\boldsymbol{\psi}}) \mathbf{z}, \end{cases} \quad (29)$$

where $\hat{\boldsymbol{\eta}}$ and $\hat{\boldsymbol{\psi}}$ are the estimates of $\boldsymbol{\eta}$ and $\boldsymbol{\psi}$, respectively; λ is a nonnegative number; A and B are defined in (7); Γ is a 6×6 positive definite matrix; $\hat{\boldsymbol{\theta}}$ is the estimate of the inertia matrix J_0 ; $\boldsymbol{\alpha}(\boldsymbol{\sigma}, \hat{\boldsymbol{\eta}}, \hat{\boldsymbol{\psi}})$ is defined in (11); \mathbf{z} is defined in (24); and $k_1, k_2, k_4, \varepsilon_1$, and ε_2 are positive tuning parameters satisfying the following inequalities:

$$\begin{aligned} k_2 \lambda^{-1} K - \frac{1}{2} \varepsilon_2 I - \frac{1}{2} l_1^2 I &> 0, \\ k_2 \lambda^{-1} I - \varepsilon_1 I - \frac{1}{2} l_2^2 I &> 0. \end{aligned} \quad (30)$$

$F(\boldsymbol{\sigma}, \boldsymbol{\omega}, \hat{\boldsymbol{\eta}}, \hat{\boldsymbol{\psi}})$ is defined as

$$F(\boldsymbol{\sigma}, \boldsymbol{\omega}, \hat{\boldsymbol{\eta}}, \hat{\boldsymbol{\psi}}) := -\boldsymbol{\omega}^\times L(\boldsymbol{\omega}) + L(\mathbf{f}), \quad (31)$$

where $L(\mathbf{a}): \mathbb{R}^3 \longrightarrow \mathbb{R}^{3 \times 6}$ is a linear operator defined as

$$L(\mathbf{a}) := \begin{bmatrix} a_1 & 0 & 0 & a_2 & a_3 & 0 \\ 0 & a_2 & 0 & a_1 & 0 & a_3 \\ 0 & 0 & a_3 & 0 & a_1 & a_2 \end{bmatrix}, \quad (32)$$

with $\mathbf{a} = [a_1 \ a_2 \ a_3]^T \in \mathbb{R}^3$.

Proof. Consider the following Lyapunov function candidate,

$$V_3 := V_2 + \frac{1}{2k_4} \tilde{\boldsymbol{\theta}}^T \Gamma \tilde{\boldsymbol{\theta}}, \quad (33)$$

where $\tilde{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}} - \boldsymbol{\theta}$ is the estimation error. Suppose

$$J_0 = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{12} & J_{22} & J_{23} \\ J_{13} & J_{23} & J_{33} \end{bmatrix}. \quad (34)$$

Then, there holds

$$J_0 \mathbf{a} = L(\mathbf{a}) \boldsymbol{\theta}, \quad (35)$$

where $L(\mathbf{a})$ is defined in (32) and

$$\boldsymbol{\theta} = [J_{11} \ J_{22} \ J_{33} \ J_{12} \ J_{13} \ J_{23}]^T. \quad (36)$$

By using (35), the item that contain unknown inertia matrix J_0 in (27) can be integrated and rewritten as

$$-\boldsymbol{\omega}^\times J_0 \boldsymbol{\omega} + J_0 k_1 \dot{\mathbf{f}}_1(\boldsymbol{\sigma}) + J_0 k_2 \dot{\mathbf{f}}_2(\hat{\boldsymbol{\eta}}, \hat{\boldsymbol{\psi}}) = -\boldsymbol{\omega}^\times L(\boldsymbol{\omega}) \boldsymbol{\theta} + L(\mathbf{f}) \boldsymbol{\theta}, \quad (37)$$

where

$$\mathbf{f} = k_1 \dot{\mathbf{f}}_1(\boldsymbol{\sigma}) + k_2 \dot{\mathbf{f}}_2(\hat{\boldsymbol{\eta}}, \hat{\boldsymbol{\psi}}). \quad (38)$$

By replacing $\boldsymbol{\eta}$ and $\boldsymbol{\psi}$ with $(\hat{\boldsymbol{\eta}}, \tilde{\boldsymbol{\eta}})$ and $(\hat{\boldsymbol{\psi}}, \tilde{\boldsymbol{\psi}})$, respectively, the items that contain $\boldsymbol{\eta}$ and $\boldsymbol{\psi}$ in (27) can be integrated and written as follows:

$$\begin{aligned} & \mathbf{z}^T \left(-\boldsymbol{\omega}^\times \delta^T \boldsymbol{\psi} + \delta^T C \boldsymbol{\psi} + \delta^T K \boldsymbol{\eta} \right) \\ & \leq \frac{1}{2\varepsilon_1} \mathbf{z}^T \boldsymbol{\omega}^\times \delta^T \delta(\boldsymbol{\omega}^\times)^T \mathbf{z} + \frac{1}{2\varepsilon_1} \mathbf{z}^T \delta^T C C \delta \mathbf{z} + \frac{1}{2\varepsilon_2} \mathbf{z}^T \delta^T K K \delta \mathbf{z} \\ & \quad + \varepsilon_1 \tilde{\boldsymbol{\psi}}^T \tilde{\boldsymbol{\psi}} + \frac{\varepsilon_2}{2} \tilde{\boldsymbol{\eta}}^T \tilde{\boldsymbol{\eta}} + \mathbf{z}^T \left(-\boldsymbol{\omega}^\times \delta^T \hat{\boldsymbol{\psi}} + \delta^T C \hat{\boldsymbol{\psi}} + \delta^T K \hat{\boldsymbol{\eta}} \right). \end{aligned} \quad (39)$$

Consider the following inequality,

$$\mathbf{z}^T \mathbf{d} \leq \frac{1}{2\gamma^2} \mathbf{z}^T \mathbf{z} + \frac{\gamma^2}{2} \mathbf{d}^T \mathbf{d}. \quad (40)$$

It follows from (31), (37), (39), and (40) that (27) can be rewritten as

$$\begin{aligned} \dot{V}_2 \leq & -k_2 \Psi^T C \Psi - k_2 \eta^T C K \eta - \tilde{\eta}^T \left(k_2 \lambda^{-1} K - \frac{\varepsilon_2}{2} I \right) \tilde{\eta} \\ & - \tilde{\Psi}^T \left(k_2 \lambda^{-1} I - \varepsilon_1 I \right) \tilde{\Psi} - \alpha^T (\sigma, \hat{\eta}, \hat{\Psi}) \alpha (\sigma, \hat{\eta}, \hat{\Psi}) \\ & + \mathbf{z}^T \left[-\alpha (\sigma, \hat{\eta}, \hat{\Psi}) - \delta^T C \delta \omega - \omega^\times \delta^T \hat{\Psi} \right. \\ & + \delta^T C \hat{\Psi} + \delta^T K \hat{\eta} + \frac{1}{2\varepsilon_1} \omega^\times \delta^T \delta (\omega^\times)^T \mathbf{z} \\ & + \frac{1}{2\varepsilon_1} \delta^T C C \delta \mathbf{z} + \frac{1}{2\varepsilon_2} \delta^T K K \delta \mathbf{z} + \frac{1}{2\gamma^2} \mathbf{z} + \mathbf{u}(t) \\ & \left. + F(\sigma, \omega, \hat{\eta}, \hat{\Psi}) \theta \right] + \frac{\gamma^2}{2} \mathbf{d}^T \mathbf{d}. \end{aligned} \quad (41)$$

Define the following evaluating function for the flexible spacecraft (6),

$$\mathbf{y} := \left[l_1 \tilde{\eta}^T \ l_2 \tilde{\Psi}^T \ l_3 \mathbf{z}^T \right]^T, \quad (42)$$

where l_1, l_2 , and l_3 are positive constants.

Substituting the control law (29) and the evaluating function (42) into (41), we have

$$\dot{V}_2 \leq -\mathbf{w} - \frac{1}{2} \mathbf{y}^T \mathbf{y} + \frac{\gamma^2}{2} \mathbf{d}^T \mathbf{d} - \mathbf{z}^T F(\sigma, \omega, \hat{\eta}, \hat{\Psi}) \tilde{\theta}, \quad (43)$$

with

$$\begin{aligned} \mathbf{w} = & k_2 \Psi^T C \Psi + k_2 \eta^T C K \eta + \alpha^T (\sigma, \hat{\eta}, \hat{\Psi}) \alpha (\sigma, \hat{\eta}, \hat{\Psi}) \\ & + k_3 \mathbf{z}^T \mathbf{z} + \tilde{\eta}^T \left(k_2 \lambda^{-1} K - \frac{\varepsilon_2}{2} I - \frac{1}{2} l_1^2 I \right) \tilde{\eta} \\ & + \tilde{\Psi}^T \left(k_2 \lambda^{-1} I - \varepsilon_1 I - \frac{1}{2} l_2^2 I \right) \tilde{\Psi}, \end{aligned} \quad (44)$$

where k_3 is a positive constant. In view of the second condition of (30), it can be obtained that \mathbf{w} is positive definite.

Substituting the adaptive law (29) and the conditions in (30) into (43), it can be deduced from (33) that

$$\begin{aligned} \dot{V}_3 = & \dot{V}_2 + \frac{1}{k_4} \tilde{\theta}^T \Gamma \dot{\tilde{\theta}} \\ \leq & -\mathbf{w} - \frac{1}{2} \mathbf{y}^T \mathbf{y} + \frac{\gamma^2}{2} \mathbf{d}^T \mathbf{d}. \end{aligned} \quad (45)$$

Next, the following two cases are considered to complete the proof.

Case 1. When the external disturbance torque $\mathbf{d} = 0$, there holds

$$\dot{V}_3 \leq -Q(\sigma, \mathbf{z}, \eta, \Psi, \hat{\eta}, \hat{\Psi}, \tilde{\eta}, \tilde{\Psi}), \quad (46)$$

where

$$Q(\sigma, \mathbf{z}, \eta, \Psi, \hat{\eta}, \hat{\Psi}, \tilde{\eta}, \tilde{\Psi}) = \mathbf{w} + \frac{1}{2} \mathbf{y}^T \mathbf{y}. \quad (47)$$

Apparently, $Q(\sigma, \mathbf{z}, \eta, \Psi, \hat{\eta}, \hat{\Psi}, \tilde{\eta}, \tilde{\Psi})$ is a positive semidefinite function. This means $\dot{V}_3 \leq 0$. According to the LaSalle invariance principle, we can conclude that

$$\begin{aligned} \lim_{t \rightarrow \infty} \sigma &= 0, \\ \lim_{t \rightarrow \infty} \omega &= 0, \\ \lim_{t \rightarrow \infty} \tilde{\theta} &= 0, \\ \lim_{t \rightarrow \infty} \eta &= 0, \\ \lim_{t \rightarrow \infty} \Psi &= 0, \\ \lim_{t \rightarrow \infty} \tilde{\eta} &= 0, \\ \lim_{t \rightarrow \infty} \tilde{\Psi} &= 0. \end{aligned} \quad (48)$$

Thus, the stability of the closed-loop flexible spacecraft system (6) with $\mathbf{d} = 0$ can be guaranteed.

Case 2. When the external disturbance torque $\mathbf{d} \neq 0$, (45) can be equivalently written as

$$\dot{V}_3 \leq \frac{\gamma^2}{2} \mathbf{d}^T \mathbf{d} - \frac{1}{2} \mathbf{y}^T \mathbf{y}. \quad (49)$$

Taking integral of both sides of (49) over $[0, T)$, ($\forall T > 0$), yields

$$2[V_3(t) - V_3(0)] \leq \gamma^2 \int_0^T \|\mathbf{d}(t)\|^2 dt - \int_0^T \|\mathbf{y}(t)\|^2 dt. \quad (50)$$

In view of (8), it can be found from the above relation that the closed-loop flexible spacecraft system (6) achieves the L_2 -gain performance with an attenuation level of γ . Thus, the attitude variable σ of the flexible spacecraft system (6) and the estimated flexible modal variables η, ϕ of the observer (29) will converge into a neighborhood of the origin, which means that these states are uniformly ultimately bounded stable.

It can be known from Theorem 2 that the state of the closed-loop system can be driven to origin by adopting the control law (29). In order to further improve the robustness

of the proposed adaptive backstepping control law (29), we introduce project operator [20] to keep the estimates of unknown parameters satisfying:

$$\Omega = \left\{ \hat{\boldsymbol{\theta}} : \boldsymbol{\theta}_{\min} < \hat{\boldsymbol{\theta}}_i < \boldsymbol{\theta}_{\max}, (i = 1, 2, \dots, 6) \right\}. \quad (51)$$

Define

$$E(\boldsymbol{\sigma}, \boldsymbol{\omega}, \hat{\boldsymbol{\eta}}, \hat{\boldsymbol{\psi}}) = k_4 \Gamma^{-1} F^T(\boldsymbol{\sigma}, \boldsymbol{\omega}, \hat{\boldsymbol{\eta}}, \hat{\boldsymbol{\psi}}) \mathbf{z}, \quad (52)$$

and the project operator [27]

$$\text{Proj}(\hat{\boldsymbol{\theta}}, E)_i = \begin{cases} 0, & \text{if } \hat{\boldsymbol{\theta}}_i = \boldsymbol{\theta}_{\max}, E_i(\boldsymbol{\sigma}, \boldsymbol{\omega}, \hat{\boldsymbol{\eta}}, \hat{\boldsymbol{\psi}}) > 0 \text{ or if } \hat{\boldsymbol{\theta}}_i = \boldsymbol{\theta}_{\min}, E_i(\boldsymbol{\sigma}, \boldsymbol{\omega}, \hat{\boldsymbol{\eta}}, \hat{\boldsymbol{\psi}}) < 0, \\ E_i, & \text{otherwise,} \end{cases} \quad (53)$$

with the following properties:

- (1) $\hat{\boldsymbol{\theta}} \in \Omega$
- (2) $\hat{\boldsymbol{\theta}}^T \{ \Gamma \text{Proj}[\hat{\boldsymbol{\theta}}, E(\boldsymbol{\sigma}, \boldsymbol{\omega}, \hat{\boldsymbol{\eta}}, \hat{\boldsymbol{\psi}})] + k_4 F^T(\boldsymbol{\sigma}, \boldsymbol{\omega}, \hat{\boldsymbol{\eta}}, \hat{\boldsymbol{\psi}}) \mathbf{z} \} \geq 0$

By using the project operator (53), the adaptive law (29) can be replaced by

$$\dot{\hat{\boldsymbol{\theta}}} = \text{Proj}[\hat{\boldsymbol{\theta}}, E(\boldsymbol{\sigma}, \boldsymbol{\omega}, \hat{\boldsymbol{\eta}}, \hat{\boldsymbol{\psi}})]. \quad (54)$$

Thus, the following observer-based adaptive backstepping control law can be obtained:

$$\begin{cases} \begin{bmatrix} \dot{\hat{\boldsymbol{\eta}}} \\ \dot{\hat{\boldsymbol{\psi}}} \end{bmatrix} = A \begin{bmatrix} \hat{\boldsymbol{\eta}} \\ \hat{\boldsymbol{\psi}} \end{bmatrix} + B(I + \lambda C) \delta \boldsymbol{\omega}, \\ \mathbf{u} = \boldsymbol{\alpha}(\boldsymbol{\sigma}, \hat{\boldsymbol{\eta}}, \hat{\boldsymbol{\psi}}) - F(\boldsymbol{\sigma}, \boldsymbol{\omega}, \hat{\boldsymbol{\eta}}, \hat{\boldsymbol{\psi}}) \hat{\boldsymbol{\theta}} + \delta^T C \delta \boldsymbol{\omega} + \boldsymbol{\omega}^\times \delta^T \hat{\boldsymbol{\psi}} - \delta^T C \hat{\boldsymbol{\psi}} - \delta^T K \hat{\boldsymbol{\eta}} - \frac{1}{2\varepsilon_1} \boldsymbol{\omega}^\times \delta^T \delta(\boldsymbol{\omega}^\times)^T \mathbf{z} - \frac{1}{2\varepsilon_1} \delta^T C C \delta \mathbf{z} - \frac{1}{2\varepsilon_2} \delta^T K K \delta \mathbf{z} - \left(\frac{1}{2\gamma^2} + \frac{1}{2} l_3^2 + k_3 \right) \mathbf{z}, \\ \dot{\hat{\boldsymbol{\theta}}} = \text{Proj}[\hat{\boldsymbol{\theta}}, E(\boldsymbol{\sigma}, \boldsymbol{\omega}, \hat{\boldsymbol{\eta}}, \hat{\boldsymbol{\psi}})], \end{cases} \quad (55)$$

where $k_1, k_2, k_3, k_4, \varepsilon_1$, and ε_2 are adjustable positive parameters and satisfy (30); $\gamma > 0$ is the given vibration-suppressing value; $\hat{\boldsymbol{\theta}}$ is the estimate of unknown parameters of inertia matrix J_0 ; $\text{Proj}[\hat{\boldsymbol{\theta}}, E(\boldsymbol{\sigma}, \boldsymbol{\omega}, \hat{\boldsymbol{\eta}}, \hat{\boldsymbol{\psi}})]$ is defined in (53); $\boldsymbol{\alpha}(\boldsymbol{\sigma}, \hat{\boldsymbol{\eta}}, \hat{\boldsymbol{\psi}})$ is presented in (11); \mathbf{z} is a function vector of system state $(\boldsymbol{\sigma}, \boldsymbol{\omega}, \hat{\boldsymbol{\eta}}, \hat{\boldsymbol{\psi}})$ denoted in (24); $F(\boldsymbol{\sigma}, \boldsymbol{\omega}, \hat{\boldsymbol{\eta}}, \hat{\boldsymbol{\psi}})$ is a function matrix of system state variable $(\boldsymbol{\sigma}, \boldsymbol{\omega}, \hat{\boldsymbol{\eta}}, \hat{\boldsymbol{\psi}})$ defined in (31); and $L(a): \mathbb{R}^3 \rightarrow \mathbb{R}^{3 \times 6}$ is a linear operator defined in (32).

Compared with the proposed OBABC law (29), the improved OBABC law (55) can ensure the obtained estimated parameters being in an interval and avoid the parameter drift.

4. Example

To demonstrate the effectiveness of the proposed OBABC law (55), a rest-to-rest attitude maneuver problem for the flexible spacecraft (6) is investigated in this section. The

flexible spacecraft is characterized by a main body inertia matrix [22]

$$J = \begin{bmatrix} 350 & 3 & 4 \\ 3 & 270 & 10 \\ 4 & 10 & 190 \end{bmatrix} \text{ kg} \cdot \text{m}^2, \quad (56)$$

and the coupling matrix

$$\delta = \begin{bmatrix} 6.45637 & 1.27814 & 2.15629 \\ -1.25619 & 0.91756 & -1.67264 \\ 1.11687 & 2.48901 & -0.83674 \\ 1.23637 & -2.6581 & -1.12503 \end{bmatrix} \text{ kg}^{1/2} \cdot \text{m}, \quad (57)$$

respectively. The following first four flexible modes have been taken into consideration:

$$\begin{aligned}\omega_{n1} &= 0.7681 \text{ rad/s}, \\ \omega_{n2} &= 1.1038 \text{ rad/s}, \\ \omega_{n3} &= 1.8733 \text{ rad/s}, \\ \omega_{n4} &= 2.5496 \text{ rad/s},\end{aligned}\quad (58)$$

with the associated damping ratios:

$$\begin{aligned}\xi_1 &= 0.005607, \\ \xi_2 &= 0.00862, \\ \xi_3 &= 0.01283, \\ \xi_4 &= 0.02516.\end{aligned}\quad (59)$$

The external disturbance $\mathbf{d}(t)$ is

$$\mathbf{d}(t) = \begin{bmatrix} 0.3 \cos(0.1t) + 0.1 \\ 0.15 \sin(0.1t) + 0.3 \cos(0.1t) \\ 0.3 \sin(0.1t) + 0.1 \end{bmatrix} \text{ N} \cdot \text{m}. \quad (60)$$

The initial attitude condition is $\sigma_1(0) = -0.22425$, $\sigma_2(0) = 0.67278$, and $\sigma_3(0) = -0.44852$, which means that the flexible spacecraft is maneuvering in a rotation of 160° . The controller regulates the MRP to the equilibrium point $[0 \ 0 \ 0]^T$. The initial attitude angular velocity is $\boldsymbol{\omega}(0) = [0 \ 0 \ 0]^T$. The initial values of the flexible modes are $\eta_i(0) = 0.001$, $\psi_i(0) = 0.001$, $i = 1, 2, 3, 4$. The initial values of the observer are assumed to zero, i.e., $\hat{\eta}_i(0) = 0$, $\hat{\psi}_i(0) = 0$, $i = 1, 2, 3, 4$. The initial vector of the adaptive controller is

$$\hat{\boldsymbol{\theta}}(0) = [42 \ 30 \ 35 \ 0.7 - 1.5 \ 2]^T. \quad (61)$$

The tuning parameters of the control law (55) are chosen as

$$\begin{aligned}k_1 &= 0.35, \\ k_2 &= 0.01, \\ k_3 &= 0.1, \\ k_4 &= 0.5, \\ \varepsilon_1 &= 100, \\ \varepsilon_2 &= 100, \\ \gamma &= 0.01, \\ \lambda &= 10^{-5}, \\ l_1 &= l_2 = l_3 = 1,\end{aligned}\quad (62)$$

$$\Gamma = \text{diag} \{0.1, 0.1, 0.1, 0.1, 0.1, 0.1\}.$$

To examine the robustness of the proposed OBABC controller (55), the compared simulation results of the designed controller in [27] and the developed observer-based adaptive backstepping controller are given in Figure 1. The coupling items considered in [27] are less than that in this paper. In addition, it is assumed that there are only three unknown inertia matrix parameters in [27], but in this paper, six unknown parameters are considered. In Figures 1(a)–1(d), the solid red lines (marked by A) represent the response of the flexible spacecraft (6) under the developed OBABC (55) and the dashed blue lines (marked by B) represent the response of the flexible spacecraft (6) under the control law proposed in [27]. The time response of attitude MRP and the angular velocity are shown in Figures 1(a) and 1(b), respectively. In addition, the estimated flexible modal coordinates and the estimated errors are illustrated in Figures 1(c) and 1(d), respectively.

It can be seen from Figures 1(a) and 1(b) that the attitude variable response curves of OBABC (55) law are smoother than the controller proposed in [27]. In addition, the overshoot and the steady-state error of the proposed OBABC law (55) are smaller than that of the proposed controller in [27]. It can be seen from these two pictures that $\boldsymbol{\sigma}$ and $\boldsymbol{\omega}$ converge to the neighborhood of $(0, 0, 0)$ in 60 s. From Figure 1(c), the control torque of the proposed controller in this paper is smaller than that of the controller presented in [27]. It can be observed from Figure 1(d) that the flexible modal displacement of the proposed controller is the smallest.

Furthermore, in order to verify the performance of the constructed observer in this paper, the observer designed in [28] is adopted to estimate the flexible modal variables. The flexible modal estimation errors are shown in Figure 2. The solid red line and the dashed blue line represent the response of the flexible spacecraft under the OBABC law (55) and the developed controller in [28], (named Controller D for convenient), respectively. It can be observed from Figure 2 that the flexible modal estimation errors of the constructed observer in this paper is smaller than that of the observer proposed in [28]. This illustrates the effectiveness of the constructed observer in this paper.

Moreover, the estimation of the unknown elements of the inertia matrix J are shown in Figure 3. It can be concluded from Figure 3 that the estimates of the unknown elements of the inertia matrix converge to a steady level in about 25 s. From the previous comparison results, the proposed OBABC controller (55) can accomplish the attitude control during maneuvers. Moreover, the information of the external disturbance, the inertia matrix, and also the flexible modal variable are not required beforehand. It can be concluded from Figures 1 and 2 that the control performance of the proposed OBABC controller (55) in this paper is better than the compared controller in [27].

5. Conclusion

In this work, an observer-based adaptive backstepping control law is presented for the attitude maneuver control issue of a flexible spacecraft with unknown inertia matrix and

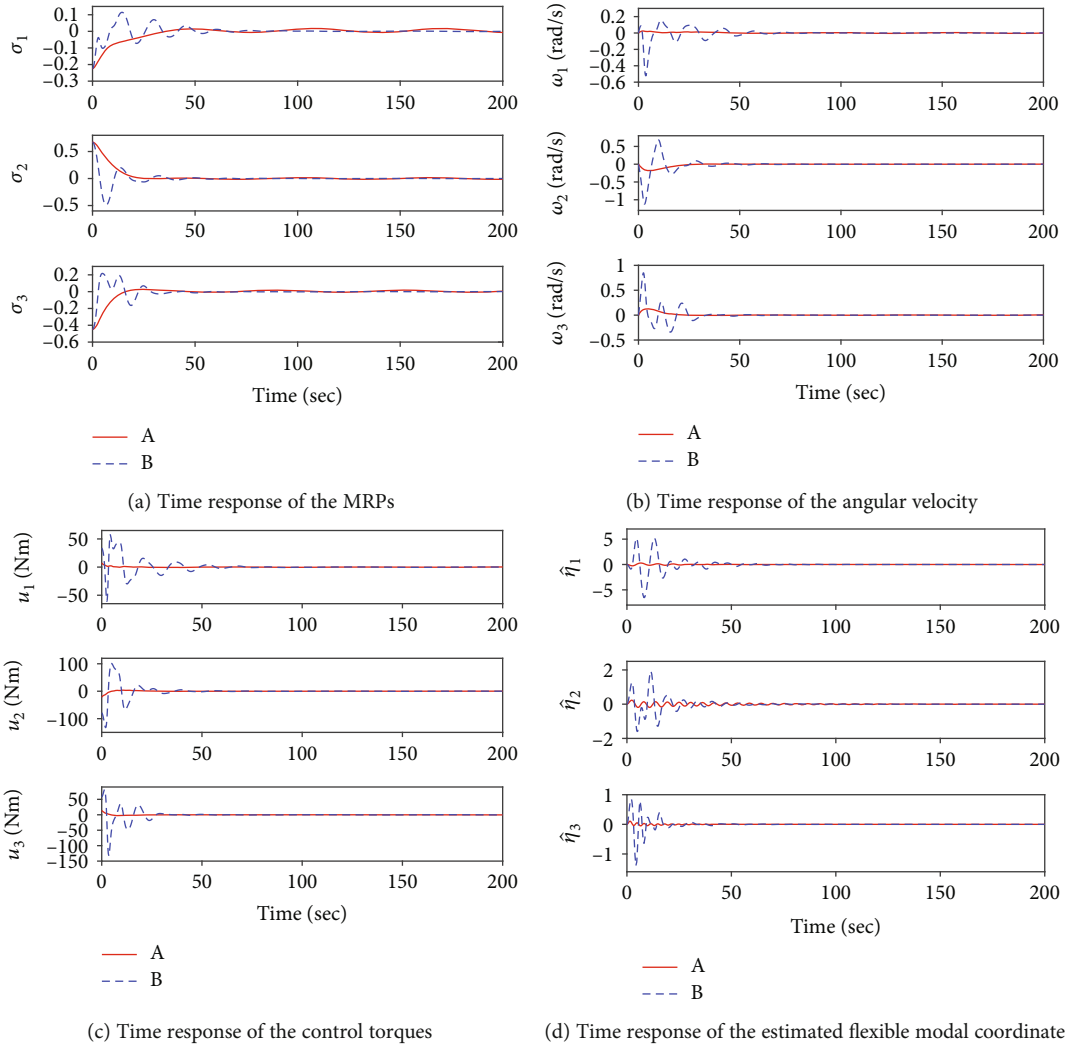


FIGURE 1: Comparison results of OBABC law (55) and controller in [27].

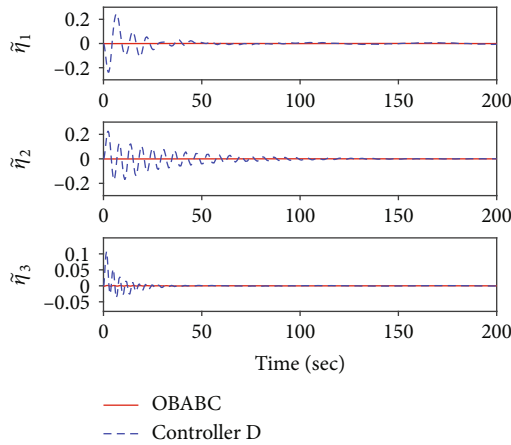
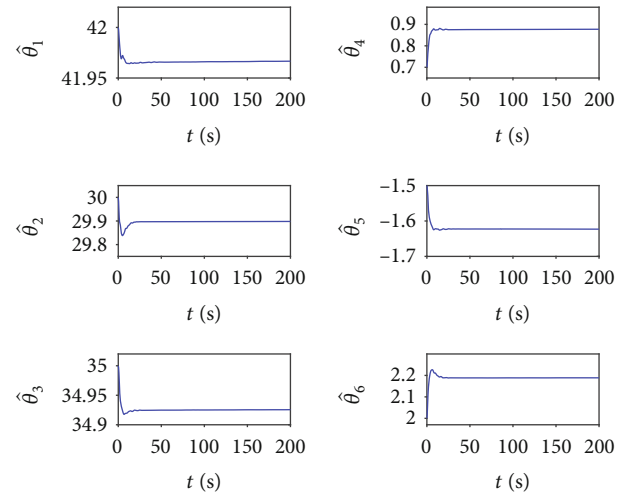


FIGURE 2: Comparison result of OBABC law (55) and controller in [28]

FIGURE 3: The estimates of the unknown elements of inertia matrix J .

flexible modal variables, which is subjected to external disturbance. In the designed control law, the observer is designed to estimate the flexible modal variables. The backstepping control technique is adopted to improve the transient response of the attitude variables of the flexible spacecraft. The adaptive law is constructed to estimate the unknown inertia parameters of the flexible spacecraft. The asymptotical stability of the closed-loop flexible spacecraft system is proven by using Lyapunov theory. Numerical simulations illustrate that the proposed controller can accomplish the attitude maneuver control for the flexible spacecraft in the presence of external disturbance. Further research will consider the attitude tracking or large angle maneuver control issue for flexible spacecraft under actuator saturation problem and design an active vibration suppression controller to suppress the flexible modal vibrations.

Data Availability

All data included in this study are available upon request by contact with the corresponding author.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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