Research Article

Robust H_∞ Consensus Control for Linear Discrete-Time Swarm Systems with Parameter Uncertainties and Time-Varying Delays

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Received 19 November 2018; Accepted 27 June 2019; Published 24 July 2019

Academic Editor: Paolo Gasbarri

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Robust H_∞ consensus control problems of linear swarm systems with parameter uncertainties and time-varying delays are investigated. In this literature, a linear consensus protocol for high-order discrete-time swarm systems is proposed. Firstly, the robust H_∞ consensus control problem of discrete-time swarm systems is transformed into a robust H_∞ control problem of a set of independent uncertain systems. Secondly, sufficient linear matrix inequality conditions for robust H_∞ consensus analysis of discrete-time swarm systems are given by the stability theory, and a H_∞ performance level γ is determined meanwhile. Thirdly, the convergence result is derived as a final consensus value of swarm systems. Finally, numerical examples are presented to demonstrate theoretical results.

1. Introduction

As a hot topic of swarm systems, distributed cooperative control for swarm systems has gained an increasing attention from many communities because of its wide applications in various areas including rendezvous [1–3], filtering [4–6], formation control [7–9], synchronization of power-driven machine [10–12], and other areas [13–16]. It should be also pointed out that some relevant novel nearest results in aerospace have been reported in [17–19]. Furthermore, as an essential research aspect of distributed cooperative control for swarm systems, the consensus problem, which is referred to an agreement on some common values by all agents, has received extensive research in recent years [20–22].

According to the difference of the system dynamics, the research objective of the consensus problem for swarm systems can be classified into three types: first-order swarm systems, second-order swarm systems, and high-order swarm systems. For first-order swarm systems, the consensus problem with input and communication delays was studied by the frequency analysis in [23]. The consensus of second-order agents with multiple time-varying delays was investi-

gated based on the graph theory in [24]. In combination with realistic applications, high-order swarm systems are more valuable and general than low-order swarm systems due to the practicality and complexity of the former. Some significant theoretical achievements about the consensus of continuous-time high-order swarm systems have been discovered in [25]. However, digital signals are processed rather than analog signals under engineering background, so discrete-time swarm systems should be paid more attention. The distributed output feedback consensus problem of general linear model discrete-time swarm systems with fixed topology was presented in [26, 27]. Based upon the leader-following model, the consensus problem of discrete-time swarm systems with directed graphs and external disturbances was investigated in [28]. Furthermore, the consensus problem of discrete-time swarm systems with switching topologies based on the matrix inequality theory was presented in [29–31].

As the industrial technology continues to develop, the research on the control theory becomes more mature. Due to operating state changes, external disturbances and model errors of swarm systems in the real world, robust H_∞ consensus control problems of swarm systems were wildly

explored by researchers, as shown in [32–34]. Robust H_∞ consensus control problems of first-order swarm systems with external disturbances and model uncertainties were presented in [32]. The second-order robust H_∞ consensus control problem of swarm systems with measurement noises and asymmetric delays was studied in [33]. Distributed H_2 and H_∞ consensus control problems of high-order linear swarm systems with (x, y, z) -dependent noise were investigated in [34]. However, due to the general existence of parameter uncertainties and time delays of swarm systems in practical application, it is more meaningful and important to take these constraints into consideration in the theoretical analysis of robust H_∞ consensus control problems of high-order linear discrete-time swarm systems, which can be deemed as inadequately available in current literature.

As is well known, precise dynamics models of swarm systems can be hardly established in practice, and many kinds of faults in systems can also contribute to uncertainties of the dynamics model. Besides, components of systems will be out of order or aged with the increase of working hours. Therefore, researches on the robust H_∞ consensus problem of high-order linear discrete-time swarm systems with parameter uncertainties are very valuable and significant. Moreover, time delays cannot be avoided in nature and they take place so commonly in biological, physical, and electrical systems. The acquisition of parameters becomes more difficult when time delays exist in systems. If time delays are ignored when the consensus problem of swarm systems with parameter uncertainties is discussed, the stability of the system is hard to be guaranteed. Motivated by the above analysis, for high-order linear discrete-time swarm systems with parameter uncertainties to achieve consensus by the robust H_∞ control method, time delays are necessary to be considered and the problem remains the most open as we know.

This paper studies robust H_∞ control consensus problems of high-order linear discrete-time swarm systems with parameter uncertainties and time-varying delays, and the innovations of this paper are threefold: (i) a sufficient linear matrix inequality (LMI) condition such that swarm systems can achieve the robust H_∞ consensus subject to parameter uncertainties and time-varying delays is derived; (ii) a H_∞ performance index γ of swarm systems is obtained; and (iii) the final consensus value of swarm systems is developed. Compared with the previous work on the robust H_∞ consensus of high-order linear swarm systems, the novelties of this literature are also threefold. Firstly, only the robust H_∞ consensus of high-order linear swarm systems was investigated in [35–39], where the H_∞ performance index was not obtained. However, in order to show the performance of the robust consensus of high-order linear discrete-time swarm systems, the H_∞ performance index, which is necessary and important, was involved in this paper. Secondly, the final consensus value was obtained by LMI theory in this paper, which could better show the advantages of the proposed consensus protocol, but most existing works for the robust H_∞ consensus of high-order linear swarm systems subject to parameter

uncertainties and time delays, including [35–38], did not consider this useful point. Thirdly, to better correspond with the practical application, influences of time delays on the robust consensus of swarm systems were sufficiently considered in this paper, which did not appear in [37–39].

The remainder of the paper is organized as follows. Section 2 states several important conclusions of the graph theory and the problem description in which the robust H_∞ consensus protocol is proposed. Section 4 shows a sufficient LMI condition that can guarantee the robust H_∞ -consensus of swarm systems and a final consensus value of swarm systems. The effectiveness of theoretical results is illustrated via simulation examples in Section 5. Finally, Section 5 draws the conclusion.

Notations. A matrix can be defined to be positive if each entry of the matrix is positive. A square matrix can be known as Schur stable if each eigenvalue of the matrix lies in the open unit disk $\Theta = \{\lambda \in \mathbb{C} : |\lambda| < 1\}$. $\text{diag}\{a_{11}, a_{22}, \dots, a_{nn}\}$ denotes the diagonal matrix, where $a_{11}, a_{22}, \dots, a_{nn}$ are diagonal entries. $\mathbb{E}$ refers to the expectation. The symbol $\otimes$ represents the Kronecker product. The transpose of a matrix M is denoted as M^T . I stands for the identity matrix with appropriate dimension. Let $\mathbf{1}_N = [1, \dots, 1]^T$ be the vector of N ones with all components 1.

2. Preliminaries and Problem Description

2.1. Graph Theory. In general, exchanges of information among N agents in swarm systems can be shown as a directed graph $G = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, where $\mathcal{V} = \{v_1, \dots, v_N\}$ denotes the node set, $\mathcal{E} \subseteq \{(v_i, v_j) : v_i, v_j \in \mathcal{V}\}$ represents the edge set, and $\mathcal{A} = [a_{ij}]$ refers to the weighted adjacency matrix with the element $a_{ij} > 0$ if $(v_j, v_i) \in \mathcal{E}$ while $a_{ij} = 0$ if $(v_j, v_i) \notin \mathcal{E}$. Define the node index of graph G as a finite index set $\mathcal{J} = \{1, 2, \dots, N\}$. Let $N_i = \{v_j \in \mathcal{V} : (v_j, v_i) \in \mathcal{E}\}$ stand for the set of all neighbours of node v_i , where the information flows from node v_j to node v_i . Let subset $J \subseteq \mathcal{V}$ of the nodes in graph G be a cluster. Suppose that $N_J = \cup_{v_i \in J} N_i = \{v_j \in \mathcal{V} : v_i \in J, (v_i, v_j) \in \mathcal{E}\}$ represents the neighbour set of a cluster N_J . It can be defined that the in-degree matrix of node v_i is $\deg_{\text{in}}(v_i) = \sum_{j=1}^N a_{ji}$, and the out-degree matrix of node v_i is $\deg_{\text{out}}(v_i) = \sum_{j=1}^N a_{ij}$. Note that a diagonal matrix $\Delta = [\Delta_{ij}]$ can be provided to describe the degree matrix of digraph G , which is defined as follows:

$$\Delta_{ij} = \begin{cases} 0, & i \neq j, \\ \deg_{\text{out}}(v_i), & i = j. \end{cases} \quad (1)$$

The definition of the Laplacian matrix in connection with digraph G can be shown in matrix form as $\mathcal{L}(G) = L = \Delta - \mathcal{A}$.

2.2. Problem Description. Consider a linear swarm system composed of N homogeneous agents marked as $1, 2, \dots, N$,

in which the i th agent's dynamics can be expressed as a state update equation

$$\begin{cases} \mathbf{x}_i(k+1) = (A + \Delta A)\mathbf{x}_i(k) + (B + \Delta B)\mathbf{u}_i(k) + B_\omega \boldsymbol{\omega}_i(k), \\ \mathbf{z}_i(k) = C\mathbf{x}_i(k), \end{cases} \quad (2)$$

where $\mathbf{x}_i(k) \in \mathbb{R}^d$ indicates the state vector; $\mathbf{u}_i(k) = [u_{i,1}(k), u_{i,2}(k), \dots, u_{i,m_1}(k)]^T \in \mathbb{R}^{m_1}$ and $\boldsymbol{\omega}_i(k) = [\omega_{i,1}(k), \omega_{i,2}(k), \dots, \omega_{i,m_2}(k)]^T \in \mathbb{R}^{m_2}$, respectively, represent the input matrix and the external disturbance; $A \in \mathbb{R}^{d \times d}$, $B \in \mathbb{R}^{d \times m_1}$, $B_\omega \in \mathbb{R}^{d \times m_2}$, and $C \in \mathbb{R}^{l \times d}$ are the given constant matrices; $\mathbf{z}_i(k) \in \mathbb{R}^l$ is the control output; ΔA and ΔB stand for uncertainties in the system matrix and input channel, respectively, such that

$$[\Delta A \ \Delta B] = DF[E_1 \ E_2], \quad (3)$$

where D denotes a positive constant matrix and F refers to an unknown matrix function which meets constraints as follows:

$$F^T F \leq I. \quad (4)$$

The parameter uncertainties ΔA and ΔB are considered permissible when equations (3) and (4) hold meanwhile.

Remark 1. When there is the external disturbance in system (2), external disturbance $\boldsymbol{\omega}_i(k) = [\omega_{i,1}(k), \omega_{i,2}(k), \dots, \omega_{i,m_2}(k)]^T$

$(k)^T \in \mathbb{R}^{m_2}$ belongs to l_2 signal shown in [33, 40], which satisfies $\sum_{k=1}^n |\omega_{i,j}(k)|^2 < \infty (i = 1, \dots, N, j = 1, \dots, m_2)$, that is, the energy of the external disturbance in system (2) is bounded on any finite interval. In this case, the robust H_∞ control problem can be analysed in an appropriate and meaningful way.

To achieve consensus for linear swarm system (2), the following control protocols are adopted:

$$\mathbf{u}_i(k) = K \sum_{j \in N_i} a_{ij} (\mathbf{x}_j(k - \tau(k)) - \mathbf{x}_i(k)), \quad (5)$$

where $0 \leq \tau(k) \leq \tau_{\max}$, K is a constant gain matrix with appropriate dimensions, and $\tau(k) \in \mathbb{Z}^*$ denotes time-varying delays of swarm systems. Then, to show the consensus analysis and design for swarm system (2), the definition is presented as follows:

Definition 1. For the given gain matrix K and any given bounded initial states, swarm system (2) with consensus protocol (5) are assured to arrive consensus if there exists a vector-valued $\mathbf{c}(k)$ depending on the initial states such that $\lim_{k \rightarrow \infty} (\mathbf{x}(k) - \mathbf{1}_N \otimes \mathbf{c}(k)) = \mathbf{0}$, where $\mathbf{c}(k)$ denotes the final consensus value.

Let $\mathbf{x}(k) = [\mathbf{x}_1^T(k), \dots, \mathbf{x}_N^T(k)]^T$, $\boldsymbol{\omega}_x(k) = [\boldsymbol{\omega}_1^T(k), \dots, \boldsymbol{\omega}_N^T(k)]^T$, and $\mathbf{z}(k) = [\mathbf{z}_1^T(k), \dots, \mathbf{z}_N^T(k)]^T$, and we can write swarm system (2) with equation (5) into a compact form as follows:

$$\begin{cases} \mathbf{x}(k+1) = (I_N \otimes (A + \Delta A) - \Delta \otimes B_{\Delta BK})\mathbf{x}(k) + ((\Delta - L) \otimes B_{\Delta BK})\mathbf{x}(k - \tau(k)) + (I_N \otimes B_\omega)\boldsymbol{\omega}_x(k), \\ \mathbf{x}(k) = \boldsymbol{\phi}(k), k \in \{-\tau_{\max}, -\tau_{\max} + 1, \dots, 0\}, \\ \mathbf{z}(k) = (I_N \otimes C)\mathbf{x}(k), \end{cases} \quad (6)$$

where $B_{\Delta BK} = (B + \Delta B)K$ and $\boldsymbol{\phi}(k)$ is an initial value at k .

The suboptimal robust H_∞ consensus control problem of system (6) is stated to find a distributed protocol (5) such that

- (i) with $\boldsymbol{\omega}_x(k) = \mathbf{0}$, system (6) is asymptotically stable for all acceptable uncertain matrices F
- (ii) with $\boldsymbol{\omega}_x(k)$ interpreted as deterministic l_2 signal, the transfer function from $\boldsymbol{\omega}_x(k)$ to $\mathbf{z}(k)$ of system (6), which is denoted by $T_{\omega z}$, satisfies $\|T_{\omega z}\|_\infty < \gamma$ for all acceptable uncertain matrices F and a given allowable scalar $\gamma > 0$, where $\|T_{\omega z}\|_\infty$ is the H_∞ norm of $T_{\omega z}$, defined by $\|T_{\omega z}\|_\infty = \sup_{\omega_x \in \mathbb{R}^{Nd}} \bar{\sigma}(T_{\omega z}(j\omega))$

In order to make an in-depth analysis for the robust H_∞ consensus control problem of system (6), we presume henceforth that the communication graph G is connected and give the following lemma about the graph theory.

Lemma 1 (see [41]). *For the Laplacian matrix L associated with an undirected graph G , the following description holds: (i) 0 is an eigenvalue of L , and its corresponding eigenvector is $\mathbf{1}_N$, and (ii) 0 is a simple eigenvalue of L , and all the other eigenvalues are positive and real if G is connected.*

Assume that $\lambda_i (i = 1, 2, \dots, N)$ denotes eigenvalues of matrix $L \in \mathbb{R}^N$ of an undirected graph G . Moreover, $\lambda_1 = 0$,

which satisfies $\lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_N$, is related to its corresponding eigenvector $\bar{\mathbf{u}}_1 = (1/(N)^{-1/2})\mathbf{1}_N$. Thus, an orthogonal matrix can be found in the form

$$U = \begin{bmatrix} \frac{1}{\sqrt{N}} & \frac{\mathbf{1}_{N-1}^T}{\sqrt{N}} \\ \frac{\mathbf{1}_{N-1}^T}{\sqrt{N}} & \bar{U} \end{bmatrix}, \quad (7)$$

such that $U^T L U = \text{diag} \{\lambda_1, \lambda_2, \dots, \lambda_N\}$.

Theorem 1. For a given $\gamma > 0$, system (6) is asymptotically stable and $\|T_{\omega z}\|_\infty < \gamma$, if and only if the following N systems are simultaneously and asymptotically stable and all of the H_∞ norms of their transfer function matrices are less than γ :

$$\begin{cases} \tilde{\mathbf{x}}_i(k+1) = A_{\Delta_i BK} \tilde{\mathbf{x}}_i(k) + B_{\Delta \lambda_i K} \tilde{\mathbf{x}}_i(k - \tau(k)) + B_\omega \tilde{\omega}_{i,x}(k), \\ \tilde{\mathbf{x}}(k) = \tilde{\phi}(k), \quad k \in \{-\tau_{\max}, -\tau_{\max} + 1, \dots, 0\}, \\ \tilde{\mathbf{z}}_i(k) = C \tilde{\mathbf{x}}_i(k), \quad (i = 1, 2, \dots, N), \end{cases} \quad (8)$$

where $A_{\Delta_i BK} = (A + \Delta A) - \Delta_i B_{\Delta BK}$ and $B_{\Delta \lambda_i K} = (\Delta_{ii} - \lambda_i)(B + \Delta B)K$.

Proof. Let $\lambda_i (i = 1, 2, \dots, N)$ be the eigenvalues of matrix $L \in \mathbb{R}^N$ for an undirected topology G , where $\lambda_1 = 0$ with the corresponding eigenvector $\bar{\mathbf{u}}_1 = (1/(N)^{-1/2})\mathbf{1}_N$, and $\lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_N$. An orthogonal matrix can be found in the form

$$U = \begin{bmatrix} \frac{1}{\sqrt{N}} & \frac{\mathbf{1}_{N-1}^T}{\sqrt{N}} \\ \frac{\mathbf{1}_{N-1}^T}{\sqrt{N}} & \bar{U} \end{bmatrix}, \quad (9)$$

such that $U^T L U = \text{diag} \{\lambda_1, \lambda_2, \dots, \lambda_N\} = \Lambda$. Assume that

$$\begin{aligned} \tilde{\mathbf{x}}(k) &= (U^T \otimes I_d) \mathbf{x}(k) = [\tilde{\mathbf{x}}_c^T(k), \tilde{\mathbf{x}}_r^T(k)]^T \\ &= [\tilde{\mathbf{x}}_1^T(k), \tilde{\mathbf{x}}_2^T(k), \dots, \tilde{\mathbf{x}}_N^T(k)]^T. \end{aligned} \quad (10)$$

Then, system (6) can be rewritten in terms of $\tilde{\mathbf{x}}(k)$ as

$$\begin{cases} \tilde{\mathbf{x}}(k+1) = (I_N \otimes (A + \Delta A) - \Delta \otimes B_{\Delta BK}) \tilde{\mathbf{x}}(k) - ((\Delta - \Lambda) \otimes B_{\Delta BK}) \tilde{\mathbf{x}}(k - \tau(k)) + (U^T \otimes B_\omega) \omega_x(k), \\ \tilde{\mathbf{x}}(k) = \tilde{\phi}(k), \quad k \in \{-\tau_{\max}, -\tau_{\max} + 1, \dots, 0\}, \\ \mathbf{z}(k) = (U \otimes C) \tilde{\mathbf{x}}(k). \end{cases} \quad (11)$$

Furthermore, reformulate the disturbance variable $\omega_x(k)$ and the performance variable $\mathbf{z}(k)$ via

$$\tilde{\omega}_x(k) = (U^T \otimes I_{m_2}) \omega_x(k) = [\tilde{\omega}_{1,x}^T(k), \tilde{\omega}_{2,x}^T(k), \dots, \tilde{\omega}_{N,x}^T(k)]^T, \quad (12)$$

$$\tilde{\mathbf{z}}(k) = (U^T \otimes I_l) \mathbf{z}(k) = [\tilde{z}_1^T(k), \tilde{z}_2^T(k), \dots, \tilde{z}_N^T(k)]^T. \quad (13)$$

Subsequently, substituting (12) and (13) into (11) gives

$$\begin{cases} \tilde{\mathbf{x}}(k+1) = (I_N \otimes (A + \Delta A) - \Delta \otimes B_{\Delta BK}) \tilde{\mathbf{x}}(k) - ((\Delta - \Lambda) \otimes (B + \Delta B)K) \tilde{\mathbf{x}}(k - \tau(k)) + (I_N \otimes B_\omega) \tilde{\omega}_x(k), \\ \tilde{\mathbf{x}}(k) = \tilde{\phi}(k), \quad k \in \{-\tau_{\max}, -\tau_{\max} + 1, \dots, 0\}, \\ \tilde{\mathbf{z}}(k) = (I_N \otimes C) \tilde{\mathbf{x}}(k). \end{cases} \quad (14)$$

In fact, system (14) consists of N individual systems in (6). Denote by $\|T_{\tilde{\omega} \tilde{\mathbf{z}}}\|_\infty$ and $\|T_{\tilde{\omega} \tilde{\mathbf{z}}_i}\|_\infty$ the transfer function matrices of system (14) and (6), respectively. Afterwards, we can deduce from equations (6), (12), (13), and (14) that

$$T_{\tilde{\omega} \tilde{\mathbf{z}}} = \text{diag} (T_{\tilde{\omega}_1 \tilde{\mathbf{z}}_1}, T_{\tilde{\omega}_2 \tilde{\mathbf{z}}_2}, \dots, T_{\tilde{\omega}_N \tilde{\mathbf{z}}_N}) = (U^T \otimes I_l) T_{\omega z} (U \otimes I_{m_2}), \quad (15)$$

which implies that

$$\|T_{\tilde{\omega} \tilde{\mathbf{z}}}\|_\infty = \max_{i=2,3,\dots,N} \|T_{\tilde{\omega} \tilde{\mathbf{z}}_i}\|_\infty = \|T_{\omega z}\|_\infty. \quad (16)$$

Therefore, the proof of Theorem 1 is completed.

In addition, it is worth mentioning that, let

$$\begin{aligned}\tilde{\mathbf{x}}(k) &= [\tilde{\mathbf{x}}_c^T(k), \tilde{\mathbf{x}}_r^T(k)]^T = [\tilde{\mathbf{x}}_c^T(k), \tilde{\mathbf{x}}_{r,2}^T(k), \dots, \tilde{\mathbf{x}}_{r,N}^T(k)]^T, \\ \tilde{\boldsymbol{\omega}}_x(k) &= [\tilde{\boldsymbol{\omega}}_{c,x}^T(k), \tilde{\boldsymbol{\omega}}_{r,x}^T(k)]^T = [\tilde{\boldsymbol{\omega}}_{c,x}^T(k), \tilde{\boldsymbol{\omega}}_{r,2x}^T(k), \dots, \tilde{\boldsymbol{\omega}}_{r,Nx}^T(k)]^T,\end{aligned}$$

$$\tilde{\mathbf{z}} = [\tilde{\mathbf{z}}_c^T(k), \tilde{\mathbf{z}}_r^T(k)]^T = [\tilde{\mathbf{z}}_c^T(k), \tilde{\mathbf{z}}_{r,2}^T(k), \dots, \tilde{\mathbf{z}}_{r,N}^T(k)]^T. \quad (17)$$

By Lemma 1, we can convert system (14) into N subsystems as follows:

$$\begin{cases} \tilde{\mathbf{x}}_c(k+1) = ((A + \Delta A) - \Delta_{11} B_{\Delta BK}) \tilde{\mathbf{x}}_c(k) + \Delta_{11} (B + \Delta B) K \tilde{\mathbf{x}}_c(k - \tau(k)) + B_{\omega} \tilde{\boldsymbol{\omega}}_{c,x}(k), \\ \tilde{\mathbf{x}}_c(k) = \tilde{\boldsymbol{\phi}}_c(k), \quad k \in \{-\tau_{\max}, -\tau_{\max} + 1, \dots, 0\}, \\ \tilde{\mathbf{z}}_c(k) = C \tilde{\mathbf{x}}_c(k), \end{cases} \quad (18)$$

$$\begin{cases} \tilde{\mathbf{x}}_{r,i}(k+1) = A_{\Delta_{ii}BK} \tilde{\mathbf{x}}_{r,i}(k) - B_{\Delta_{ii}K} \tilde{\mathbf{x}}_{r,i}(k - \tau(k)) + B_{\omega} \tilde{\boldsymbol{\omega}}_{i,x}(k), \\ \tilde{\mathbf{x}}_{r,i}(k) = \tilde{\boldsymbol{\phi}}_{r,i}(k), \quad k \in \{-\tau_{\max}, -\tau_{\max} + 1, \dots, 0\}, \\ \tilde{\mathbf{z}}_{r,i}(k) = C \tilde{\mathbf{x}}_{r,i}(k), \end{cases} \quad (i = 2, 3, \dots, N). \quad (19)$$

Obviously, swarm system (6) can achieve consensus if subsystem (19) is asymptotically stable. Note that subsystem (18) determines the final consensus value of swarm system (6), and the details of it will be discussed below.

Remark 2. By Theorem 1, the distributed H_{∞} consensus control problem of swarm system (6) is transformed into the robust H_{∞} control problems of N subsystems (8), so that a set of independent systems which have the same dimensions as a single agent in (2) is obtained. Therefore, the computational complexity is decreased evidently and the stability theory of swarm systems can be used. The essential tools producing this result by right of the state space decomposition approach.

3. Main Results

In this section, the robust stability and existence conditions of the H_{∞} consensus control problem for swarm system (2) with protocol (5) is provided by using a basis-dependent Lyapunov function and the LMI approach. Above all, we should introduce the following two lemmas.

Lemma 2 (Jensen inequality [42]). *For any constant matrix $M \geq 0$, $M \in \mathbb{R}^{n \times n}$, and $\boldsymbol{\psi}(i) \in \mathbb{R}^n$, positive integers α_1 and α_2 satisfying $\alpha_2 \geq \alpha_1 \geq 1$, an inequality holds as follows:*

$$-(\alpha_2 - \alpha_1 + 1) \sum_{i=\alpha_1}^{\alpha_2} \boldsymbol{\psi}^T(i) M \boldsymbol{\psi}(i) \leq - \left(\sum_{i=\alpha_1}^{\alpha_2} \boldsymbol{\psi}(i) \right)^T M \left(\sum_{i=\alpha_1}^{\alpha_2} \boldsymbol{\psi}(i) \right). \quad (20)$$

Lemma 3. *Suppose that equal positive integer time-varying delays $\tau(k)$ exist in whole communication network of swarm*

system (2). Meanwhile, the undirected graph G is fixed and connected. Then, by use of control protocol (5), the robust consensus of swarm system (2) with H_{∞} -norm consensus performance bound γ can be achieved globally asymptotically if we can prove that, there are a feedback matrix K with suitable dimensions and four positive-definite matrices P , R , Q_1 , and Q_2 with $d \times d$ dimensions satisfying

$$\tilde{\Omega}_i = \begin{bmatrix} \tilde{\Omega}_{i11} & \tilde{\Omega}_{i12} & \tilde{\Omega}_{i13} & \tilde{\Omega}_{i14} \\ * & \tilde{\Omega}_{i22} & 0 & \tilde{\Omega}_{i24} \\ * & * & \tilde{\Omega}_{i33} & 0 \\ * & * & * & \tilde{\Omega}_{i44} \end{bmatrix} < 0, \quad i = 2, \dots, N, \quad (21)$$

where

$$\begin{aligned}\tilde{\Omega}_{i11} &= A_{\Delta_{ii}BK}^T P A_{\Delta_{ii}BK} - P + \tau_{\max} Q_1 + Q_2 \\ &\quad + \tau_{\max} (A_{\Delta_{ii}BK} - I)^T R (A_{\Delta_{ii}BK} - I) + C^T C - \frac{R}{\tau_{\max}}, \\ \tilde{\Omega}_{i12} &= \tau_{\max} (A_{\Delta_{ii}BK} - I)^T R B_{\Delta_{ii}K} + A_{\Delta_{ii}BK}^T P B_{\Delta_{ii}K}, \\ \tilde{\Omega}_{i13} &= \frac{R}{\tau_{\max}}, \quad \tilde{\Omega}_{i14} = A_{\Delta_{ii}BK}^T P B_{\omega} + \tau_{\max} (A_{\Delta_{ii}BK} - I)^T R B_{\omega}, \\ \tilde{\Omega}_{i22} &= B_{\Delta_{ii}K}^T P B_{\Delta_{ii}K} + \tau_{\max} B_{\Delta_{ii}K}^T R B_{\Delta_{ii}K} - Q_1, \\ \tilde{\Omega}_{i24} &= B_{\Delta_{ii}K}^T P B_{\omega} + \tau_{\max} B_{\Delta_{ii}K}^T R B_{\omega}, \\ \tilde{\Omega}_{i33} &= -Q_2 - \frac{R}{\tau_{\max}}, \quad \tilde{\Omega}_{i44} = B_{\omega}^T P B_{\omega} + \tau_{\max} B_{\omega}^T R B_{\omega} - \gamma^2 I.\end{aligned} \quad (22)$$

Proof. For purpose of consensus analysis, we construct a Lyapunov function as follows:

$$V(k) = \sum_{i=1}^5 V_i(k), \quad (23)$$

where

$$\begin{aligned} V_1(k) &= \sum_{i=2}^N \tilde{\mathbf{x}}_{r,i}^T(k) P \tilde{\mathbf{x}}_{r,i}(k), \\ V_2(k) &= \sum_{i=2}^N \sum_{s=k-\tau(k)}^{k-1} \tilde{\mathbf{x}}_{r,i}^T(s) Q_1 \tilde{\mathbf{x}}_{r,i}(s), \\ V_3(k) &= \sum_{i=2}^N \sum_{s=k-\tau_{\max}}^{k-1} \tilde{\mathbf{x}}_{r,i}^T(s) Q_2 \tilde{\mathbf{x}}_{r,i}(s), \\ V_4(k) &= \sum_{i=2}^N \sum_{s=-\tau_{\max}+1}^{-1} \sum_{l=k+s}^{k-1} \tilde{\mathbf{x}}_{r,i}^T(l) Q_1 \tilde{\mathbf{x}}_{r,i}(l), \\ V_5(k) &= \sum_{i=2}^N \sum_{s=-\tau_{\max}}^{-1} \sum_{l=k+s}^{k-1} \tilde{\boldsymbol{\eta}}_{r,i}^T(l) R \tilde{\boldsymbol{\eta}}_{r,i}(l), \end{aligned} \quad (24)$$

for $\tilde{\boldsymbol{\eta}}_{r,i}(l) = \tilde{\mathbf{x}}_{r,i}(l+1) - \tilde{\mathbf{x}}_{r,i}(l)$.

Then, we can define the derivative of $V(k)$ as

$$\begin{aligned} \Delta V(k) &= V(k+1) - V(k) = \sum_{i=1}^5 \Delta V_i(k) \\ &= \sum_{i=1}^5 (V_i(k+1) - V_i(k)). \end{aligned} \quad (25)$$

From the solution of (8), $\Delta V_1(k)$ can be rewritten as follows:

$$\begin{aligned} \Delta V_1(k) &= V_1(k+1) - V_1(k) \\ &= \sum_{i=2}^N (\tilde{\mathbf{x}}_{r,i}^T(k+1) P \tilde{\mathbf{x}}_{r,i}(k+1) - \tilde{\mathbf{x}}_{r,i}^T(k) P \tilde{\mathbf{x}}_{r,i}(k)) \\ &= \sum_{i=2}^N \left(\tilde{\mathbf{x}}_{r,i}^T(k) \left(A_{\Delta_{ii}BK}^T P A_{\Delta_{ii}BK} - P \right) \tilde{\mathbf{x}}_{r,i}(k) \right) \\ &\quad - 2 \sum_{i=2}^N \left(\tilde{\mathbf{x}}_{r,i}^T(k) A_{\Delta_{ii}BK}^T P B_{\Delta_{\lambda_i K}} \tilde{\mathbf{x}}_{r,i}(k - \tau(k)) \right) \\ &\quad + 2 \sum_{i=2}^N \left(\tilde{\mathbf{x}}_{r,i}^T(k) A_{\Delta_{ii}BK}^T P B_{\omega} \tilde{\boldsymbol{\omega}}_{i,x}(k) \right) \\ &\quad - 2 \sum_{i=2}^N \left(\tilde{\mathbf{x}}_{r,i}^T(k - \tau(k)) B_{\Delta_{\lambda_i K}}^T P B_{\omega} \tilde{\boldsymbol{\omega}}_{i,x}(k) \right) \\ &\quad + \sum_{i=2}^N \left(\tilde{\mathbf{x}}_{r,i}^T(k - \tau(k)) B_{\Delta_{\lambda_i K}}^T P B_{\Delta_{\lambda_i K}} \tilde{\mathbf{x}}_{r,i}(k - \tau(k)) \right) \\ &\quad + \sum_{i=2}^N \left(\tilde{\boldsymbol{\omega}}_{i,x}^T(k) B_{\omega}^T P B_{\omega} \tilde{\boldsymbol{\omega}}_{i,x}(k) \right). \end{aligned} \quad (26)$$

$\Delta V_2(k)$ can be reformulated as

$$\begin{aligned} \Delta V_2(k) &= V_2(k+1) - V_2(k) \\ &= \sum_{i=2}^N (\tilde{\mathbf{x}}_{r,i}^T(k) Q_1 \tilde{\mathbf{x}}_{r,i}(k)) \\ &\quad - \sum_{i=2}^N (\tilde{\mathbf{x}}_{r,i}^T(k - \tau(k)) Q_1 \tilde{\mathbf{x}}_{r,i}(k - \tau(k))) \\ &\quad + \sum_{i=2}^N \sum_{s=k+1-\tau(k+1)}^{k-1} \tilde{\mathbf{x}}_{r,i}^T(s) Q_1 \tilde{\mathbf{x}}_{r,i}(s) \\ &\quad - \sum_{i=2}^N \sum_{s=k+1-\tau(k)}^{k-1} \tilde{\mathbf{x}}_{r,i}^T(s) Q_1 \tilde{\mathbf{x}}_{r,i}(s) \\ &\leq \sum_{i=2}^N (\tilde{\mathbf{x}}_{r,i}^T(k) Q_1 \tilde{\mathbf{x}}_{r,i}(k)) \\ &\quad - \sum_{i=2}^N (\tilde{\mathbf{x}}_{r,i}^T(k - \tau(k)) Q_1 \tilde{\mathbf{x}}_{r,i}(k - \tau(k))) \\ &\quad + \sum_{i=2}^N \sum_{s=k+1-\tau_{\max}}^{k-1} \tilde{\mathbf{x}}_{r,i}^T(s) Q_1 \tilde{\mathbf{x}}_{r,i}(s). \end{aligned} \quad (27)$$

$\Delta V_3(k)$ can be rewritten as

$$\begin{aligned} \Delta V_3(k) &= V_3(k+1) - V_3(k) \\ &= \sum_{i=2}^N (\tilde{\mathbf{x}}_{r,i}^T(k) Q_2 \tilde{\mathbf{x}}_{r,i}(k) - \tilde{\mathbf{x}}_{r,i}^T(k - \tau_{\max}) Q_2 \tilde{\mathbf{x}}_{r,i}(k - \tau_{\max})). \end{aligned} \quad (28)$$

$\Delta V_4(k)$ can be reformulated as

$$\begin{aligned} \Delta V_4(k) &= V_4(k+1) - V_4(k) = \sum_{i=2}^N \sum_{s=-\tau_{\max}+1}^{-1} \sum_{l=k+1+s}^{k-1} \tilde{\mathbf{x}}_{r,i}^T(l) Q_1 \tilde{\mathbf{x}}_{r,i}(l) \\ &\quad - \sum_{i=2}^N \sum_{s=-\tau_{\max}+1}^{-1} \sum_{l=k+s}^{k-1} \tilde{\mathbf{x}}_{r,i}^T(l) Q_1 \tilde{\mathbf{x}}_{r,i}(l) \\ &= \sum_{i=2}^N \sum_{s=-\tau_{\max}+1}^{-1} (\tilde{\mathbf{x}}_{r,i}^T(k) Q_1 \tilde{\mathbf{x}}_{r,i}(k) - \tilde{\mathbf{x}}_{r,i}^T(k+s) Q_1 \tilde{\mathbf{x}}_{r,i}(k+s)) \\ &= \sum_{i=2}^N \left((\tau_{\max} - 1) \tilde{\mathbf{x}}_{r,i}^T(k) Q_1 \tilde{\mathbf{x}}_{r,i}(k) \right. \\ &\quad \left. - \sum_{l=k-\tau_{\max}+1}^{k-1} \tilde{\mathbf{x}}_{r,i}^T(l) Q_1 \tilde{\mathbf{x}}_{r,i}(l) \right). \end{aligned} \quad (29)$$

$\Delta V_5(k)$ can be rewritten as

$$\begin{aligned}
\Delta V_5(k) &= V_5(k+1) - V_5(k) \\
&= \sum_{i=2}^N \sum_{s=-\tau_{\max}}^{-1} \tilde{\eta}_{r,i}^T(k) R \tilde{\eta}_{r,i}(k) - \tilde{\eta}_{r,i}^T(k+s) R \tilde{\eta}_{r,i}(k+s) \\
&= \sum_{i=2}^N \left(\tau_{\max} \tilde{\eta}_{r,i}^T(k) R \tilde{\eta}_{r,i}(k) - \sum_{l=k-\tau_{\max}}^{k-1} \tilde{\eta}_{r,i}^T(l) R \tilde{\eta}_{r,i}(l) \right). \quad (30)
\end{aligned}$$

Furthermore, the term $\tau_{\max} \tilde{\eta}_{r,i}^T(k) R \tilde{\eta}_{r,i}(k)$ can be shown that

$$\begin{aligned}
\tau_{\max} \tilde{\eta}_{r,i}^T(k) R \tilde{\eta}_{r,i}(k) &= \tau_{\max} (\tilde{\mathbf{x}}_{r,i}(k+1) - \tilde{\mathbf{x}}_{r,i}(k))^T R (\tilde{\mathbf{x}}_{r,i}(k+1) \\
&\quad - \tilde{\mathbf{x}}_{r,i}(k)) \tau_{\max} ((A_{\Delta_{ii}BK} - I) \tilde{\mathbf{x}}_{r,i}(k) \\
&\quad + B_{\Delta_{\lambda_i K}} \tilde{\mathbf{x}}_{r,i}(k - \tau(k)) + B_{\omega} \tilde{\omega}_{i,x}(k))^T R \\
&\quad \times ((A_{\Delta_{ii}BK} - I) \tilde{\mathbf{x}}_{r,i}(k) + B_{\Delta_{\lambda_i K}} K \tilde{\mathbf{x}}_{r,i}(k \\
&\quad - \tau(k)) + B_{\omega} \tilde{\omega}_{i,x}(k)) = \tau_{\max} \tilde{\mathbf{x}}_{r,i}^T(k) \\
&\quad \times ((A_{\Delta_{ii}BK} - I)^T R (A_{\Delta_{ii}BK} - I)) \tilde{\mathbf{x}}_{r,i}(k) \\
&\quad + 2 \tilde{\mathbf{x}}_{r,i}^T(k) (A_{\Delta_{ii}BK} - I)^T R B_{\Delta_{\lambda_i K}} \tilde{\mathbf{x}}_{r,i}(k - \tau(k)) \\
&\quad + 2 \tau_{\max} \tilde{\mathbf{x}}_{r,i}^T(k) (A_{\Delta_{ii}BK} - I)^T R B_{\omega} \tilde{\omega}_{i,x}(k) \\
&\quad + 2 \tau_{\max} \tilde{\mathbf{x}}_{r,i}^T(k - \tau(k)) B_{\Delta_{\lambda_i K}}^T R B_{\omega} \tilde{\omega}_{i,x}(k) \\
&\quad + \tau_{\max} \tilde{\mathbf{x}}_{r,i}^T(k - \tau(k)) B_{\Delta_{\lambda_i K}}^T R B_{\Delta_{\lambda_i K}} \tilde{\mathbf{x}}_{r,i} \\
&\quad \times (k - \tau(k)) + \tau_{\max} \tilde{\omega}_{i,x}^T(k) B_{\omega}^T R B_{\omega} \tilde{\omega}_{i,x}(k). \quad (31)
\end{aligned}$$

At the same time, according to Lemma 2, the following inequality holds:

$$\begin{aligned}
- \sum_{l=k-\tau_{\max}}^{k-1} \tilde{\eta}_{r,i}^T(l) R \tilde{\eta}_{r,i}(l) &\leq - \left(\sum_{l=k-\tau_{\max}}^{k-1} \tilde{\eta}_{r,i}(l) \right)^T \frac{R}{\tau_{\max}} \left(\sum_{l=k-\tau_{\max}}^{k-1} \tilde{\eta}_{r,i}(l) \right) \\
&= - (\tilde{\mathbf{x}}_{r,i}(k) - \tilde{\mathbf{x}}_{r,i}(k - \tau_{\max}))^T \frac{R}{\tau_{\max}} (\tilde{\mathbf{x}}_{r,i}(k) \\
&\quad - \tilde{\mathbf{x}}_{r,i}(k - \tau_{\max})). \quad (32)
\end{aligned}$$

Let $\tilde{\xi}_i(k) = [\tilde{\mathbf{x}}_{r,i}^T(k), \tilde{\mathbf{x}}_{r,i}^T(k - \tau(k)), \tilde{\mathbf{x}}_{r,i}^T(k - \tau_{\max}), \tilde{\omega}_{i,x}^T(k)]^T$. Then, it follows from (8) and (25)–(32) that

$$\begin{aligned}
&\sum_{i=2}^N \left(\tilde{\mathbf{z}}_{r,i}^T(k) \tilde{\mathbf{z}}_{r,i}(k) - \gamma^2 \tilde{\omega}_{i,x}^T(k) \tilde{\omega}_{i,x}(k) \right) + \Delta V(k) \\
&= \sum_{i=2}^N \left(\tilde{\mathbf{x}}_{r,i}^T(k) C^T C \tilde{\mathbf{x}}_{r,i}(k) - \gamma^2 \tilde{\omega}_{i,x}^T(k) \tilde{\omega}_{i,x}(k) \right) \quad (33) \\
&\quad + \sum_{i=1}^5 \Delta V_i(k) \leq \sum_{i=2}^N \left(\tilde{\xi}_i^T(k) \tilde{\Omega}_i \tilde{\xi}_i(k) \right) < 0,
\end{aligned}$$

if system (8) is asymptotically stable. This implies that the robust consensus of swarm system (2) with H_{∞} -norm con-

sensus performance bound γ can be achieved globally asymptotically by the use of control protocol (5) if matrix inequality (21) holds.

Therefore, the conclusion of Lemma 3 is drawn.

Remark 3. In Lemma 3, for making sure that system (8) can achieve robust consensus with a H_{∞} performance index γ , a sufficient condition is obtained. Nevertheless, it is not difficult to find that (21) is a nonlinear matrix inequality (NMI) and therein lies parameter uncertainties.

To cope with the uncertain matrices F and the nonlinear terms of (21), the following two lemmas are given.

Lemma 4 (see [43]). For given matrices Y , D , and E of appropriate dimensions where Y is symmetric, a matrix inequality holds as follows:

$$Y + DFE + E^T F^T D^T < 0, \quad (34)$$

for all F subject to $F^T F \leq I$, if and only if there exists a scalar $\varepsilon > 0$ such that

$$Y + \varepsilon DD^T + \varepsilon^{-1} E^T E < 0. \quad (35)$$

Lemma 5 (Schur complement [44]). The linear matrix inequality

$$\begin{pmatrix} Q(x) & S(x) \\ S(x)^T & R(x) \end{pmatrix} > 0, \quad (36)$$

where $Q(x) = Q(x)^T$, $R(x) = R(x)^T$, and $S(x)$ are dependent on x , is the identical meaning as anyone of the following two descriptions:

$$(1) \quad Q(x) > 0, \quad R(x) - S(x)^T Q(x)^{-1} S(x) > 0$$

$$(2) \quad R(x) > 0, \quad Q(x) - S(x) R(x)^{-1} S(x)^T > 0$$

Theorem 2. Consider swarm system (2) with a fixed, undirected, and connected communication topology G . The robust consensus of swarm system (2) with H_{∞} -norm consensus performance bound γ can be achieved globally asymptotically by use of control protocol (5) if we can find two scaling factors $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$, a feedback matrix K with suitable dimensions and six positive-definite matrices P , S , R , M , Q_1 , and Q_2 with $d \times d$ dimensions which are a settlement of the minimization problem as follows:

$$\min \quad \text{tr}[PS + RM], \quad (37)$$

subject to

$$\begin{bmatrix}
-S + \varepsilon_1 DD^T & 0 & A_{iBK} & B_{\lambda_i K} & 0 & B_\omega & 0 & 0 \\
0 & -\frac{1}{\tau_{\max}} M + \varepsilon_2 DD^T & A_{iBK} - I & B_{\lambda_i K} & 0 & B_\omega & 0 & 0 \\
A^T & (A - I)^T & -P + \tau_{\max} Q_1 + Q_2 + C^T C - \frac{R}{\tau_{\max}} & 0 & \frac{R}{\tau_{\max}} & 0 & E_1^T & E_1^T \\
B_{\lambda_i K}^T & B_{\lambda_i K}^T & 0 & -Q_1 & 0 & 0 & ((\Delta_{ii} - \lambda_i) E_2 K)^T & ((\Delta_{ii} - \lambda_i) E_2 K)^T \\
0 & 0 & \frac{R^T}{\tau_{\max}} & 0 & -Q_2 - \frac{R}{\tau_{\max}} & 0 & 0 & 0 \\
0 & 0 & E_1 & (\Delta_{ii} - \lambda_i) E_2 K & 0 & -\gamma^2 I & 0 & 0 \\
0 & 0 & E_1 & (\Delta_{ii} - \lambda_i) E_2 K & 0 & 0 & -\varepsilon_1 I & 0 \\
0 & 0 & 0 & K & 0 & 0 & 0 & -\varepsilon_2 I
\end{bmatrix} < 0, \quad i = 2, 3, \dots, N, \quad (38)$$

$$\begin{bmatrix} P & I \\ I & S \end{bmatrix} > 0, P > 0, S > 0, \quad (39)$$

$$\begin{bmatrix} R & I \\ I & M \end{bmatrix} > 0, R > 0, M > 0, \quad (40)$$

where $A_{iBK} = A - \Delta_{ii} BK$ and $B_{\lambda_i K} = (\Delta_{ii} - \lambda_i) BK$. It can also be said if the minimization problem (37)–(40) has an available settlement P, S, R, M, Q_1, Q_2 , and K , the robust consensus of swarm system (2) with H_∞ -norm consensus performance

bound γ can be achieved globally asymptotically by the use of control protocol (5).

Proof. Inequality (21) can be expressed as

$$\begin{bmatrix} A_{\Delta_{ii}BK}^T \\ B_{\Delta\lambda_i K}^T \\ 0 \\ B_\omega^T \end{bmatrix} P \begin{bmatrix} A_{\Delta_{ii}BK} & B_{\Delta\lambda_i K} & 0 & B_\omega \end{bmatrix} + \tau_{\max} \begin{bmatrix} (A_{\Delta_{ii}BK} - I)^T \\ B_{\Delta\lambda_i K}^T \\ 0 \\ B_\omega^T \end{bmatrix} R \begin{bmatrix} (A_{\Delta_{ii}BK} - I) & B_{\Delta\lambda_i K} & 0 & B_\omega \end{bmatrix} \\
+ \begin{bmatrix} -P + \tau_{\max} Q_1 + Q_2 + C^T C - \frac{R}{\tau_{\max}} & 0 & \frac{R}{\tau_{\max}} & 0 \\ * & -Q_1 & 0 & 0 \\ * & * & -Q_2 - \frac{R}{\tau_{\max}} & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} \leq 0. \quad (41)$$

According to Lemma 5, one can obtain that inequality (41) can be rewritten as follows:

$$\begin{bmatrix}
-P^{-1} & 0 & A_{\Delta_{ii}BK} & B_{\Delta\lambda_i K} & 0 & B_\omega \\
* & -\frac{1}{\tau_{\max}}R^{-1} & A_{\Delta_{ii}BK} - I & B_{\Delta\lambda_i K} & 0 & B_\omega \\
* & * & -P + \tau_{\max}Q_1 + Q_2 + C^T C - \frac{R}{\tau_{\max}} & 0 & \frac{R}{\tau_{\max}} & 0 \\
* & * & * & -Q_1 & 0 & 0 \\
* & * & * & * & -Q_2 - \frac{R}{\tau_{\max}} & 0 \\
* & * & * & * & * & -\gamma^2 I
\end{bmatrix} < 0. \quad (42)$$

By replacing $\Delta A = DFE_1$ and $\Delta B = DFE_2$, inequality (42) is equivalently written as follows:

$$\begin{bmatrix}
-P^{-1} & 0 & A_{iBK} & B_{\lambda_i K} & 0 & B_\omega \\
* & -\frac{1}{\tau_{\max}}R^{-1} & A_{iBK} - I & B_{\lambda_i K} & 0 & B_\omega \\
* & * & -P + \tau_{\max}Q_1 + Q_2 + C^T C - \frac{R}{\tau_{\max}} & 0 & \frac{R}{\tau_{\max}} & 0 \\
* & * & * & -Q_1 & 0 & 0 \\
* & * & * & * & -Q_2 - \frac{R}{\tau_{\max}} & 0 \\
* & * & * & * & * & -\gamma^2 I
\end{bmatrix}
+ \begin{bmatrix} D \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} F \begin{bmatrix} 0 & 0 & E_1 & (\Delta_{ii} - \lambda_i)E_2 K & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ D \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} F \begin{bmatrix} 0 & 0 & E_1 & (\Delta_{ii} - \lambda_i)E_2 K & 0 & 0 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & E_1 & (\Delta_{ii} - \lambda_i)E_2 K & 0 & 0 \end{bmatrix}^T F^T \begin{bmatrix} D \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T + \begin{bmatrix} 0 & 0 & E_1 & (\Delta_{ii} - \lambda_i)E_2 K & 0 & 0 \end{bmatrix}^T F^T \begin{bmatrix} 0 \\ D \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T \leq 0. \quad (43)$$

Step 1: Seek out a set of available values of P_0, S_0, R_0 , and M_0 for LMI (38)–(40), if there does not exist, then exit. Set $k = 0$.
 Step 2: Set $P_k = P_0, S_k = S_0, R_k = R_0$, and $M_k = M_0$ and seek out P_k, S_k, R_k , and M_k which are a settlement for the LMI problem $\min \text{tr}(P_k S + S_k P + R_k M + M_k R)$ subject to LMI (38)–(40).
 Step 3: If $\text{tr}(P_k S + S_k P + R_k M + M_k R) < 4d + \varepsilon$, then exit. Otherwise, set $k = k + 1$ and switch to Step 2.

ALGORITHM 1: A possible way to the optimization problem (37)–(40).

In virtue of Lemma 4 and Lemma 5, inequality (43) comes into existence for all acceptable uncertain matrices F

if and only if we can prove that there are two scaling factors $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ satisfying

$$\begin{bmatrix} -P^{-1} + \varepsilon_1 D D^T & 0 & A_{iBK} & B_{\lambda_i K} & 0 & B_\omega & 0 & 0 \\ 0 & -\frac{1}{\tau_{\max}} R^{-1} + \varepsilon_2 D D^T & A_{iBK} - I & B_{\lambda_i K} & 0 & B_\omega & 0 & 0 \\ A^T & (A - I)^T & -P + \tau_{\max} Q_1 + Q_2 + C^T C - \frac{R}{\tau_{\max}} & 0 & \frac{R}{\tau_{\max}} & 0 & E_1^T & E_1^T \\ B_{\lambda_i K}^T & B_{\lambda_i K}^T & 0 & -Q_1 & 0 & 0 & ((\Delta_{ii} - \lambda_i) E_2 K)^T & ((\Delta_{ii} - \lambda_i) E_2 K)^T \\ 0 & 0 & \frac{R^T}{\tau_{\max}} & 0 & -Q_2 - \frac{R}{\tau_{\max}} & 0 & 0 & 0 \\ 0 & 0 & E_1 & (\Delta_{ii} - \lambda_i) E_2 K & 0 & -\gamma^2 I & 0 & 0 \\ 0 & 0 & E_1 & (\Delta_{ii} - \lambda_i) E_2 K & 0 & 0 & -\varepsilon_1 I & 0 \\ 0 & 0 & 0 & K & 0 & 0 & 0 & -\varepsilon_2 I \end{bmatrix} < 0, \quad i = 2, 3, \dots, N. \quad (44)$$

By the Cone Complementarity Linearization (CCL) algorithm in [45], we set $S = P^{-1}$ and $M = R^{-1}$. Then, the optimization problem (37)–(40) can be reasonably settled by transforming the nonlinear matrix inequality (NMI) (21).

Thus, the proof of Theorem 2 is completed.

Remark 4. According to Theorem 2, notice that the optimization problem (40)–(45) transformed from the NMI (21) can be reasonably solved based upon Algorithm 1. Hereafter, the linear swarm system (2) with the control protocol (5) achieves robust H_∞ consensus.

It should be pointed out that the final outcome of first “exit” in Step 1 is that available initial values of P_0, S_0, R_0 , and M_0 cannot be obtained. In this case, we set $k = 0$ and

repeat Step 1. As long as the available values are obtained, we switch to Step 2. Then, second “exit” in Step 3 means that suitable values of P_k, S_k, R_k , and M_k are obtained.

Theorem 3. With $\omega_x(k)$ interpreted as deterministic l_2 signal, when swarm system (6) achieves robust H_∞ consensus with parameter uncertainties and time-varying delays, the final consensus value $c(k)$ satisfies

$$\lim_{k \rightarrow \infty} (c(k) - x_C(k)) = 0, \quad (45)$$

where $x_C(k)$ can be the following form:

$$\begin{cases} x_C(k) = \mathbf{1}_N \otimes \left(\left(\frac{1}{N} A_{\Delta_{11}BK}^k \sum_{i=1}^N x_i(0) \right) + \sum_{i=0}^{k-1} \sum_{l=0}^{k-i-1} \sum_{j=1}^N \left(\frac{1}{N} A_{\Delta_{11}BK}^i B_\omega \omega_{x,j}(l) \right) \right), & (k \leq \tau(k)), \\ x_C(k+1) = A_{\Delta_{11}BK} x_C(k) + (B + \Delta B) K x_C(k - \tau(k)) + \mathbf{1}_N^T \otimes \left(\frac{1}{N} \sum_{i=1}^N B_\omega \omega_{x,i}(k) \right), & (k > \tau(k)), \end{cases} \quad (46)$$

for $A_{\Delta_{11}BK} = (A + \Delta A) - \Delta_{11} B_{\Delta_{11}BK}$.

Proof. In the following, we consider two cases for the proof.

Case 1. ($k \leq \tau(k)$).

In this case, system (18) can be equivalently written as follows:

$$\begin{cases} \tilde{\mathbf{x}}_c(k+1) = A_{\Delta_{11}BK} \tilde{\mathbf{x}}_c(k) + B_{\omega} \tilde{\omega}_{c,x}(k), \\ \tilde{\mathbf{z}}_c(k) = C \tilde{\mathbf{x}}_c(k). \end{cases} \quad (47)$$

Assume that $\mathbf{x}_c(k) = (U \otimes I_d) [\tilde{\mathbf{x}}_c^T(k), 0]^T$ and $\mathbf{x}_{\bar{c}}(k) = (U \otimes I_d) [0, \tilde{\mathbf{x}}_r^T(k)]^T$, then from (10), we can uniquely decompose $\mathbf{x}(k)$ into $\mathbf{x}(k) = \mathbf{x}_c(k) + \mathbf{x}_{\bar{c}}(k)$. Considering the mentioned above, we can also conclude that if the robust H_{∞} consensus of system (6) is achieved, the subsystem (19) should be *Schur stable*, which implies that the response of system (19) due to $\mathbf{x}_{\bar{c}}(0)$ complies with $\lim_{k \rightarrow \infty} \mathbf{x}_{\bar{c}}(k) = 0$. Therefore, the final consensus value $\mathbf{c}(k)$ depends totally on $\mathbf{x}_c(k)$. Since $[\tilde{\mathbf{x}}_c^T(k), 0]^T = \mathbf{e}_1 \otimes \tilde{\mathbf{x}}(k)$, we have $\mathbf{x}_c(0) = \bar{\mathbf{u}}_1 \otimes \tilde{\mathbf{x}}_c(0) = \bar{\mathbf{u}}_1 \otimes ((\mathbf{e}_1^T \otimes I_d) \tilde{\mathbf{x}}(0))$, and because $\tilde{\mathbf{x}}(0) = (U^T \otimes I_d) \mathbf{x}(0)$, then we can obtain $\mathbf{x}_c(0) = \bar{\mathbf{u}}_1 \otimes \tilde{\mathbf{x}}_c(0) = \bar{\mathbf{u}}_1 \otimes ((\mathbf{e}_1^T \otimes I_d) \tilde{\mathbf{x}}(0))$, that is to say

$$\begin{aligned} \mathbf{x}_c(0) &= \bar{\mathbf{u}}_1 \otimes ((\mathbf{e}_1^T \otimes I_d) * (U^T \otimes I_d) \mathbf{x}(0)) \\ &= \bar{\mathbf{u}}_1 \otimes ((\mathbf{e}_1^T U^T \otimes I_d) \mathbf{x}(0)) \\ &= \bar{\mathbf{u}}_1 \otimes \left(\left(\frac{1}{\sqrt{N}} \mathbf{1}_N^T \otimes I_d \right) \mathbf{x}(0) \right) = \mathbf{1}_N^T \otimes \left(\frac{1}{N} \sum_{i=1}^N \mathbf{x}_i(0) \right). \end{aligned} \quad (48)$$

Likewise, let $\omega_{c,x}(k) = (U \otimes I_d) [\tilde{\omega}_{c,x}^T(k), 0]^T$ and $\tilde{\omega}_{\bar{c},x}(k) = (U \otimes I_d) [0, \tilde{\omega}_{r,x}^T(k)]^T$, then by (12), $\mathbf{x}(k)$ can be uniquely decomposed as $\omega_x(k) = \omega_{c,x}(k) + \omega_{\bar{c},x}(k)$. If system (6) achieves the robust H_{∞} consensus, the response of system (19) due to $\omega_{\bar{c},x}(0)$ also should comply with $\lim_{k \rightarrow \infty} \omega_{\bar{c},x}(k) = 0$. Since $[\tilde{\omega}_{c,x}^T(k), 0]^T = \mathbf{e}_1 \otimes \tilde{\omega}_x(k)$, we have $\tilde{\omega}_{c,x}(k) = \bar{\mathbf{u}}_1 \otimes \tilde{\omega}_x(k) = \bar{\mathbf{u}}_1 \otimes ((\mathbf{e}_1^T \otimes I_d) \tilde{\omega}_x(k))$, and because $\tilde{\omega}_x(k) = (U^T \otimes I_d) \omega_x(k)$, then we can obtain $\omega_{c,x}(k) = \bar{\mathbf{u}}_1 \otimes \tilde{\omega}_{c,x}(k) = \bar{\mathbf{u}}_1 \otimes ((\mathbf{e}_1^T \otimes I_d) \tilde{\omega}_x(k))$, that is

$$\begin{aligned} \omega_{c,x}(k) &= \bar{\mathbf{u}}_1 \otimes ((\mathbf{e}_1^T \otimes I_d) * (U^T \otimes I_d) \omega_x(k)) \\ &= \bar{\mathbf{u}}_1 \otimes ((\mathbf{e}_1^T U^T \otimes I_d) \omega_x(k)) \\ &= \bar{\mathbf{u}}_1 \otimes \left(\left(\frac{1}{\sqrt{N}} \mathbf{1}_N^T \otimes I_d \right) \omega_x(k) \right) \\ &= \mathbf{1}_N^T \otimes \left(\frac{1}{N} \sum_{i=1}^N \omega_{x,i}(k) \right). \end{aligned} \quad (49)$$

Hence, we have

$$\begin{aligned} \mathbf{x}_c(k) &= A_{\Delta_{11}BK}^k \mathbf{x}_c(0) + \mathbf{1}_N \otimes \sum_{i=0}^{k-1} \left(A_{\Delta_{11}BK}^i B_{\omega} \tilde{\omega}_{c,x}(k-i-1) \right) \\ &= \mathbf{1}_N \otimes \left(\left(\frac{1}{N} A_{\Delta_{11}BK}^k \sum_{i=1}^N \mathbf{x}_i(0) \right) \right. \end{aligned}$$

$$\begin{aligned} &+ \sum_{i=0}^{k-1} \sum_{l=0}^{l=k-i-1} \left(A_{\Delta_{11}BK}^i B_{\omega} \omega_{c,x}(l) \right) \Bigg) \\ &= \mathbf{1}_N \otimes \left(\left(\frac{1}{N} A_{\Delta_{11}BK}^k \sum_{i=1}^N \mathbf{x}_i(0) \right) \right. \\ &+ \sum_{i=0}^{k-1} \sum_{l=0}^{l=k-i-1} \sum_{j=1}^{j=N} \left(\frac{1}{N} A_{\Delta_{11}BK}^i B_{\omega} \omega_{x,j}(l) \right) \Bigg). \end{aligned} \quad (50)$$

Case 2. ($k > \tau(k)$).

In this case, we also define $\mathbf{x}_c(k) = (U \otimes I_d) [\tilde{\mathbf{x}}_c^T(k), 0]^T$ and $\mathbf{x}_{\bar{c}}(k) = (U \otimes I_d) [0, \tilde{\mathbf{x}}_r^T(k)]^T$. Similarly, we can have $\mathbf{x}_c(k) = \mathbf{1}_N^T \otimes ((1/N) \sum_{i=1}^N \mathbf{x}_i(k))$ and $\omega_{c,x}(k) = \mathbf{1}_N^T \otimes ((1/N) \sum_{i=1}^N \omega_{x,i}(k))$. By (18), we can obtain

$$\begin{aligned} \mathbf{x}_c(k+1) &= A_{\Delta_{11}BK} \mathbf{x}_c(k) + (B + \Delta B) K * \mathbf{x}_c(k - \tau(k)) \\ &+ \mathbf{1}_N^T \otimes \left(\frac{1}{N} \sum_{i=1}^N B_{\omega} \omega_{x,i}(k) \right), \quad (k > \tau(k)). \end{aligned} \quad (51)$$

Thus, the final consensus value $\mathbf{c}(k)$ complies with $\lim_{k \rightarrow \infty} (\mathbf{c}(k) - \mathbf{x}_c(k)) = 0, k = 0, 1, 2, \dots$.

The conclusion of Theorem 3 is obtained.

Remark 5. With $\omega_x(k)$ interpreted as deterministic l_2 signal, the final consensus value $\mathbf{c}(k)$ of system (6) is presented by Theorem 3. In the $k \leq \tau(k)$ case, the final consensus value $\mathbf{c}(k)$ has two divisions, one is $(1/N) A_{\Delta_{11}BK}^k \sum_{i=1}^N \mathbf{x}_i(0)$, which is associated with the system matrix $A + \Delta A$, the neighbouring feedback matrix K , and the initial state $\mathbf{x}(0)$; and the other is $\sum_{i=0}^{k-1} \sum_{l=0}^{l=k-i-1} \sum_{j=1}^{j=N} ((1/N) A_{\Delta_{11}BK}^i B_{\omega} \omega_{x,j}(l))$, which is related to the external disturbance $\omega_x(k)$. This implies that the external disturbance $\omega_x(k)$ has an effect on the final consensus value, which is also related to the system matrix $A + \Delta A$, the neighbouring feedback matrix K , and the initial state $\mathbf{x}(0)$. The result is different from that of the swarm system without external disturbances and parameter uncertainties, which is discussed in [29]. In the $k > \tau(k)$ case, we get a linear recursion formula (51) for $\mathbf{x}_c(k)$, which is the limit of the final consensus value $\mathbf{c}(k)$ as $k \rightarrow \infty$. This implies that the final consensus value $\mathbf{c}(k)$ is associated with the states $\mathbf{x}_c(k)$, $\mathbf{x}_c(k - \tau(k))$, and the external disturbance $\omega_x(k)$ when $k > \tau(k)$.

4. Results and Discussion

In this section, we firstly use a simulation example to validate the effectiveness of the theoretical results, and then apply the control protocol (5) to achieve state consensus for a swarm system which consists of 8 agents. The dynamics of these agents are represented by (2), and relevant system matrices is given as follows:

$$\begin{aligned}
A &= \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \\
B &= \begin{bmatrix} 0.2 \\ -0.4 \\ 1 \end{bmatrix}, \\
D &= \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.3 \end{bmatrix}, \\
E_1 &= \begin{bmatrix} 0.1 & 0.3 & 0 \\ 0.2 & 0.4 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \\
E_2 &= \begin{bmatrix} 0.2 \\ 0.1 \\ 0.3 \end{bmatrix}, \\
F &= \begin{bmatrix} r_1 & 0 & 0 \\ 0 & r_2 & 0 \\ 0 & 0 & r_3 \end{bmatrix},
\end{aligned} \tag{52}$$

where r_1 , r_2 , and r_3 are uncertain parameters satisfying $-1 \leq r_1 \leq 1$, $-1 \leq r_2 \leq 1$, and $-1 \leq r_3 \leq 1$, respectively. Then, swarm system (2) can be reformulated as

$$\mathbf{x}_i(k+1) = (A + DFE_1)\mathbf{x}_i(k) + (B + DFE_2)\mathbf{u}_i(k). \tag{53}$$

We analyze the robust H_∞ consensus among the above 8 agents with an undirected topology G as shown in Figure 1.

Assume that the initial state of each agent is randomly given as $x_1(0) = [1, 5, -2]^T$, $x_2(0) = [2, 4, 3]^T$, $x_3(0) = [1, 1, 2]^T$, $x_4(0) = [3, 2, 1]^T$, $x_5(0) = [5, 6, -2]^T$, $x_6(0) = [-3, 3, 4]^T$, $x_7(0) = [-2, -4, -3]^T$, and $x_8(0) = [-5, -2, -1]^T$ and set $r_1 = 0.15$, $r_2 = 0.25$, $r_3 = 0.15$, and $\tau_{\max} = 2$. The dynamics of each agent is combined with protocol (5). Let $\gamma = 2$ and suppose that the exogenous disturbance inputs are selected as $\omega_{i,x}(k) = 0.1e^{-0.5k} \sin(k)$. By Theorem 2, we can get that

$$\begin{aligned}
K &= [0.0408 \quad -0.0470 \quad 0.1564], \\
P &= \begin{bmatrix} 2.2469 & 0.1228 & 0.0750 \\ 0.1228 & 2.5584 & 0.0513 \\ 0.0750 & 0.0513 & 2.1872 \end{bmatrix}, \\
S &= \begin{bmatrix} 1.1140 & 0.0002 & 0.0216 \\ 0.0002 & 1.1336 & 0.0315 \\ 0.0216 & 0.0513 & 1.3031 \end{bmatrix},
\end{aligned}$$

FIGURE 1: Communication topology G .

$$\begin{aligned}
R &= \begin{bmatrix} 0.5206 & 0.0090 & -0.0160 \\ 0.0090 & 0.4821 & 0.0249 \\ -0.0160 & 0.0249 & 1.1160 \end{bmatrix}, \\
M &= \begin{bmatrix} 1.9259 & -0.0384 & 0.0426 \\ -0.0384 & 2.0822 & -0.0706 \\ 0.0426 & -0.0706 & 1.3476 \end{bmatrix}, \\
Q_1 &= \begin{bmatrix} 0.2527 & 0.0038 & 0.0864 \\ 0.0038 & 0.2092 & 0.0099 \\ 0.0864 & 0.0099 & 0.3334 \end{bmatrix}, \\
Q_1 &= \begin{bmatrix} 0.9301 & -0.0003 & 0.0061 \\ -0.0003 & 0.9222 & 0.0023 \\ 0.0061 & 0.0023 & 0.9131 \end{bmatrix}.
\end{aligned} \tag{54}$$

The simulation results are produced from Figures 2–4. The state trajectories of swarm system (2) are shown. In addition, the final consensus value $c(k)$ deduced from Theorem 3 is marked with the red asterisk.

In view of Figures 2–4, we can see easily that the state trajectories of swarm system (2) can be asymptotically convergent to the common value $c(k)$, associated with $r_j (j=1, 2, 3)$. From Definition 1, we can also find clearly that the robust H_∞ consensus of swarm system (2) with control protocol (5) is achieved. Therefore, the effectiveness of the theoretical results is demonstrated.

5. Conclusions

Robust H_∞ consensus control problems of linear high-order discrete-time swarm systems with parameter uncertainties and time-varying delays were investigated. A robust H_∞ consensus control protocol was proposed while the impact of time-varying delays is involved. LMI conditions for robust H_∞ consensus analysis were given, and a final consensus value was obtained from the derivation process. If swarm systems satisfy the sufficient LMI conditions, then we can ensure that swarm systems with parameter uncertainties and time-varying delays could achieve robust H_∞ consensus, and the convergence result could be determined as a final consensus value. Finally, the effectiveness of the theoretical results was verified by relevant simulation examples. Further studies will focus on the impact of switching topologies and directed

FIGURE 2: State 1 trajectories of swarm system (2).

FIGURE 3: State 2 trajectories of swarm system (2).

FIGURE 4: State 3 trajectories of swarm system (2).

graphs on the consensus problem of linear discrete-time swarm systems.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

Acknowledgments

This work is supported by National Natural Science Foundation under grant #61763040 and grant #61867005. The authors would like to express my faithful gratitude to all those who helped me during the writing of this paper. We gratefully acknowledge the help of our supervisors, Mister Xiaogang Yang and Mister Jianxiang Xi, who have offered us patient instruction in the academic studies.

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