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Research Article

Adaptive Fault Compensation and Disturbance Suppression Design for Nonlinear Systems with an Aircraft Control Application

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A comprehensive adaptive compensation control strategy based on feedback linearization design is proposed for multivariable nonlinear systems with uncertain actuator fault and unknown mismatched disturbances. Firstly, the linear dynamic system is obtained through nonlinear feedback linearization, and the dynamic model of the mismatched disturbances as well as its relevance to the nonlinear system is given. The effect of disturbances on the system output is suppressed with the basic controller of the linearized system. Then, a direct adaptive controller is developed for the multiple uncertain actuator faults. Finally, an integrated algorithm based on adaptive weighted fusion could provide an effective compensation for the effect of multiple uncertain faults and mismatched disturbances. Thus, the stability and asymptotic tracking performance of the closed-loop system are ensured. The feasibility and performance of the proposed control strategy are validated by the numerical simulation results.

1. Introduction

Actuator faults are common in performance-critical systems. The occurrence of faults will cause severe deterioration in performance or even catastrophic problems of system instability. Actuator faults are featured with multiple essential uncertainties, including the fault mode, time, value, and type. Therefore, it is necessary to develop the effective fault-tolerant control technology to address the problem associated with the multiple uncertainties of actuator faults, so as to sustain reliability and safety of the closed-loop system.

In recent years, the problem of actuator faults compensation control has attracted more and more attention. A variety of control methods are tested with several profound achievements. Many effective fault-tolerant control methods were reviewed in literatures [1–5]. Multimodel adaptive control methods were employed as a fault compensation in literatures [6–8]. Literatures [9–11] applied neural network to the design of reconfigurable aircraft control in the case of sensors or actuator faults. The fault recognition and fault-tolerant control strategies of the near space vehicle are designed base on the adaptive sliding mode control method in reference [12]. For the spacecraft attitude control system with external disturbances, two kinds of effective fault-tolerant control method were proposed in literature [13]. To enhance the overall performance of the multisensor measurement system and reduce the influence of faults of each sensor on the system, a new multisensor information fusion design framework was proposed in reference [14]. Fault detection and diagnosis methods are also widely used to for the problems of component faults in the control system [15]. In literatures [16, 17], the adaptive observer design was used to reconstruct actuator faults and a fault-tolerant controller was designed based on estimated information for fault. Besides, adaptive control is also an effective tool with widespread application in fault-tolerant control for both linear and nonlinear systems [18–21]. Although great practical progress has been made in actuator fault compensation for the nonlinear system, there are still many unresolved problems for control system with uncertain dynamics and actuator faults. For example, the problems of multiple-actuator fault compensation control in the general nonlinear system can be further investigated to improve closed-loop system stability and asymptotic tracking control.

The so-called feedback linearized system refers to a kind of nonlinear system linearized by appropriate nonlinear feedback control [22]. Based on the feedback linearization, the control objectives such as models match, pole assignment, and tracking can be further realized. References [23, 24] combined feedback linearization theory with adaptive control and effectively solved the parameter uncertainty and fault-tolerant control problems of nonlinear systems. In addition, the performance of the controlled system suffers from quite different influences due to the various disturbances during the actual operation of the nonlinear system. Therefore, the disturbance suppression problems should be given adequate attention. In literatures [25–27], disturbance decoupling for the measurable disturbances in linear systems provides a potential approach for disturbance suppression problems. However, this method is not suitable for nonmeasurable disturbances. The robust control method is proposed for the nonmeasurable disturbances in literatures [28, 29], without implementation for control objective of asymptotic tracking. The disturbances suppression method based on adaptive control design can effectively estimate the unknown system parameters and disturbance parameters. In literature [30], the adaptive internal model control method was applied in the spacecraft system to realize the attitude tracking with external disturbances. For general hypersonic vehicles with uncertain system parameters and external disturbances, a new sliding mode control method was proposed in literature [31]. The problem of asymptotic tracking of nonlinear systems under sinusoidal disturbances was investigated in literature [32]. And a disturbance suppression algorithm was proposed for single-input single-output nonlinear systems, but the algorithm is inappropriate for multi-input multi-output nonlinear systems with mismatched disturbances. In addition, the suppression of mismatched disturbances in multi-input and multioutput nonlinear systems were studied in literatures [33–35].

Unknown disturbances and uncertain actuator faults may occur simultaneously in the actual operation, which increases the difficulties in asymptotic tracking control for multi-input and multioutput nonlinear systems. Although some theoretical achievements have been made in disturbance suppression and actuator fault compensation for multi-input and multioutput nonlinear systems, some critical problems are left open. The problem of unmatched disturbance suppression in nonlinear systems with uncertain multivariable is solved in literature [35]. On this basis, the problem of multiple uncertain actuator fault compensation and mismatched input disturbance suppression is further studied in this paper for the case of a feedback linearized multivariable nonlinear systems. Compared with some available fault-tolerant control methods, the currently proposed control method presents the following improvement: (1) a new adaptive actuator failure compensation and disturbance rejection scheme with relaxed design conditions is designed for general multivariable nonlinear systems; (2) a new composite fault-tolerant control approach is developed to handle a set of uncertain actuator failures, by using a complete parametrization for estimation of both the failure pattern parameters and the failure value parameters; (3) an adaptive disturbance rejection scheme is developed in details, including error equations, adaptive laws, and stability analysis, for multivariable nonlinear systems with uncertainties from both the actuator failure and unmatched disturbances, such that desired closed-loop performances are ensured including signals boundedness and asymptotic output tracking; and (4) an important aircraft flight control application is conducted.

2. Problem Description and Knowledge Preparation

This chapter first describes the problem of actuator fault compensation and disturbance suppression of the systems with redundant actuators and then introduces some basic concepts involved in this paper.

2.1. Control Problem Statement. Consider the nonlinear system as below

$$\dot{x} = f(x) + g(x)u + p(x)d(t) \tag{1}$$

$$y = h(x), \tag{2}$$

where $x \in R^n$ is state vector, $y = [y_1, y_2, \cdots, y_q]^T \in R^q$ is system output, $u = [u_1, u_2, \cdots, u_m]^T \in R^m$ is system input, and $d(t) \in R^p$ is the uncertain external disturbance. $f(x) \in R^n$, $g(x) = [g_1(x), g_2(x), \cdots, g_m(x)] \in R^{n\times m}$, $p(x) \in R^{n\times p}$, and $h(x) \in R^q$ are known.

2.1.1. Actuator Fault Model. The classical model of the actuator fault can be represented as [19]

$$u_j(t) = \bar{u}_j(t) = \bar{u}_{j0} + \sum_{i=1}^{q_j} \bar{u}_{ji} f_{ji}(t), \quad t \geq t_j, \tag{3}$$

where $j \in \{1, 2, \cdots, m\}$, $t_j > 0$, $\bar{u}_{j0}$, and $\bar{u}_{ji}$ represent the parameters of the uncertain fault. $f_{ji}(t), i = 1, 2, \cdots, q_j$ are known. The fault model (3) is written in the following parameterized form

$$\bar{u}_j(t) = \theta_j^T \varpi_j(t), \tag{4}$$

where $\theta_j = [\bar{u}_{j0}, \bar{u}_{j1}, \cdots, \bar{u}_{jq_j}]^T \in R^{q_j+1}$, $\varpi_j(t) = [1, f_{j1}(t), \cdots, f_{jq_j}(t)]^T \in R^{q_j+1}$.When the uncertain actuator fault occurs in the system, the actual input $u(t)$ acting on the system can be expressed as

$$u(t) = (I - \sigma(t))v(t) + \sigma(t)\bar{u}(t), \tag{5}$$

where $v(t)$ is the control input signal to be designed. $\bar{u}(t) = [\bar{u}_1(t), \bar{u}_2(t), \cdots, \bar{u}_m(t)]^T$. $\sigma(t) = \text{diag}\{\sigma_1(t), \sigma_2(t), \cdots, \sigma_m(t)\}$ is the corresponding actuator fault mode matrix. If the j actuator fails, then $\sigma_i(t) = 1$; otherwise, $\sigma_i(t) = 0$. Considering actuator fault (5), the system model can be expressed as

$$\begin{aligned} \dot{x} &= f(x) + g(x)\sigma(t)\bar{u}(t) + g(x)(1-\sigma(t))v(t) + p(x)d(t) \\ y &= h(x). \end{aligned} \tag{6}$$

2.1.2. External Disturbance Model. The disturbance term $p(x)d(t)$ in this paper has the following characteristics: (1) $p(x) \neq g(x)\alpha$ $\alpha \in R^{m\times p}$ indicates that the disturbance signal $d(t)$ is incompatible with the control signal $u(t)$; (2) the component of the disturbance vector $d(t) = [d_1(t), d_2(t), \cdots, d_p(t)]^p \in R^p$ can be expressed as [36]:

$$d_j(t) = d_{j0} + \sum_{k=1}^{q_j} d_{jk}\Phi_{jk}(t), \ \ j = 1, 2, \cdots, p, \tag{7}$$

and it also can be rewritten in the parameterized form as

$$d_j(t) = \theta_{dj}^{*T}\varpi_{dj}(t), \tag{8}$$

where $\theta_{dj}^{*T} = [d_{j0}, d_{j1}, \cdots, d_{jq_j}] \in R^{q_j+1}$, $\varpi_{dj}(t) = [1, \Phi_{j1}(t), \cdots, \Phi_{jq_j}(t)]^T \in R^{q_j+1}$, $j = 1, 2, \cdots, p$, d_{j0},and d_{jk} are unknown while $\Phi_{jk}(t)$ are known. By selecting appropriate q_j and basic function $\Phi_{jk}(t)$, the disturbance model (7) can offer an approximate description for many practical disturbance signals, such as constant value, sinusoidal signal, and non-sinusoidal time-varying disturbance.

Remark 1. When the disturbance is consistent with the control input, i.e., $p(x) = g(x)\alpha$ and $\alpha \in R^{m\times p}$, the control signal can be derived as $u(t) = u_1(t) + u_2(t)$, where $u_1(t)$ is the basic control variable that can stabilize the nonlinear multivariable system, and $u_2(t) = -\alpha d(t)$ is the disturbance suppression component. Without such match, i.e., $p(x) \neq g(x)\alpha$ and $\alpha \in R^{m\times p}$, the above control method cannot eliminate the influence of disturbances. Therefore, a new control input $u(t)$ needs to be designed to suppress disturbance.

2.1.3. Control Objective. For system (1) with uncertain actuator faults (3) up to $m - q_a q \leq q_a \leq m$ and unmatched external disturbance $d(t)$, the number of the faults depends on the actual application. In this paper, $q_a = m - 1$. That is, the total actuator faults are no more than $m - q_a = 1$, but it is impossible to identify in advance the exact amount of faults. The actuator fault compensation method designed in this case can be applied to the problem of simultaneous or alternating faults of multiple actuators. The mathematical expressions of the corresponding fault modes are

$$\sigma_{(1)} = \text{diag}\ \{0, 0, \cdots, 0\}, \sigma_{(2)} = \text{diag}\ \{1, 0, \cdots, 0\}. \tag{9}$$

In this paper, a fault compensation control algorithm is developed based on the following assumptions to achieve the above control objectives.

Assumption 2. When at most one actuator of system (1) fails and the fault information is available, it is still possible to design effective control methods to adjust the residual actuators adaptively so that the system still fulfills the desired control objective.

The goal of this paper is to design an adaptive controller $v(t)$ to solve the issue due to multiple uncertainties of faults and disturbances, especially the uncertain fault mode, in order to guarantee the stability of the closed-loop system and asymptotical tracking performance of system output.

2.2. Feedback Linearization. For a multi-input and multioutput nonlinear system

$$\dot{x} = f(x) + g(x)u, \quad y = h(x) \tag{10}$$

where $u \in R^m$, $y \in R^q$.

Assumption 3. Supposing the correlation vector as $\{\rho_1, \rho_2, \cdots, \rho_q\}$, $1 \leq \rho_i \leq n$ in a neighborhood Ω_0 at $x_0 \in R^n$, if for $\forall x \in \Omega_0$, $L_{g_j}L_f^k h_i(x) = 0$, $1 \leq j \leq m$, $0 \leq k < \rho_i - 1$, $1 \leq i \leq q$, and $L_{g_j}L_f^{\rho_i - 1}h_i(x_0) \neq 0$ for $j \in \{1, 2, \cdots, m\}$.

Similarly, for a nonlinear system with input disturbances

$$\dot{x} = f(x) + p(x)d(t), \quad y = h(x), \tag{11}$$

where $d(t) \in R^p$, $y \in R^q$, and it has a correlation set $\{v_1, v_2, \cdots, v_q\}$, $1 \leq v_i \leq n$. The disturbance suppression design of the multivariable nonlinear system in this paper involves the following assumption:

Assumption 4. If $i = i_1, i_2, \cdots, i_p \in \{1, 2, \cdots, m\}$, then $\rho_i = v_i$; If $i \neq i_1, i_2, \cdots, i_p$, then $\rho_i < v_i$.

2.2.1. Strict Feedback Linearization. If $\rho_1 + \rho_2 + \cdots + \rho_q = n$, the system (1) can be transformed into a strict feedback subsystem through strict feedback linearization and differential homeomorphic mapping $\xi = T(x) \in R^n$, where

$$\begin{aligned} T(x) &= \left[h_1(x), L_f h_1(x), \cdots, L_f^{\rho_1 - 1}h_1(x), \cdots, h_q(x), \cdots, L_f^{\rho_q - 1}h_q(x)\right]^T \\ \xi &= \left[\xi_{1,1}, \xi_{1,2}, \cdots, \xi_{1,\rho_1}, \xi_{2,1}, \cdots, \xi_{q,\rho_q}\right]^T \end{aligned} \tag{12}$$

Then the dynamic equation becomes

$$
\begin{aligned}
\dot{\xi}_{1,1} &= \xi_{1,2} \\
\dot{\xi}_{1,2} &= \xi_{1,3} \\
&\vdots \\
\dot{\xi}_{1,\rho 1} &= b_1(\xi) + A_{1\sigma}(x)v + \bar{A}_{1\sigma}(x)\bar{u}(t) + \Delta_{d1}(x,t) \\
&\vdots \\
\dot{\xi}_{q,1} &= \xi_{q,2} \\
&\vdots \\
\dot{\xi}_{q,\rho_q} &= b_q(\xi) + A_{q\sigma}(x)v + \bar{A}_{q\sigma}(x)\bar{u}(t)\Delta_{dq}(x,t)
\end{aligned} \tag{13}
$$

where $b_i(\xi) = L_f^{\rho_i} h_i(x)$, $A_{i\sigma}(x)$, $\bar{A}_{i\sigma}(x)$, and Δ_{di} are the ith row of $A_\sigma(x)$, $\bar{A}_\sigma(x)$, and Δ_d, respectively, $A_\sigma(x) = A(x)(I - \sigma(t))v(t)$, $\bar{A}_\sigma(x) = \sigma(t)\bar{u}(t)$, with

$$
A(x) = \begin{bmatrix} L_{g_1} L_f^{\rho_1 - 1} h_1(x) & \cdots & L_{g_m} L_f^{\rho_1 - 1} h_1(x) \\ L_{g_1} L_f^{\rho_2 - 1} h_2(x) & \cdots & L_{g_m} L_f^{\rho_2 - 1} h_2(x) \\ \cdots & & \cdots \\ L_{g_1} L_f^{\rho_q - 1} h_q(x) & \cdots & L_{g_m} L_f^{\rho_q - 1} h_q(x) \end{bmatrix}
$$

$$
\begin{aligned}
\Delta_d(x,t) &= \begin{bmatrix} L_{p_1} L_f^{\rho_1 - 1} h_1(x) & \cdots & L_{p_p} L_f^{\rho_1 - 1} h_1(x) \\ L_{p_1} L_f^{\rho_2 - 1} h_2(x) & \cdots & L_{p_p} L_f^{\rho_2 - 1} h_2(x) \\ \cdots & & \cdots \\ L_{p_1} L_f^{\rho_q - 1} h_q(x) & \cdots & L_{p_p} L_f^{\rho_q - 1} h_q(x) \end{bmatrix} d(t) \\
&= \left[\delta_1(x), \delta_2(x), \cdots, \delta_q(x)\right]^T.
\end{aligned} \tag{14}
$$

The system output is expressed as

$$
y_1 = \xi_{1,1}, \ y_2 = \xi_{2,1}, \ \cdots, \ y_q = \xi_{q,1}. \tag{15}
$$

2.2.2. Partial Feedback Linearization. If $\rho_1 + \rho_2 + \cdots + \rho_q < n$, only partial feedback linearization can be carried out in system (1) by means of coordinate transformation within the neighborhood of x_0. Supposing $T_c(x)$ is a smooth function with the following form

$$
T_c(x) = \left[h_1(x), \cdots, L_f^{\rho_1 - 1} h_1(x), \cdots, L_f^{\rho_q - 1} h_q(x)\right]^T. \tag{16}
$$

Literature [24] indicates that there is always smooth mapping

$$
T_z(x) = \left[T_1(x), \cdots, T_{n-(\rho_1 + \cdots + \rho_q)}(x)\right]^T. \tag{17}
$$

To form the differential homeomorphism $[\xi^T, \eta^T]^T = T(x) = [T_c(x)^T, T_z(x)^T]^T$, $\xi \in R^{\rho_1 + \rho_2 + \cdots + \rho_q}$, $\eta \in R^{n-(\rho_1 + \rho_2 + \cdots + \rho_q)}$, system (1) is converted into

$$
\begin{aligned}
\dot{\xi}_{1,1} &= \xi_{1,2} \\
\dot{\xi}_{1,2} &= \xi_{1,3} \\
&\vdots \\
\dot{\xi}_{1,\rho_1} &= b_1(\xi,\eta) + A_{1\sigma}(x)v + \bar{A}_{1\sigma}(x)\bar{u}(t) + \Delta_{d1}(x,t) \\
&\vdots \\
\dot{\xi}_{q,1} &= \xi_{q,2} \\
&\vdots \\
\dot{\xi}_{q,\rho_q} &= b_q(\xi,\eta) + A_{q\sigma}(x)v + \bar{A}_{q\sigma}(x)\bar{u}(t) + \Delta_{dq}(x,t)
\end{aligned} \tag{18}
$$

$$
\dot{\eta} = \psi(\xi,\eta) + \Psi_g(\xi,\eta)u + \Psi_p(\xi,\eta)d(t), \tag{19}
$$

where $b_i(\xi,\eta)$, $A_{i\sigma}(x)$, $\bar{A}_{i\sigma}(x)$, $i = 1, 2, \cdots, q$ have the same definition with the equation (13). $\Delta_{di}(x)$, $i = 1, 2, \cdots, q$ is the unmatched disturbance. $\psi(\xi,\eta) = (\partial T_z(x)/\partial x) f(x)|_{x=T^{-1}(\xi,\eta)}$, $\Psi_g(\xi,\eta) = (\partial T_z(x)/\partial x) g(x)|_{x=T^{-1}(\xi,\eta)}$, and $\Psi_p(\xi,\eta) = (\partial T_z(x)/\partial x) p(x)|_{x=T^{-1}(\xi,\eta)}$. The system (19) is termed as the zero dynamic form of the multivariable nonlinear system (1).

2.2.3. Nonlinear Feedback Control Law. Based on Assumptions 3 and 4, if the system parameters and fault parameters of nonlinear system (1) are accessible, feedback linearization design can be used to design an ideal controller. By taking y_i derivatives of ρ_i in system (1), we can obtain the following equation:

$$
y_i^{(\rho_i)} = L_f^{\rho_i} h_i(x) + \sum_{j=1}^{m} L_{g_j} L_f^{\rho_i - 1} h_i(x) u_j + \delta_i(x), \tag{20}
$$

where $\delta_i(x) = \sum_{j=1}^{p} L_{p_j} L_f^{\rho_i - 1} h_i(x) d_j(t)$. We can further obtain

$$
\begin{bmatrix} y_1^{(\rho_1)} \\ y_2^{(\rho_2)} \\ \vdots \\ y_q^{(\rho_q)} \end{bmatrix} = \begin{bmatrix} L_f^{\rho_1} h_1(x) \\ L_f^{\rho_2} h_2(x) \\ \vdots \\ L_f^{\rho_q} h_q(x) \end{bmatrix} + A(x)u + \Delta_d(x,t). \tag{21}
$$

When $m = q$ and assuming $A(x)$ is nonsingular in x_0, the control input signal could be rearranged as

$$
u(t) = -A^{-1}(x) \begin{bmatrix} L_f^{\rho_1} h_1(x) \\ L_f^{\rho_2} h_2(x) \\ \vdots \\ L_f^{\rho_q} h_q(x) \end{bmatrix} + A^{-1}(x) u_L. \tag{22}
$$

The linearized system can be obtained

$$\begin{bmatrix} y_1^{(\rho_1)} \\ y_2^{(\rho_2)} \\ \vdots \\ y_q^{(\rho_q)} \end{bmatrix} = \begin{bmatrix} u_{L1} \\ u_{L2} \\ \vdots \\ u_{Lq} \end{bmatrix} + \Delta_d(x, t), \quad (23)$$

where u_L is the linear feedback control law to be designed.

2.2.4. Linear Feedback Control. The control law from the linearized system provides the possibility to guarantee the output tracking performance of the system. The control law $u_L = [u_{L1}, u_{L2}, \cdots, u_{Lq}]^T$ is

$$u_{Li} = y_{mi}^{(\rho_i)} + \alpha_{i1}\left(y_{mi}^{(\rho_i-1)} - y_i^{(\rho_i-1)}\right) + \cdots + \alpha_{i\rho_i}(y_{mi} - y_i) + \delta_{ci}(x, t), \quad (24)$$

where $\delta_{ci}(x, t) = -\sum_{j=1}^{p} L_{p_j} L_f^{\rho_i-1} h_i(x) d_j(t)$. With substitution of u_{Li} and equation (24) into equation (23), the dynamic equation of the tracking error $e_i = y_i - y_{mi}$ is obtained as

$$e_i^{(\rho_i)} + \alpha_{i1} e_i^{(\rho_i-1)} + \cdots + \alpha_{i\rho_i} e_i = 0. \quad (25)$$

By selecting appropriate value for $\alpha_{i\rho_i}$, $i = 1, 2, \cdots, q$, $s^{\rho_i} + \alpha_{i1} s^{\rho_i-1} + \cdots + \alpha_{i\rho_i}$ becomes the Hurwitz polynomial. The output error and its higher derivative $e_i, \dot{e}_i, \cdots, e_i^{(\rho_i-1)}$ asymptotically approach to zero as $t \longrightarrow \infty$. If $y_{mi}, \dot{y}_{mi}, \cdots, y_{mi}^{(\rho_i-1)}$ is bounded, then boundedness could be expected for $y_i, \dot{y}_i, \cdots, y_i^{(\rho_i-1)}$.

Remark 5. If for all $i = 1, 2, \cdots, m$, $\rho_i < \nu_i$, then $\delta_i(x, t) = 0$, in equation (26), i.e., $\Delta_{di}(x) = 0$, $i = 1, 2, \cdots, m$. Thus, equation (21) can be simplified as

$$\begin{bmatrix} y_1^{(\rho_1)} \\ y_2^{(\rho_2)} \\ \vdots \\ y_q^{(\rho_q)} \end{bmatrix} = \begin{bmatrix} L_f^{\rho_1} h_1(x) \\ L_f^{\rho_2} h_2(x) \\ \vdots \\ L_f^{\rho q} h_q(x) \end{bmatrix} + A(x)u. \quad (26)$$

Linearized system becomes disturbance-free, and disturbance suppression is unnecessary in the basic feedback control system. In addition, in combination with equation (24), equation (21) can be further expressed as $[y_1^{(\rho_1)}, y_1^{(\rho_2)}, \cdots, y_q^{(\rho_q)}]^T = u_L(t)$, where $u_{Li}(t)$ can assume the following simplification $u_{Li} = y_{mi}^{(\rho_i)} + \alpha_{i1}(y_{mi}^{(\rho_i-1)} - y_i^{(\rho_i-1)}) + \cdots + \alpha_{i\rho_i}(y_{mi} - y_i)$, $i = 1, 2, \cdots, m$.

Remark 6. If $\rho_i > \nu_i$, $i \in 1, 2, \cdots, m$, then $\delta_i(x, t)$ in equation (20) is related to the disturbance term $d(t)$ and its differential term $\dot{d}(t), \cdots, d^{\rho_i-\nu_i}(t)$, and could be expressed as a function of the differential term $\dot{d}(t), \cdots, d^{\rho_i-\nu_i}(t)$ $\delta_i(x, t) = -d^{\rho_i-\nu_i}/dt(\sum_{j=1}^{p} L_{p_j} L_f^{\rho_i-1} h_i(x) d_j(t))$. In this case, in order to achieve disturbance suppression and asymptotic tracking control, it is necessary to acquire the differential information of the disturbance in advance. However, the derivation process is rather complicated. Therefore, such situation is not considered in this design.

3. Actuator Fault Compensation and Disturbance Suppression Design

If the relevance of the system $\{\rho_1, \rho_2, \cdots, \rho_q\}$ satisfies $\rho_1 + \rho_2 + \cdots + \rho_q = n$, the system with uncertain actuator fault can be linearized by strict feedback and converted into

$$\begin{aligned} \dot{\xi}_{1,1} &= \xi_{1,2}, \\ \dot{\xi}_{1,2} &= \xi_{1,3}, \\ &\vdots \\ \dot{\xi}_{1,\rho_1} &= b_1(\xi) + A_{1\sigma}(x)v + \bar{A}_{1\sigma}(x)\bar{u}(t) + \Delta_{d1}(x, t) \\ &\vdots \\ \dot{\xi}_{q,1} &= \xi_{q,2}, \\ &\vdots \\ \dot{\xi}_{q,\rho_q} &= b_q(\xi) + A_{q\sigma}(x)v + \bar{A}_{q\sigma}(x)\bar{u}(t) + \Delta_{dq}(x, t) \\ y_1 &= \xi_{1,1}, \quad y_2 = \xi_{2,1}, \quad \cdots, \quad y_q = \xi_{q,1}. \end{aligned} \quad (27)$$

Based on u_{Li}, $i = 1, 2, \cdots, q$ in equation (24), the control signal $w_d(t) \in R^q$of the system (27) could be determined through nonlinear feedback, if

$$A_\sigma(x)v(t) + \bar{A}_\sigma(x)\bar{u}(t) = w_d(t). \quad (28)$$

The control signal can guarantee asymptotic output tracking, i.e., $\lim_{t\to\infty}(y(t) - y_m(t)) = 0$. With occurrence of uncertain actuator fault, the control input signal $v(t)$ could be calculated according to equation (28).

3.1. Adaptive Disturbance Suppression Design. The control signal $w_d(t)$ and the feedback linearization are determined in this chapter. The detailed derivation includes adaptive controller, error equation, parameter adaptive updating law, and stability analysis.

3.1.1. Adaptive Feedback Linearization Design. In the disturbance model $d_j(t) = \theta_{dj}^{*T} \varpi_{dj}(t)$, $\varpi_{dj}(t) j = 1, 2, \cdots, p$ are known functions while θ_{dj}^* are unknown parameters. The unknown parameters could be estimated with $\widehat{d}_j(t) = \widehat{\theta}_{dj}^T \varpi_{dj}(t)$ in the disturbance suppression design, where $\widehat{\theta}_{dj}$ is the estimate of the disturbance parameter θ_{dj}^*, $j = 1, 2, \cdots, p$. Based on the estimation, the adaptive linear control law is obtained as

$$u(t) = -A^{-1}(x)\begin{bmatrix} L_f^{\rho_1}h_1(x) \\ L_f^{\rho_2}h_2(x) \\ \vdots \\ L_f^{\rho_q}h_q(x) \end{bmatrix} + A^{-1}(x)\hat{u}_L, \tag{29}$$

where $\hat{u}_L$ is the estimated value of u_L, and its estimated component is

$$\begin{aligned} \hat{u}_{Li} &= y_{mi}^{(\rho_i)} + \alpha_{i1}\left(y_{mi}^{(\rho_i-1)} - y_i^{(\rho_i-1)}\right) \\ &\quad + \alpha_{i\rho_i}(y_{mi} - y_i) + \hat{\delta}_{ci}(x,t), \\ \hat{\delta}_{ci}(x,t) &= -\sum_{j=1}^{p} L_{p_j} L_f^{\rho_i-1} h_i(x)\hat{d}_j(t). \end{aligned} \tag{30}$$

3.1.2. Error Model. Let $z_{1,1} = \xi_{1,1} - y_{m1}$, $z_{1,2} = \xi_{1,2} - \dot{y}_{m1}$, $\cdots$, $z_{1,\rho_1} = \xi_{1,\rho_1} - y_{m1}^{(\rho_1-1)}$, $\cdots z_{i,\rho_i} = \xi_{i,\rho_i} - y_{mi}^{(\rho_i-1)}$, $\cdots$, $z_{q,\rho_q} = \xi_{q,\rho_q} - y_{mq}^{(\rho_q-1)}$, $z \in R^{\rho_1+\rho_2+\cdots+\rho_q} = [z_1^T, z_2^T, \cdots, z_q^T]^T = [z_{1,1}, \cdots, z_{1,\rho_1}, \cdots, z_{i,\rho_i}, \cdots, z_{q,\rho_q}]^T$. Combining the system output in equation (15): $y_1 = \xi_{1,1}$, $y_2 = \xi_{2,1}$, $\cdots$, $y_q = \xi_{q,1}$ and $e_i = y_i - y_{mi}$, one can obtain $z_{1,1} = e_1$, $z_{1,2} = \dot{e}_1$, $\cdots$, $z_{1,\rho_1} = e_1^{(\rho_1-1)}$, $\cdots$, $z_{i,\rho_i} = e_i^{(\rho_i-1)}$, $\cdots$, $z_{q,\rho_q} = e_q^{(\rho_q-1)}$. And the state error equation of the multi-input multioutput system is calculated by

$$\dot{z} = A_z z + B_{z\tilde{E}_d}, \tag{31}$$

where $A_z = \operatorname{diag}\{A_{z_1}, A_{z_2}, \cdots, A_{z_q}\}$, $B_{z\tilde{E}_d} = [B_{z_1}^T \tilde{E}_{d1}, \ B_{z_2}^T \tilde{E}_{d2}, \cdots, B_{z_q}^T \tilde{E}_{dq}]^T$, and $\tilde{E}_{di} = \sum_{j=1}^{p} L_{p_j} L_f^{\rho_i-1} h_i(x)\tilde{\theta}_{dj}^T(t)\varpi_{dj}$,

$$\begin{aligned} A_{z_i} &= \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ & \cdots & & & \cdots & \\ -\alpha_{i1} & -\alpha_{i2} & \cdots & -\alpha_{i(\rho_i-2)} & -\alpha_{i(\rho_i-1)} & -\alpha_{i\rho_i} \end{bmatrix}, \\ B_{z_i} &= [0, 0, \cdots, 1]^T \in R^{\rho_i}, \quad i = 1, 2, \cdots, q. \end{aligned} \tag{32}$$

3.1.3. Adaptive Laws. Based on error system (31), an adaptive law is incorporated to update unknown disturbance parameters $\hat{\theta}_{dj}, j = 1, 2, \cdots, p$. Lyapunov function is designed in following form

$$V_d = \frac{1}{2} z^T P z + \frac{1}{2}\sum_{j=1}^{p} \tilde{\theta}_{dj}^T \Gamma_{dj}^{-1} \tilde{\theta}_{dj}, \tag{33}$$

where adaptive gain matrix $\Gamma_{dj} = \Gamma_{dj}^T > 0$, $P \in R^{n\times n}$ is positive definite symmetric matrix and satisfies the following equation

$$PA_z + A_z^T P = -Q, \tag{34}$$

where $Q = Q^T > 0$. Taking the derivative with respect to V_d gives

$$\begin{aligned} \dot{V}_d &= \frac{1}{2}\dot{z}^T P z + \frac{1}{2} z^T P \dot{z} + \sum_{j=1}^{p} \tilde{\theta}_{dj}^T \Gamma_{dj}^{-1} \dot{\tilde{\theta}}_{dj} \\ &= \frac{1}{2} z^T \left(PA_z + A_z^T P\right) z + z^T P B_{z\tilde{E}_d} + \sum_{j=1}^{p} \tilde{\theta}_{dj}^T \Gamma_{dj}^{-1} \dot{\tilde{\theta}}_{dj} \\ &= -\frac{1}{2} z^T Q z + Z_P \left[\tilde{E}_{d1}, \cdots, \tilde{E}_{dq}\right]^T + \sum_{j=1}^{p} \tilde{\theta}_{dj}^T \Gamma_{dj}^{-1} \dot{\tilde{\theta}}_{dj} \\ &= -\frac{1}{2} z^T Q z + \sum_{j=1}^{p}\sum_{i=1}^{q} Z_{Pi} \tilde{\theta}_{dj}^T \varpi_{\rho_i,j} + \sum_{j=1}^{p} \tilde{\theta}_{dj}^T \Gamma_{dj}^{-1} \dot{\tilde{\theta}}_{dj}, \end{aligned} \tag{35}$$

where $Z_P = [Z_{P1}, Z_{P2}, \cdots, Z_{Pq}]^T$, Z_{Pi}, $i = 1, 2, \cdots, q$ are the components of $z^T P \in R^{1\times(\rho_1+\rho_2+\cdots+\rho_q)}$, $\varpi_{\rho_i,j} = L_{p_j} L_f^{\rho_i-1} h_i(x) \varpi_{dj}$. Design control equation is given by

$$w_d(t) \triangleq -\begin{bmatrix} L_f^{\rho_1}h_1(x) \\ L_f^{\rho_2}h_2(x) \\ \vdots \\ L_f^{\rho_q}h_q(x) \end{bmatrix} + \hat{u}_L, \tag{36}$$

and the adaptive law of the parameter $\hat{\theta}_{dj}$ is

$$\dot{\hat{\theta}}_{dj} = \sum_{i=1}^{q} \Gamma_{dj} Z_{Pi} \varpi_{\rho_i,j}, \quad j = 1, 2, \cdots, p. \tag{37}$$

With a substitution into equation (33), the following could be obtained

$$\dot{V}_d = -\frac{1}{2} z^T Q z \le 0. \tag{38}$$

So it is ensured that $\hat{\theta}_{dj} \in L^\infty$, $\dot{\hat{\theta}}_{dj} \in L^2 \cap L^\infty$. The stability of the closed-loop system can be determined from the negative definition of $\dot{V}_d$ and $\lim_{t\to\infty} z_{i,1}(t) = \lim_{t\to\infty}(y_i(t) - y_m(t)) = 0$, $i = 1, 2, \cdots, q$. It indicates that a desired performance is achieved with the control system.

3.2. Adaptive Fault Compensation Control Design. Supposing the fault information (fault mode, fault value, and fault time) is known. Two ideal controllers $v_{(1)}^*(t)$ and $v_{(2)}^*(t)$ are designed for the two cases (without fault and actuator u_1 fault). Through weighted fusion design, an integrated controller $v^*(t)$ is obtained, which can deal with the simultaneous coexistence of two fault modes mentioned above.

3.2.1. Fault Free Condition. In this case, $\sigma = diag\{0, 0, \cdots, 0\}$, the control equation (28) is $A(x)v(t) = w_d(t)$. By selecting an appropriate $h_{21}(x) \in R^{m\times(m-1)}$, the following equation could be satisfied

$$v(t) = v^*_{(1)}(t) = h_{21}(x)v^*_{0(1)}(t) \tag{39}$$

By solving the equation $A(x)h_{21}(x)v^*_{0(1)}(t) = w_d(t)$, we could obtain the following equation

$$v^*_{0(1)}(t) = K_{21}(x)w_d(t), \tag{40}$$

where $K_{21}(x) \in R^{(m-1)\times q}$.

3.2.2. $\boldsymbol{u_1}$ Fault Condition. In case of u_1 fault condition, $\sigma = diag\{1, 0, \cdots, 0\}$, $u_1 = \bar{u}_1$, $u_i = v_i$, $i = 2, \cdots, m$, $A(x) = [A_1, A_2, \cdots, A_m] = [A_1, A_{(2)}] \in R^{q\times m}$, where $A_{(2)} = [A_2, \cdots, A_m] \in R^{q\times(m-1)}$, $v = [v_1, v^T_{a(2)}]^T \in R^m$, where $v_{a(2)} = [v_2, \cdots, v_m]^T \in R^{m-1}$, by selecting the appropriate matrix equation $h_{22}(x) \in R^{(m-1)\times(m-1)}$, one could have $v^*_{a(2)}(t) = h_{22}(x)v^*_{0(2)}(t)$. And the equation could be solved

$$A_1\bar{u}_1(t) + A_{(2)}h_{22}(x)v^*_{0(2)}(t) = w_d(t). \tag{41}$$

The ideal controller under this condition is

$$v^*_{0(2)}(t) = K_{22}(x)w_d(t) + K_{221}(x)\bar{u}_1(t). \tag{42}$$

3.2.3. Integrated Control Law. The fault index function is defined as

$$\begin{aligned} \chi^*_1 &= \begin{cases} 1 & \text{fault free condition} \\ 0 & \text{others,} \end{cases} \\ \chi^*_2 &= \begin{cases} 1 & u_1 \text{ fault condition} \\ 0 & \text{others.} \end{cases} \end{aligned} \tag{43}$$

With a weighted fusion of controller $v^*_{(1)}(t)$ and $v^*_{(2)}(t)$, an ideal integrated controller structure is achieved.

$$v^*(t) = \chi^*_1(t)v^*_{(1)}(t) + \chi^*_2(t)v^*_{(2)}(t) = v^*_{\chi_1(1)}(t) + \left[0, v^{*T}_{\chi_2 a(2)}(t)\right]^T, \tag{44}$$

where $v^*_{\chi_2 a(2)}(t) = \chi^*_2(t)h_{22}K_{22}(x)w_d(t) + \chi^*_2(t)h_{22}K_{221}(x)\bar{u}_1(t)$.

3.2.4. Adaptive Controller Structure. From equation (44), the structure of adaptive controller can be deduced as

$$v(t) = v_{\chi_1(1)}(t) + v_{\chi_2(2)}(t) = v_{\chi_1(1)}(t) + \left[0, v^T_{\chi_2 a(2)}(t)\right]^T, \tag{45}$$

where

$$v_{\chi_1(1)} \triangleq diag\{\chi_{1,1}, \cdots, \chi_{1,m}\}h_{21}K_{21}w_d \tag{46}$$

$$v_{\chi_2 a(2)} \triangleq diag\{\chi_{2,1}, \cdots, \chi_{2,m-1}\}h_{22}K_{22}w_d + \begin{bmatrix} \theta^T_{1(1)}\varpi_1\phi_{2,1} \\ \theta^T_{1(2)}\varpi_1\phi_{2,2} \\ \vdots \\ \theta^T_{1(m-1)}\varpi_1\phi_{2,m-1} \end{bmatrix}. \tag{47}$$

$\chi_{j,i}$ and $\theta_{1(i)}$ are the estimated value of $\chi^*_{j,i}$ and $\theta^*_{1(i)}$, $\chi^*_{1,i} = \chi^*_1$, $i = 1, \cdots, m$, $\chi^*_{2,i} = \chi^*_2$, $\theta^*_{1(i)} = \chi^*_2\theta^*_1$, $i = 1, 2, \cdots, m-1$.

Remark 7. As the number of $f_{ji}(t)$ increase, the parameters of the actuator failure (including the parameters of failure indicator function χ^*_i and χ^*_2, failure model θ^*_1 also increases. In our proposed actuator failure compensation design, all the unknown parameters will be estimated multiple (m or $m-1$) times based on $\chi^*_{1,i} = \chi^*_1$, $i = 1, \cdots, m$, $\chi^*_{2,i} = \chi^*_2$, $\theta^*_{1(i)} = \chi^*_2\theta^*_1$, $i = 1, 2, \cdots, m-1$. With the development of science and technology, the computers have become more advanced, the computation complexity can be solved effectively.

3.2.5. Error Equations. Equation (21) could be rewritten as

$$\begin{aligned} \begin{bmatrix} y_1^{(\rho_1)} \\ y_1^{(\rho_2)} \\ \vdots \\ y_q^{(\rho_q)} \end{bmatrix} &= \begin{bmatrix} L_f^{\rho_1}h_1(x) \\ L_f^{\rho_2}h_2(x) \\ \vdots \\ L_f^{\rho_q}h_q(x) \end{bmatrix} + A(x)u - w_d(t) + w_d(t) + \begin{bmatrix} \delta_1(x) \\ \delta_2(x) \\ \vdots \\ \delta_q(x) \end{bmatrix} \\ &= \begin{bmatrix} y_{m1}^{(\rho_1)} + \alpha_{11}\left(y_{m1}^{(\rho_1-1)} - y_1^{(\rho_1-1)}\right) + \alpha_{1\rho_1}(y_{m1} - y_1) \\ y_{m2}^{(\rho_2)} + \alpha_{21}\left(y_{m2}^{(\rho_2-1)} - y_2^{(\rho_2-1)}\right) + \alpha_{2\rho_2}(y_{m2} - y_2) \\ \vdots \\ y_{mq}^{(\rho_q)} + \alpha_{q1}\left(y_{mq}^{(\rho_q-1)} - y_q^{(\rho_q-1)}\right) + \alpha_{q\rho_q}\left(y_{mq} - y_q\right) \end{bmatrix} \\ &\quad + A(x)(I - \sigma(t))(v(t) - v^*(t)) + \begin{bmatrix} \delta_1(x) \\ \delta_2(x) \\ \vdots \\ \delta_q(x) \end{bmatrix} - \begin{bmatrix} \hat{\delta}_1(x) \\ \hat{\delta}_2(x) \\ \vdots \\ \hat{\delta}_q(x) \end{bmatrix}. \end{aligned} \tag{48}$$

To obtain the output error $e(t)=y(t)-y_m(t)$ and the parameter estimation error $\tilde{\chi}_{1,i}(t)$, $\tilde{\chi}_{2,i}(t)$, the dynamic formula between $\tilde{\theta}_{1(i)}(t)$ and $\tilde{\theta}_{dj}$ is reformulated as

$$\begin{bmatrix} e_1^{(\rho_1)}+\cdots+\alpha_{1\rho_1}e_1 \\ e_2^{(\rho_2)}+\cdots+\alpha_{2\rho_2}e_2 \\ \vdots \\ e_q^{(\rho_q)}+\cdots+\alpha_{q\rho_q}e_q \end{bmatrix} = A(x)(I-\sigma(t))(v-v^*) + \left[\tilde{E}_{d1}, \tilde{E}_{d2}, \cdots, \tilde{E}_{dq}\right]^T. \tag{49}$$

If $\sigma=\sigma_{(1)}=\mathrm{d}iag\{0,\cdots,0\}$

$$\begin{aligned}\begin{bmatrix} e_1^{(\rho_1)}+\cdots+\alpha_{1\rho_1}e_1 \\ e_2^{(\rho_2)}+\cdots+\alpha_{2\rho_2}e_2 \\ \vdots \\ e_q^{(\rho_q)}+\cdots+\alpha_{q\rho_q}e_q \end{bmatrix} &= \sum_{i=1}^{m} A_i\tilde{\chi}_{1,i}v_{1,i} + \sum_{i=1}^{m-1} A_{i+1}\tilde{\chi}_{2,i}v_{2,i} \\ &\quad + \sum_{i=1}^{m-1} A_{i+1}\tilde{\theta}_{1(i)}^T\varpi_1\phi_{2,i} + \left[\tilde{E}_{d1}, \tilde{E}_{d2}, \cdots, \tilde{E}_{dq}\right]^T \\ &= \sum_{i=1}^{m} A_i\tilde{\chi}_{1,i}v_{1,i} + \sum_{i=1}^{m-1} A_{i+1}\tilde{\chi}_{2,i}v_{2,i} \\ &\quad + \sum_{i=1}^{m-1} A_{i+1}\tilde{\theta}_{1(i)}^T\varpi_1\phi_{2,i} + \begin{bmatrix} \sum_{j=1}^{p}\tilde{\theta}_{dj}^T\varpi_{\rho_1,j} \\ \sum_{j=1}^{p}\tilde{\theta}_{dj}^T\varpi_{\rho_2,j} \\ \vdots \\ \sum_{j=1}^{p}\tilde{\theta}_{dj}^T\varpi_{\rho_q,j} \end{bmatrix} \triangleq \tilde{E}_1,\end{aligned} \tag{50}$$

where $h_{21}K_{21}w_d=[v_{1,1},\cdots,v_{1,m}]^T$, $h_{22}K_{22}w_d=[v_{2,1},\cdots,v_{2,m-1}]^T$, $\varpi_{\rho_i,j}=L_{p_j}L_f^{\rho_i-1}h_i(x)\varpi_{dj}$.

If $\sigma=\sigma_{(2)}=\mathrm{d}iag\{1,\cdots,0\}$, equation (49) can be expressed as

$$\begin{aligned}\begin{bmatrix} e_1^{(\rho_1)}+\cdots+\alpha_{1\rho_1}e_1 \\ e_2^{(\rho_2)}+\cdots+\alpha_{2\rho_2}e_2 \\ \vdots \\ e_q^{(\rho_q)}+\cdots+\alpha_{q\rho_q}e_q \end{bmatrix} &= \sum_{i=2}^{m} A_i\tilde{\chi}_{1,i}v_{1,i} + \sum_{i=1}^{m-1} A_{i+1}\tilde{\chi}_{2,i}v_{2,i} \\ &\quad + \sum_{i=1}^{m-1} A_{i+1}\tilde{\theta}_{1(i)}^T\varpi_1\phi_{2,i} \\ &\quad + \left[\sum_{j=1}^{p}\tilde{\theta}_{dj}^T\varpi_{\rho_1,j}, \sum_{j=1}^{p}\tilde{\theta}_{dj}^T\varpi_{\rho_2,j}, \cdots, \sum_{j=1}^{p}\tilde{\theta}_{dj}^T\varpi_{\rho_q,j}\right]^T \\ &= ^{\Delta}\tilde{E}_2.\end{aligned} \tag{51}$$

The state error equation can be obtained from equations ((50), (51)).

$$\dot{z}=A_z z + B_{z\tilde{E}_k}, \tag{52}$$

where $A_z=\mathrm{d}iag\{A_{z_1},\cdots,A_{z_q}\}\in R^{(\rho_1+\cdots+\rho_q)\times(\rho_1+\cdots+\rho_q)}$, $\tilde{E}_{kj}$ is the jth component of $\tilde{E}_k$, $k=1,2$, $B_{z\tilde{E}_k}=[B_{z_1}^T\tilde{E}_{k1}, B_{z_2}^T\tilde{E}_{k2},\cdots, B_{z_q}^T\tilde{E}_{kq}]^T\in R^{(\rho_1+\cdots+\rho_q)}$.

3.2.6. Adaptive Laws. Based on state error equation (52), the adaptive laws could be derived with projection algorithm, parameters $\chi_{1,i}$, $i=1,\cdots,m$, $\chi_{2,i}$ and $\theta_{1(i)}$, $i=1,2,\cdots,m-1$, are estimated as

$$\dot{\chi}_{1,i}(t)=\begin{cases} -\gamma_{1i}z^TPB_ZA_iv_{1,i} & i=2,\cdots,m \\ -\gamma_{1i}z^TPB_ZA_iv_{1,i}+f_{\chi_{1,i}} & i=1 \end{cases} \tag{53}$$

$$\dot{\chi}_{2,i}(t)=-\gamma_{2i}z^TPB_zA_{i+1}v_{2,i}, \quad i=1,\cdots,m-1 \tag{54}$$

$$\dot{\theta}_{1(i)}(t)=-z^TPB_zA_{i+1}\Gamma_{1i}\varpi_1\phi_{2,i}, \quad i=1,\cdots,m-1 \tag{55}$$

where $\Gamma_{1i}=\Gamma_{1i}^T>0$, $\gamma_{1i}>0$ and $\gamma_{2i}>0$ are the adaptive gains, $f_{\chi_{1,i}}$ is the projection algorithm. Consequently, according to adaptive laws $\dot{\chi}_{1,1}=-\gamma_{11}z^TPB_zA_1v_{1,1}+f_{\chi_{1,1}}$, we can derive that $0\le\chi_{1,1}\le 1$ and

$$(\chi_{1,1}-\chi_{1,1}^*)f_{\chi_{1,1}}\le 0. \tag{56}$$

3.2.7. Performance Analysis. (I) For time period $t\in[T_0,T_1)$, $T_1=\infty$, $\sigma=\sigma_{(1)}=\mathrm{diag}\{0,\cdots,0\}$. The Lyapunov function is defined as

$$\begin{aligned}V_0 &= \frac{1}{2}z^TPz + \frac{1}{2}\left[\sum_{i=1}^{m}\tilde{\chi}_{1,i}^2\gamma_{1i}^{-1} + \sum_{i=1}^{m-1}\tilde{\chi}_{2,i}^2\gamma_{2i}^{-1} + \sum_{i=1}^{m-1}\tilde{\theta}_{1(i)}^T\Gamma_{1i}^{-1}\tilde{\theta}_{1(i)}\right] \\ &\quad + \frac{1}{2}\sum_{j=1}^{p}\tilde{\theta}_{dj}^T\Gamma_{dj}^{-1}\tilde{\theta}_{dj}.\end{aligned} \tag{57}$$

Combining equations ((37), (50), (51), (52), (53), (54), (55)) one would have the derivative of V_0

$$\dot{V}_0=-z^TQz\le 0, \quad t\in[T_0,T_1). \tag{58}$$

Thus, it can be proved that the designed adaptive controller and its parameter adaptive laws could ensure the desired system performance under the free fault condition, i.e., ξ, $\tilde{\chi}_{1,i}(t)$, $\tilde{\chi}_{2,i}(t)$, $\tilde{\theta}_{1(i)}(t)$, and $\tilde{\theta}_{dj}$ are all bounded, and the output error asymptotes to zero as time going on.

(II) If actuator u_1 has faults in time period (T_1,T_2) $(T_2=\infty)$, i.e., $\sigma=\sigma_{(2)}=\mathrm{diag}\{1,\cdots,0\}$, the Lyapunov function is defined as

$$V_1 = \frac{1}{2}z^T Pz + \frac{1}{2}\left[\sum_{i=2}^{m}\tilde{\chi}_{1,i}^2\gamma_{1i}^{-1} + \sum_{i=1}^{m-1}\tilde{\chi}_{2,i}^2\gamma_{2i}^{-1} + \sum_{i=1}^{m-1}\tilde{\theta}_{1(i)}^T\Gamma_{1i}^{-1}\tilde{\theta}_{1(i)}\right] + \frac{1}{2}\sum_{j=1}^{p}\tilde{\theta}_{dj}^T\Gamma_{dj}^{-1}\tilde{\theta}_{dj}. \tag{59}$$

Combining equations ((37), (50), (51), (52), (53), (54), (55)) gives the derivative of V_1,

$$\dot{V}_1 = -z^T Qz \le 0,\ \ t \in [T_1, T_2). \tag{60}$$

The above equation indicates that ξ, $\tilde{\chi}_{1,i}(t)$, $i = 2, \cdots, m$, $\tilde{\chi}_{2,i}(t)$, $\tilde{\theta}_{1(i)}(t)$, $i = 1, \cdots, m-1$ and $\tilde{\theta}_{dj}$, $j = 1, 2, \cdots, p$ are bounded when actuator u_1 is failed. In addition, the adaptive projection algorithm $\dot{\chi}_{1,1}(t) = -\gamma_{1i}z^T PB_z A_i v_{1,i} + f_{\chi_{1,i}}$ can ensure $0 \le \chi_{1,1}(t) \le 1$. Thus, it can be verified that with increase in t, the closed-loop system is stable and the output asymptotically approaches zero: $\lim_{t\to\infty}(y(t) - y_m(t)) = 0$. In conclusion, the following theorem can be obtained.

Theorem 8. *For the multivariable nonlinear system (1) with potential uncertain actuator fault (3) and mismatched disturbance $d(t)$, controller (45) and its parameter adaptive laws can ensure the closed-loop system stability and asymptotic tracking output:* $\lim_{t\to\infty}(y - y_m) = 0$, *if* $\rho_1 + \rho_2 + \cdots + \rho_q = n$ *and the equivalent control matrix $A_\sigma(x) = A(x)(I - \sigma(t))v(t)$ in uncertain fault condition has full rank in the domain U (definition is $U \subset R^n \longrightarrow V \subset R^q$).*

3.3. Fault Compensation Design of Zero Dynamic System. If $\rho_1 + \rho_2 + \cdots + \rho_q < n$, there is differential homeomorphism

$$\left[\xi^T, \eta^T\right]^T = T(x) = \left[T_c(x)^T, T_z(x)^T\right]^T. \tag{61}$$

The nonlinear system with uncertain actuator fault $\dot{x}(t) = f(x) + g(x)\sigma(t)\bar{u}(t) + g(x)(1 - \sigma(t))v(t) + p(x)d(t)$, $y = h(x)$ is converted into

$$\begin{aligned}
\dot{\xi}_{1,1} &= \xi_{1,2}\\
\dot{\xi}_{1,2} &= \xi_{1,3}\\
&\cdots\\
\dot{\xi}_{1,\rho_1} &= b_1(\xi,\eta) + A_1(x)(I-\sigma)v + A_1(x)\sigma\bar{u} + \Delta_{d1}(x,t)\\
&\cdots\\
\dot{\xi}_{i,\rho_i} &= \xi_{i,\rho_i+1}\\
&\cdots\\
\dot{\xi}_{i,\rho_i} &= b_i(\xi,\eta) + A_i(x)(I-\sigma)v + A_i(x)\sigma\bar{u} + \Delta_{di}(x,t)\\
\dot{\xi}_{q,1} &= \xi_{q,2}\\
&\cdots\\
\dot{\xi}_{q,\rho_q} &= b_q(\xi,\eta) + A_q(x)(I-\sigma)v + A_q(x)\sigma\bar{u} + \Delta_{dq}(x,t),
\end{aligned} \tag{62}$$

and zero dynamic subsystem

$$\dot{\eta} = \psi(\xi,\eta) + \Psi_\sigma(\xi,\eta)\bar{u} + \bar{\Psi}_\sigma(\xi,\eta)v + \Psi_p(\xi,\eta)d(t), \tag{63}$$

where $T_c(x) = \left[h_1(x), \cdots, L_f^{\rho_1-1}h_1(x), \cdots, L_f^{\rho_q-1}h_q(x)\right]^T$, $T_z(x)$ definitely exists and is nonunique. $\Psi_\sigma(\xi,\eta) = (\partial T_z(x)/\partial x)g(x)\sigma$, $\bar{\Psi}_\sigma(\xi,\eta) = (\partial T_z(x)/\partial x)g(x)(I - \sigma)$ is related to the fault mode σ.

3.3.1. Stable Zero Dynamic Assumption. To ensure the stability of the closed-loop system and output $y_i(t)$ asymptotic tracking reference signal $y_{mi}(t)$, the differentials of ρ_i, $i = 1, 2, \cdots, q$ of $y_{mi}(t)$ are bounded and piecewise continuous. In this paper, the controller is developed based on the following assumption:

Assumption 9. The nonlinear system (1) still belongs to the minimum phase system under condition of centralized arbitrary fault, which is considered as the fault mode of this paper. That is, with input of $u(t)$, $d(t)$, and ξ, the zero dynamic subsystem given by

$$\begin{aligned}\dot{\eta} = {}&\psi(\xi,\eta) + \Psi_\sigma(\xi,\eta)\bar{u}) + \Psi_p(\xi,\eta)d\\ &+ \bar{\Psi}_\sigma(\xi,\eta)v\left(\xi,\eta,\tilde{\chi}_{1,i},\tilde{\chi}_{2,i},\tilde{\theta}_{1(i)}\right)\end{aligned} \tag{64}$$

could guarantee input state stability.

Remark 10. Based on Assumption 9, if $\sigma \in \Sigma$ in any fault case, the state ξ, fault signal $\bar{u}$, and the designed feedback control signal $v(\xi, \eta, \tilde{\chi}_{1,i}, \tilde{\chi}_{2,i}, \tilde{\theta}_{1(i)})$ are all bounded while $d(t)$ is bounded disturbance. According to the input state stability condition of the zero dynamic system, η is bounded. Combined with the performance analysis results in Section 3.2, it can be inferred that the nonlinear feedback control signal designed in this paper $v(\xi, \eta, \tilde{\chi}_{1,i}, \tilde{\chi}_{2,i}, \tilde{\theta}_{1(i)})$ is bounded.

Combined with Assumption 3, the signal $v(t)$ of adaptive fault compensation designed for the partial feedback linearization system (18) is similar to that for full feedback linearization system in Section 3.2. The detailed derivation is not rendered. The closed-loop system has the following desired control performance.

Theorem 11. *Based on the input state stability condition of zero dynamic (Assumption 9) and the equivalent control matrix $A_\sigma(x)$ in uncertain fault with row full ranks in domain of U, the adaptive controller (45) and its parameter adaptive law can achieve desired stability for closed-loop system (3) and asymptotic tracking output:* $\lim_{t\to\infty}(y(t) - y_m(t)) = 0$ *in the case of multiple uncertain actuator faults (3) and unknown disturbances.*

Proof. Assuming one of the actuators failed at time T_1, and the system has no fault during time period (T_1, T_2), it can be derived according to the performance analysis in Section 3.2 that the estimated parameters ξ, $\tilde{\chi}_{1,i}(t)$, $\tilde{\chi}_{2,i}(t)$, and $\tilde{\theta}_{1(i)}$

(t) are bounded and the state error z asymptotically approaches to zero as t tends towards infinity. Boundedness of $v(t)$ could be further confirmed from equations ((45), (46), (47)). Input state stability and row full rank constitute an estimation criterion for performance of a closed-loop system in terms of stability and asymptotical tracking capability.

4. Applications in Aircraft Control System

In this section, the proposed control method is applied to the aircraft control system, so that the developed control algorithm could be comprehensively validated. The numerical simulation results show that this method can offer an effective compensation for uncertain actuator fault in the case of gust disturbance.

4.1. Aircraft Dynamics in Turbulent Flow. The research of aircraft dynamic model under turbulence conditions in reference [37] shows that the longitudinal nonlinear dynamic model of the aircraft can be expressed as [38, 39]

$$\begin{aligned} \dot{V} &= \frac{F_x \cos(\alpha) + F_z \sin(\alpha)}{m_r} + d_1, \\ \dot{\alpha} &= q_r + \frac{-F_x \sin(\alpha) + F_z \cos(\alpha)}{m} + d_2, \\ \dot{\theta} &= q_r, \\ \dot{q} &= \frac{M}{I_y} + d_3, \end{aligned} \tag{65}$$

where V is the aircraft speed, α is the attack angle, θ is the angle of pitch, q_r is the pitch rate, m_r is the mass, I_y is the rotational inertia, M is the pitch moment, and d_1, d_2, and d_3 are turbulence disturbance signals

$$\begin{aligned} F_x &= q_r S C_x(\alpha, q_r, \delta_{e1}, \delta_{e2}) + T_1 \cos\gamma_1 + T_2 \cos\gamma_2 - m_r g \sin(\theta), \\ F_z &= \bar{q} S C_z(\alpha, q_r, \delta_{e1}, \delta_{e2}) + T_1 \sin\gamma_1 + T_2 \sin\gamma_2 + m_r g \cos(\theta), \\ M &= \bar{q} c S C_m(\alpha, q_r, \delta_{e1}, \delta_{e2}), \end{aligned} \tag{66}$$

$\bar{q} = 1/2\rho V_2$ is the dynamic pressure, ρ is the air density, S is the density of the wing, c is the average chord, and T_1 and T_2 are thrusters. C_x, C_z, and C_m are given by

$$\begin{aligned} C_x &= C_{x1}\alpha + C_{x2}\alpha^2 + C_{x3} + C_{x4}(k_1\delta_{e1} + k_2\delta_{e2}), \\ C_z &= C_{z1}\alpha + C_{z2}\alpha^2 + C_{z3} + C_{z4}(k_1\delta_{e1} + k_2\delta_{e2}) + C_{z5}q, \\ C_m &= C_{m1}\alpha + C_{m2}\alpha^2 + C_{m3} + C_{m4}(k_1\delta_{e1} + k_2\delta_{e2}) + C_{m5}q, \end{aligned} \tag{67}$$

where δ_{e1} and δ_{e2} are the two actuators that require fault compensation.

4.1.1. State Space Representation. The state variables x_1, x_2, x_3, and x_4 are represented by V, α, θ, and q, respectively. The input variables δ_{e1}, δ_{e2}, T_1, and T_2 are represented by u_1, u_2, u_3, and u_4. Nonlinear system (1) can be expressed as

$$\begin{aligned} \dot{x}_1 &= \left(c_1^T \varphi_0(x_2) x_1^2 + \varphi_1(x)\right) \cos(x_2) + \left(c_2^T \varphi_0(x_2) x_1^2 + \varphi_2(x)\right) \sin(x_2) \\ &\quad + k_1 g_1(x) u_1 + k_2 g_1 u_2 + g_{31}(x) u_3 + g_{41}(x) u_4 + d_1, \dot{x}_2 \\ &= x_4 - \left(c_1^T \varphi_0(x_2) x_1 + \varphi_1(x) \frac{1}{x_1}\right) \sin(x_2) \\ &\quad + \left(c_2^T \varphi_0(x_2) x_1 + \varphi_2(x) \frac{1}{x_1}\right) \cos(x_2) + k_1 g_2(x) u_1 \\ &\quad + k_2 g_2(x) u_2 + g_{32}(x) u_3 + g_{42}(x) u_4 + d_2, \\ \dot{x}_3 &= x_4, \\ \dot{x}_4 &= \phi(x) + b_1 x_1^2 u_1 + b_2 x_2^2 u_2 + d_3, \end{aligned} \tag{68}$$

where $\varphi_0(x_2) = [x_2, x_2^2, 1]^T$, $\varphi_1(x) = p_0 \sin(x_3)$, $\varphi_2(x) = p_1 x_4 x_1^2 + p_0 \cos(x_3)$, $g_1(x) = a_1 x_1^2 \cos(x_2) + a_2 x_1^2 \sin(x_2)$, $g_2(x) = -a_1 x_1 \sin(x_2) + a_2 x_1 \cos(x_2)$, $g_{31}(x) = \cos r_1 \cos(x_2) + \sin r_2 \sin(x_2)$, $g_{32}(x) = -\cos r_1 (\sin(x_2)/x_1) + \sin r_1 (\cos(x_2)/x_1)$, $g_{41}(x) = \cos r_1 \cos(x_2) + \sin r_2 \sin(x_2)$, $g_{42}(x) = -\cos r_2 (\sin(x_2)/x_1) + \sin r_2 (\cos(x_2)/x_1)$, and $\phi(x) = [x_1^2 x_2, x_1^2 x_2^2, x_1^2, x_1^2 x_4]^T$. B, k_1, k_2, c_1, c_2, p_1, a_1, a_2, r_1, r_2, b_1, and b_2 are known constants.

4.1.2. Control Objectives. For the aircraft control system (68) with uncertain turbulent disturbance and actuator faults, an adaptive fault compensation controller is designed to ensure that the stability of the closed-loop system is satisfied and that the system output $y(t) = [x_1, x_2, x_3]^T$ could track the desired control instruction $y_m(t) = [y_{m1}, y_{m2}, y_{m3}]^T = [3\sin(0.1t) + 88, 1.2\sin(0.1t), 3\sin(0.1t)]^T$. According to Theorem 11, $\rho_1 = v_1 = 1$, $\rho_2 = v_2 = 1$, $\rho_3 = v_3 = 2$, and $\rho_1 + \rho_2 + \rho_3 = 4$. The system satisfies Assumption 3 without zero dynamic subsystem after feedback linearization. The following fault modes corresponding to the requirements of fault compensation can be obtained:

$$\begin{gathered} \mathrm{d}iag\{1,0,0,0\}, \mathrm{d}iag\{0,1,0,0\}, \mathrm{d}iag\{0,0,1,0\}, \\ \mathrm{d}iag\{0,0,0,1\}, \text{and } \mathrm{d}iag\{0,0,0,0\}. \end{gathered} \tag{69}$$

4.1.3. Numerical Simulation Conditions. The aircraft parameters in reference [36] are as follows: $m_r = 4600$kg, $g = 9.80665 m/s^2$, $S = 39.02 m^2$, $c = 1.98 m$, $r_1 = \arctan 53/1216$, $r_2 = \arctan 2/45$, $\rho = 0.7377 kg/m^3$, $I_y = 31027 kg \cdot m^2$, $C_{x1} = 0.39$, $C_{x2} = 2.9099$, $C_{x3} = -0.0758$, $C_{x4} = 0.0961$, $C_{z1} = -7.0186$, $C_{z2} = 4.1109$, $C_{z3} = -0.3112$, $C_{z4} = -0.2340$, $C_{z5} = -0.1023$, $C_{m1} = -0.8789$, $C_{m2} = -3.8520$, $C_{m3} = -0.0108$, $C_{m4} = -1.8987$, and $C_{m5} = -0.6266$. The disturbances are given by $d_1(t) = 30\cos(5t) + 50N$, $d_2(t) = 15\sin(8t) + 30\cos(4t)N$, and $d_3(t) = 20\sin(10t) + 10N.m$.

During the simulation verification, the following fault conditions are incorporated: (i) When $t < 150s$, the system is in the absence of faults: $u_i(t) = v_i(t)$, $i = 1, 2, 3, 4$; (ii) When $150s \le t \le 300s$, actuator u_1 is stuck: $u_1(t) = 0$ deg, $u_i(t) = v_i(t)$, $i = 2, 3, 4$; (iii) When $300s \le t \le 400s$, actuator u_1 returns to normal: $u_i(t) = v_i(t)$, $i = 1, 2, 3, 4$; (iv) When $t \ge 400s$, actuator u_4 is stuck: $u_4(t) = 300$ N, $u_i(t) = v_i(t)$, $i = 1, 2, 3$.

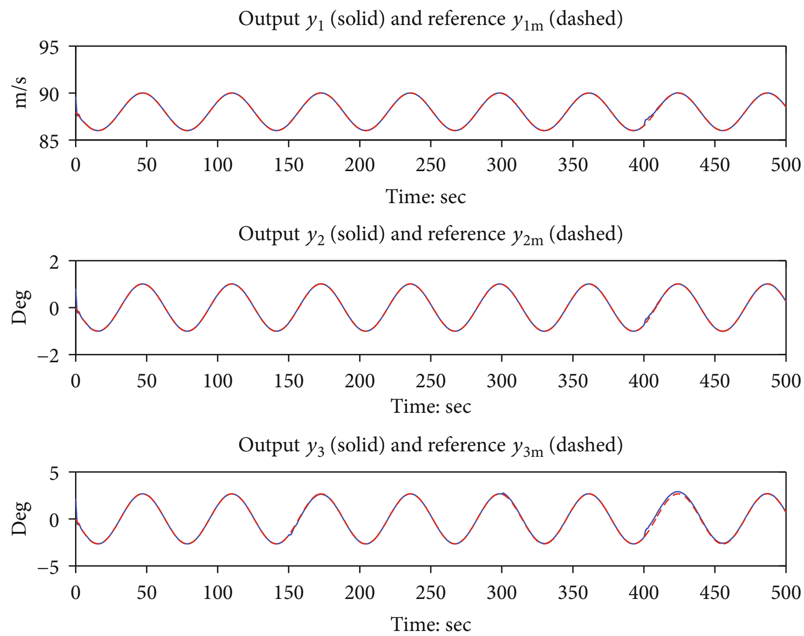

Figure 1: System outputs and reference outputs.

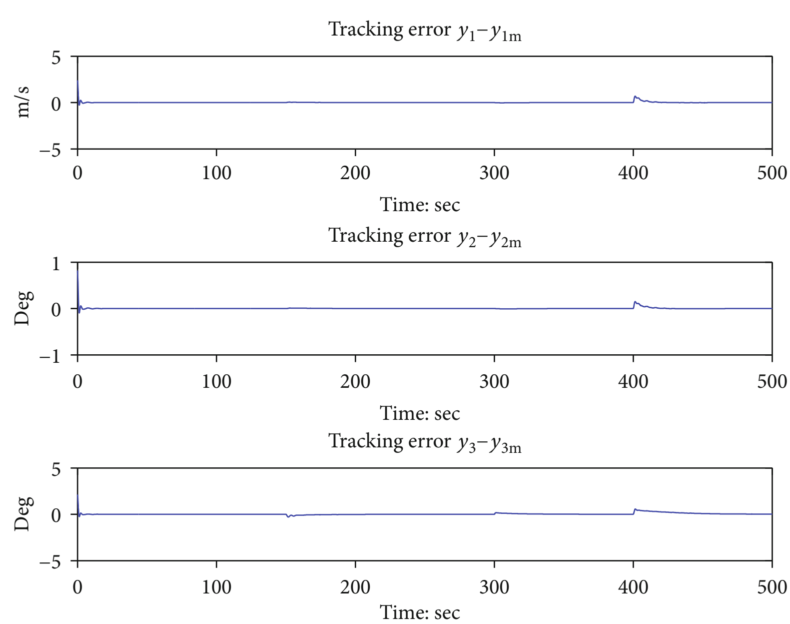

Figure 2: Tracking errors.

4.2. Simulation Results. In the simulation, the parameters of the adaptive controller $\hat{u}_L$ are $\alpha_{11} = \alpha_{21} = \alpha_{31} = \alpha_{32} = 0.75$, and the other design parameters are as follows:

(1) Initial state: $x_0 = [0.008, 0.72, 0.008, 0.008]^T$

(2) The base function in disturbance model (8), initial disturbance parameter, and the adaptive gain are $\varpi_{d1} = \varpi_{d2} = \varpi_{d3} = [1, \sin(8t), \cos(4t), \sin(10t), \cos(5t)]^T$, $\hat{\theta}_{d1}(0) = \hat{\theta}_{d2}(0) = \hat{\theta}_{d2}(0) = [0, 0, 0, 0, 0]$, and $\Gamma_{d1} = \Gamma_{d2} = \Gamma_{d3} = 10I_5$, respectively

(3) The base function in actuator failure model (3), initial failure parameter, and the adaptive gain are $\varpi_1(t) = \varpi_4(t) = [1, \sin(t)]^T \in R^2$, $[\chi_{11}(0), \chi_{12}(0), \chi_{13}(0), \chi_{14}(0)] = [1, 1, 1, 1]$, $[\chi_{21}(0), \chi_{22}(0), \chi_{23}(0)] = [0, 0, 0]$, $[\chi_{31}(0), \chi_{32}(0), \chi_{33}(0)] = [0, 0, 0]$, $\theta_{1(1)}(0) = \theta_{1(2)}(0) = \theta_{1(3)}(0) = [0, 0]^T$, $\theta_{4(1)}(0) = \theta_{4(2)}(0) = \theta_{4(3)}(0) = [0, 0]^T$, $\gamma_{11} = \gamma_{12} = \gamma_{13} = \gamma_{14} = 1$, $\gamma_{21} = \gamma_{22} = \gamma_{23} = 1$, $\gamma_{31} = \gamma_{32} = \gamma_{33} = 1$, and $\Gamma_{1i} = \Gamma_{4i} = 5I_2$

Simulation results are shown in Figures 1–3, including a comparison between the actual output of the system and

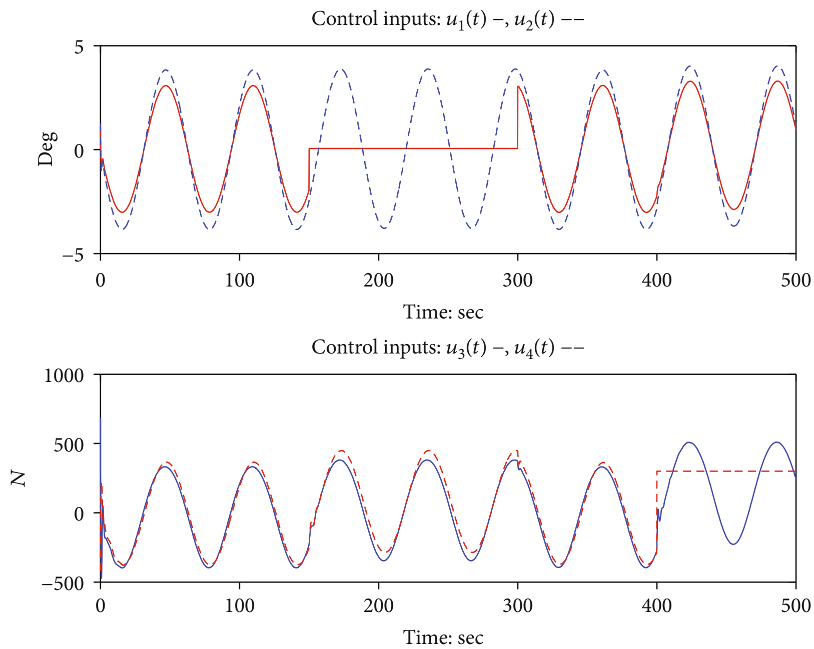

FIGURE 3: Control inputs.

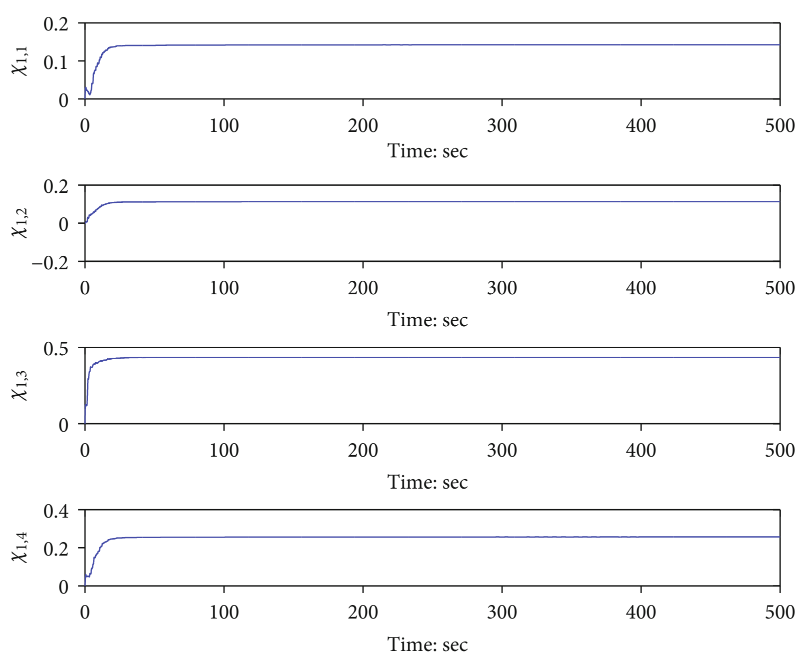

FIGURE 4: Parameter estimates $\chi_{1,i}$ of $\chi^*_{1,i}$.

the corresponding reference signal, the tracking error of the system, and the control input signals of four actuators acting on the system in the aircraft.

It can be seen from Figures 1 and 2 that during the actual operation, the designed control algorithm can always fulfill the control objective of system stability and asymptotic tracking, irrespective of normal operation or uncertainties in time, value, or fault model. The results in Figure 3 show that the system has external disturbance and no actuator fault during the period $t \in [0,150s)$. In the process of the asymptotic tracking of a given instruction, a transient response appears and decreases with time. The robustness of the proposed control method is verified through the results. When the actuator u_1 fails at $t = 150s$ and actuator u_4 fails at $t = 400\,s$ (shown in Figure 3), the simulation results demonstrate the effectiveness of the proposed adaptive compensation algorithm for both actuator fault and the disturbance. Moreover, the estimates of the adaptive controller parameters $\chi_{1,i}$, $\chi_{2,i}$, $\chi_{3,i}$,

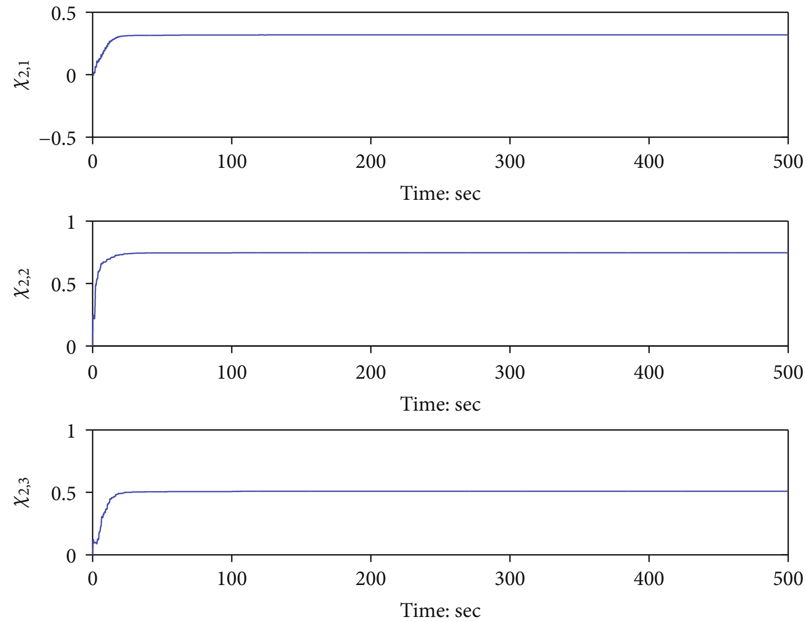

Figure 5: Parameter estimates $\chi_{2,i}$ of $\chi^*_{2,i}$.

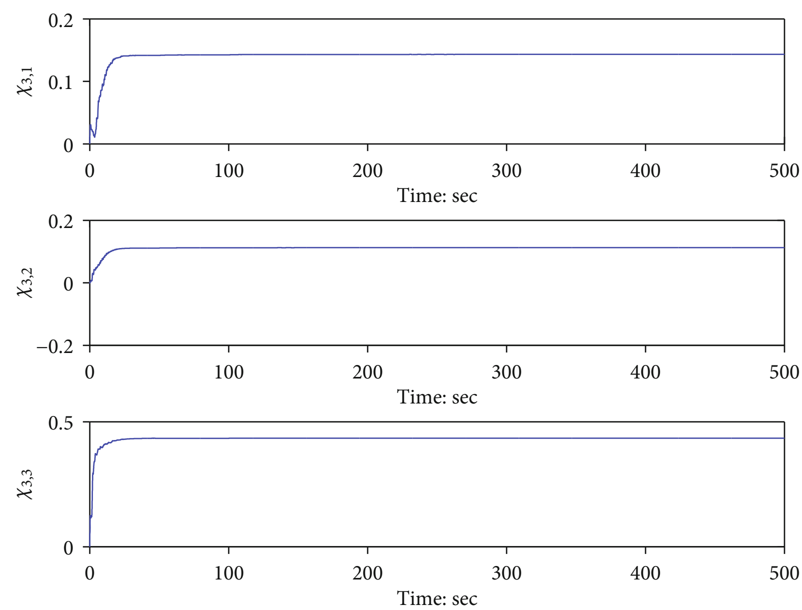

Figure 6: Parameter estimates $\chi_{3,i}$ of $\chi^*_{3,i}$.

$\theta_{1(i)}$, and $\theta_{4(i)}$ of $\chi^*_{1,i}\chi^*_{2,i}$, $\chi^*_{3,i}$, $\theta^*_{1(i)}$, and $\theta^*_{4(i)}$ for $y_m(t)$ are shown in Figures 4–8, which indicate that all signals in the adaptive control system are bounded, and the desired performance is met.

5. Conclusions

For multivariable nonlinear systems with multiple uncertain actuator faults and mismatched input disturbances, a control method of adaptive fault and disturbance compensation is proposed in this paper, with the following main conclusions. (1) An adaptive algorithm is adopted to establish the relation, and a set of adaptive fault compensation controllers is constructed based on parameter estimation. Then, a weighted algorithm is used to fuse multiple controllers into a comprehensive controller, so as to solve multiple uncertain actuator faults. (2) Under the condition of uncertain fault, a new parametric design method is adopted to obtain the parameter adaptive law of the fault compensation controller, so that the desired

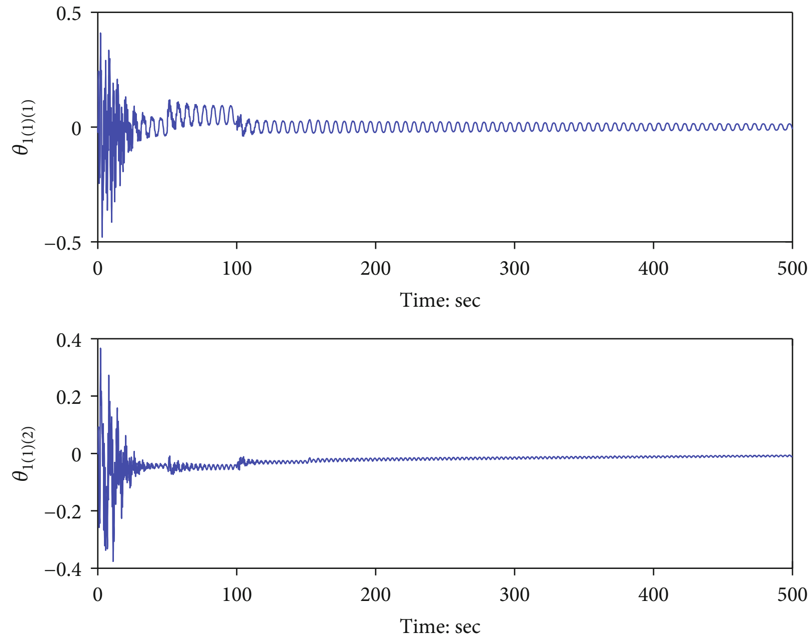

Figure 7: Parameter estimates $\theta_{1(i)}$ of $\theta^*_{1(i)}$.

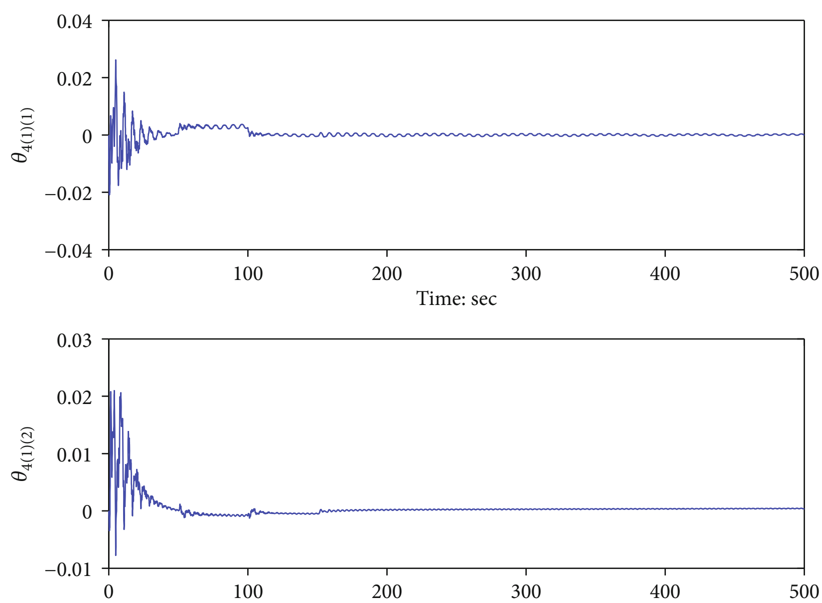

Figure 8: Parameter estimates $\theta_{4(i)}$ of $\theta^*_{4(i)}$.

performance of the closed-loop system can be guaranteed. (3) The effectiveness of the proposed theoretical method is verified by the simulation results of aircraft control under fault and disturbance conditions. The problem of fault compensation control for multivariable nonlinear system with known parameters is studied in this paper. (4) The proposed method can be further extended to solve the problem of fault compensation of the system with unknown parameters.

Data Availability

The data (System parameters and Simulation parameters) used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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References

[1] Z. Gao, C. Cecati, and S. X. Ding, "A survey of fault diagnosis and fault-tolerant Techniques—Part I: fault diagnosis with model-based and signal-based approaches," *IEEE Transactions on Industrial Electronics*, vol. 62, no. 6, pp. 3757–3767, 2015.

[2] M. Witczak, "Fault diagnosis and fault-tolerant control strategies for non-linear systems," in *Analytical and Soft Computing Approaches*, vol. 266 of Lecture Notes in Electrical Engineering, , Springer, 2014.

[3] J. Lan and R. J. Patton, "A new strategy for integration of fault estimation within fault-tolerant control," *Automatica*, vol. 69, pp. 48–59, 2016.

[4] G. Tao, "Direct adaptive actuator failure compensation control: a tutorial," *Journal of Control and Decision*, vol. 1, no. 1, pp. 75–101, 2014.

[5] M. Hashemi, J. Askari, J. Ghaisari, and M. Kamali, "Robust adaptive actuator failure compensation for a class of uncertain nonlinear systems," *International Journal of Automation and Computing*, vol. 14, no. 6, pp. 719–728, 2017.

[6] C. Tan, G. Tao, and R. Qi, "A discrete-time indirect adaptive multiple-model actuator failure compensation scheme," *International Journal of Adaptive Control and Signal Processing*, vol. 29, no. 6, pp. 685–704, 2015.

[7] C. Liu, B. Jiang, and K. Zhang, "Integrated multiple-model adaptive fault identification and reconfigurable fault-tolerant control for Lead-wing close formation systems," *International Journal of Systems Science*, vol. 49, no. 4, pp. 701–717, 2018.

[8] L. Liu, Y. J. Liu, and S. Tong, "Neural networks-based adaptive finite-time fault-tolerant control for a class of strict-feedback switched nonlinear systems," *IEEE Transactions on Cybernetics*, vol. 49, no. 7, pp. 2536–2545, 2019.

[9] A. Bounemeur, M. Chemachema, and N. Essounbouli, "Indirect adaptive fuzzy faulttolerant tracking control for MIMO nonlinear systems with actuator and sensor failures," *ISA Transactions*, vol. 79, pp. 45–61, 2018.

[10] H. Shen, Z. G. Wu, J. H. Park, and Z. Zhang, "Extended dissipativity-based synchronization of uncertain chaotic neural networks with actuator failures," *Journal of the Franklin Institute*, vol. 352, no. 4, pp. 1722–1738, 2015.

[11] B. Li, Q. Hu, Y. Yang, and O. A. Postolache, "Finite-time disturbance observer based integral sliding mode control for attitude stabilisation under actuator failure," *IET Control Theory and Applications*, vol. 13, no. 1, pp. 50–58, 2019.

[12] Z. Jing, J. Bin, X. Fei, G. Zhifeng, and X. Yufei, "Adaptive sliding mode backstepping control for near space vehicles considering engine faults," *Journal of Systems Engineering and Electronics*, vol. 29, no. 2, pp. 343–351, 2018.

[13] Q. Shen, D. Wang, S. Zhu, and E. K. Poh, "Integral-type sliding mode fault-tolerant control for attitude stabilization of spacecraft," *IEEE Transactions on Control Systems Technology*, vol. 23, no. 3, pp. 1131–1138, 2015.

[14] Y. Gu, J. N. Gross, M. B. Rhudy, and K. Lassak, "A fault-tolerant multiple sensor fusion approach applied to UAV attitude estimation," *International Journal of Aerospace Engineering*, vol. 2016, Article ID 6217428, 12 pages, 2016.

[15] L. Yao, L. Li, Y. Guan, and H. Wang, "Fault diagnosis and fault-tolerant control for non-gaussian nonlinear stochastic systems via entropy optimisation," *International Journal of Systems Science*, vol. 50, no. 13, pp. 2552–2564, 2019.

[16] X. Yao, L. Wu, and L. Guo, "Disturbance-observer-based fault tolerant control of high-speed trains: a Markovian jump system model approach," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 99, pp. 1–10, 2019.

[17] M. S. Mahmoud, A. M. Memon, and P. Shi, "Observer-based fault-tolerant control for a class of nonlinear networked control systems," *International Journal of Control*, vol. 87, no. 8, pp. 1707–1715, 2014.

[18] H. Sobhanipour and A. A. Afzalian, "Active fault tolerant control for switched positive linear systems," *International Journal of Robust and Nonlinear Control*, vol. 29, no. 14, pp. 4971–4984, 2019.

[19] G. Tao, S. Chen, X. Tang, and S. M. Joshi, *Adaptive Control of Systems with Actuator Failures*, Springer, 2004.

[20] L.-B. Wu, J. H. Park, and N.-N. Zhao, "Robust adaptive fault-tolerant tracking control for nonlinear systems with full-state constraints," *IEEE Transactions on Cybernetics*, pp. 1–13, 2019.

[21] C. Wang, C. Wen, and Y. Lin, "Decentralized adaptive backstepping control for a class of interconnected nonlinear systems with unknown actuator failures," *Journal of the Franklin Institute*, vol. 352, no. 3, pp. 835–850, 2015.

[22] W. Ao, Y. Song, and C. Wen, "Adaptive robust fault tolerant control design for a class of nonlinear uncertain MIMO systems with quantization," *ISA Transactions*, vol. 68, pp. 63–72, 2017.

[23] X. Tang, G. Tao, and S. M. Joshi, "Adaptive actuator failure compensation for nonlinear MIMO systems with an aircraft control application," *Automatica*, vol. 43, no. 11, pp. 1869–1883, 2007.

[24] A. Isidori, *Nonlinear control systems*, Springer, 3rd edition, 1995.

[25] S. S. Sastry and P. V. Kokotovic, "Feedback linearization in the presence of uncertainties," *International Journal of Adaptive Control and Signal Processing*, vol. 2, no. 4, pp. 327–346, 1988.

[26] J. He, R. Qi, B. Jiang, and J. Qian, "Adaptive output feedback fault-tolerant control design for hypersonic flight vehicles," *Journal of the Franklin Institute*, vol. 352, no. 5, pp. 1811–1835, 2015.

[27] Y. Liu and Y. Jing, "Practical finite-time almost disturbance decoupling strategy for uncertain nonlinear systems," *Nonlinear Dynamics*, vol. 95, no. 1, pp. 117–128, 2019.

[28] H. Li, Y. Wang, L. Xie, and D. Cheng, "Disturbance decoupling control design for switched Boolean control networks," *Systems & Control Letters*, vol. 72, no. 72, pp. 1–6, 2014.

[29] U. Başer, M. K. K. Cevik, and J. M. Schumacher, "Disturbance decoupling and robustness of stability," *International Journal of Robust and Nonlinear Control*, vol. 10, no. 15, pp. 1317–1336, 2000.

[30] J. B. Hoagg and T. M. Seigler, "Filtered-dynamic-inversion control for unknown minimum-phase systems with

unknown-and-unmeasured disturbances," *International Journal of Control*, vol. 86, no. 3, pp. 449–468, 2013.

[31] B. Ren, Q. C. Zhong, and J. Chen, "Robust control for a class of nonaffine nonlinear systems based on the uncertainty and disturbance estimator," *IEEE Transactions on Industrial Electronics*, vol. 62, no. 9, pp. 5881–5888, 2015.

[32] Z. Chen and J. Huang, "Attitude tracking and disturbance rejection of rigid spacecraft by adaptive control," *IEEE Transactions on Automatic Control*, vol. 54, no. 3, pp. 600–605, 2009.

[33] J. Guo, H. Zhang, X. Lu, and J. Zhou, "Nonlinear disturbance observer-based adaptive sliding mode control for a generic hypersonic vehicle," *International Journal of Aerospace Engineering*, vol. 2018, Article ID 6978170, 19 pages, 2018.

[34] Y. Park, "Robust and optimal attitude control of spacecraft with disturbances," *International Journal of Systems Science*, vol. 46, no. 7, pp. 1222–1233, 2015.

[35] J. Yang, S. Li, and W. H. Chen, "Nonlinear disturbance observer-based control for multi-input multi-output nonlinear systems subject to mismatching condition," *International Journal of Control*, vol. 85, no. 8, pp. 1071–1082, 2012.

[36] G. Tao, *Adaptive Control Design and Analysis*, Wiley, New York, NY, USA, 2003.

[37] A. E. Bryson, *Control of Spacecraft and Aircraft*, Princeton University Press, Princeton, NJ, USA, 1994.

[38] L. Wen, G. Tao, H. Yang, and Y. Zhang, "An adaptive disturbance rejection control scheme for multivariable nonlinear systems," *International Journal of Control*, vol. 89, no. 3, pp. 594–610, 2015.

[39] X. Tang, G. Tao, and S. M. Joshi, "Adaptive actuator failure compensation for parametric strict feedback systems and an aircraft application," *Automatica*, vol. 39, no. 11, pp. 1975–1982, 2003.