

Research Article Effects of Chamber Width on H₂/Air Rotating Detonations

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To get the effects of chamber width on the H_2 /Air rotating detonations, several models with different widths have been investigated. By using a one-step chemical reaction model, one wave is induced in all models. The chamber width has a significant effect on the flow field. When the chamber width is small, the variation of the flow field with the radius is not obvious. But when the width increases, the curvature of the detonation wave reflecting between the inner and outer walls at the head would become enlarged. The height of the detonation wave both on the inner wall and the outer wall has been presented. When the width reaches a limited value, the detonation wave cannot sustain on the inner wall. The normal velocity is used to characterize the detonation wave. The normal velocities on the outer wall and average diameter are almost the same. The former one is approximately the CJ value.

1. Introduction

Tangential combustion instability is the most serious problem in the development of liquid rocket engine. Due to the bad mixing efficiency near the face plate, a high-temperature combustible mixing layer may exist in front of the flame. This provides the condition to form a rotating detonation wave. So rotating detonation may be one of the causations inducing the tangential combustion instability.

Rotating detonation is firstly found by Voitsekhovskii [1] in the 1950s. After that, Nicholls et al. [2] designed a model engine using this kind of combustion mode to produce thrust. Early in the 1970s, similarities between the tangential instability of the liquid rocket engine and rotating detonation are brought forward by Ar'kov et al. [3]. Later, Shen [4] made a pilot study on two-phase detonation in a liquid rocket engine. A new type of tangential instability was found. The rotating detonation in a tangential direction induced by the leading wave during the process of combustion is an important factor to form this type of tangential instability. Due to the difficulties of realization, the relative investigations disappear for a while until the time comes into the 21st century.

So far, many researchers performed experimental and numerical investigations on rotating detonation engine. But

almost all the studies are focused on the coaxial annular combustor model in a rocket-based engine. Recently, Smirnov et al. [5, 6] studied on the rotating detonation in a ramjet engine. They investigated on the peculiarities of the ignition process and transition stage towards the rotating detonation wave mode in a D unsteady-state problem statement. The results validate the possibility of rotating detonation in a ramjet engine and enrich the detonation wave theory. But their model is a coaxial annular combustor all the same. During the process of investigations, few researchers combine the rotating detonation and combustion instability. The limited investigations on the detonation in the hollow chamber [7–12] show the possibility of detonation in the chamber without an inner cylinder. But the inner mechanism has not been understood yet. To compare the differences of wave between the annular and hollow chambers, it is very important to capture the flow field in the transition geometry. There are two methods for converting an annular chamber to a hollow one gradually: decrease the length or diameter of the inner cylinder. Thus, the chamber width effect is one of the key points to understand the behavior in transition geometry.

The operation of rotating detonation is contingent on many effects. In the processes of detonation onset and stabilization in a high-speed flow, the ignition delays are crucial to

determine the scenario of the process [13]. Current researches indicate that operation of rotating detonation is highly contingent on the chamber width [14-22]. Nakayama et al. [17-19] focused on studying the detonation propagation phenomena in curved channels and present the relationship of v_n and curved tube in detail. But the structure is different from the rotating detonation chamber (RDC). Schwer et al. [20] investigated the effect of the chamber width on the flow field and compare the specific impulse. Based on the two-dimensional configuration, Lee et al. [21] demonstrated flow feature variation with the radius of curvature, such as cell structures and pressure variations. Eude et al. [22] and Zhou and Wang [16] described the effects of shock wave reflections near the head and give the effects of the chamber width. The overall rotating detonation engine geometry and combustor channel width variation have been studied. However, the width is not large enough. The flow features in the annular chamber with big channel are still lacking. The differences in these channels have not been understood yet. Further study is worthy and needed.

In the present study, to reveal the physical mechanism more accurately, simulations are carried out to investigate the effect of chamber width on H_2/Air rotating detonation. The basic flow structures are given firstly to help understand the rotating detonation in the annular chamber. Then, effects of the chamber width are presented by comparing differences in all geometries. Through this work, the propagation process of the detonation waves with different chamber widths is better understood. It is hoped to enrich the detonation wave theory and make a contribution to understanding the structure in the transition chamber.

2. Physical Model and Numerical Scheme

2.1. Physical Model. In the present study, the model of the combustion chamber is a coaxial cylinder, shown in Figure 1. The inner radius is not fixed, the outer radius is $R_{out} = 50$ mm, and the total length is L = 80 mm. The distance between the inner and outer wall is the chamber width *H*. A premixed stoichiometric mixture of hydrogen-air is filled with the front of the chamber. The downstream area of the chamber is full of combustion products. In order to depress the initial pressure, a section of ignitable gas with low temperature is formed downstream of the inlet. In this section, there is an ignition region with Chapman-Jouguet (CJ) theoretic pressure and temperature to induce the rotating detonation wave.

2.2. Numerical Scheme. Detonation onset is strongly dependent on turbulence, which is testified by both theoretical and experimental studies [23, 24]. But just as the former studies [25–27] have shown, shock waves are usually not affected by viscous effects. Compared with convection terms in detonation propagation process, the effect of transport phenomena is usually small. Thus, the transport properties such as the viscosity, thermal conduction, and mass diffusion could be ignored. When using a mathematical model disregarding turbulence effect, the stabilized detonation in stoichiometric mixtures could be captured more easily. So in



FIGURE 1: Sketch map of the physical model.

the present study, ignoring the viscous effects, 3D Euler reaction equations are used as control equations which can be written as in a Cartesian coordinate:

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{E}}{\partial x} + \frac{\partial \mathbf{F}}{\partial y} + \frac{\partial G}{\partial z} = \mathbf{Q}_c, \qquad (1)$$

where $\mathbf{U} = (\rho, \rho u, \rho v, \rho w, \rho e_t, \rho_1, \dots, \rho_{NS-1})^T$ and

$$\begin{split} Q_{e} &= (0, 0, 0, 0, \sigma_{1}, \cdots, \sigma_{NS-1})^{T}, \\ E &= \begin{pmatrix} \rho u \\ \rho u^{2} + p \\ \rho uv \\ \rho uw \\ (\rho e_{t} + p)u \\ \rho_{1} u \\ \vdots \\ \rho_{NS-1} u \end{pmatrix}, \\ F &= \begin{pmatrix} \rho v \\ \rho uv \\ \rho v^{2} + p \\ \rho vw \\ (\rho e_{t} + p)v \\ \rho_{1} v \\ \vdots \\ \rho_{NS-1} v \end{pmatrix}, \end{split}$$



FIGURE 2: Pressure and outlet mass flow rate with two grid systems.

$$G = \begin{pmatrix} \rho w \\ \rho u w \\ \rho v w \\ \rho w^{2} + p \\ (\rho e_{t} + p) w \\ \rho_{1} w \\ \vdots \\ \rho_{NS-1} w \end{pmatrix}.$$
 (2)

Here, u and v are velocity components in x- and y-directions, respectively. ρ is the density, and p is the pressure. U is the conservation variable vector, \mathbf{Q}_c is the source term due to reaction, and E, F, and G are convective flux vectors.

Since only the gas dynamic properties in the combustor are considered, to save the simulation cost, a one-step chemical reaction model for stoichiometric hydrogen/air is used to describe the reaction in the present study. The model has been used by Yi et al. [28], Shao et al. [29, 30], and Liu et al. [25] to compute the flow-field structure of rotating detonation successfully. The results indicate that this model is particularly suitable for rotating detonation simulation.

The spatial terms are discretized by the fifth-order WENO scheme. In order to improve the efficiency of calculation, the two-order Runge-Kutta method that possesses the character of TVD is used for time integration. The boundary of the inlet is inflow, and the exit is outlet boundary condition. The inner and outer walls are using wall condition.

The injection boundary is specified according to the local pressure p_w at the head [26].

- (1) Choked: in the case of $p_w \ge p_0$, the pressure at the head is larger than the total injection pressure, and the reaction mixture cannot be injected into the chamber. The inlet is choked and a wall condition is set
- (2) Subsonic injection: in the case of p_{cr} ≤ p_w ≤ p₀, the injection is subsonic. The parameters are calculated as follows:

$$p = p_w, T = T_0 \left(\frac{p}{p_0}\right)^{\gamma - 1/\gamma},$$

$$u = \sqrt{\frac{2\gamma}{\gamma - 1} R T_0 \left[1 - \left(\frac{p}{p_0}\right)^{\gamma - 1/\gamma}\right]},$$

$$p_{cr} = p_0 \left(\frac{2}{\gamma + 1}\right)^{\gamma/\gamma - 1},$$
(3)

where *T* is the temperature of mixture, *u* is the axial injection velocity, and p_{cr} is the critical pressure

(3) Sonic injection: in the case of p_w ≤ p_{cr}, the injection is not affected by the wall pressure. The injection is sonic and the parameters are calculated as

$$p = p_{cr},$$

$$T = T_0 \left(\frac{p}{p_0}\right)^{\gamma - 1/\gamma},$$

$$u = \sqrt{\frac{2\gamma}{\gamma + 1}RT_0}$$
(4)



FIGURE 3: Three-dimensional structure of the continuous rotating detonation wave (H = 15 mm).



FIGURE 4: Pressures at the head.

2.3. Grid Dependence. The present code has been used in our previous work [25, 26] and shows a great performance on capturing the RDC's main propagation characteristics. To validate the accuracy and grid dependency of the simplified reaction model, Liu et al. [26] calculated a onedimensional detonation case based on this reaction model. The results show that the propagation velocity and pressure were in good agreement with the theoretical values, with the error ranges of propagation velocity and pressure of 0.04–0.38% and 2.2–2.52%, respectively. But the calculated temperature is approximately 200 K lower than the theoretical value, with the error range of 6.62-6.72%. According to the results of Yi et al. [28], who have also calculated this case based on both this reaction model and a detailed reaction model, the temperature error of the one-step reaction model was mainly caused by its simplicity. During the experiments of rotating detonation, only the propagation velocity and pressures could be obtained. The simplified reaction model is accurate in simulating the velocity and pressure. According to the method for accumulation of error estimations in aerodynamic simulations described in papers [31, 32], it is reasonable to adopt the simplified reaction model. In the present study, two grid systems (0.5 mm and 0.25 mm) were taken to simulate to validate grid dependency. Just as Figure 2 has shown, the pressures with the same point and outlet mass flow rates have the similar trend and stabilization processes that coincide with each other as well. To catch the flow field in detail, the average grid size of 0.25 mm is used in the present study.



FIGURE 5: Pressure contours on the inner, middle, and outer radius (H = 15 mm).

3. Results and Analysis

3.1. Basic Flow Structures

3.1.1. Detonation Structure. As Figure 3(a) has shown, the basic flow structures of rotating detonation are similar to the former research [26]. Since the fresh mixtures are injected from the top side continuously, while rotating, ① the detonation wave is formed near the inlet and ② fresh mixture zone ahead of it. With the detonated products expanding to the exit, ③ an oblique shock wave is set up at the bottom of the detonation wave. There is ④ a contact surface between combustion products of different cycles. Besides, ⑤ a contact surface also exists between fresh mixture zone and old prod-

ucts. When the combustion products flow out of the exit, the thrust is provided.

The detonation wave is propagating in a clockwise direction. Based on the flow structures at the head, repeated shock wave reflection between the inner and outer walls in the channel can be easily seen. It shows that a detonation wave front marked 1 and reflection waves marked 2, 3, and 4 exist in the channel of the inlet. Apart from the regular reflection, Mach reflections exist in the marked circles *A* and *B*. The detonation wave front 1 is reflected on the outer wall. Because of the Mach reflection behavior, a Mach steam and reflection wave 2 are generated on the outer wall, as shown in the marked circle *A*. After that, reflection shock wave 2 propagates at the head. In marked circle *B*, another Mach stem is



FIGURE 6: 3D distribution of detonation wave.

generated and reflection wave 3 propagates in the channel until it disappears. But both the lengths of the Mach steams are small.

From the inner wall to the outer wall, three different points located on the inner wall, average diameter, and outer wall, respectively, are marked to record the pressure. Figure 4 shows the pressures at the head. Affected by the reflection of the wave in the channel, a little peak appears behind the dominant peak. The detonation wave is compressed on the concave outer wall while expanded on the convex inner wall. Due to detonation wave curvature, the value of the dominant peak on the inner wall is the smallest.

In order to analyze the flow field clearly, the chambers are extended on a two-dimensional plane along the annulus direction. Figure 5 shows the pressure contours on the inner, middle, and outer radius of the chambers when the detonation wave propagated stably. The structures on the three slices are quite different. Because of the reflection of the wave in the channel, there are two strong waves on the inner wall. The first one is detonation wave front 1, and the other is the reflection wave 2. On the outer wall, there are also two waves and the later one is the reflection wave 3. On the middle radius slice, there are three strong waves which are detonation wave front 1 and reflection waves 2 and 3, respectively.

The detonation heights on the three slices are not the same. Figure 6 shows the 3D distribution of the detonation wave. It is easily seen that the detonation wave front at the head is curved. The detonation height on the inner wall is larger than that on the outer wall. The results show a great consistency with the work by Zhou and Wang [15]. In order to characterize the curvature of the wave, angular difference θ as the central angle of the detonation from the inner wall to the outer wall is defined.

Figure 7 shows the variation of the detonation height and angular difference when H = 15 mm. It can be easily seen that the internal height is larger than the external wall. The smallest detonation height appears at the radius from 42 mm to 44 mm. The variation of the height with the radius is not linear. The angular difference θ on the detonation front at the inlet from the inner wall to the outer wall is -8.47° .

3.1.2. Propagation Velocity. By measuring the interval Δt_i of each two ordinal waves, as shown in Figure 8(a), an instantaneous average frequency $f_i = 1/\Delta t_i$ in one cycle can be com-



FIGURE 7: Variation of detonation height and angular difference (H = 15 mm).

puted. Counting all the N cycles, an average frequency of rotating detonation waves can be obtained, i.e.,

$$f_{\rm av} = \frac{\sum_{i=1}^{N} f}{N}.$$
 (5)

By this method, when the width is 15 mm, the computed average result is 7.49 kHz with the range of 7.46-7.51 kHz. Propagation frequency-time and velocity-time distributions of pressure signal are shown in Figure 8(b). Based on the frequency distribution and $v_{out} = \pi D_{out} \bar{f}$, the corresponding propagation average velocity is 2353 m/s. With constant pressure of 1 atm and temperature of 300 K, respectively, the CJ velocity (ER = 1) is computed as 1969 m/s. The detonation on the outer wall seems to be overdriven. At the same time, the velocities on the average radius and inner wall are calculated as 2000 m/s and 1647 m/s, respectively. The propagation velocities on the outer wall is larger than the CJ value while the velocity on the inner wall is smaller than the CJ value.



FIGURE 8: Variation of the detonation wave (H = 15 mm).



FIGURE 9: Sketch map of the normal velocity.

According to the theory by Lee [33], the actual propagation velocity is smaller than the CJ value. Because of big curvature in the chamber with a wide channel, the planar detonation wave should transform into a curved detonation wave. Figure 9 shows the sketch map of the normal velocity (H = 15 mm). It is evidently seen that the detonation front is curved, and the inner front is ahead of the outer front. Usually the normal detonation velocity v_n is used to characterize the detonation. In the present study, just as shown in Figure 9, the angular difference ϕ between the tangential direction of the detonation front and the tangential direction is used to modify the velocity. The dashed line is the location



FIGURE 10: Variation of the angular difference ϕ .

of the average radius. Although there is a Mach stem which is overdriven at the outer wall, the velocity near the Mach stem should be modified. The normal velocity on the outer wall is computed as $v_n = v \times \sin \phi$. The variation of angular difference with the radius is shown in Figure 10. The angular difference varies with the radius. On the inner wall, the detonation wave is almost perpendicular to the tangential direction. On the outer wall, the angular difference ϕ is 56.2°. So the normal velocity on the outer wall is $v_n = 1956$ m/s. It is almost 99.3% of the CJ value. The angular difference at the average radius is 69°. And the corresponding normal velocity is 1867 m/s.



FIGURE 11: Pressure contours at the entrance with different chamber widths.

3.2. Effects of the Chamber Width

3.2.1. Effects on Detonation Wave Geometry. By keeping the outer radius constant, seven chambers with different widths (H = 5, 10, 15, 20, 30, 40, and 49 mm) are simulated. According to the studies [5, 6], a stoichiometric hydrogen-air

mixture is a high sensitivity of combustible mixture, which makes it possible for transverse waves initiating another wave rotating in the same direction due to their interaction with the curved external wall. So during the propagation process of one wave, the mode may evolve into a two-headed or multiheaded mode near stoichiometric conditions, which could



FIGURE 12: 3D distribution of the detonation wave.

affect the measured frequency. But in the present study, one wave is induced in all models. Figure 11 shows the pressure contours at the entrance with different chamber widths after the detonation propagated in a stable mode. All the detonation waves are propagating in a clockwise direction. A reflection shock wave is generated and propagates at the head until it disappears. The difference of the detonation wave is obvious. With the increase of the chamber width, the curvature of the wave and the length of the Mach stem on the inner wall become enlarged. It is obviously seen that when H = 30 mm, there is a Mach steam on the inner wall because of the Mach reflection behavior.

Figure 12 shows the 3D distribution of the detonation wave in different chambers. The structures of the flow flied are similar. In contrast to the former results by Zhou and Wang [15], the height of the detonation wave becomes decreased with the increase of width. It can be clearly seen that the external height of the detonation wave is smaller than that in the inner wall when H < 30 mm.

Since the angular velocity is constant, the propagation velocity of the detonation wave varies with radius. The velocity on the outer wall is larger than that on the inner wall. It consumed fuel more rapidly on the outer wall than on the inner wall. Thus, the height of the detonation wave is smaller than that in the inner wall. But with the chamber width H increased to 30 mm, the internal height is smaller than the external height. It contrasts finely with the former phenomena. Comparing the maximum pressure of the three lines,

the largest is on the inner wall when H > 30 mm. But when H < 30 mm, the largest pressure is on the outer wall. It is obviously seen that the value of the angular difference θ becomes enlarged with the increase of the chamber width. When the chamber width increases to 40 mm, the angular difference θ reaches to -130 deg.

3.2.2. Effects on the Pressure and Propagation Velocity. Figure 13 shows the pressure on the inner wall at the entrance of different chambers. As the effect of the transverse wave, similar to the experimental results [10], a little peak appears behind the dominant peak. But when H = 5 mm and H = 40 mm, the effect of the transverse is weak and no little peak appears. Besides, when H = 40 mm, the value of the pressure is not stable.

The appearing causation of these two peak pressure signals has been illustrated in the former publication [10, 15, 20]. With the increase of width, the interval between the dominant peak and the little peak becomes enlarged. It shows a great coincidence with the reflection.

Figure 14 shows the largest value of pressures on different radii. Comparing the maximum pressure of the dominant peaks on the inner wall, the largest value occurs when H = 40 mm. On the outer wall, the largest value appears when H = 15 mm. Besides, the largest pressure value of the little peak on the inner wall happens when H = 30 mm. It is related to the Mach steam which makes the pressure enlarged.



FIGURE 13: Pressure at the entrance of different chambers.

According to equation $v = \pi D \overline{f}$, the corresponding velocity at the three lines (the inner wall, the outer wall, and average diameter) is simulated, respectively. The results indicate that the frequencies become enlarged along with the chamber

width increase. Just as shown in Figure 15, the propagation velocities on the outer wall in all geometries are larger than the CJ value. The velocities on the inner wall is smaller than the CJ value. When the width between the outer and inner



FIGURE 14: Pressures distribution of different chambers.



FIGURE 15: Propagation velocity distribution with different chamber widths.

walls is large, this phenomenon becomes more evident. The phenomenon has been reported in several numerical and experimental works [15, 20, 34]. It shows a great consistency with the work by Bykovskii et al. [34]. But different from the conclusion that the propagation velocity at the average radius is approximately the CJ value, in the present study, the propagation velocity has a significant difference with the CJ value.

TABLE 1: Variation of angular difference ϕ and normal detonation velocity.

Chamber width <i>H</i> (mm)	Angular difference on the outer wall	Normal velocity on the outer wall v_n	Angular difference at the average diameter ϕ_{av}	Normal velocity at the average diameter v_n
	ϕ_{out} (°)	(m/s)	(°)	(m/s)
5	90	1960	90	1861
10	70	1954	88	1864
15	56.5	1955	69	1867
20	45.3	1955	62	1865
30	32	1952	45.8	1865
40	19	1951	31.2	1862

When H = 30 mm and 40 mm, the velocities on the average radius are computed as 2593 m/s and 3596 m/s, respectively. They are much larger than the CJ value.

Just as formerly said, the smaller the width is, the smaller the curve is. When the width H varies, the corresponding angular difference ϕ is captured, respectively. Table 1 shows the variation of angular difference ϕ and normal detonation velocity. It obviously shows that the angular difference ϕ decreases when the width increases. The normal velocities on the outer wall are in the range of 1951-1960 m/s. It is the largest when H = 5 mm while it is the smallest when H= 40 mm. But they are almost the same, the error within 0.1%. So the normal velocity on the outer wall is approximately the CJ value.

The same as getting the normal velocity on the outer wall, the normal velocity on the average diameter is computed. The angular difference ϕ_{av} on the average diameter is larger than the corresponding value on the outer wall. But when H = 5 mm, it is constant. The normal velocities on the average radius are almost the same. It is found that the normal propagation speeds of the wave on average radius are also almost the same.

3.2.3. Effects on Propagation Mode. When the chamber width increases to 49 mm, the detonation wave cannot propagate stably in the channel. The detonation wave cannot sustain on the inner wall. After a long time of transition, the detonation comes into a special propagation mode. Figure 16 shows the propagation process of detonation wave at the head. It can be easily seen that at times t_0 , there is only one wave propagating in the channel. With the time going to times t_1 , a point with high pressure and temperature is generated when the detonation wave passes by the inner wall. When the detonation wave reaches to the outer wall, new waves are generated at times t_2 . Wave 1 and wave 3 rotate in the opposite direction and will collide with each other until they disappear at times t_3 . At the time interval of t_3 - t_5 , the process is quite similar to t_0 - t_2 . There is only one wave propagating in the head at times t_3 . A zone with high pressure and temperature is generated when the detonation wave passes by the inner wall at times t_4 . When the detonation wave reaches to the outer wall at times t_5 , new waves are generated. This process repeats with the time going. During the process, some



FIGURE 16: The process of the propagation at the head.

detonation waves are generated or extinguished, some waves collide and disappear, while some directions are changed. The waves inside keep the stabilization process above, and finally, the detonation wave propagates in this special mode. The detonation height on the outer wall is nearly not changed which is only 7 mm.

4. Conclusion

In the present study, to investigate the effects of chamber width on H_2/Air rotating detonation wave, several models with different chamber widths are simulated. The results show that the chamber width has a significant effect on the detonation wave.

- (1) When the chamber width is small, the variation of the flow flied is not obvious. When the width increases, the wave is reflected between the inner and outer walls at the head. The curvature of the wave becomes enlarged, and the distance between the two waves on the inner wall also increases
- (2) The detonation height on the inner wall and outer wall decreased with the increase of chamber width. When H < 30 mm, the internal detonation height is larger than the external height while it is in contrast when H > 30 mm
- (3) The propagation velocities on the outer wall in all geometries are larger than the CJ value while the velocities on the inner wall are smaller than the CJ value. The normal velocity is used to characterize the detonation wave. The normal velocities on the

outer wall and average diameter are almost the same, and the former is approximately the CJ value

(4) When the chamber width is large enough, the detonation wave cannot sustain on the inner wall. The effect of the transverse wave makes the detonation wave propagate unstably. In order to get a stable detonation wave, a smaller inlet of the fresh air is necessary when the chamber width is large enough

Data Availability

All data generated or analyzed during this study are included in this article.

Conflicts of Interest

The authors declare that they have no financial and personal relationship with other people or organizations that can inappropriately influence their work. There is no professional or other personal interest of any nature or kind in any product, service, and company that could be construed as influencing the position presented in or the review of the paper.

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