Design of Composite Feedback and Feedforward Control Law for Aircraft Inertially Stabilized Platforms

Valerii Azarskov, Anatoly Tunik, and Olha Sushchenko

Aerospace Control Systems Department, National Aviation University, Liubomyra Huzara, ave. 1, Kyiv 03058, Ukraine

Correspondence should be addressed to Olha Sushchenko; sushoa@ukr.net

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The design of the control systems of the inertially stabilized platforms (ISPs) as part of airborne equipment for the majority of aircraft has its peculiarity. The presence of rate gyros in the inertial measurement unit gives the possibility to measure the rotation rate of the ISP base, which is the main disturbance interfering with the ISP accuracy. Inclusion of the feedforward disturbance gain in the control law with the simplest PI feedback significantly improves the accuracy of stabilization by the invariance theory. A combination of feedback and feedforward controllers produces a synergetic effect, thus, improving ISP accuracy. This article deals with the design of the airborne ISP control systems consisting of two stages: the parametric optimization of the PI feedback control based on composite “performance-robustness” criterion and the augmentation of the obtained system with feedforward gain. To prove the efficiency of the proposed control laws, the simulation of the ISP was undertaken. We have used a simulation of the heading-hold system of the commuter aircraft Beaver and the yaw rate output of this closed-loop system we have used as a source of the disturbance. The results of modeling proved the efficiency of the proposed design method.

1. Introduction

Nowadays, the inertially stabilized platforms (ISPs) are the most powerful means of stabilizing high-precision optical and optical-electronic equipment on moving vehicles. They may be used as the onboard equipment of various vehicles: aerial, terrestrial, and marine [1, 2]. Their applications for piloted aircraft and unmanned vehicles (UAVs) are significantly important, as they could be the main part of commercial payload [3] as well as an important part of navigation equipment, especially for UAV landing [4], stabilization, and trajectory tracking [5]. In these cases, UAVs are unable to perform their flight missions without inertially stabilized visual equipment. The application of ISPs in aerial vehicles has its peculiarities that distinguish it from terrestrial and marine applications.

Firstly, the ISPs in aerial vehicles must be lighter, smaller, cheaper, and more power-efficient than their terrestrial and marine counterparts. This imposes further restrictions on their design parameters, particularly, on the computational power of the on-board computers. These restrictions are absent in the terrestrial and marine ISPs [6]. Taking this fact into account, ISP control laws must be as simple as possible to implement them in the airborne computers.

Secondly, practically every aircraft is equipped with the Strapdown Inertial Navigation System (SINS), which is the kernel of the aircraft navigation system [7]. This fact gives the possibility of direct measurement of the aircraft’s body rotation rate, which is the main source of the ISP disturbance. It is possible to design the ISP control laws based on feedback and feedforward (or combined) structure as far as the disturbance is measurable [1]. Such an approach was intensively developed in the former Soviet literature as “invariance theory” [8]. In American literature, it is known as the “disturbance rejection” [9]. The simplest verbal description of this approach is given in [10]. Some applications of this approach in other areas are given in [11, 12]. As it is known [1, 2], the entire ISP consists of two main contours: high-bandwidth rate inner loop for image stabilization and a lower-bandwidth outer loop for some target pointing or tracking.
The first one is frequently used as an independent system for manual target tracking, and this article deals with the control law design for such a system. Aside from this introduction, the structure of the article consists of the second item, where the problem statement and the ISP linearized mathematical model are considered; the third item deals with the design of feedback and feedforward control law based on robustness and invariance theories; the fourth item deals with the simulation of the designed control system in the Simulink tool box; and the fifth item is the conclusion.

2. Problem Statement and Mathematical Model of ISP

The ISP consists of the biaxial gimbals (for the control of the azimuth and elevation angles) driven by two electric motors. We will consider the design of the control system for the azimuth axis only because the elevation angle control is similar. Figure 1 represents the block diagram of ISP combined control system, consisting of a feedback controller "Fb.con." and feedforward controller "Ff.con."

Other elements in Figure 1 stand for as follows: "Dist.mod." is the disturbance model, "ISP model" represents a model of the DC motor with the azimuth frame of gimbals, \( \omega, \omega_d \) are the ISP output signal and the ISP command signal in the tracking mode, respectively, \( \omega_d \) is the disturbance signal, and \( L_{PWM} \) is the pulse width of the PWM transistor, which is the control input for ISP. Here and further, we will assume the linear model of the pulse-width modulator. So, in this case, there are two channels of disturbance propagation. The first one is through the disturbance input of the controlled plant, and the second one is through the control input. For the sake of brevity, we will concisely describe the disturbance model, which is the series connection of two systems. The first one is the turbulence Dryden model [13, 14] of the lateral turbulent wind, and the second one is the model of the closed-loop heading angle stabilization system for commuter aircraft Beaver. We have used only the yaw rate output of this system as the source of the disturbance \( \omega_d \) for ISP. As far as, in our case, this disturbance can be directly measured, it is possible to obtain the rejection of the measured disturbance by feedforward controller, and suppressing the fluctuations of the output variable by the feedback controller. Under [8–10], the feedback controller is used for suppression of all possible measurable and nonmeasurable disturbances, and the feedforward controller is used for main disturbance \( \omega_d \) rejection.

Now, we can formulate the problem statement for the feedback controller as follows. Let the dynamics of the controlled plant be described by the standard state-space model [15].

\[
\frac{dx}{dt}(t) = Ax(t) + Bu(t) + B_d(t)d(t),
\]
\[
y(t) = Cx(t),
\]

where \( A, B_u, B_d, C \) are the state propagation, control input, disturbance input, and observation matrices, respectively.

Other variables in (1) are control input \( u = L_{PWM} \), disturbance \( d = \omega_d \), and output \( y = \omega \). The state vector consists of components \( x = [\omega, T]^T \), where \( T \) is the torque of the DC electric motor. The most relevant approach to solve the disturbance suppression problem for the system (1) is the minimization of the bounded \( L_2 - \text{gain} \) [16, 17]. In the general case, this gain is defined as follows [16, 17]:

\[
L_2 = \frac{\int_0^\infty (x^TQx + u^TRu)dt}{\int_0^\infty d^Tdtdt} \leq \gamma^2,
\]

where \( Q, R \) are corresponding weighting matrices. The paper [17] describes the procedure of the \( L_2 - \text{gain minimization} \) for static output feedback only. It was shown in [16] that for the more general case of static and dynamic output feedback, it is possible to use parametric optimization procedure for minimization of the \( L_2 - \text{gain and simultaneously } H_\infty \)-norm of the transfer function \( W_{df}(s) \) between the disturbance \( d \) and ISP output \( y \)

\[
\|W_{df}(s)\|_\infty < \gamma^*,
\]

where \( \gamma^* \) is the possible minimal value \( \gamma \), thus guaranteeing certain robustness of system [16, 17]. Controllers, based on results [16, 17], must have the structure of the static output feedback and also the observer-based dynamic output feedback [16]. In our case, it is possible to use simple controllers’ proportional-integral type recommended in [1, 2], which can be designed by a less sophisticated parametric optimization procedure. For achieving this goal, it is necessary to define the ISP mathematical model using concrete numerical parameters. Analysis of its dynamic properties gives the possibility to make demands on the optimization procedure.

For considered ISP complied with initial specifications, we have chosen the DC servomotor EDU-02 [18] with reducer ratio \( n = 518 \). Total moment of inertia of servomotor loaded with gimbals equals \( J = 0.0013 \text{kg} \cdot \text{m}^2 \). Other parameters have the following values: time constant of electric motor \( \tau = 0.01 \text{sec} \), viscous friction coefficient \( f_v = 0.081 \text{N} \cdot \text{m} \cdot \text{sec} \). Coulomb friction moment equals \( M_c = M_c \text{ sign} (\omega) \), where \( M_c = 0.09 \text{N} \cdot \text{m} \). The speed control of the servomotor uses a PWM transducer with maximal pulse width 2.5 msec. Suppose that the model of PWM transducer is linear, then the transfer function of servomotor from this transducer to the motor torque will have the following form

![Block diagram of ISP combined control system.](image-url)
\[ W_M(s) = \frac{K_M}{s^5 + 1}, \tag{4} \]

where \( K_M = 2.6 \text{N} \cdot \text{msec}^{-1} \). Taking into account these facts, it is possible to write down the following equations of dynamics \([1, 2, 6]\):

\[
\begin{align*}
\tau \frac{dT}{dt} + T &= K_M L_{PWM} - M_{CF}(\bar{\omega}), \\
J \frac{d\bar{\omega}}{dt} &= T - M_{VF}(\bar{\omega}),
\end{align*}
\tag{5}
\]

where \( M_{VF}(\bar{\omega}) = f_{\omega}\bar{\omega} \) is the viscous friction moment, and \( \bar{\omega} = \omega + \omega_d \). Here, \( \omega \) is the rotation rate of the motor with the motionless base, and \( \omega_d \) is the vehicle rotation rate (the yaw rate in our case). The last one is the main disturbance of this control system. We suppose that high-frequency vibrations of the vehicle body are suppressed by the vibration isolation means \([1]\). One of the difficulties of the control system design is the nonlinear term in the first equation of system (1) due to Coulomb friction \( M_{CF}(\bar{\omega}) \). As far as the disturbance \( \omega_d \) and the rotation rate of gimbals \( \omega \) are the random processes, it is necessary to use the random-input describing function \([19, 20]\) for the statistical linearization of the nonlinear term \( M_{CF} \) of \( \bar{\omega} \). As it is known \([19, 20]\), the coefficients of linearization depend on the ratio \( m_{\bar{\omega}}/\sigma_{\bar{\omega}} \), where \( m_{\bar{\omega}} \) and \( \sigma_{\bar{\omega}} \) are the mean value and the root mean square (rms) of the input random process \( \omega \), respectively. Due to the ISP’s main goal, the control system must maintain the rotation rate of the gimbals frame near zero in the steady-state mode; therefore, it can be possible to accept as the first approximation that \( m_{\bar{\omega}}/\sigma_{\bar{\omega}} = 0 \). In this case, it will be enough to use only an equivalent statistical gain of nonlinear elements respective to the root mean squares \( k = \sigma_y/\sigma_x \), where \( \sigma_y \) and \( \sigma_x \) are the root mean squares of output and input signals of this element. Taking into account this circumstance, the equivalent statistical gain of a nonlinear element \( M_{CF} = M_C \text{ sign (}\bar{\omega}) \) equals \([19]\):

\[ K_{CF} = \frac{M_C}{\sigma_{\bar{\omega}}}. \tag{6} \]

The root mean square \( \sigma_{\bar{\omega}} \) depends on the type of aircraft used as the ISP carrier and its flight conditions. As it was mentioned above, we have chosen the small commuter aircraft Beaver as a carrier. All aerodynamic characteristics, which are necessary to create its mathematical model, are circumstantially presented in \([13]\) along with the Dryden model of atmospheric turbulence (see also \([14]\)). The yaw rate of this carrier in the automatic heading-hold flight mode in the stochastically disturbing atmosphere was accepted as the model of disturbance \( \omega_d \). The Dryden filter was created for the following flight conditions: altitude \( H = 1500 \text{m} \), true airspeed \( V = 64 \text{m/sec} \), and rms of instant wind velocity \( \sigma_V = 4 \text{m/sec} \). And its output was applied to the correspondent input of the mathematical model of the closed-loop heading angle stabilization system. The root mean square of the yaw rate, in this case, obtained as a result of system modeling was \( \sigma_{\bar{\omega}} = 0.0115 \text{rad/sec} \). Substituting this value in (6), we can estimate \( K_{CF} = 7.82 \). Using \( X = (\omega, T) \) as the state vector, \( u = L_{PWM} \) as the control input, and \( d = \omega_d \) as the disturbance, we can rewrite system (5) in the standard Cauchy form (1), or

\[
\begin{align*}
\frac{d}{dt} \begin{bmatrix} \omega \\ T \end{bmatrix} &= \begin{bmatrix} -f_{\omega} / T & 1 \\ -K_{CF} / \tau & 1 / \tau \end{bmatrix} \begin{bmatrix} \omega \\ T \end{bmatrix} + \begin{bmatrix} 0 \\ K_M / \tau \end{bmatrix} \cdot L_{PWM} - \begin{bmatrix} f_{\omega} / T \\ K_{CF} / \tau \end{bmatrix} \cdot \omega_d, \\
Y &= \begin{bmatrix} 10 \end{bmatrix} \cdot \begin{bmatrix} \omega \\ T \end{bmatrix}^T.
\end{align*}
\tag{7}
\]

Substituting numerical values of coefficients in (7), we obtain model of the open loop system

\[
\begin{align*}
\frac{d}{dt} \begin{bmatrix} \omega \\ T \end{bmatrix} &= \begin{bmatrix} -62.31 & 769.29 \\ -782 & -100 \end{bmatrix} \begin{bmatrix} \omega \\ T \end{bmatrix} + \begin{bmatrix} 0 \\ 260 \end{bmatrix} \cdot L_{PWM} - \begin{bmatrix} 62.31 \\ 782 \end{bmatrix} \cdot \omega_d, \\
y &= \omega.
\end{align*}
\tag{8}
\]

Note that two complex eigenvalues of the state propagation matrix \( A \) in (8) \((-81.15 \pm 775.4i)\) have very large eigenfrequency: 779 rad/sec (124 Hz). This fact must be taken into account in the procedure of control law design. As it was stated in \([1, 2]\), the most popular control law for such controlled plants is the simplest PI controller. The transient processes in the closed-loop systems with such types of controlled plants have very high-frequency and low-frequency modes; therefore, such systems from a mathematical point of view can be classified as “stiff” systems. Differentiating the transient processes in real time requires a very high computational burden. Therefore, PID controllers are not well suited for these situations. Taking into account these considerations, we can specify the initial general problem statement for ISP control law design as follows.

1. Design of ISP feedback control law for the disturbance suppression based on the bounded \( L_2 \)–gain approach. As far as the type of the PI feedback was determined from the very beginning \([1, 2]\), it is necessary to design a parametric optimization procedure for this simplest control law.

2. Design of the ISP feedforward control law for the closed-loop system, determined at the previous step. This procedure based on the invariance theory will be described below.

3. Simulation of the designed system in order to evaluate the efficiency of the proposed control law.

2.1. Design of ISP Control Law. As it was stated before, we have chosen the PI control law for ISP feedback loop as

\[ K_{con} = K_p + K_i \cdot s^{-1}. \tag{9} \]
Therefore, the goal of the optimization procedure is the determination of the proportional $K_p$ and integral $K_i$ gains as a result of running some optimization procedure.

From item 1 of the problem statement, it is obvious that, first of all, we have to find some optimal gains in the control law (9) without feedforward, i.e., the gain $K_{ff}$ of “Ff.con” (see Figure 1) must be equal to zero at this stage. Following aforementioned the $L_2$ gain minimization approach and expressions (2) and (3), we proposed cost function for optimization procedure including \( \|W_d(s)\|_2 \) and \( \|W_{df}(s)\|_\infty \) with weighting coefficients allowing to achieve the trade-off between robustness and performance of control system. Also, sometimes, it is necessary to impose the limitations on the values of the control gains in order to avoid the saturation of the amplifiers of digital to analog converters (like PWM, for example) [21]. Eventually, the composite performance-robustness index based on the above-mentioned considerations looks like

\[
J = \left[ \lambda_2 \|W_d(j\omega, K_{con}) \cdot W_{w/f}(j\omega)\|_2 \right. \\
\left. + \lambda_\infty \|W_{df}(j\omega, K_{con}) \cdot W_{w/f}(j\omega)\|_\infty \right] + PF,
\]

where $\lambda_2, \lambda_\infty$ are weighting coefficients, and $\omega$ is the argument of the frequency response characteristic. We have to use a weighting transfer function $W_{w/f}(j\omega)$ in order to estimate norms in the realistic frequency bandwidth (no more than 50 Hz), so this transfer function is chosen as the second-order filter with eigenfrequency 314 rad/sec and unit damping ratio. Weighting coefficients were accepted as $\lambda_2 = \lambda_\infty = 200$. These values were obtained by trial and error method, which is the standard approach for many optimization problems [6, 22]. PF is the penalty function, which is necessary to preserve closed-loop system stability during executing the optimization procedure on the one hand and to limit the absolute values of the gains in (6) in order to avoid the aforementioned saturation. It consists of three parts [6, 22]

\[
PF = \sum_{i=1}^{3} PF_i.
\]

The first one $PF_1(d_m)$ as a function of minimum distance to the imaginary axis could be graphically shown in Figure 2 and defined over the area $D$ for its first border (see Figure 2) as follows [6, 22].

\[
PF(d_m) = \begin{cases} 
0, & \text{if } d_m \geq d_{m1} \\
\frac{P}{2} \left[ 1 + \cos \left( \frac{\pi(d_m - d_0)}{d_{m1} - d_0} \right) \right], & \text{if } d_m \leq d_0
\end{cases}
\]

where $P$ is a very large value (for example $P = 10^4 \div 10^6$), $D$ is the distance between the maximum and the minimum real parts of the poles. $PF_1(d_m)$ defines minimum stability margin $d_0$ of the closed-loop system.

The penalty function $PF_2(d_m)$ for the second border, which defines the closed-loop system bandwidth, looks like (12). The third component $PF_3$ has to restrict too large values of gains in (6), and it was chosen in the following form.

\[
PF_3 = 0.0035 \cdot K_p^2 + 0.002 \cdot K_i^4.
\]

This component plays the same role as the weighting matrix $R$ in expression (2) for standard $L_\infty$ gain minimization procedure [17]. It is useful to mention that restriction imposed on controller (9) gains $K_p$ and $K_i$ by cost function (13) constrains the bandwidth of the closed-loop system. Eventually, we can formulate the optimization procedure as follows.

\[
\text{Findargmin}_{K_{con}} J(K_{con}) \text{ ; subjectto } K_{con} \in D,
\]

where $D$ is the admissible domain in the controller parameters plane determined by PF, and $J(K_{con})$ is the performance-robustness index defined by (10), which is minimized by the function "fminsearch.m" in MATLAB. As it is known, this function uses Nelder-Mead minimization procedure. After running this procedure, we obtained the following parameters of the controller $K_{con}$: $K_p = 62.25, K_i = 9.64$. The values of norms are equal to $H_2 = 0.43, H_\infty = 0.047$. These norms demonstrate a good ability to suppress external disturbance $\omega_d$ [16, 17, 23]. The small value of the $H_\infty$ norm of the transfer function $W_{dy}(j\omega, K_{con})$ shows high degree of robustness of ISP control. Note that these values are upper estimates, and simulation demonstrates even higher efficiency of the controller (6). The eigenvalues of the closed-loop system equal $-81.1 \pm j3584, -0.2$. Then, this system is the very "stiff" one, as we noted before. Poles of the plant (5) substantiate the high-frequency complex poles of the closed-loop system, and low-frequency real pole appears due to the integral gain of controller.

Now, we will determine the gain $K_{ff}$ of the feedforward controller “Ff.con.” (see Figure 1). Under [8, 9], the invariance condition concludes in the following: two aforementioned channels of the disturbance propagation must be equal and have opposite signs. Applying this condition to the system (7), one can obtain

\[
B_d K_{ff} = -B_d.
\]
Figure 3: ISP functioning in stabilization mode without feedforward: (a) plot of disturbance $\omega_d$, (b) plot of ISP rotation rate $\omega$. 
Introducing the pseudoinverse matrix \((B^T_u \cdot B_u)^{-1} B^T_u\), it is possible to find gain \(K_{FF}\) as follows

\[
K_{FF} = (B^T_u \cdot B_u)^{-1} \cdot B^T_u \cdot B_d. \tag{16}
\]

Substituting expressions for matrices \(B_d\) and \(B_u\) from system (7) and their numerical values from system (8), we obtain

\[
K_{FF} = \frac{K_{CF}}{K_M} = \frac{7.82}{2.6} = 3. \tag{17}
\]

2.2. Simulation of ISP with and without Feedforward Gain. The first stage of this process is the estimation of efficiency of ISP disturbance suppression in stabilization mode with a zero reference signal. Figure 3(a) represents the disturbance plot, and Figure 3(b) represents the plot of output ISP signal or the rotation rate of the azimuth frame of the gimbals. The rms of the input disturbance \(\omega_d\) equals \(\sigma_{\omega_d} = 0.0112\) rad/sec; meanwhile, the rms of the ISP rotation rate \(\omega\) is much less: \(\sigma_\omega = 4.031 \cdot 10^{-4}\) rad/sec. Therefore, the efficiency of this ISP is good enough, because it suppresses the disturbance \(k_s = \sigma_{\omega_d}/\sigma_\omega = 27.81\) times. However, including the feedforward gain in the control law much more improves ISP efficiency for suppressing input disturbance. Figure 4(a) represents the same plot of disturbance \(\omega_d\), and Figure 4(b) represents the plot of the rotation rate of the ISP azimuth frame of gimbals with feedforward controller. The rms of input disturbance
\( \omega_d \) equals the same \( \sigma_{\omega d} = 0.0112 \text{rad/sec} \), but the rms of ISP rotation rate \( \omega \) is much smaller: \( \sigma_{\omega} = 5.65 \times 10^{-6} \text{rad/sec} \). The input disturbance is suppressed \( k_s = \sigma_{\omega d}/\sigma_{\omega} = 1978 \) times, so the efficiency of ISP increases at more than 71 times by the simplest mean. The experimentally obtained plot \( k_s = f(K_{FF}) \) demonstrates a sharp peak in the proximity of point \( (k_s = 1978, K_{FF} = 3) \). So, using feedforward/feedback control leads to improvement of stabilization accuracy.

The second stage of the modeling is the estimation of the ISP efficiency in the tracking mode with the nonzero reference signal, which is accepted as step function having magnitude 0.25 rad/sec (14.325 deg/sec). The transient processes, in this case, are shown in Figures 5 and 6. Plots in Figure 5 demonstrate the fast response of the ISP control systems (Figure 5(a)) and the absence of saturation of the control input (Figure 5(b)). Plots in Figure 6 show the ISP rotation rate error in the tracking mode without feedforward (Figure 6(a)) and with feedforward (Figure 6(b)). It is obvious that including feedforward gain practically eliminates the jitter of the target image in the field of sight of the stabilized camera in the tracking mode, thus essentially improving the ISP performance.

Of course, applying modern inertial sensors and electronics [11, 12, 24] can improve practical implementation of the obtained results.
3. Discussion

The design of effective but simple control laws for the airborne ISP was one of the initial goals of this research. However, further progress in the microelectronics opens up the possibility to use more effective ISP control laws. The new studies and papers will be devoted to the possibility of using other types of controllers for disturbance rejection and comparing them with the represented controller.

The most widespread sensors applicable to the designed controller are MEMS gyroscopes. For more expensive applications, it is possible to use Coriolis vibratory gyroscopes. The choice of actuators depends on a mass of loading. For the small mass, it is convenient to use moment electric motors; in other cases, it is necessary to use electric motors with reducers. In both cases, electric motors of direct current are used.

4. Conclusions

Usage of the rate gyroscopes in the inertial measurement unit (IMU) of the airborne navigation equipment gives an additional source to increase the ISP accuracy because it gives the possibility to include the feedforward disturbance gain in the control law following the main principles of the invariance theory.

The design of a simple PI control law based on the parametric robust optimization provides a quite acceptable accuracy and robustness of the ISP feedback control system.

Figure 6: Rotation rate error in the tracking mode: (a) without feedforward, (b) with feedforward.
Augmenting PI feedback control law with feedforward disturbance gain significantly improves the accuracy of inertial platform stabilization to such degree, which is inaccessible for feedback only even when the more sophisticated control law is used [6].

The results of the simulation of the ISP control systems with and without the feedforward disturbance gain prove the efficiency of the proposed algorithms for the stabilization of the target image in the camera’s field of view.

Data Availability

The paper “Design of Composite Feedback and Feedforward Control Law for Aircraft Inertially Stabilized Platforms” data used to support the findings of this study are included within the article in the list of references.

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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References