

## Research Article

# Experimental Study of Aircraft Achieving Dutch Roll Mode Stability without Weathercock Stability

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Weathercock stability is usually considered essential to achieve normal flight, while the Dutch roll mode stability can still be achieved without weathercock stability which has been algebraically proved. This paper proposed a flight experiment to investigate the characteristics of an airplane with Dutch roll mode stability but no weathercock stability. Firstly, the algebraic analysis based on a standard lateral-directional mode approximation was made to demonstrate the effect of yawing stability derivative  $C_{n\beta}$  on the Dutch roll mode characteristics. The flight experiment was organized after that using a model glider which was modified to have zero  $C_{n\beta}$  but with marginal change on  $C_{y\beta}$ . The convergence of Dutch roll mode in flight meets the algebraic and numerical analysis as expected. However, the difference of handling characteristics between the original and modified configurations indicates some other roles the weathercock stability plays in flight as well as some limitations of utilizing mode criterion in flight quality analysis.

## 1. Introduction

Weathercock stability, also known as directional static stability or yaw stiffness, is the ability of the aircraft to turn in to wind in order to maintain directional equilibrium. The static stability principle suggests that weathercock stability is essential for a stable airplane [1]. The derivative  $C_{n\beta}$  that corresponds to the yawing moment due to sideslip is designed to be a positive value for most conventional airplanes as a result of weathercock stability assumption. The main contribution to  $C_{n\beta}$  comes from vertical stabilizer. The fuselage and wing are also major contributors to the value of this derivative [2].

From the perspective of lateral-directional stability study in flight dynamics, the perturbed state problems were first introduced in 1911 to investigate whether the response to the external disturbance was acceptable [3, 4]. After the airplane stability quartic in terms of its polynomial coefficients was rooted by Bairstow, the Routh stability criterion was the first used to study the dynamic stability behavior of an airplane; then, the term Dutch roll was used by Hunsaker to identify the oscillatory roots of the lateral-directional char-

acteristic equations [5–8]. For the lateral-directional dynamic stability, a hypothetical small airplane with various modifications of the fin area and dihedral setting was used to investigate the motions caused by the lateral disturbance before 1938 [9]. As demonstrated in Ref. [9], with different combinations of  $C_{n\beta}$  and  $C_{l\beta}$ , types of spiral and oscillatory divergence limits on these two derivatives were found.

To make specifications of the flight quality for new military airplanes, MIL-8785B (ASG), which includes specifications of lateral-directional mode characteristics, was published in 1969 based on the data from typical existing aircrafts [10]. It was continuously validated and revised by the US military, institutes, and manufacturers; then, the final revision of MIL-F-8785C was issued in 1980 [11]. Later in 1995, this military specification was expanded and revised to MIL-STD-1797A and designated as a handbook for guidance only, where the flight quality specifications for the lateral-directional mode were involved identically [12].

For the lateral-directional stability and control study from the designer's perspective, in the preliminary configuration design stage, the tail-volume method is often used in the

first-cut estimation of vertical stabilizer sizing. According to the statistics of the tail-volume range for specific categories of existing aircrafts, the appropriate tail volumes are determined, and the area of the vertical stabilizer can be obtained [2, 13]. As suggested in Ref. [14], for most airplanes, the derivative  $C_{n\beta}$  has a lower limit of 0.001-0.0025.

The vertical stabilizer appears indispensable for an aircraft. However, in aviation history before the stability augment system was invented, several aircrafts with flying wing configurations such as Ho.229 and N-9M appeared which have no vertical stabilizer to provide enough directional static stability. On the other hand, it has been proven that the lateral-directional stability of this configuration can be achieved by adjusting the spanwise dihedral layout without installing a stability augment system [15, 16]. Instead of focusing on the required lateral-directional stability derivative  $C_{l\beta}$  or  $C_{n\beta}$ , a criterion of dynamic stability was included in the aerodynamic design for flying wing, which considered the Dutch roll mode characteristics as the critical element for flight qualities [17]. Moreover, a unified algebraic approach was developed to approximate aircraft lateral-directional modes and predict departure susceptibility [18]. And in Ref. [19], the relationship between vertical stabilizer and lateral-directional stability was studied and illustrated by some representative experiments. Above all, the study on flying wing aircrafts and the Dutch roll mode stability characteristics illustrates the possibility of normal flight with no sufficient weathercock stability. But the role that weathercock stability played in normal flight is still worth investigation.

This paper is aimed at demonstrating that the weathercock stability is not essential to achieve Dutch roll mode stability for a conventional layout airplane but has its influence on flight quality. The demonstration is made from three aspects: algebraic analysis, numerical study, and flight experiments. The flight test result may indicate some other significance of the derivative  $C_{n\beta}$  in flight quality which are not demonstrated in Dutch roll mode flight quality specifications.

## 2. The Essential Factor for Dutch Roll Mode Stability

To demonstrate the weathercock stability unessential and find the crucial factors to achieve the Dutch roll mode stability, the classic lateral-directional mode literal approximation was considered for theoretical research. In fact, such literal expression involves aerodynamic and inertial data and it provides valuable insight into the connection between the relative values of certain key derivatives and behavior of aircraft motion [18].

Commonly, the derivation of the approximation contains omissions of relatively unimportant terms, to make the relationship between the cause and effect apparent. But for the purpose of studying the crucial factor for Dutch roll stability, some certain omissions need examination again in the developing of approximation. The simplified model for the approximation is obtained by separating slow mode from fast modes. That is, for lateral-directional motions, the fourth-order equation approximation is made by sepa-

rating the spiral mode (first-order system) from the roll and Dutch roll mode (third-order system).

To develop the Dutch roll mode natural frequency approximation, we designate the stability axis reference frame and assume a condition of level flight; the dynamic matrix from the lateral-directional linearization small disturbance equations is given by [20]

$$A_{\text{lat}} = \begin{bmatrix} Y_{\beta}/V_0 & 0 & -1 & g/V_0 \\ L'_{\beta} & L'_p & L'_r & 0 \\ N'_{\beta} & N'_p & N'_r & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad (1)$$

where  $L'_i$  and  $N'_i$  are standard primed derivatives,  $V_0$  refers to trimmed level flight velocity, and  $g$  refers the gravitational acceleration. Consider the standard factorization of the characteristic polynomial for lateral-directional modes,

$$P_{\text{lat}}(s) = (s + \lambda_S)(s + \lambda_R)(s^2 + 2\zeta_{\text{DR}}\omega_{\text{DR}}s + \omega_{\text{DR}}^2). \quad (2)$$

The subscripts  $_{S,R,DR}$  refer to the spiral, roll, and Dutch roll modes, respectively. Letting

$$P_{\text{lat}}(s) = s^4 + Bs^3 + Cs^2 + Ds + E, \quad (3)$$

one has

$$\begin{aligned} B &= \lambda_R + \lambda_S + 2\zeta_{\text{DR}}\omega_{\text{DR}}, \\ C &= \omega_{\text{DR}}^2 + 2\zeta_{\text{DR}}\omega_{\text{DR}}(\lambda_R + \lambda_S) + \lambda_R\lambda_S, \\ D &= (\lambda_R + \lambda_S)\omega_{\text{DR}}^2 + 2\zeta_{\text{DR}}\omega_{\text{DR}}\lambda_R\lambda_S, \\ E &= \lambda_R\lambda_S\omega_{\text{DR}}^2. \end{aligned} \quad (4)$$

The fourth-order polynomial coefficients can be determined by derivatives in matrix  $A_{\text{lat}}$ . Consider the four coefficients in the characteristic polynomial for the lateral-directional system matrix, they are

$$\begin{aligned} B &= -\frac{Y_{\beta}}{V_0} - N'_r - L'_p, \\ C &= N'_{\beta} + L'_p \left( \frac{Y_{\beta}}{V_0} + N'_r \right) + \frac{Y_{\beta}N'_r}{V_0} - N'_pL'_r, \\ D &= N'_p \left( \frac{Y_{\beta}L'_r}{V_0} + L'_{\beta} \right) - L'_p \left( \frac{N'_rY_{\beta}}{V_0} + N'_{\beta} \right) - \frac{L'_{\beta}g}{V_0}, \\ E &= \left( N'_rL'_{\beta} - N'_{\beta}L'_r \right) \frac{g}{V_0}. \end{aligned} \quad (5)$$

First to consider first-order spiral mode. The time constant  $\lambda_S$  is customary assumed to be  $|\lambda_S| \ll \min(\omega_{\text{DR}}, |\lambda_R|)$ . From that,

$$D \cong \omega_{\text{DR}}^2 \lambda_R. \quad (6)$$

Consider the classic roll approximation which is given by

$$\lambda_R = -L'_p \quad (7)$$

that is found by considering a single DOF roll subsidence model [20]. Accordingly, as long as the relationship  $|\lambda_S| \ll \min(\omega_{DR}, |\lambda_R|)$  can be satisfied, from equations (5), (6), and (7), the classic Dutch roll natural frequency approximation yields

$$\omega_{DR}^2 = N'_\beta + \frac{Y_\beta N'_r}{V_0} - \frac{N'_p}{L'_p} \left( L'_\beta + \frac{Y_\beta L'_r}{V_0} \right) + \frac{L'_\beta g}{L'_p V_0}. \quad (8)$$

In order to demonstrate that weathercock stability is not essential to Dutch roll stability and investigate the crucial factor, we move the focus on the right terms of equation (8) for the result. As can be seen in the right polynomial, the first three terms are positive. For most of the aircraft configurations and flight conditions, weathercock stability aerodynamic coefficient  $N'_\beta$  is relatively big among the right four terms, where the third term on the right side is usually very small. Therefore, when simply setting  $N'_\beta$  to zero, the second term on the right always remains positive to guarantee the residual Dutch roll stability, and this can be achieved by a relatively big  $Y_\beta$ .

The vertical tail has been widely recognized as essential for most fixed-wing aircrafts to achieve lateral-directional stability, particularly in the Dutch roll mode stability. From the perspective of reducing sideslip and keeping steady, the yawing moment and side force produced by vertical tail both make effort. The related two aerodynamic coefficients are  $N'_\beta$  and  $Y_\beta$ . This amounts to saying that although the weathercock stability can affect the Dutch roll mode, it is not essential for the Dutch roll stability, and the side-force derivative  $Y_\beta$  should be another crucial factor. In fact, with respect to the flight quality specifications, zero weathercock stability but with sufficient  $N'_\beta$  the lateral-directional mode characteristics can satisfy the flight quality specification.

### 3. Flight Experiment Verification with Zero Weathercock Stability

To verify the algebraic approximation result, a model glider with the conventional configuration was selected as the flight test object with reduced weathercock stability but maintained side force stability. The glider has a wingspan of 1.68 m, a length of 0.87 m, and a mass of 0.714 kg, and it is powered by an electric motor-driven propeller. The inertia moment parameters relative to lateral-directional motions can be found in Table 1. To have zero  $N'_\beta$  but a marginal effect on  $Y_\beta$ , the aircraft was modified by reducing the area of the vertical stabilizer to less than one-fourth and attaching a fin that was almost three times larger in front of the center of gravity.

The original and modified aircrafts are shown in Figures 1 and 2. The typical level flight condition for this glider is 18 m/s at 100 m in altitude.

TABLE 1: Inertia moments involving the lateral-directional stability for the modified configuration.

Parameters	Values (kg · m <sup>2</sup> )
$I_x$	0.046
$I_z$	0.074
$I_{xz}$	0.001

The aerodynamic parameters were obtained from a vortex lattice method program [21]. The vortex layout of the modified configuration is shown in Figure 3. In Table 2, several representative aerodynamic derivatives involving lateral-directional stability are listed to show the difference between original and modified configurations. Then, three groups of eigenvalues of system matrix  $A_{lat}$  were obtained by substituting the aerodynamic derivatives and inertial parameters. Here, the standard primed derivatives  $L'_\beta$  and  $N'_\beta$  as well as other relevant derivatives are given by [20]

$$\begin{aligned} L'_i &= \frac{L_i + (I_{xz}/I_z)N_i}{I_x - I_{xz}^2/I_z}, \\ N'_i &= \frac{N_i + (I_{xz}/I_x)L_i}{I_z - I_{xz}^2/I_x}, \\ Y_\beta &= \frac{C_{y\beta}q * S}{mV_0}, \\ L_\beta &= C_{l\beta}q * Sb, \\ L_p &= C_{lp}q * Sb, \\ L_r &= C_{lr}q * Sb \frac{b}{2V_0}, \\ N_\beta &= C_{n\beta}q * Sb, \\ N_p &= C_{np}q * Sb \frac{b}{2V_0}, \\ N_r &= C_{nr}q * Sb \frac{b}{2V_0}. \end{aligned} \quad (9)$$

The calculated three groups of lateral-directional mode eigenvalues and the corresponding parameters are shown in Table 3. According to the Level 1 criterion in MIL-F-8785C, the time constant of roll mode  $\tau_R$  should be less than 1.4 s, and the time  $\tau_{double}$  to double the amplitude of the spiral mode to 20 degrees should be longer than 20 s. For the Dutch roll mode, its damping ratio  $\zeta_{DR}$ , natural frequency  $\omega_{DR}$ , and their product  $\zeta_{DR}\omega_{DR}$  should be larger than 0.08, 0.4, and 0.15, respectively. All three lateral-directional mode characteristics of the modified configuration including Dutch roll mode satisfy the Level 1 specification.

As can be seen in Table 3, the Dutch roll mode frequency of modified configuration is smaller compared with that of original configuration. This can be attributed to the significantly reduced  $C_{n\beta}$ . From equation (8), the reduction of  $C_{n\beta}$  is equivalent to the decrease of  $N'_\beta$  on the right side,

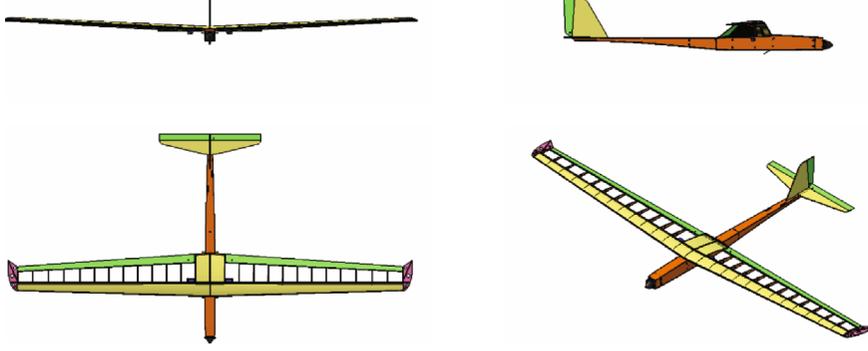


FIGURE 1: 3-dimensional view of the original configuration.

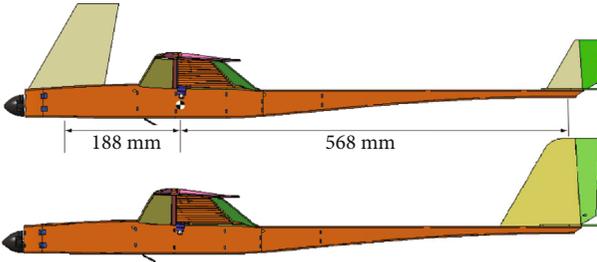


FIGURE 2: Contrast between the modified and original configurations from the side view.

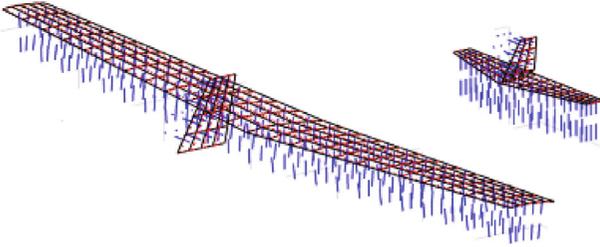


FIGURE 3: The aerodynamic calculation vortex lattice layout.

TABLE 2: Stability derivatives of lateral-directional mode characteristics for the original and modified glider configurations.

Lateral-directional stability derivative	Original configuration	Modified configuration
$C_{n\beta}$	0.0593	0.0001
$C_{y\beta}$	-0.1835	-0.1807
$C_{nr}$	-0.0477	-0.0188

thence caused the oscillation frequency  $\omega_{DR}$  decrease. The trend of frequency between these configurations is reasonable from this perspective.

Before the flight test, a lateral-directional motion simulation of the modified configuration is made and  $\beta$  and  $\phi$  after a  $10^\circ$  sideslip disturbance are shown in Figure 4.

The glider with the modified configuration was flight-tested by remote control with no stability augmentation. Several rudder doublet excitations were conducted, and the convergence of the Dutch roll motion demonstrates the consistency between the flight test and the numerical

TABLE 3: Lateral-directional mode characteristics of the modified and original configurations.

Configuration	Dutch roll mode			
	$\lambda_{DR}$	$\zeta_{DR}$	$\zeta_{DR}\omega_{DR}$	$\omega_{DR}$
Modified	$-0.3772 \pm 1.535i$	0.239	0.377	1.581
Original	$-0.773 \pm 3.748i$	0.202	0.773	3.826

Configuration	Spiral mode		Roll mode	
	$\lambda_S$	$\tau_{double}$	$\lambda_R$	$\tau_R$
Modified	-0.0368	N/A	-1.959	0.0557 s
Original	0.181	3.823	-18.120	0.0552 s

result. A representative sequential shooting of the aircraft response to the rudder doublet is shown in Figure 5. This convergence indicates that the aircraft can achieve Dutch roll mode stability without the weathercock stability.

Two noticeable outcomes besides appeared in the flight test according to the controller's feedback. Firstly, the modified glider requires additional rudder control input in turning compared with the original configuration. For conventional aircrafts, sideslip arises but within a limit at a relatively small value with no rudder control conducted in turning maneuvers. But for the modified glider, obvious sideslip was observed when conducting a turning maneuver if rudder control was not involved, and the nose heading did not change much until rudder control input is towards the same direction. This can be explained as no yawing moment to reduce the sideslip because of the lack of weathercock stability. The simulation of the sideslip and rolling angle and the rolling and yawing angular velocity after the glider encounter a  $10^\circ$  right rolling angle disturbance is shown in Figure 6 to make a comparison between original and modified gliders on this issue. As can be seen in Figure 6, for the original aircraft, the  $10^\circ$  rolling angle soon results in a relatively big yawing angle velocity up to  $15^\circ/s$  and goes on increasing to the same direction but the sideslip angle remains under  $2^\circ$ . On the contrary, the modified one is in big sideslip angle up to 8 at the beginning but yawing angular velocity increases slowly and remains under  $10^\circ/s$ .

Secondly, the modified glider was easier to enter spins than the original configuration. The flat spin occurred twice in climbing soon after the aircraft took off. During the climbing and turn at a slow airspeed, the sideslip of the glider was

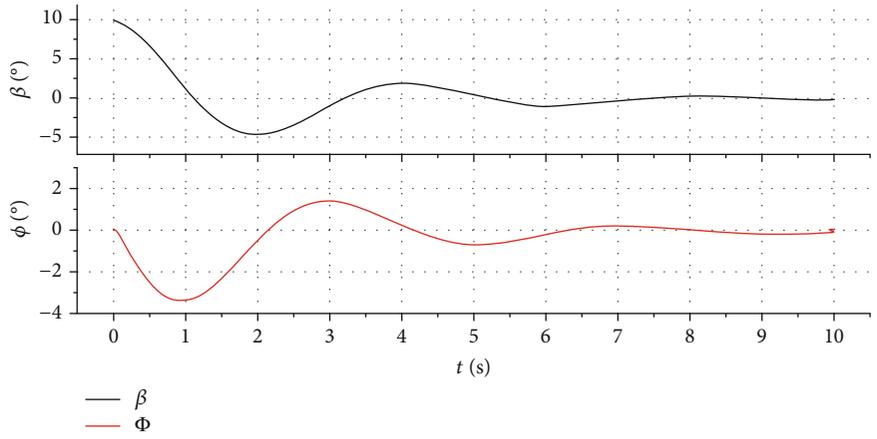


FIGURE 4:  $\beta$  and  $\phi$  of the modified glider after  $10^\circ$  sideslip disturbance in lateral-directional simulation.

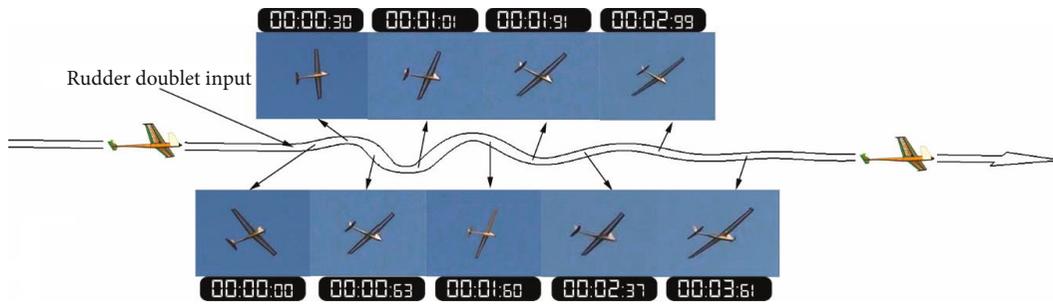


FIGURE 5: Configuration modified aircraft response to the rudder doublet Dutch roll mode excitation.

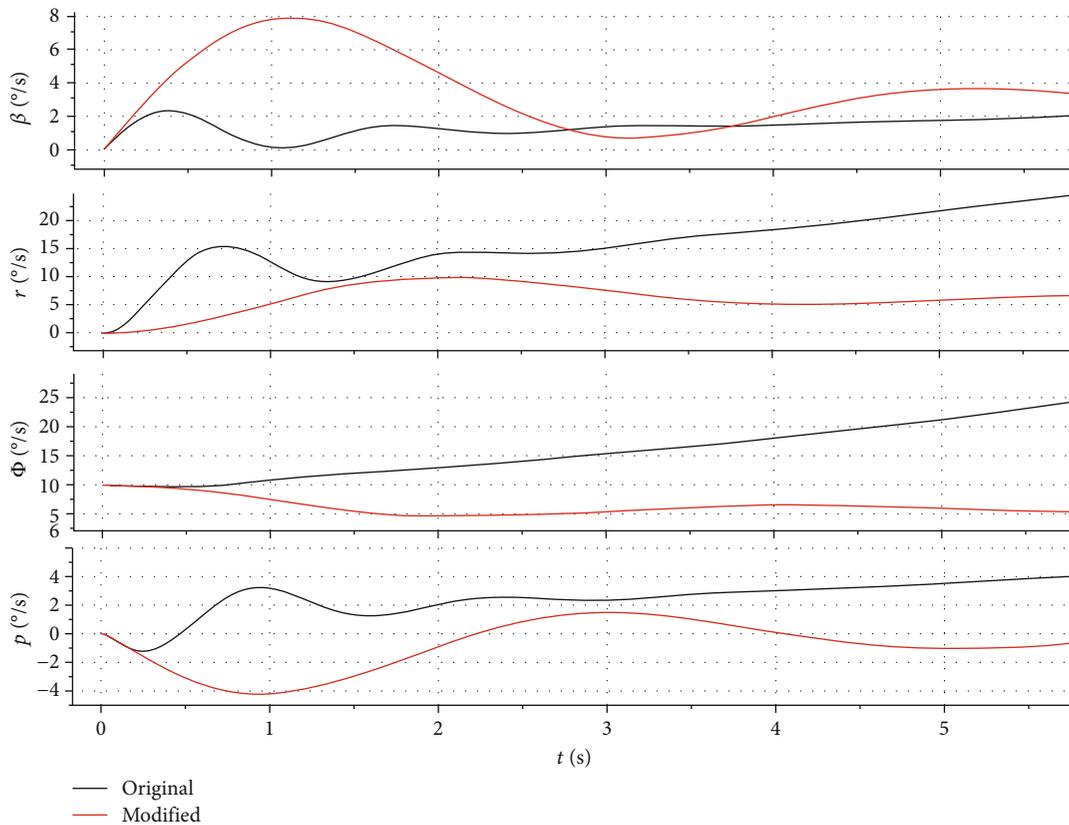


FIGURE 6: Lateral-directional motions of modified and original gliders after  $10^\circ$  rolling angle disturbance.

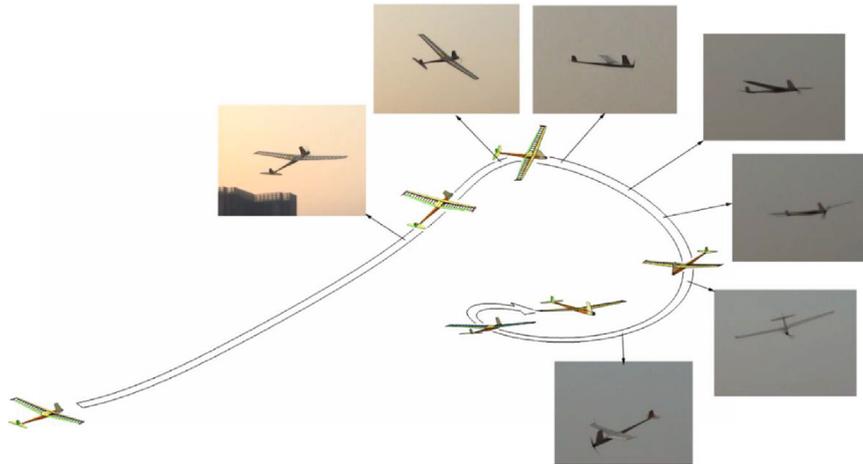


FIGURE 7: Flat spin in the takeoff and climb phase.

difficult to realize and eliminate in time. Thus, with the increase in sideslip angle and big attack angle in climbing at low airspeed, one of the wings stalled; then, the glider entered a spin and quickly developed to a flat spin because there was no weathercock stability to reduce sideslip, as shown in Figure 7. From the sequential photos, obvious left sideslip during the right flat spin can be observed and this was difficult to recover under conventional spin recover operation, that is, same aileron and opposite rudder. The reduced rudder should cause the result.

#### 4. Discussion on the Phenomena and Relevant Inspirations

Both algebraic analysis and flight experiment illustrate that the weathercock stability is not essential to the Dutch roll mode stability. However, in flight appeared the phenomena that additional rudder control requirement in turning maneuvers and easy occurrence of spins reflect the important roles the weathercock stability plays in flight. The convergence of the Dutch roll mode demonstrates that, within the flight quality judgment criterion based on the lateral-directional mode characteristic specifications defined in MIL-F-8785C or MIL-STD-1797A, the Dutch roll dynamic stability can be achieved with zero weathercock stability. But those two phenomena can bring bad effect in flight and they cannot be avoided by just satisfying the mode criterions in MIL-8785C and MIL-STD-1797A.

In turning maneuvers, the additional rudder control requirement of the modified glider reflects that  $C_{n\beta}$  plays a more important role in limiting the sideslip angle during the maneuvers rather than in the Dutch roll mode stability. In addition, with less or even zero weathercock stability, the characteristics of the rudder control requirement in coordinated turns should also be investigated and set a reference standard.

Meanwhile, the lack of weathercock stability makes it more difficult to recover from the beginning of spin. Sufficient weathercock stability can restrict the sideslip from

going serious when spin happens; thus, the aerodynamic state on wings will be improved. As can be inferred this will help to prevent the spin from developing and recover to normal flight.

While neither MIL-F-8785C nor MIL-STD-1797A involves such relevant specifications on weathercock stability, the phenomena found in flight experiment cannot be demonstrated basically by satisfying the mode criterions according to flight quality specification. From the view of the aircraft design, the sufficient weathercock stability has its meanings, regardless by the means of aerodynamics or flight control system. Besides the way like arranging appropriate vertical stabilizer, the split spoilers on the wing tips controlled by flight control system according to the simultaneously detected sideslip angle can also make the aircraft possess sufficient weathercock stability when needed. The method from the view of flight control system has its special signification for flying wing configuration and this is worth further detailed research.

The impact of gust wind on flight has been investigated from different aspects like overload control. For the impact of lateral wind gust, it is closely related to the residual lateral-directional stability characteristics of the aircraft. To investigate the influence of zeroed  $C_{n\beta}$  on the response of aircraft to the lateral gust wind, the numerical method was used. The simulation is based on the lateral-directional linearization model of the original and modified gliders. The lateral gust wind was simplified to a disturbance in sideslip angle at  $10^\circ$ . By comparing the motions after the sideslip angle disturbance between different models, the influence of  $C_{n\beta}$  on the response to lateral gust wind can be studied. The four typical lateral-directional motion parameters of the response motions are shown in Figure 8.

From Figure 8, it apparently shows that the modified glider with zeroed  $C_{n\beta}$  has less directional and rolling angular rate  $r$  and  $p$  response after the  $10^\circ$  sideslip disturbance with zero control input compared with those of the original glider. Both configurations have sideslip angle convergence, though the modified glider expresses lower oscillation frequency which is consistent with the previous numerical

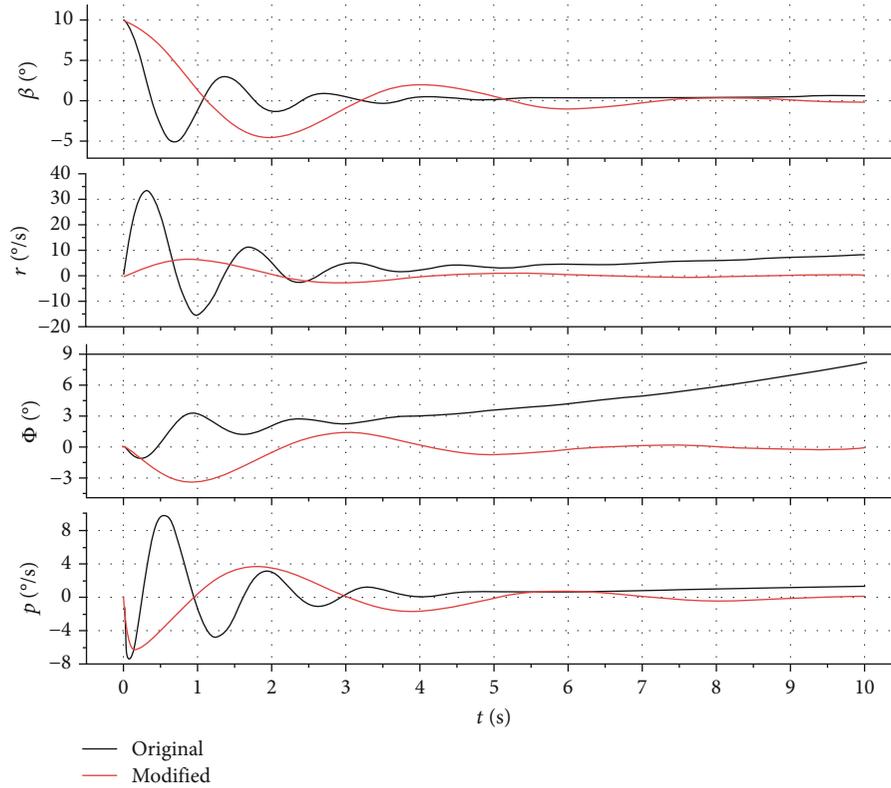


FIGURE 8: Lateral-directional motions of modified and original gliders after 10° sideslip angle disturbance.

and experimental study. From this perspective, the zeroed  $C_{n\beta}$  can help to reduce the impact of lateral gust wind especially on directional motions. For flying wing and other aircraft configurations with less weathercock stability, they can be less affected by the impact of lateral gust wind compared with bigger weathercock stability configurations.

## 5. Conclusion

In this paper, the algebraic analysis and flight experiment are used to illustrate that the weathercock stability is unessential to the Dutch roll mode stability but plays some roles in normal flight. First, an algebraic approach was developed to approximate Dutch roll mode natural frequency and the literal expression demonstrates that the side force derivative  $C_{y\beta}$  can also guarantee the residual Dutch roll stability at zeroed  $C_{n\beta}$ . Then, a flight experiment was organized based on a model glider, which was modified to have zero  $C_{n\beta}$  but with a marginal change on  $C_{y\beta}$ . The flight test showed the convergence of the Dutch roll after procedural mode excitation and some other phenomena that are thought as a result of lack of weathercock stability. The phenomena include the extra requirement of rudder input in the turning maneuvers compared with original configuration and easy occurrence of spin. The relationship between weathercock stability and the impact of lateral gust wind is also investigated based on the simulation of the two glider configurations. The result shows that with less weathercock stability the impact of lateral gust wind can be weakened.

The convergence of Dutch roll in the flight experiment showed the consistence with the algebraic and numerical analysis. With the lateral-directional mode criterions in specifications like MIL-F-8785C, they can be satisfied without the weathercock stability. From the view of flight experiment, the side force derivative  $C_{y\beta}$  also plays an important role in Dutch roll mode stability.

Even though a residual Dutch roll mode stability can be achieved with zero  $C_{n\beta}$ , the weathercock stability still shows its roles played in flight. The extra rudder control input in turning maneuvers and easier occurrence of spin demonstrated its effects. These problems cannot be demonstrated or avoided by merely satisfaction of mode criterion in flight quality specifications. Hence, the appropriate weathercock stability has its meanings no matter by means of aerodynamics or directional stability augmentation. Moreover, the reference criterion of weathercock stability in flight quality is worth further investigation with the development of flying wing and other unconventional aircraft design.

## Nomenclature

$A_{lat}$ :	Matrix of system stability derivative in lateral-directional linearization small disturbance equations
$b$ :	Span
$C_{l\beta}$ :	Coefficient of rolling moment due to sideslip
$C_{lp}$ :	Rolling damping moment coefficient
$C_{lr}$ :	Coefficient of rolling moment due to the yawing angular rate

$C_{n\beta}$ :	Coefficient of yawing moment due to sideslip
$C_{np}$ :	Coefficient of yawing moment due to the rolling angular rate
$C_{nr}$ :	Yawing damping moment coefficient
$C_{y\beta}$ :	Coefficient of side force due to sideslip
$I_x$ :	Inertial moment of the X component
$I_z$ :	Inertial moment of the Z component
$I_{xz}$ :	Inertial product of the XZ component
$L_{\beta}$ :	Rolling moment derivative due to sideslip angle
$L_p$ :	Rolling moment derivative due to rolling angular velocity
$L_r$ :	Rolling moment derivative due to yawing angular velocity
$m$ :	Mass
$N_{\beta}$ :	Yawing moment derivative due to sideslip angle
$N_p$ :	Yawing moment derivative due to rolling angular velocity
$N_r$ :	Yawing moment derivative due to yawing angular velocity
$q$ *:	Dynamic air pressure
$S$ :	Reference wing area
$V_0$ :	Speed
$\zeta_{DR}$ :	Dutch roll mode damping ratio
$\lambda_{DR}$ :	Dutch roll mode eigenvalue
$\lambda_R$ :	Roll mode eigenvalue
$\lambda_S$ :	Spiral mode eigenvalue
$\tau_{double}$ :	Time to double amplitude
$\tau_R$ :	Roll mode time constant
$\omega_{DR}$ :	Natural frequency of Dutch roll
$Y_{\beta}$ :	Side force derivative due to sideslip angle.

## Data Availability

The aerodynamic coefficient data used to support the findings of this study are available from the corresponding author upon request.

## Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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