Research Article

Numerical Investigation of Inlet Thermodynamic Conditions on Solid Fuel Ramjet Performances

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In this work, 2D numerical RANS (Reynolds Average Navier-Stokes) simulations were carried out to investigate the thermodynamic performance of a solid fuel ramjet (SFRJ) with different inlet conditions. This is achieved by using an in-house FORTRAN code to simulate a 2D turbulent, reacting, unsteady flow in the ramjet engine. The inlet conditions are characterized by three key parameters: (1) swirl number (S_N), (2) mass flow rate ($\dot{m}_{\text{air}}$), and (3) inlet temperature (T_{in}). With the code numerically validated by benchmarking with a number of computed cases, it is applied to perform systematic studies on the turbulent flow recirculation, combustion, and heat transfer characteristics. It is found that increasing S_N , $\dot{m}_{\text{air}}$, or T_{in} can dramatically enhance the combustion heat release rate, regression rate, and combustor average temperature. Furthermore, the analysis on the chemical reaction intermediate (CO) reveals that the chemical reaction is more sufficient with increased $\dot{m}_{\text{air}}$, but $S_N = 0$. In addition, a secondary vortex is generated at the corner of the backward facing step in the presence of a swirl flow resulting from the instability of the shear layer. Finally, the nonlinear correlations between the heat transfer, combustion characteristics, and flow field characteristics and the corresponding inlet thermodynamic parameters are identified.

1. Introduction

The ramjet is one of the most popular air-breathing propulsion systems due to its high flight speed and high reliability. The flight of a ramjet is achieved by ram air compression [1]. The compressed incoming air flows through the combustor and is mixed with fuel, and then it is ignited by the high-temperature gas.

Ramjets could be divided into three main types in terms of the type of fuel, namely, the gaseous fuel ramjet (GFRJ), the liquid fuel ramjet (LFRJ), and the solid fuel ramjet (SFRJ). As for the ramjet with gaseous and liquid fuels, a fuel supply system is required for the application of the ramjet, which will lead to an increase in cost and complexity of the propulsion system. Thus, comparing with GFRJ and LFRJ, the ramjet with solid fuel has the simplest structure and the highest reliability, because it does not require a fuel supply system and contains no moving parts as well.

The performance of SFRJ could be mainly determined by the regression rate of the solid fuel. However, due to the lack of a fuel supply system, the solid fuel production rate (regression rate) could not be controlled actively like gaseous and liquid fuels. A previous study [2] suggested that the regression rate was sensitive to the geometries [3–5], flight conditions [6], and the fuel types [7–9]. The thermodynamic performance of SFRJ, scramjet, and hybrid rockets were extensively investigated and reported in the literature [10–20]. A systematic investigation on a small-scale solid fuel ramjet and the combustion characteristic of boron-fueled SFRJ were conducted in the Israel Institute of Technology, via both experimental and numerical approaches [21–24]. Research of geometry effects on the performance of SFRJ was investigated in Ref. [4]. It is reported that the local regression rate is closely related to the heat flux. And as described in Ref. [25], the lobe geometry also plays an important role in the heat release and characteristics of

a scramjet combustor. Liou et al. [26] carried out a series of numerical studies on the combustion behaviors in SFRJ. In their studies, the correlation between the engine geometry, mass flow rate, and sustained combustion is obtained. Gany [27, 28] conducted a theoretical examination of ideal ramjet performance. The results showed that the specific thrust is increased with an increase of fuel/air ratio up to the stoichiometric ratio. Li et al. [29] preformed an investigation on the low-temperature ignition characteristics of a supersonic combustor with and without struts, and the results showed that flame stability can be sensitive to the addition of the struts. Li et al. [30] performed an investigation on the combustion characteristics of SFRJ. It was found that variation in engine geometry could lead to a different combustion behavior.

The foregoing studies confirmed that the sensitivity between the SFRJ performance and the geometry, fuel type, and flight conditions is owed mainly to the variation of the thermodynamic parameters. A previous study [31] reported the performance of a solid fuel ramjet with different types of fuel. The results indicated that the thermodynamic constants of the solid fuel could be sensitive to the fuel type. Li et al. [32] conducted a research on fuel reactivity-controlled self-starting and propulsion performance of a scramjet, and the results showed that the lowest flight Mach number for self-starting of a hydrogen-fueled scramjet is reduced from 6.2 to 5.1 when the activation energy is decreased by 50%. Li et al. [33] conducted a research of the effects of pressure and oxygen concentration on the combustion characteristics of Al/Mg fuel-rich propellants, and the result indicated that the heat conduction and heat feedback to the burning surface could be enhanced by higher environmental pressure, and the combustion performance could be sensitive to the ambient oxygen concentration. Hashim [34] investigated the effects of Ti and Mg particles on the combustion characteristics of boron-HTPB-based solid fuels in a hybrid rocket, and according to their paper, the fuel regression rate is found to be closely related to the thermal conductivity. Reference [35] conducted an investigation on thermodynamic analysis of the specific thrust of a hydrocarbon-fueled scramjet, and the results showed that the specific thrust initially can be enhanced with the fuel equivalence ratio, then it can reach a maximum, and finally it can reduce rapidly for a given flight Mach number. Tian et al. [36] numerically investigated the combustion performance of a hybrid rocket motor with segmented grain, and they found that the heat transfer could be sensitive to the flow conditions of the engine. The regression rate was shown to be closely related to the heat transfer of the engine internal flow field. However, the literature of thermal analysis is mainly focused on hybrid rockets and fuels with metal particles. The detailed thermal analysis of a solid fuel ramjet with HDPE is quite limited. This partially motivated the present research.

In this work, a detailed analysis of the effect of the inlet condition on heat transfer and combustion characteristics of SFRJ is investigated. The governing equations and the numerical model are described in Section 2. A CFD code is developed to simulate the two-dimensional, turbulent, reacting, unsteady flow in SFRJ. The effects of the swirl

TABLE 1: Chemistry model of ethylene.

	Reaction	E_a (J/mol)	A_k ($\text{cm}^3 \cdot \text{mol}^{-1} \cdot \text{s}^{-1}$)
1	$\text{C}_2\text{H}_4 + \text{O}_2 \rightarrow 2\text{CO} + 2\text{H}_2$	149779.2	2.10E14
2	$2\text{CO} + \text{O}_2 \rightarrow 2\text{CO}_2$	84261.5	3.48E11
3	$2\text{H}_2 + \text{O}_2 \rightarrow 2\text{H}_2\text{O}$	0.0	3.00E20

number, mass flow rate, and inlet temperature on SFRJ's thermodynamic performance are discussed in Section 3. Key findings are summarized in Section 4.

2. Numerical Method

2.1. Governing Equations of Fluid Domain. Equation (1) describes the governing equations of the fluid domain [37]:

$$\frac{\partial \mathbf{Q}}{\partial t} + \frac{\partial \mathbf{E}}{\partial x} + \frac{\partial \mathbf{F}}{\partial y} = \frac{\partial \mathbf{E}_V}{\partial x} + \frac{\partial \mathbf{F}_V}{\partial y} + \mathbf{H} + \mathbf{H}_V + \mathbf{S}, \quad (1)$$

where $\mathbf{Q}$, $\mathbf{E}$, $\mathbf{F}$, $\mathbf{E}_V$, $\mathbf{F}_V$, $\mathbf{H}$, $\mathbf{H}_V$, and $\mathbf{S}$ are expressed as follows:

$$\mathbf{Q} = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ \rho E \\ \rho_i \end{bmatrix}, \quad \mathbf{E} = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho vu \\ \rho uw \\ (\rho E + p)u \\ \rho_i u \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} \rho v \\ \rho uv \\ \rho v^2 + p \\ \rho vw \\ (\rho E + p)v \\ \rho_i v \end{bmatrix}, \quad \mathbf{E}_V = \begin{bmatrix} 0 \\ \tau_{xx} \\ \tau_{xy} \\ \tau_{x\theta} \\ u\tau_{xx} + v\tau_{xy} + q_x \\ \rho D_i \partial c_i / \partial x \end{bmatrix},$$

$$\begin{aligned}
\mathbf{F}_V &= \begin{bmatrix} 0 \\ \tau_{xy} \\ \tau_{yy} \\ \tau_{y\theta} \\ u\tau_{xy} + v\tau_{yy} + q_y \\ \rho D_i \partial c_i / \partial y \end{bmatrix}, \\
\mathbf{H} &= -\frac{1}{y} \begin{bmatrix} \rho v \\ \rho uv \\ \rho(v^2 - w^2) \\ 2\rho vw \\ (\rho E + p)v \\ \rho_i v \end{bmatrix}, \\
\mathbf{H}_V &= \frac{1}{y} \begin{bmatrix} 0 \\ \tau_{xy} \\ \tau_{yy} - \tau_{\theta\theta} \\ 2\tau_{y\theta} \\ u\tau_{xy} + v\tau_{yy} + w\tau_{y\theta} + q_y \\ \rho D_i \partial c_i / \partial y \end{bmatrix}, \\
\mathbf{S} &= \begin{bmatrix} S_\rho \\ S_{\rho u} \\ S_{\rho v} \\ S_{\rho w} \\ S_{\rho E} \\ w_i + S_{\rho i} \end{bmatrix}.
\end{aligned} \tag{2}$$

In equation (3), the shear stress τ and heat fluxes (q_x and q_y) in x and y directions can be calculated by

$$\begin{aligned}
\tau_{xx} &= \frac{2}{3}\mu \left(2\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} - \frac{v}{y} \right), \\
\tau_{yy} &= \frac{2}{3}\mu \left(2\frac{\partial v}{\partial y} - \frac{\partial u}{\partial x} - \frac{v}{y} \right), \\
\tau_{xy} &= \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right), \\
\tau_{\theta\theta} &= 2\mu \frac{v}{y} - \frac{2}{3}\mu \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{v}{y}, \\
\tau_{x\theta} &= \mu \left(\frac{\partial w}{\partial x} \right),
\end{aligned}$$

$$\tau_{y\theta} = \mu \frac{\partial w}{\partial y} + \frac{w}{y},$$

$$q_x = \lambda \frac{\partial T}{\partial x} + \rho \sum_{i=1}^7 D_i h_i \frac{\partial c_i}{\partial x},$$

$$q_y = \lambda \frac{\partial T}{\partial y} + \rho \sum_{i=1}^7 D_i h_i \frac{\partial c_i}{\partial y}, \tag{3}$$

in which ρD_i is calculated by

$$\rho D_i = \frac{1 - X_i}{1 - c_i} \left(\frac{\mu_l}{S_c} + \frac{\mu_t}{S_{ct}} \right). \tag{4}$$

The laminar and turbulent viscosities (μ_{lilj} and μ_{tilj}) of the mixture are calculated using the following [38]:

$$\begin{aligned}
\mu_l &= \sum_{i=1}^7 \frac{X_i \mu_{li}}{\phi_i}, \\
\phi_i &= \sum_{j=1}^7 \frac{X_j \left[1 + \sqrt{\mu_{li}/\mu_{lj}} (M_i/M_j)^{1/4} \right]^2}{\sqrt{8(1 + (M_i/M_j))}}, \\
\mu_t &= \frac{a_1 \rho k}{\max(a_1 \omega, f_2 \Omega)}, \\
f_2 &= \tanh \left[\max \left(2 \frac{\sqrt{k}}{0.99 \omega y'}, \frac{500 \mu}{\rho y^2 \omega} \right) \right].
\end{aligned} \tag{5}$$

The equation shown below is applied to calculate the thermal conductivity λ .

$$\lambda = \frac{\mu_l c_p}{P_{rl}} + \frac{\mu_t c_p}{P_{rt}}. \tag{6}$$

The pressure is given by equation (7), and the Newton iteration method (equation (8)) is used to calculate the temperature (T) of the flow field:

$$p = \sum_{i=1}^7 \frac{\rho_i}{M_i} R_u T, \tag{7}$$

$$\begin{aligned}
\rho E - \frac{1}{2} \rho (u^2 + v^2 + w^2) \\
= \sum_{i=1}^7 \rho_i \left(\int_{298}^T C_{pi} dT + h_i^{298} \right) - R_u T \sum_{i=1}^7 \frac{\rho_i}{M_i},
\end{aligned} \tag{8}$$

in which C_{pi} that is used in equation (8) is defined by equation (9), and the parameter α_{ki} ($k = 1, 2, 3, 4, 5$) is obtained from the chemical kinetics package [39]:

$$C_{pi} = a_{1i} + a_{2i} T + a_{3i} T^2 + a_{4i} T^3 + a_{5i} T^4. \tag{9}$$

FIGURE 1: Physical model and inlet velocity profile of the SFRJ used in this investigation.

2.2. Governing Equation of Solid Domain. The governing equation of the solid domain can be written as follows [40]:

$$\rho_s c_s \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left(\lambda_s \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(\lambda_s \frac{\partial T}{\partial y} \right) + \frac{1}{y} \left(\lambda_s \frac{\partial T}{\partial y} \right) + \rho_s \dot{r} \frac{\partial h_s}{\partial y}, \quad (10)$$

where $\rho_s \dot{r} \partial h_s / \partial y$ denotes the solid phase energy change caused by the chemical reaction [41].

2.3. Numerical Solution Method. To simulate the 2-dimensional, turbulent, reacting flow in SFRJ, the RANS equations of the fluid domain are discretized by the finite-volume method. The third-order monotone upstream-centered scheme for conservation laws (MUSCL) and advection upstream splitting method by pressure-based eight function (AUSMPW+) scheme [42] are utilized to compute convective flux quantities. To improve the accuracy and computation efficiency of the unsteady cases, the lower upper symmetric Gauss-Seidel (LU_SGS) [43] implicit method is used for temporal discretization. Meanwhile, the $k-\omega$ SST (shear-stress transport) turbulence model proposed by Menter [44] is used to simulate the turbulent flow in this research.

At the first 0.5 s of the ignition process, the ignition gas of 0.3 kg/s of mass flow rate, at 2500 K of total temperature, flows through the engine together with incoming air and ignites the fuel. After ignition, sustained combustion is achieved, and the simulation will terminate when a stable result is obtained (at the global time of 1.35 s).

2.4. Chemical Reaction Model. In our previous research, two chemical reaction models (eddy-dissipation and finite rate models) were compared in the combustion process of swirl intake SFRJ [45]. The result shows little difference between the two models. Thus, in order to save computational resources, the finite rate model is used in this work. However,

TABLE 2: Simulation conditions.

Case	S	Mach number	$\dot{m}_{air}$ (kg/s)	Inlet temperature (K)
1	0	2	0.3	540
A	2	0.6	0.3	540
3	0.74	2	0.3	540
1	0.6	2	0.3	540
B	2	0.6	0.4	540
3	0.6	2	0.5	540
1	0.6	2	0.3	500
C	2	0.6	0.3	540
3	0.6	2	0.3	580

FIGURE 2: y^+ of the validation cases.

the chemical reaction of high-density polyethylene (HDPE) in SFRJ is very complicated, and it is very difficult to describe accurately. Thus, to simplify the combustion, according to Ref. [46], the pyrolysis products of polyethylene can be approximated as C_2H_4 only, and the main products of the combustion are O_2 , CO_2 , H_2O , CO , and N_2 . Therefore, a simplified kinetic of chemical reaction [46, 47] shown in Table 1 is applied to describe the chemical reaction of HDPE. In this research, the chemical reaction process is described by the finite rate model, and the Arrhenius formula is utilized to determine reaction rate constants.

$$\dot{r}_k = A_k T_w^n \exp \left[-\frac{E_a}{R_u T_w} \right], \quad (11)$$

where A_k , n , and E_a could be obtained from Table 1. The value of n for reactions 1, 2, and 3 are 0, 2, and -1, respectively.

Moreover, to calculate the pyrolysis rate of the solid fuel more accurately, the zeroth order ($n = 0$) Arrhenius equation [48] is employed:

$$\dot{r} = A_{py} \exp \left[-\frac{E_{apy}}{R_u T_w} \right], \quad (12)$$

in which the value of A_{py} and E_{apy} are 8750 m/s and 130 kJ/mol, respectively [49].

2.5. Physical Model and Boundary Conditions. Figure 1(a) describes the physical model of the SFRJ used in this investigation. The boundary conditions can be summarized as follows: (1) mass flow inlet condition at the inlet, (2) pressure outlet condition at the engine outlet, (3) axisymmetric boundary condition at the axis, and (4) application of no-slip adiabatic wall condition at the walls.

The heat transfer between the solid and fluid domains is considered, and the coupled wall temperature is calculated by equation (13). Moreover, for the cases with 0.6 swirl number, the radial profiles of the tangential and axial components of the inlet momentum are defined by the experimental data of Dellenback et al. [50] (see Figures 1(b) and 1(c)).

$$k \frac{\partial T}{\partial y} \Big|_{\text{gas}} = k_{\text{sol}} \frac{\partial T}{\partial y} \Big|_{\text{solid}}. \quad (13)$$

2.6. Case Description. A series of simulations are conducted with an in-house code written in FORTRAN, and the investigations concentrate on the different inlet conditions of SFRJ as shown in Table 2. The simulations are performed with different swirl intensities, which are defined by equation (14) (see group A), various inlet mass flow rates (group B), and different inlet temperatures (group C). The model of SFRJ has 40 mm of inlet diameter, 70 mm of port diameter, 500 mm of the fuel length, and 28.5 mm of throat diameter.

$$S_N = \frac{\int_0^R r^2 u w dr}{R \int_0^R r u^2 dr}, \quad (14)$$

FIGURE 3: The regression rate of the validation cases.

TABLE 3

$\dot{r}$ (mm/s) Quality of the mesh	Experiment	Simulation	Deviation
Coarse		0.1692	13%
Medium	0.1936	0.1808	7%
Fine		0.182	6%

where R is the inlet diameter, and u and w represent the axial and tangential velocities, respectively.

2.7. Mesh Independency and Model Validation. To save computing resources and guarantee the accuracy of the simulation, the mesh-independent studies are conducted at first. The simulations with three different mesh numbers (161580, 113566 and 78569) of case A2 are conducted, and the mesh is clustered near the wall boundary and the solid fuel surface, as shown in Figure 1. The y^+ of the cases with different mesh numbers is shown in Figure 2. It could be seen from Figure 2 that y^+ values for the fine mesh and the medium mesh are controlled under the value of 5, while the y^+ of the coarse mesh is higher than 5. The result of the local regression rate is shown in Figure 3, and the regression rate for the fine mesh and the medium mesh shows little difference with each other but shows significant difference with the case of the coarse mesh. The deviations between the calculated and experimental average regression rates are shown in Table 3; as shown in Table 3, the deviation between the coarse mesh and the experiment is much higher than that of the other two cases. Thus, considering saving the computational resources and improving the accuracy of the simulation, the case with the medium mesh number (113566) is used in this research.

Before the in-house code can be used for this investigation, the numerical models should be validated against the benchmark cases. An experiment performed by Nejad [51] is utilized to validate the swirl flow model, and a dump combustor model with the swirl number of 0.5 is used in the validation. The result is described in Figure 4, and the

FIGURE 4: Normalized axial velocity (a) and tangential velocity (b) for swirling flow.

FIGURE 5: Temperature distribution of the semi-infinite plate with constant temperature (a) and constant heat flux (b).

FIGURE 6: Distribution of temperature, pressure (a), and mass fractions (b) for a shock-induced case.

agreement between the predicted results of the normalized axial (W/U_0) and tangential (U/U_0) velocities and experimental results are shown to be excellent.

The heat diffusion model of the solid domain is validated by using a semi-infinite plate case. In this validation, the semi-infinite plate is represented by a plate with the size of 2 mm * 15 mm, density of 7840 kg/m³, specific heat of 465 J/(kg·K), and 49.8 W/(m·K) of thermal conductivity. In addition, the simulated heat transfer is calculated by equation (8), and two different boundary conditions are included at the heated wall surface: (a) heat flux $q_w = 70$ kW and (b) temperature $T_w = 900$ K. Equations (15) and (16) are utilized for the analytical solution of the two cases.

$$T(x, t) = T_w + \operatorname{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right)(T_0 - T_w), \quad (15)$$

$$T(x, t) = T_0 + \frac{2q_w\sqrt{\alpha t/\pi}}{\lambda} \exp\left(\frac{-x^2}{4\alpha t}\right) - \frac{q_w x}{\lambda} \left(1 - \operatorname{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right)\right). \quad (16)$$

The error function is represented by $\operatorname{erf}()$, and the thermal diffusivity of the material is denoted by $\alpha = \lambda/\rho c$. The results are displayed in Figure 5, and the excellent agreements are achieved.

A shock-induced combustion benchmark case performed by Lehr [52] is used for the validation of the finite rate chemical reaction model. This validation is performed with a 15 mm diameter spherical projectile moving through a stoichiometric mixture (H_2 and air) under the pressure of 42662 Pa, temperature of 250 K, and speed of 1685 m/s. The chemical reaction mechanism involving seven species and eight-element reactions is employed. The results are shown in Figure 6, and the experimental data are well reproduced by the results obtained via the in-house code.

A firing experiment on case A2 is performed with a connected pipe facility to validate the accuracy of the in-house code. In this experiment, the engine (fueled by HDPE) with 70 mm of port diameter, 40 mm of inlet, and 500 mm of fuel length is employed. The comparison between the regression rate of the experiment (measured by a 3D scanner) and that of the simulation is shown in Figure 7. It could be seen from Figure 7 that the trends of the experiment and the simulation are the same, but the location of the reattachment point (the location with the maximum regression rate) has a slight difference in axial direction. The reason behind this is the complicated flow conditions and higher velocity decay near the backward facing step in the experiment. In general, a good agreement is obtained for the predicted results and the experimental result, which indicates that the SFRJ combustion process can be well predicted by the in-house code.

3. Results and Discussion

3.1. Flow Field Characteristics. Figure 8 illustrates the temperature contours of different cases. It can be seen from Figure 8 that the flow field is divided into two zones by the flame

FIGURE 7: The regression rate obtained from the experiment and the simulation for the SFRJ case.

surface (the regions with high temperature over 2200 K), where the core flow area with low temperature is an air-rich zone and the regions with high temperature can be regarded as a fuel-rich zone. The phenomenon mentioned above is also observed experimentally in Ref. [8]. In the case with a swirl number of 0.6, compared to the nonswirl flow case, the area of the air-rich zone is dramatically decreased. However, when the swirl number is increased to 0.74, the air-rich zone will be further reduced, but with an insignificant decrease. In addition, under the effect of a swirling inlet flow, the flame surface is pushed forward to the solid domain where the temperatures around the reattachment point and the shear layer are significantly promoted.

For the purpose of better describing the temperature change of the flame surface in the engine, the temperature profile of the flame surface is obtained (see Figure 9). It can be seen from Figure 9 that the introduction of swirling flow into the engine can increase the temperature of the shear layer and the reattachment point. This corresponds with the flame surface temperature distribution shown in the contour; however, the temperature is not enhanced downstream of the flow field. The reason behind this may be due to the attenuation of the swirl intensity at the end of the engine. The temperature profiles of cases B1-B3 indicate that with the increase of the inlet mass flow rate, the temperature is also enhanced but with little difference. As for the cases with different T_{in} (cases C1-C3), the temperatures in the shear layer are almost the same. However, in the reattachment point and the recirculation zone, the temperatures are proportional to the inlet temperature. It is interesting that a temperature fluctuation exists in the shear layer, which can indicate that the instability behavior appears in the shear layer.

Figure 10 shows the streamlines of the recirculation zone. The length of the recirculation zone is decreased with the increase of S_N (see Figure 10(a)). The reason behind this is mainly due to the attenuated axial velocity and the enhancement of the tangential velocity caused by the increase of swirl intensity. However, the effect of T_{in} and $\dot{m}_{air}$ on the area of the recirculation zone is not significant.

FIGURE 8: Temperature contours for the cases with different inlet conditions.

3.2. Heat Transfer Characteristics. Figure 11 describes the regression rates of the cases with various working conditions. As shown in Figure 11, the local regression rates are all climbing rapidly in the recirculation zone and reach the maximum value at the reattachment point; then, they decrease gradually at the turbulent redevelopment zone. It could be seen from Figure 11 that the regression rate can be enhanced with the increase of S_N , $\dot{m}_{\text{air}}$, and T_{in} . However, the degree of the enhancement on the regression rate with the different working conditions is not the same.

The heat flux of each case is shown in Figure 12, corresponding to the profile of the regression rate; the heat flux has the same trend, and both increase rapidly in the recirculation zone and reach their maximum values at the reattachment point. Moreover, the heat flux can be dramatically

improved with a higher swirl number and a higher inlet mass flow rate. Combining with the analysis of the regression rate, it is found that the regression can significantly depend on the surface heat flux.

The time trace of the heat transfer coefficients is shown in Figure 13. In Figure 13, X/L represents the normalized axial distance, which is calculated by equations (17) and (18). As described in equations (17) and (18), a negative value means that the fuel is in an endothermic state; on the contrary, positive heat transfer coefficients mean that the fuel is in an exothermic state. As shown in Figure 13, at the period of 0-0.5 s, ignition occurs, and for the cases with a swirling inlet flow, the absolute value of h is decreased with the increase of X/L . When the ignition gas is quenched at 0.5 s, the heat transfer coefficient changes rapidly to

FIGURE 9: Temperature distributions of the flame surface.

FIGURE 10: Streamlines for the cases with different inlet conditions.

FIGURE 11: Profiles of the local regression rates.

positive and reaches its a peak. At the period of 0.5-0.6 s, h is decreased with the increase of X/L . After 0.7 s, the heat transfer coefficient at $X/L = 0.5$ is decreased gradually until it is lower than the value at $X/L = 0.14$. However, for the case with a nonswirl flow, during the ignition process, the absolute value of the heat transfer coefficient reaches the maximum value at the position of $X/L = 0.14$, and the same trend is observed when the self-sustained combustion is achieved.

Based on the streamlines shown in Figure 10, it is known that the position of $X/L = 0.05$ is within the recirculation zone, and the location of $X/L = 0.14$ is near the reattachment point. As described in Ref. [53], the heat transfer coefficient can be promoted with the effect of vortices. Thus, the aforementioned phenomenon could be explained as follows. For the swirl flow cases, the tangential momentum is generated with the effect of the swirl flow, which leads to the instability of the shear layer near the backward facing step. At the time of 0.5 s, the quench of the ignition gas enhances the instability of the shear layer. The unstable shear layer then causes the separated boundary layer to roll

up into the secondary vortex [54]. As shown in Figure 14(b), at the time of 0.5 s, a secondary vortex appears at the position near $X/L = 0.05$ for the cases with a swirl flow, and the heat transfer coefficient may be enhanced by the vortices. After the time of 0.6 s, as the combustion process goes by, the secondary vortices vanish. However, as described in Figure 14(a), for the case with a nonswirl flow, the secondary vortex does not exist from the start. Thus, it is believed that the existence of the secondary vortex may lead to the heat transfer coefficient behavior shown in Figure 13.

Moreover, it could be concluded from Figure 13 that the overall heat transfer coefficient is increased with the increase of S_N , $\dot{m}_{\text{air}}$, and T_{in} .

$$\frac{\lambda_{\text{fluid}}(\partial T / \partial n)|_w}{T_w - T_{\text{aw}}}, \quad (17)$$

$$T_{\text{aw}} = T_{\infty} \left\{ 1 + \sqrt{P_{rl}} \left[\frac{\gamma_{\text{fluids}} - 1}{2} \right] Ma_{\infty}^2 \right\}. \quad (18)$$

FIGURE 12: Profiles of the local heat flux.

3.3. Combustion Chrematistics. Figure 15 introduces the time trace of the mass fraction of C_2H_4 ($C_2H_{4\%}$) near the fuel surface. It could be seen from Figure 15(a), for the case with a nonswirl flow during the ignition process (0-0.5 s), that the mass fraction of C_2H_4 increases rapidly over the ignition time and reaches the value of 33% at the location of $X/L = 0.05$ (recirculation zone) and 0.14 (reattachment point); however, at the redevelopment zone, the concentration of C_2H_4 climbs gradually up to the value of 35%. As the ignition stops, $C_2H_{4\%}$ drops suddenly and increases gradually until the combustion reaches a stable state, and the maximum value occurs at the reattachment point. For the case with a 0.6 swirl number, after the ignition, the mass fraction of C_2H_4 grows up with the combustion time; then, $C_2H_{4\%}$ at the recirculation zone starts to be higher than that at the reattachment point. When the swirl number further increases to 0.74, after the ignition, $C_2H_{4\%}$ at the recirculation zone maintains the maximum value (followed by the C_2H_4 concentration at the reattachment point) among all of the positions until the

end of the combustion. The same phenomenon could be observed in Figures 15(b) and 15(c).

Figure 16 reflects the contour of the mass fraction of CO_2 , O_2 , and CO , and it could be seen from Figure 16 that CO_2 is mainly concentrated in the fuel-rich zone and O_2 is concentrated in the air-rich zone, and these two zones are divided by the flame surface (see Figure 8). As analyzed in Ref. [3], the combustion in the recirculation zone is taken as a chemical process controlled by the longer residence time of the fuel, and as for the reaction in the shear layer and redevelopment zone, the chemical reaction is subject to a diffusion process. As shown in Figure 16(c), the higher mass fraction of the intermediate of the chemical reaction (CO) appears in the recirculation zone, and it is believed that this is because the chemical reaction in the recirculation zone is incomplete. And this situation occurs in other cases as well.

In order to further analyze the distribution characteristics of the aforementioned components in the combustor, the radial profiles of the mass fraction of CO_2 ($CO_{2\%}$), O_2

FIGURE 13: Time trace of heat transfer coefficients for the cases with different inlet conditions.

FIGURE 14: Streamlines for the cases with and without swirling flow at different times.

($O_{2\%}$), and CO ($CO_{\%}$) are shown in Figures 17–19. As shown in Figure 17, the maximum value of $CO_{2\%}$ occurs around the flame surface and fuel-rich zone (see Figure 16), and the flame surface is moving towards the fuel surface with the increase of the swirling number, mainly owing to the centrifugal force

generated by the swirl flow; however, at the outlet of the combustor, no obvious difference is detected for the position of the flame surface between the cases with and without swirl flow; the reason behind this is the attenuation of the swirl intensity at the end of the combustor. And in the core flow area (air-rich

FIGURE 15: Time trace of the mass fraction of C_2H_4 at the fuel surface for the cases with different inlet conditions.

zone), for the case of a nonswirl flow, the mass fraction of CO_2 is 0, and as shown in Figure 16(c), no CO is detected in the core flow area, which indicates that the combustion mainly takes place at the fuel-rich zone near the fuel surface. As for the distribution of O_2 concentration, in the core flow area, the mass fraction of O_2 is increased with a decreasing swirl number, and it is believed that introducing the swirl flow can increase the mixing degree of fuel and incoming air. According to the chemical mechanism mentioned in Table 1, the concentration of the intermediate could be used to determine whether the chemical reaction is sufficient. As shown in Figure 16(c), the CO_2 has the highest value in the recirculation zone, and gradually reduces downstream of the flow field. Moreover, the mass fraction of CO is promoted with the enhancement of the swirl number at the flame surface and fuel-rich zone, which indicates that the combustion is more sufficient for the case with a nonswirl flow, and the combustion controlled by the chemical process in the recirculation zone is incomplete.

Moreover, as shown in Figure 18, the distribution of CO_2 and O_2 has an insignificant difference among the three cases. However, the mass fraction of CO in the fuel-

rich zone can be promoted with a lower inlet mass flow rate. As described in Figure 19, the combustion affected by the inlet temperature is not obvious.

The heat release rate (Q) is shown in Figure 20, and it could be seen that the flame surface is the place where the chemical reaction mainly occurs. This corresponds to the high-temperature zone (shown in Figure 8). However, a gap is detected in the region of heat release around the mixing plate between the combustor and the aft-burning chamber, and it is thought that the compression structure in the flow field of SFRJ may inhibit the chemical reactions. In addition, the maximum value of the heat release rate occurs in the shear layer (see Figure 21); thus, it is considered that the combustion may be enhanced with the expansion structure in SFRJ.

As shown in Figure 21, the heat release rate in the shear layer is increased with a higher swirl number, and in the reattachment point and redevelopment zone, no significant difference is detected. Moreover, as illustrated in Figure 20(b), with a higher inlet mass flow rate, the value of Q is promoted. It could be seen from Figure 21, for the cases with various inlet temperatures, that the heat release rate is enhanced in the shear layer.

FIGURE 16: Contours of CO_2 (a), O_2 (b), and CO (c) mass fraction. FIGURE 17: Radial profiles of the mass fraction of CO_2 (a), O_2 (b), and CO (c) for the cases with different inlet swirl numbers.

The parameters of ramjet performance and the combustion characteristics of the test cases are shown in Table 4, in which a/f , c^* , I_{sp} , and F are calculated by the equations below. It could be seen from Table 4 that the thrust, the

characteristic velocity, and the average regression rate can all be enhanced with the increase of the inlet swirl number, mass flow rate, and temperature. However, the specific impulse could be reduced with a higher swirl number and

FIGURE 18: Radial profiles of the mass fraction of CO_2 (a), O_2 (b), and CO (c) for the cases with different inlet mass flow rates.

FIGURE 19: Radial profiles of the mass fraction of CO_2 (a), O_2 (b), and CO (c) for the cases with different inlet temperatures.

FIGURE 20: Contours of the heat release rate for the cases with different inlet conditions.

temperature. The reason behind this may be the enhanced regression rate with a relatively lower thrust.

$$a/f = \frac{\dot{m}_{\text{air}}}{\dot{m}_f}, \quad (19)$$

$$c^* = \frac{A_t p_0}{\dot{m}_{\text{air}} + \dot{m}_f}, \quad (20)$$

$$I_{\text{sp}} = \frac{F_{\text{engine}}}{\dot{m}_f}, \quad (21)$$

$$\kappa = \frac{a/f}{a/f_{\text{stoich}}}, \quad (22)$$

$$F_{\text{engine}} = \dot{m}_{\text{out}} v_{\text{out}} + (p_1 - 101352) A_{\text{out}} - \dot{m}_{\text{air}} v_{\text{air}}. \quad (23)$$

The air equivalence ratio is also calculated based on equation (22); the combustion is in a highly oxidized state with the value of κ exceeding one, and the fuel-rich status on the contrary, if the value of κ equals one, which means the combustion occurs in stoichiometry. As Table 4 describes, the combustion occurs in a fuel-rich state with a higher swirl number and inlet temperature. In addition, based on the evaluation of all the cases with different inlet conditions, case C3, with a 0.5 kg/s inlet mass flow rate, has a relatively higher thrust, specific impulse, and characteristic velocity, and the air equivalence ratio is approximately equal to 1, which indicate that the engine has optimal performance and the combustion is close to the stoichiometric ratio.

4. Conclusions

In this paper, 2D numerical studies were conducted by using an in-house FORTRAN code to simulate the axisymmetric,

FIGURE 21: Distributions of the heat release rate at the flame surface for the cases with different inlet conditions.

TABLE 4: Thermodynamic parameters for the cases with different inlet conditions.

Case		F (N)	I_{sp} (s)	c^* (m/s)	$\dot{r}$ (mm/s)	a/f	a/f_{stoich}	$\dot{m}_f$ (kg/s)	κ
A	1	165	1265	983	0.12	23.07	14.8	0.013	1.56
	2	198	1038	1069	0.18	15.73	14.8	0.0191	1.06
	3	211	997	1089	0.20	14.12	14.8	0.0212	0.954
B	1	198	1038	1069	0.18	15.73	14.8	0.0191	1.06
	2	303	1156	1395	0.25	15.27	14.8	0.0262	1.03
	3	406	1246	1712	0.31	15.34	14.8	0.0326	1.04
C	1	193	1076	1055	0.17	16.73	14.8	0.0179	1.13
	2	198	1038	1069	0.18	15.73	14.8	0.0191	1.06
	3	204	1004	1074	0.19	14.77	14.8	0.0203	0.998

unsteady, turbulent, reacting, swirling flow in SFRJ (solid fuel ramjet). The effects of inlet thermodynamic conditions on SFRJ performance have been investigated by varying S_N , $\dot{m}_{air}$, and T_{in} . The main conclusions are summarized as follows:

- (i) The turbulent flow field is separated into two zones by the flame surface: (1) the fuel-rich zone near the fuel surface where the chemical reaction mainly occurs and (2) the air-rich zone in the core flow area
- (ii) A secondary vortex at the corner of the backward facing step is generated with the introduction of the swirl flow owing to the instability of the shear layer
- (iii) Two chemical combustion mechanisms are detected in SFRJ: the chemical process in the recirculation zone and the diffusion process in the redevelopment zone and shear layer. According to the analysis of the CO distribution, the reaction controlled by the diffusion process is more sufficient
- (iv) Increasing S_N , $\dot{m}_{air}$, and T_{in} is shown to increase the combustor average temperature and the regression

rate. In addition, at the reattachment point, the local heat transfer is enhanced, and so the overall heat transfer can be promoted with the increase in the parameters mentioned above

- (v) The chemical process of SFRJ is analyzed in detail. The pyrolysis behavior of solid fuel depends strongly on the inlet conditions. Based on the analysis of the distribution of the chemical reaction intermediate (CO), the reaction is more sufficient with $S_N = 0$ and higher $\dot{m}_{air}$
- (vi) The local heat release rate achieves the maximum value in the shear layer, which corresponds to the high-temperature zone. Moreover, the overall heat release can be enhanced with a higher S_N , $\dot{m}_{air}$, and T_{in}
- (vii) Based on the elevation of the engine performance, it could be concluded that the best overall performance is achieved when $\dot{m}_{air} = 0.5$ kg/s (i.e., case B3)

Nomenclature

- A_k : Chemical reaction preexponential factor ($\text{cm}^3 \cdot \text{mol}^{-1} \cdot \text{s}^{-1}$)
 A_{out} : Area of outlet (m^2)

A_{py} : Pyrolytic reaction preexponential factor (m/s)
 A_t : Area of throat (m^2)
 a/f : Air-fuel ratio
 a_1 : Constant ($a_1 = 0.31$)
 c^* : Characteristic velocity (m/s)
 c_i : Mass fraction (component i)
 c_p : Specific heat at constant pressure (J/(kg·K))
 C_{pi} : Specific heat at constant pressure of component i (J/(kg·K))
 c_s : Heat capacity of unit mass of solid fuel (J/(kg·K))
 D_i : Diffusion coefficient for component i (m^2/s)
 E : Convective flux vectors
 E : Energy (J)
 E_a : Chemical reaction activation energy (J/mol)
 E_{apy} : Pyrolytic reaction activation energy (kJ/mol)
 E_V : Vectors of viscous flux
 F : Vectors of convective flux
 F_{engine} : Thrust of SFRJ (N)
 F_V : Vectors of viscous flux
 H : Vectors of axisymmetric source terms (convective flux)
 h : Heat transfer coefficient
 h_i : Enthalpy of unit mass for component i (J/kg)
 h_s : Energy of unit mass solid fuel (J/kg)
 h_i^{298} : Formation heat (component i) (J/kg)
 H_V : Vectors of axisymmetric source terms (viscous flux)
 I_{sp} : Specific impulse of SFRJ (N·s/kg)
 k : Turbulent kinetic energy
 L : Length of the solid fuel (mm)
 Ma_{∞} : Inlet Mach number
 M : Molecular weight (kg/mol)
 $\dot{m}$: Mass flow rate (kg/s)
 n : Temperature exponent
 P : Pressure (Pa)
 p_0 : Pressure of afterburning chamber (Mpa)
 P_r : Laminar Prandtl numbers
 Q : Conservative vectors
 Q : Heat release rate (w/m^2)
 q : Heat fluxes (w/m^2)
 R_u : Universal gas constant ($J \cdot mol^{-1} \cdot K^{-1}$)
 $\dot{r}$: Regression rate (mm/s)
 $\bar{\dot{r}}$: Average regression rate (mm/s)
 $\dot{r}_k$: Pyrolysis rate (m/s)
 S : Chemical reaction source term
 S_N : Swirl number
 Sc : Laminar Schmidt number
 Sct : Turbulent Schmidt number
 T : Temperature (K)
 T_0 : Initial temperature (K)
 T_{∞} : Inlet temperature (K)
 T_{aw} : Recovery temperature (K)
 u : Axial velocity (m/s)
 v : Radial velocity (m/s)
 w : Tangential velocity (m/s)
 w_i : Mass source for component i
 X_i : Mole fraction of component i

X_j : Mole fraction of component j
 y : Distance to the wall surface (m)
 γ : Specific heat ratio
 κ : Air equivalence ratio
 λ : Thermal conductivity ($w/(m \cdot K)$)
 λ_s : Solid fuel thermal conductivity ($w/(m \cdot K)$)
 μ : Viscosity (Pa·s)
 ρ : Total density (kg/m^3)
 ρ_{fuel} : Density of solid fuel (kg/m^3)
 ρ_i : Density for component i (kg/m^3)
 ρ_s : Density of solid fuel (kg/m^3)
 τ : Shear stress (Pa)
 ω : Turbulent dissipation rate
 Ω : Absolute value of vorticity (s^{-1}).

Subscripts

ave: Average
 f: Fuel
 fluid: Fluid domain
 i: Component i
 in: Engine inlet
 j: Component j
 l: Laminar flow
 out: Outlet surface
 stoich: Stoichiometry
 t: Turbulent flow
 w: Wall
 x: X direction
 y: Y direction
 θ : Θ direction
 %: Mass fraction of the components.

Abbreviation

HDPE: High-density polyethylene
 SFRJ: Solid fuel ramjet.

Data Availability

Some or all data, models, or code generated or used during the study are available from the corresponding author by request.

Conflicts of Interest

The authors declare that they have no conflict of interest.

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