

Research Article

Aeroengine High-Attitude/Low Mach Number Oscillations: Mechanism and Prevention Design

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Received 14 July 2020; Accepted 6 July 2021; Published 20 July 2021

Academic Editor: Antonio Viviani

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The issue of aeroengine oscillations over high-attitude and low-speed flight envelope has been an unsolved problem due to their classified nature and hard reproduction in simulated altitude test stand. Efforts have been sought for either structural integrity or component damage. However, it is rarely realized that the oscillations can be an inherent property of the engine itself. Consequently, a dynamical system approach is proposed in this paper to demonstrate that engine oscillations are recurring over high-attitude and low-speed flight envelope, yet they can be suppressed through appropriate control designs. However, the resulting design can be compromised with the conventional high-gain control where the transient and steady-state performance must be balanced with disturbance attenuation performance. Examples are given to illustrate and validate the claims made through the en route analysis.

1. Introduction

Aeroengine oscillations can cause detrimental effects on engine components, and they can also propagate through the pylon to aircraft suffering from structural vibration and passenger discomfort [1, 2]. Yet, controlling aeroengine oscillations has been challenging due to the fact that they can be caused by a variety of resources, e.g., assembly tolerance, aerodynamic disturbance (including stall and surge), combustion shock, and bearing damage [3–6]. Conventionally, preventing engine oscillations has been implemented through high-precision assembly (often requiring extensive dynamic balance experimentations) or passive control such as vibration absorbers attenuating oscillations along the transmission path [7–12].

From a control-theoretic perspective, engine controls must provide required control capability while respecting the corresponding limits. The design procedure usually works as follows: the flight envelope is divided into several regions, and within each region, a linear model is obtained for the operational condition, e.g., small perturbation state space model or finite impulse response model; then, the con-

troller is designed for each linear model using, e.g., PID control, LQR/LTR, or H_{∞} optimal control [13–15]; finally the full flight envelope control is achieved through gain scheduling (usually, scheduling variables are T1 and P1, and the PI controller parameters are corrected based on the scheduling variables). This has been the standard practice in modern engine control designs and indeed has resulted in fairly acceptable performance. However, to capture the characteristics of the nonlinear aeroengine models, system identification approaches are usually adopted due to the real-time nature for control purposes. In this respect, neural networks [16], generic algorithms [17], NARMAX [18], generalized describe function [19], and Hammerstein-Wiener representation [20] methods are all utilized for nonlinear model identifications of aeroengines. The corresponding nonlinear design approaches are then deployed for aeroengine control, see [21, 22] and references therein.

Yet, even after these inexhaustible efforts to prevent aeroengine oscillations, one situation is still recurring in practice where the engine oscillates violently over high-attitude and low Mach number flight envelope. This has been troubling particularly for military engines since they operate frequently

on high attitude with low speed, where *the engine inlet has a very small air flow (3-5 kg/s) and very low temperature (-40 degree Celsius) and pressure (5-10 kPa)*. Many efforts have been exercised to find out the reasons causing oscillations under such flight conditions. Unfortunately, no consensus has been reached due to the following reasons: (1) flight data are classified henceforth difficult for dissemination; (2) environmental conditions with 3-5 kg/s air flow, -40 degree Celsius temperature, and 5-10 kPa pressure are out of the scope of simulated altitude test stand (SATS), and henceforth, the oscillations are difficult to be “reproduced” in SATS for close scrutiny. As a consequence, even the analysis of aeroengine oscillations over high-attitude/low Mach number flight envelope is not documented in literature. The aim of the paper is to investigate the mechanism and further provide solutions to address this difficult issue. The roadmap is as follows: Section 2 provides a perspective or rationale to look at the attempted problem before explicitly attacking it; Section 3 investigates and addresses the formulated problem presenting the main results; Section 4 will target the engine oscillation with the proposed method before the discussions in Section 5.

2. Problem Formulation and Preliminaries

The fundamental rationale comes from a crucial observation: *the engine oscillates over low-speed flight condition, but somehow, it is suppressed over high-speed envelope at high attitude*. This implies that the oscillation is not caused by structural disintegrity such as assembly tolerance or components damage but may likely be an inherent property. It is thus suggested to regard the aeroengine itself as a dynamical system, henceforth treating oscillations as responses of dynamical system to exogenous disturbances. That is, it is the disturbance attenuation property of dynamical system that should be looked at for addressing this engine oscillation problem.

To proceed, it is assumed that a typical two-spool turbofan engine with a generic transfer function is considered:

$$G_E(s) = \frac{d}{as^2 + bs + c}. \quad (1)$$

The key is to investigate the effect of engine control on its disturbance attenuation property. For both simplicity and clarity, a gain feedback k is taken for engine control leading to closed-loop system:

$$G(s) = \frac{dk}{as^2 + bs + c + dk}. \quad (2)$$

Conventionally, the feedback gain should be such designed so that the engine has

- (1) fast acceleration and deceleration from one state to another or good transient performance
- (2) desired rotational speeds for set-point conditions or small steady-state error

Usually, the above objective is achieved through a design of large feedback gain, which is the well-known “high-gain

control.” Indeed, at high attitude, k , is designed to be “aggressive” for fast tracking and small steady-state error, with the spoon speed approaching the mechanical limit of high-pressure rotational shaft. This has been the standard practice even “design philosophy” in industry, spreading across control designs for difference engines including turboshafts, turbofans, and turbo-props. It is rarely noticed that there can be potential “fallacies” for a long troubling issue of engine oscillations over high-attitude and small speed envelope. *It is the aim of this paper to disseminate the reasons behind the “unsolved problem,” and thus, it is now worth of another look at this protocol and to see if it should be revised.*

As explained above, the key is to note that the oscillations must be related with vibrational signal propagating through the engine transmission path. Thus, frequency spectrum information should be analysed. To proceed, the “magnification” of control is scrutinized, and the rationale is that if the vibrational frequency signal is significantly magnified along the transmission path due to control design, it must be recognized as a key factor for oscillations. Now, since the engine is an inherent inertia system, its frequency response function (FRF) can be written as

$$G_E(j\omega) = \frac{d}{c - a\omega^2 + b\omega j}. \quad (3)$$

Correspondingly, the engine with gain feedback has FRF:

$$G(j\omega) = \frac{dk}{c + dk - a\omega^2 + b\omega j}. \quad (4)$$

Thus, if a sinusoidal input $d(t) = \sin \omega t$ coming from either environmental disturbances or equipment dynamics, the effect of control is to “reshape” the transmission defined as $T_d(j\omega)$:

$$T_d(j\omega) \equiv \frac{G(j\omega)}{G_E(j\omega)} = k \frac{c - a\omega^2 + b\omega}{c + dk - a\omega^2 + b\omega j}. \quad (5)$$

Therefore, the effect of control on disturbance can be measured through its gain amplification:

$$|T_d(j\omega)| = |k| \sqrt{\frac{(c - a\omega^2)^2 + b^2\omega^2}{(c + dk - a\omega^2)^2 + b^2\omega^2}}. \quad (6)$$

Thus, the problem of aeroengine oscillations over high-attitude/low Mach number flight envelope can be tackled as analyzing the gain amplification properties from the dynamical system perspective.

3. Dynamical System Approach to Engine Oscillations

From (6), it is obvious that the control gain should be such designed that $|T_d(j\omega)|$ is less than or equal to unity, not to magnify the disturbance transmissions across the engine. That is, an unfortunate design of control gain k leading to $|T_d(j\omega)| \gg 1$ would significantly amplify the disturbance

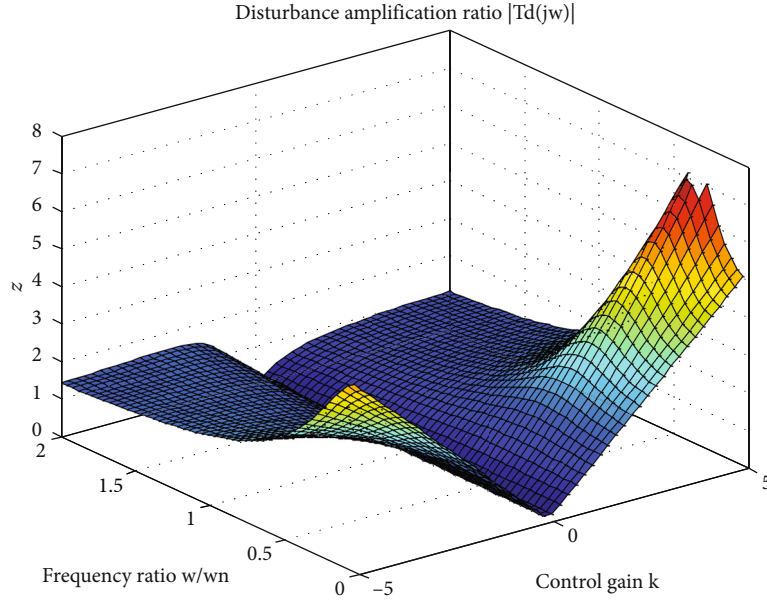


FIGURE 1: Dependence of disturbance amplification upon control gain and frequency ratio.

and this will transmit along the engine causing significant oscillations. The problem of aeroengine oscillations over high-altitude/low Mach number flight envelope is thus identified as $|T_d(j\omega)| \gg 1 \forall k$ or simply

$$|k| \sqrt{\frac{(c - a\omega^2)^2 + b^2\omega^2}{(c + dk - a\omega^2)^2 + b^2\omega^2}} > 1. \quad (7)$$

To illustrate the significance of the result, the following generic model is utilized:

$$G_E(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}, \quad (8)$$

where ω_n is the natural frequency and ξ is the damping ratio of the aeroengine.

That is, it is assumed that $a = 1$, $b = 2\xi\omega_n$, and $c = d = \omega_n^2$. Substitution into (6) results in

$$|T_d(j\omega)| = |k| \sqrt{\frac{(\omega_n^2 - \omega^2)^2 + 4\xi^2\omega_n^2\omega^2}{(\omega_n^2 + k\omega_n^2 - \omega^2)^2 + 4\xi^2\omega_n^2\omega^2}} \quad (9)$$

or

$$|T_d(j\omega)| = |k| \sqrt{\frac{((\omega/\omega_n) - (\omega_n/\omega))^2 + 4\xi^2}{[(k+1)(\omega/\omega_n) - (\omega_n/\omega)]^2 + 4\xi^2}}. \quad (10)$$

3.1. Control Gain k vs. Frequency ω . It is seen clearly that given engine dynamics ω_n and ξ , the property of $|T_d(j\omega)|$ is determined by control gain k and disturbance signal frequency ω . Now, a scenario is assumed where the engine slows down from high speed to low speed resulting in a critical damping ratio $\xi = 1$ from an underdamped one. The depen-

dence of $|T_d(j\omega)|$ upon control gain k and frequency ratio ω/ω_n is shown in Figure 1. It is seen clearly that regions for both $|T_d(j\omega)| < 1$ and $|T_d(j\omega)| > 1$ exist, indicating that disturbance can be either suppressed or amplified. It is thus concluded that care must be taken to avoid those amplification regions that may lead to excessive vibration oscillations.

3.2. $\omega = \omega_n$. It is often problematic when the disturbance signal has the natural frequency ω_n , and from (10), one has

$$|T_d(j\omega_n)| = \frac{2}{\sqrt{(4/k^2) + (1/\xi^2)}}. \quad (11)$$

This is illuminating since at high altitude, the engine is underdamped with $0 < \xi < 1$ for high flight speed, while becoming overdamped with $\xi > 1$ for very low Mach numbers. The former case is due to the following fact:

$$|T_d(j\omega_n)| = \frac{2}{\sqrt{(4/k^2) + (1/\xi^2)}} < 2\xi. \quad (12)$$

Thus, the magnitude $|T_d(j\omega_n)|$ is bounded above by twice no matter how the control gain k is designed. With decreased flight speed, one would have

$$\frac{2}{\sqrt{(4/k^2) + 1}} \leq |T_d(j\omega_n)| \leq \sqrt{|k|\xi}. \quad (13)$$

Thus, with increased control gain and damping ratio (with very low speed), the magnitude $2 \leq |T_d(j\omega_n)| \leq \sqrt{|k|\xi}$ can cause significantly excessive oscillations. For example, for $\xi = 10$ and $k = 20$, one has $|T_d(j\omega_n)| = 14.4$, which is an unacceptably large enhancement of vibration. Meanwhile, it

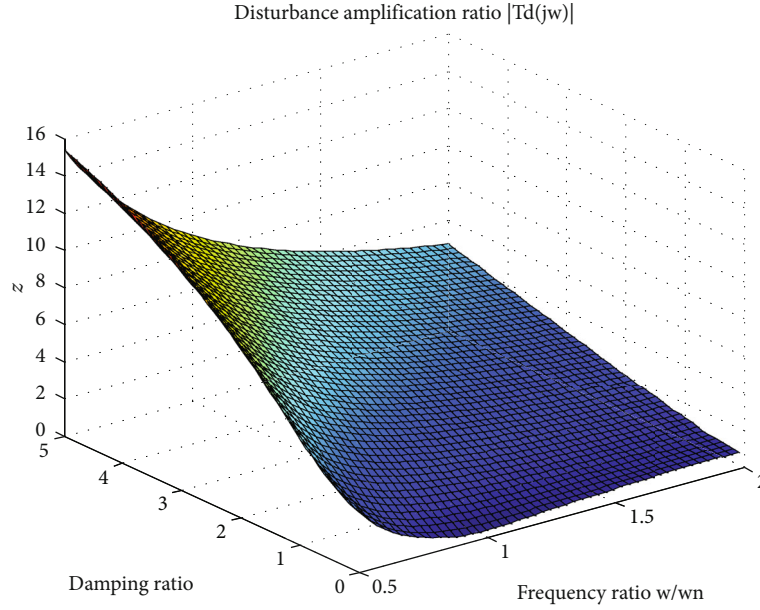


FIGURE 2: Dependence of disturbance amplification upon damping ratio and frequency ratio.

is noticed that for the same control gain $k = 20$, $|T_d(j\omega_n)| = 0.99$ for $\xi = 0.5$. Thus, the engine does not oscillate for high speed, and with decreased flight speed, the engine starts to get oscillated until an unacceptable level of vibration occurs leading to fault alert in flight control system. Put differently, this explains why the engine oscillations do not occur at high Mach number while beginning to emerge at a very low Mach number flight envelope.

3.3. *Fixed Control Gain k .* Now, further consider the situation where the control gain is fixed, e.g., $k = 20$. Then, we have

$$|T_d(j\omega)| = 20 \sqrt{\frac{((\omega/\omega_n) - (\omega_n/\omega))^2 + 4\xi^2}{((21\omega/\omega_n) - (\omega_n/\omega))^2 + 4\xi^2}}. \quad (14)$$

This is shown in Figure 2, where the following two observations can be made:

- (1) For certain frequency ratio ω/ω_n , when the damping ratio ξ increases from underdamped to overdamped or the engine decreases flight speed, the amplification effect on vibration becomes even influential, henceforth leading to excessive oscillations
- (2) For certain damping ratio ξ , with vibration frequency decreasing, the engine oscillations get larger; yet, it is exactly over low-speed flight envelope that excessive oscillations occur over a broad frequency band, indicating why the low Mach number oscillation is recurring in practice

4. Prevention Design of Engine Oscillations

The analysis in Section 3 has clearly shown that it is the property of engine dynamics that causes the oscillations over low-speed flight envelope. To prevent such oscillations, the con-

trol design must be enforced with a new constraint from (10) as $|T_d(j\omega)| \leq 1$ or

$$|k| \sqrt{\frac{((\omega/\omega_n) - (\omega_n/\omega))^2 + 4\xi^2}{[(k+1)(\omega/\omega_n) - (\omega_n/\omega)]^2 + 4\xi^2}} \leq 1. \quad (15)$$

For $\omega = \omega_n$, this will lead to

$$\frac{2}{\sqrt{(4/k^2) + (1/\xi^2)}} \leq 1. \quad (16)$$

That is,

$$|k| \leq \frac{2\xi}{\sqrt{|4\xi^2 - 1|}}. \quad (17)$$

Here comes the “apogee” of the engine oscillation prevention design—the control gain must be (significantly) restricted that high-gain control may not be possible. To fully appreciate the implication, it is noted that for closed-loop engine control, the steady-state error for step responses is

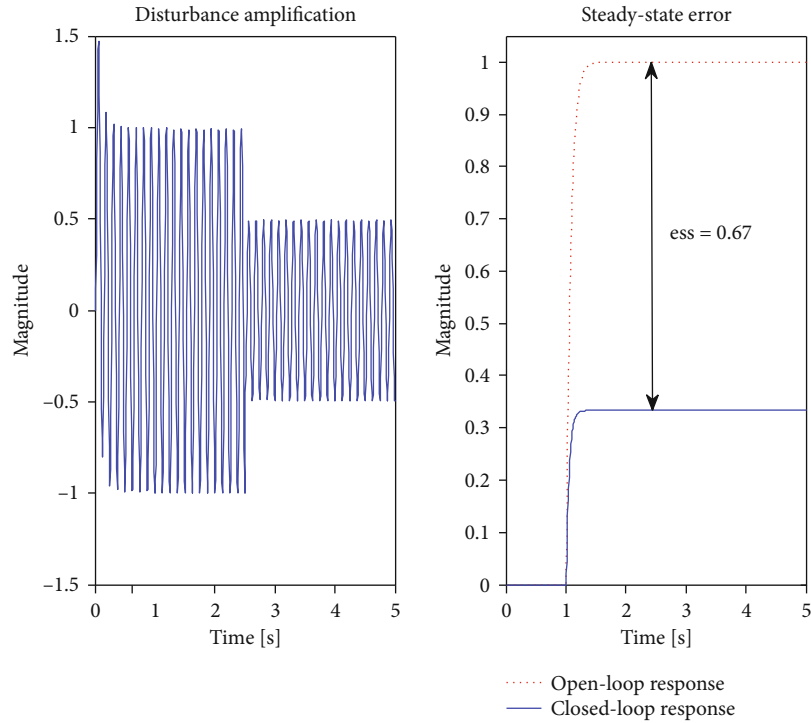
$$e_{ss} = 1 - \lim_{s \rightarrow 0} G(s) = \frac{c}{c + dk} \quad (18)$$

or

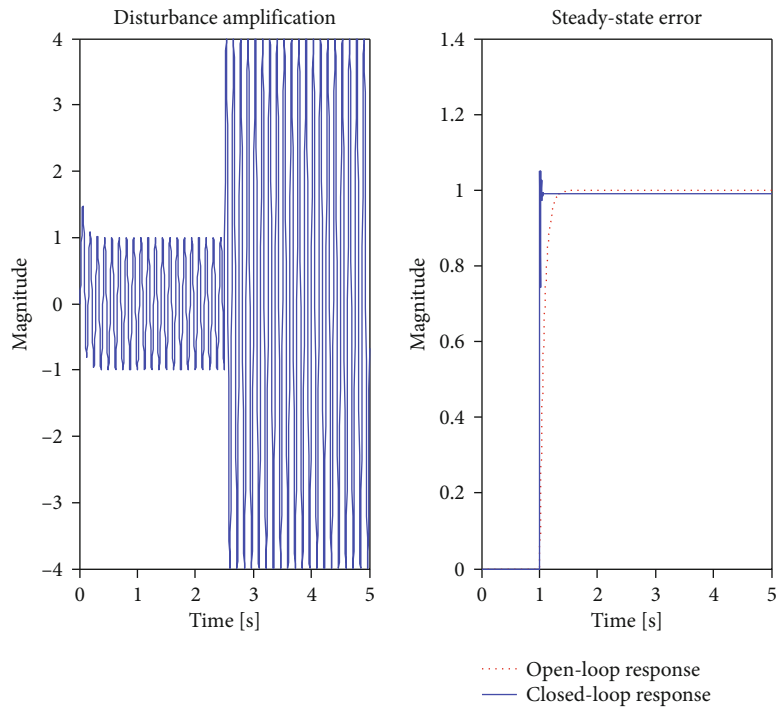
$$e_{ss} = 1 - \frac{k}{k+1} = \frac{1}{k+1}, \quad (19)$$

for the case of $G_E(s) = \omega_n^2 / (s^2 + 2\xi\omega_n s + \omega_n^2)$.

For both cases, it is seen that control gain k must be large to reduce the steady-state error. The same conclusion can be reached for improving the dynamic responses. And this



(a)



(b)

FIGURE 3: Compromise between disturbance transmission performance and steady-state performance, illustrating clearly that high-gain control design philosophy should be challenged in aeroengine control design.

has been the “design philosophy” in industry for a variety of engine control designs such as turboshafts, turbofans, and turbo-props. However, what has been demonstrated in (17) for prevention or alleviation of engine oscillations is that

the magnitude of control gain must be constrained. For example, for engine operation over low-speed flight with $\xi = 2$, the control gain is restricted to be $|k| \leq 1.03$. This is obviously a significant restriction on control design since this

will result in the steady-state error for step input (assuming control gain is positive):

$$e_{ss} = \frac{1}{k+1} \geq 0.49. \quad (20)$$

That is, at least a 49% deviation from expected rotational speed value must occur which is obviously unacceptable—indeed, a steady-state error of less than 1% is usually enforced by Airworthiness Regulations such as Federal Aviation Administration (FAA) or Civil Aviation Administration of China (CAAC) [23–25]. To meet such a regulation, there must have $k > 99$, which is clearly a contradiction to engine oscillation prevention design of $|k| \leq 1.03$. Such a contradiction is clearly demonstrated in Figure 3, where (a) presents with $k = 0.5$ satisfying the disturbance transmission condition $|T_d(j\omega)| = 0.5 \leq 1$; henceforth, the disturbance is suppressed by half and the engine oscillation is prevented, yet the steady-state error to step response is obviously too large to be accepted; (b) presents with $k = 99$; however, the steady-state error to step response is clearly satisfied, yet the disturbance transmission $|T_d(j\omega)| = 4$ can result in significant vibration enhancement, pushing to the path to engine oscillation.

Remark 1. It is also noted that for high altitude and high speed with $0 < \xi < 1$, particularly around $\xi = 0.5$, it can be derived from (16) that

$$|T_d(j\omega)| = \frac{1}{\sqrt{1 + (1/k^2)}} \leq 1. \quad (21)$$

Thus, the engine oscillation prevention design is always satisfied and this justifies the “validity” of high-gain control.

Remark 2. That is, high-gain control over high-altitude and high-speed flight envelope does not cause the engine oscillation. Yet, with decreased flight speed, the engine oscillates violently since over 14 times amplification of vibration can result simply due to the dynamics of the engine itself.

5. Discussions and Conclusions

What has been proposed in this paper is that disturbance response is an inherent property of any dynamical system; henceforth, oscillations are unavoidable for systems under disturbance excitations. Therefore, while ensuring high-precision assembly tolerance, disturbance response property must also be taken into account. For the case of control system design, the controller should not only possess acceptable dynamic/transient and steady-state performance but also have desired disturbance attenuation performance.

However, it has also been analysed that the two performance indices can be conflicting, particularly over the “problematic” region of high-altitude and low-speed flight envelope, implying that a compromise must be balanced for optimal design confirmations. It is worth pointing out that handling disturbance has been a requirement for any control

system design [26], yet, what has been shown is that the two performance indices may be so “compromising” that *no feasible solution can be obtained that would provide acceptable steady-state performance and disturbance attenuation performance. This matter of fact does not seem to be fully appreciated in industry, and this paper is an attempt to disseminate this design concept.* In conclusion, the following points are hoped to be illuminated:

- (1) Disturbance response is an inherent property of the aeroengine as a dynamical system, and it is thus not possible to fully suppress the oscillations even with high-precision assembly tolerance
- (2) Aeroengine control design should provide both acceptable dynamic/steady-state performance and desired disturbance-attenuation performance. Yet, the two performance indices are often conflicting over certain flight envelopes
- (3) For certain cases, the two performance indices can be significantly compromised, even resulting in no feasible design for aeroengine controls, or unacceptable performance as enforced by aviation regulation agencies
- (4) Therefore the “common wisdom” of high-gain control in aeroengine control design industry has to be challenged; and such a design practice must be revised to take the disturbance attenuation property into account—aeroengine transient and steady-state performance must be balanced with its disturbance attenuation performance
- (5) Consequently, over high-altitude/low-speed flight envelope, aeroengine oscillations can only be suppressed by careful structural designs and henceforth will be involved in other component designs, which is a complicated system engineering procedure

Data Availability

The underlying data supporting the results of your study can be obtained by requesting the corresponding author.

Conflicts of Interest

The author declares that there are no conflicts of interest.

Acknowledgments

This work is supported by the Central Military Commission Special Fund for Defense Science, Technology & Innovation (No. 20-163-00-TS-009-096-01); Central Military Commission Foundation to Strengthen Program Technology Fund (No. 2019-JCJQ-JJ-347); State Foreign Affairs Bureau Fund for Introduction Plan of High-end Foreign Experts (G20200010100); Aviation Science Fund of China-Xi’an 631 Research Institute (201919052001); and the Fundamental Research Funds for the Central Universities (No. NS2020017).

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