

Research Article

Comparative Assessment of Modified γ -Re $_{\theta t}$ Models for Scramjet Intake Flow Analysis

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The selection of an appropriate turbulence/transition model is critical when simulating the hypersonic flows based on the Reynolds-averaged Navier–Stokes (RANS) equation. In particular, a deep understanding of the validity, reliability, and limitations of existing models is an essential prerequisite to facilitate their further development. This paper reports on the assessment of two models dedicated to hypersonic boundary layer transition analysis. Both models are compared with two other models that are widely used in this field. The double ramp and shock wave laboratory scramjet intake cases are used for the validation and evaluation of the predictive performance of both models via comparison against experimental data. The results reveal that the appropriate selection of the transition model is critical to the attainment of accurate results. Moreover, the forebody of the scramjet combustion propulsion-01 (SCP-01) flight vehicle, which is a research model, is analyzed as a case study. The air intake performance of the SCP-01, as predicted by four models, is compared and analyzed at different flight altitude and Mach number conditions. The results reveal that the accuracy of the prediction of boundary layer and separated flow transitions significantly affects the flow field and corresponding air intake performance. Furthermore, the uncertainty of results obtained using the two models increases significantly with an increase in altitude. Finally, the reliability and limitations of the transition models considered in this study are examined.

1. Introduction

The scramjet engine is the most popular air breathing propulsion system designed for hypersonic flights. Because scramjet engines use the freestream air as an oxidizer, flight vehicles installed with such engines need not carry an oxidizer. This characteristic plays a significant role in reducing the weight of a flight vehicle compared to rocket-based propulsion systems, wherein the oxidizer and fuel contribute to the flight vehicle weight. This weight reduction is advantageous and affords improvement in flight performance. Furthermore, the resulting high payload-to-weight ratio makes scramjet-based hypersonic flight vehicles economical compared to their rocketpowered counterparts [1]. Additionally, scramjets afford the realization of a high specific impulse compared to rocket engines, thereby resulting in increased propulsion efficiency [2].

Typical scramjet engines comprise an inlet, isolator, combustor, and nozzle, as depicted in Figure 1. Unlike the turbo engine family, scramjet engines do not use moving components that compress air. Instead, they use shock waves generated from the inlet and the isolator as the air compression mechanism. The air compressed by the shock waves as it flows past the inlet and isolator is subsequently passed through a combustor and burned with injected fuel to realize supersonic combustion. Thereafter, the combustion gas expands through a nozzle to produce thrust. For a given scramjet intake geometry, the air intake performance parameters, such as the compression ratio, total pressure loss, and mass flow rate, are determined by the flight conditions (i.e., Mach number, altitude, and attitude) alone. In other words, the scramjet intake geometry (i.e., the inlet and isolator) directly affects the overall engine performance because it determines the flow field, shock structure, and resulting air intake performance under the given flight conditions.



FIGURE 1: Schematic of a scramjet engine [3].

The boundary layer transition is an important phenomenon that affects air intake performances. Despite their high flight Mach numbers, hypersonic vehicles typically operate at low-unit Reynolds number conditions owing to the low air density at high altitudes. Therefore, for a fixed Mach number, an increase in flight altitude (or reduction in Reynolds number) causes the location of the boundary layer transition to shift downstream [4–6]. This delay in boundary layer transition results in a long laminar boundary layer that extends along the inlet surface up to the transition point. However, laminar boundary layers pose a high possibility of separation under adverse pressure gradients or at junction corners formed between the inlet surface and the sidewalls. Moreover, laminar boundary layers lead to the formation of large separation bubbles in the vicinity of ramp corners.

Flow separations considerably degrade the air intake performance by significantly altering the flow and shock structures within the inlet-isolator region. Because laminar boundary layers are susceptible to flow separations, their early transition to turbulent boundary layers is desirable for the suppression of separation. Accordingly, several existing studies have considered the use of trips comprising discrete roughness elements to force the early transition of the boundary layer over the inlet surface to eliminate the uncertainty owing to the separation [7]. In contrast, from an aerothermodynamic performance perspective, the skin friction and heat transfer in the turbulent boundary layer are much higher than those in the laminar boundary layer. In particular, it is reported that at hypersonic Mach numbers, the maximum heat transfer occurs in the transitional region [8]. Therefore, the laminar state of external boundary layers is preferred in terms of the drag reduction and heat protection of hypersonic flight vehicles. Hence, the possibility and location of the occurrence of the boundary layer transition are key factors that determine the air intake performance and aerothermodynamic performance of hypersonic flight vehicles, especially at high-altitude conditions.

Moreover, the existence of a complex flow structure, including shock waves, shock train, and shock wave boundary layer interactions (SWBLI), in the inlet and isolator regions makes the analysis and prediction of the air intake performance difficult and uncertain [9]. Therefore, an appropriate method that duly considers the underlying viscous effects is required to improve the prediction and analysis of the scramjet air intake performance under various operating conditions. Based on the preceding discussions, the ability to predict the transition and separation in the analysis is crucial not only for analyzing the air intake performance but also for improving the scramjet inlet designs.

Both the experimental and numerical approaches could be employed to investigate the complex high-speed viscous flow structures within the scramjet intakes. Typically, experimental studies concerning hypersonic flow phenomena that require the use of special equipment or ground facilities encounter problems associated with high-enthalpy conditions, short test times, and the uncertainty of freestream conditions. Therefore, the computational fluid dynamics-(CFD-) based numerical approach is widely used as an effective alternative to experimentation owing to its several advantages, such as time and cost savings as well as independence from physical feasibility. Moreover, the availability of advanced computational resources has facilitated the widespread use of high-fidelity CFD-based analysis and investigation techniques, such as direct numerical simulation [10-12], large eddy simulation [13–15], and detached eddy simulation [16-18]. Nonetheless, the use of eddy viscosity modeling to solve the Reynolds-averaged Navier-Stokes (RANS) equation [19] constitutes the most common and practical computational approach. It is typically employed in engineering applications that require analysis of different configurations and operating conditions.

Although the RANS simulations are useful as an engineering tool, the accuracy of the results depends on the turbulence model used, especially for problems wherein the boundary layer transition is important and governs all the flow features. Accordingly, different turbulence models, such as the algebraic [20–22], laminar kinetic energy (k_L) [23, 24], turbulence intermittency [25–27], stability theory-based [28–31], and correlation-based [32, 33] models, have been developed to account for the effect of transition in RANS simulations. Several models have exhibited satisfactory prediction capability when applied to the simulation of transitional shear flows. However, their validity and applicability are limited to certain types of flow or over a specific

range of conditions that govern the underlying instability mechanisms leading to transition. For example, the γ -Re_{θ t} model [32], which is a widely known correlation-based transition model, guarantees high prediction accuracy when simulating predominantly two-dimensional subsonic (lowspeed) flows. However, it cannot accurately predict transitions in three-dimensional high-speed flows owing to the strong crossflow and Mack's 2nd mode instabilities, respectively [34–36]. To address these limitations of correlationbased models, several other transition models have been developed [37–39]. However, correlation-based models offer several advantages, and therefore, the demand for their improvement and development continues to increase.

The correlation-based transition model is inherently problem-specific and suffers from generality because its underlying correlations are established to fit limited target data. When a turbulence model is used to analyze the flow field inside a scramjet intake, inaccurate boundary layer transition and SWBLI predictions could cause the results obtained to differ significantly from the actual flow behavior [40]. To overcome these limitations, research pertinent to the development and improvement of new and transition models [41–43] has attracted increased attention. The existing efforts to improve the model include the modification of the correlations of the γ -Re_{θ t} model [44–46] or both the equations and the correlations [47-49]. Moreover, the abovementioned studies focused on models pertaining to the transition of the attached boundary layer with no consideration of transitions occurring in separated flows. Because the flow inside a scramjet intake depends on the transition of both the attached and separated shear flows, the accuracy and reliability of transition prediction for separated flows are also regarded as significant. However, studies concerning the investigation or development of transition prediction models for separated flows at hypersonic Mach numbers are seldom reported.

Although several transition prediction models have been proposed and investigated in existing studies based on different concepts, their validation remains problem-specific. Accordingly, there exists no generalized model capable of accurate and reliable prediction of hypersonic boundary layer transitions. Therefore, a working-level knowledge and understanding of the characteristics and limitations of existing models are essential prerequisites to facilitate the development or improvement of new or existing models, respectively, which afford high accuracy and generality [34, 50].

This paper presents a modified model based on the γ -Re_{θ t} framework to afford improved accuracy on transition prediction in both the boundary layer and separated flow transitions at hypersonic Mach numbers. The model is evaluated by analyzing the benchmark problem and scramjet intake flows, and the pertinent results of the flow field and air intake performances are compared against those from existing models. The *k*- ω shear stress transport (SST) [50] and γ -Re_{θ t} models are considered for comparison. Moreover, an additional model for the prediction of hypersonic boundary layer transition [43] is considered for comparison. The double ramp [51] and shock wave laboratory (SWL) intakes [52] are analyzed as validation cases. Subsequently, the analysis

results are compared against experimental data to examine the predictive performances and characteristics of the candidate models. Additionally, the forebody of the SCP-01 flight vehicle [53], which was designed for research purposes, is analyzed under different altitudes and Mach numbers. The observed differences in the air intake performance with respect to flow conditions and turbulence/transition models are comprehensively analyzed. The characteristics of the two transition models for the hypersonic boundary layer are examined, and the influence of the assumption of highaltitude conditions made for both models is discussed. Finally, the uncertainty and sensitivity of the scramjet air intake performance prediction depending on the models, which may arise for high-altitude conditions, are assessed.

2. Method of Analysis

2.1. Baseline and Modified γ -Re_{θt} Models

2.1.1. Baseline γ -Re_{θt} Model by Langtry and Menter. The γ -Re_{θt} model was proposed by Langtry and Menter [32, 54, 55] to predict boundary layer transitions in RANS simulations. Because this model uses local variables to determine the transition onset location, it is advantageous for use in CFD simulations based on the unstructured grid system and parallel computation. To predict the transition, the transport equations for intermittency (γ) and transition onset momentum thickness Reynolds number (Re_{θt}) are combined with the *k*- ω SST model [50]. Therefore, the model comprises four transport equations. The transport equations for γ and Re_{θt} could be expressed as

$$\frac{\partial(\rho\gamma)}{\partial t} + \frac{\partial(\rho u_j \gamma)}{\partial x_j} = P_{\gamma} - E_{\gamma} + \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_t}{\sigma_f} \right) \frac{\partial\gamma}{\partial x_j} \right], \quad (1)$$

$$\frac{\partial \left(\rho \widetilde{\operatorname{Re}}_{\theta t}\right)}{\partial t} + \frac{\partial \left(\rho u_{j} \widetilde{\operatorname{Re}}_{\theta t}\right)}{\partial x_{j}} = P_{\theta t} + \frac{\partial}{\partial x_{j}} \left[\sigma_{\theta t} (\mu + \mu_{t}) \frac{\partial \widetilde{\operatorname{Re}}_{\theta t}}{\partial x_{j}}\right].$$
(2)

The production (P_{γ}) and destruction terms (E_{γ}) in the γ transport equation (Equation (1)) can be expressed as

$$P_{\gamma} = F_{\text{length}} c_{a1} \rho S[\gamma F_{\text{onset}}]^{0.5} (1 - C_{e1} \gamma), \qquad (3)$$

$$E_{\gamma} = c_{a2} \rho \Omega \gamma F_{turb} (C_{e2} \gamma - 1), \qquad (4)$$

$$F_{\text{onset}} = \max (F_{\text{onset2}} - F_{\text{onset3}}, 0), \tag{5}$$

$$F_{\text{onset3}} = \max\left(1 - \left(\frac{R_{\text{T}}}{2.5}\right)^3, 0\right),\tag{6}$$

$$F_{\text{onset2}} = \min\left(\max\left(F_{\text{onset1}}, F_{\text{onset1}}^4\right), 2.0\right),\tag{7}$$

$$F_{\text{onset1}} = \frac{\text{Re}_{\text{v}}}{2.193 \cdot \text{Re}_{\theta c}}.$$
(8)

Equation (3) represents a production term that increases the intermittency, thereby triggering transition, whereas Equation (4) represents a destruction term that is responsible for relaminarization. The F_{length} and F_{onset} terms in Equation (3) determine the length of the transitional region and transition onset location, respectively. The Re_{θ_c} term in Equation (8) and F_{length} are defined as functions of the local transition onset momentum thickness Reynolds number, $\widetilde{\text{Re}}_{\theta_t}$ (dependent variable of the transport equation, Equation (2)), and their correlations were determined based on experimental data and a series of numerical experiments.

The production term in the transport equation for $\text{Re}_{\theta t}$ (Equation (2)) could be expressed as Equation (9). $\text{Re}_{\theta t}$ represents a function of the pressure gradient parameter (λ_{θ}) and freestream turbulence intensity (Tu_{∞}), and is given as an empirical correlation [56]. Through the transport equation, the production term cause $\widetilde{\text{Re}}_{\theta t}$ to increase to the value corresponding to the transition onset given by the $\text{Re}_{\theta t}$ correlation.

$$P_{\theta t} = c_{\theta t} \frac{\rho}{t} \left(\operatorname{Re}_{\theta t} - \widetilde{\operatorname{Re}}_{\theta t} \right) (1.0 - F_{\theta t}).$$
(9)

Following laminar separation, the predicted location of the reattachment point after the transition of a separated shear layer is located further downstream compared to experimental data. To address this discrepancy, the following equation is considered to ensure that the separated flow quickly transitions to turbulence and reattaches. This is accomplished by forcing γ to attain a value greater than 1.0 in the separated flow region [54, 55].

$$\gamma_{\text{sep}} = min\left(s_1 max\left(0, \left(\frac{Re_v}{3.235\text{Re}_{\theta c}}\right) - 1\right)F_{\text{reattach}}, 2.0\right)F_{\theta t}.$$
(10)

As described in Equation (11), the larger value of γ between those obtained from the transport equation and γ_{sep} calculated using Equation (10) is considered γ_{eff} .

$$\gamma_{\rm eff} = \max\left(\gamma, \gamma_{\rm sep}\right).$$
 (11)

The boundary layer transition is simulated by applying Equation (11) to the production and destruction terms of the *k*-transport equation of the k- ω SST model as follows:

$$\frac{\partial(\rho k)}{\partial t} + \frac{\partial(\rho u_j k)}{\partial x_j} = \gamma_{\text{eff}} \cdot P_k - \min(\max(\gamma_{\text{eff}}, 0.1), 1.0)$$
$$\cdot E_k + \frac{\partial}{\partial x_j} \left[(\mu + \sigma_k \mu_t) \frac{\partial k}{\partial x_j} \right].$$
(12)

The details, such as the model coefficients and correlations, pertinent to the above equation are not presented in this paper, but the same could be referred to in Langtry and Menter [32].



FIGURE 2: Flow chart of the blended model implemented in ANSYS Fluent.

The model proposed by Langtry and Menter [32] adopts empirical correlations based on experimental data and a series of numerical experiments on two-dimensional subsonic boundary layers on a flat plate. Accordingly, this model demonstrates excellent prediction accuracy in cases involving two-dimensional subsonic flows, wherein the Tollmien– Schlichting (T–S) instability or bypass transition [32, 54, 55] constitutes the dominant physical mechanism causing the transition. On the other hand, as mentioned above, the reliability of the model is not guaranteed when simulating flows wherein the transition is caused by other physical mechanisms or factors. Such flows include the three-dimensional boundary layer with strong crossflow (crossflow instability), the supersonic boundary layer (oblique first mode), and the hypersonic boundary layer (Mack's 2nd mode) [34–36].

2.1.2. Model Modified by Krause. It has been reported that when simulating the hypersonic boundary layers, the value of $\widetilde{\text{Re}}_{\theta t}$ can significantly exceed those typically attained for subsonic flows, and it attains values of up to 10⁵ in certain regions, such as reattachment zones [43]. Accordingly, the γ -Re_{θt} model could yield inaccurate results, such as the prediction of delayed transition. To alleviate this problem and improve the predictive capability for hypersonic flows, Krause proposed alternative correlations Re_{θc} and F_{length} (Equations (8) and (3), respectively) as functions of Tu_{co} alone, instead of $\widetilde{\text{Re}}_{\theta t}$ [43]. The proposed correlations for Re_{θc} and F_{length} could be expressed as

$$Re_{\theta c} = 967.34 \cdot Tu_{\infty}^{-1.0315},$$
 (13)

$$F_{\text{length}} = 10.435 \cdot T u_{\infty}^{2.9756}.$$
 (14)

The resulting γ -Re_{θ t} model that applies the abovedescribed correlations is referred to as the Krause model.



FIGURE 3: Configuration of the double ramp case.

Further details concerning this model could be obtained from Krause [43].

2.1.3. Blended Model. To improve the prediction accuracy concerning the transition of hypersonic shear flows, this paper presents a model based on the γ -Re_{θ t} model framework. The correlations proposed in previous studies are considered to predict the transition of attached boundary layers. Furthermore, an additional modification to Equation (10) is made to improve the prediction accuracy of transitions occurring in the region of separated flows [46]. A major highlight of the proposed model is that the values of Re_{θ c} used in Equation (8) and that used to evaluate γ_{sep} in Equation (10) are considered independent parameters. Therefore, the value of Re_{θ c} used to evaluate γ_{sep} is referred to as Re_{sep}, and a separate correlation is used for its evaluation.

Based on the results obtained in previous studies concerning the hypersonic boundary layer transition, it is understood that $\text{Re}_{\theta t}$ varies with the Mach number [57]. To reflect this relationship in the model, the $\text{Re}_{\theta t}$ correlation proposed by Zhang and Zhenghong [44] is considered. The correlation comprises the original $\text{Re}_{\theta t}$ correlation ($\text{Re}_{\theta t, \text{original}}$) multiplied by a function of the Mach number. It can be expressed as

$$Re_{\theta t, Zhang} = Re_{\theta t, original} \cdot G(M),$$
 (15)

where

$$G(M) = 0.00987M^3 - 0.14407M^2 + 0.75109M + 1.$$
 (16)

Equation (17), which was proposed by Frauholz et al. [45], can be used as the F_{length} correlation that affects the length of the transitional region. In the attached flow region, the original correlation proposed by Langtry and Menter as a function of $\text{Re}_{\theta t}$ is used for the $\text{Re}_{\theta c}$ evaluation, which affects the location of the transition onset (Equation (8)). The correlation (Equation (13)) proposed by Krause [43] is used for evaluating Re_{sep} , which is used to determine the transition location in separated flow regions. The correlations are expressed as follows:

$$F_{\text{length}} = 0.0045 T u_{\infty} - 0.0902 T u_{\infty}^{2} + 0.2343 T u_{\infty}^{3} + 1.2776 T u_{\infty}^{4},$$
(17)

TABLE 1: Flow conditions of the double ramp case.

Parameter	Value
p_{∞} (Pa)	520
$ ho_{\infty} (\text{kg/m}^3)$	0.0171
T_{∞} (K)	106
T ₀ (K)	1430
M_{∞}	8.1
$\operatorname{Re}_{x,\infty}(1/m)$	$3.8 imes 10^6$

$$Re_{sep} = 967.341 T u_{\infty}^{-1.0315}$$
. (18)

Because the correlations obtained from prior studies concerning hypersonic boundary layers are applied in combination, the proposed model is referred to as the blended model throughout this study. Further details concerning the blended model are available in [46].

2.2. Numerical Method. The ANSYS Fluent 19.0 CFD solver was used in this study. The values of $\operatorname{Re}_{\theta t}$, F_{length} , and $\operatorname{Re}_{\theta c}$, i.e., the main variables of the γ -Re_{θ t} model whose correlations need to be defined, can be accessed and modified using the user-defined function (UDF) feather within Fluent [58]. In this study, UDFs were used to implement the Krause and blended models in Fluent. The Krause model considers the $\operatorname{Re}_{\theta c}$ and F_{length} correlations as functions of a single parameter Tu_{∞} . These correlations can be easily implemented via UDF. On the other hand, the proposed blended model considers the $\operatorname{Re}_{\theta c}$ values used for γ_{sep} and F_{onset1} evaluations separately. Accordingly, the $\operatorname{Re}_{\theta c}$ used for $\gamma_{\operatorname{sep}}$ evaluation is independently defined as Re_{sep} . In Fluent, although $\text{Re}_{\theta c}$ can be modified using the UDF, its values used for F_{onset1} and γ_{sep} evaluations are not considered independent. Therefore, to ensure that the blended model is appropriately implemented, the $Re_{\theta c}$ value is applied only for the attached flow region, whereas the individually defined Re_{sep} value is applied in the separated flow region. The blended model is implemented using the UDF to ensure that $Re_{\theta c}$ and Re_{sep} are considered exclusively in cases where $\gamma_{sep} < 1$ and $\gamma_{sep} > 1$, respectively. The flow chart of the blended model implemented in Fluent is depicted in Figure 2.

A steady two-dimensional flow analysis was performed in this study using an implicit density-based solver with a cell-



FIGURE 4: Computational domain, grid system, and boundary conditions for the double ramp case.

TABLE 2: Grid information of the grid dependency test for the double ramp case.

Grid type	Number of cells	Number of grids in the streamwise direction	Number of grids in the transverse direction	Max <i>y</i> +	Average <i>y</i> +
Finest	264,000	661 (51 + 251 + 301 + 61)	401	1.291	0.418
Fine	221,400	616 (41 + 226 + 301 + 51)	361	1.291	0.42
Medium	162,000	541 (41 + 201 + 251 + 51)	301	2.106	0.792
Coarse	104,500	419 (41 + 176 + 226 + 51)	251	5.703	3.139

based formulation. The Roe scheme was chosen for flux calculations with gradient reconstruction based on least squares. The second-order upwind scheme was used to discretize all flow and turbulence variables. The ideal gas relation was used for evaluating the air properties, and the fixed value of 1006.43 J/kg·K was used for the specific heat at constant pressure (c_p) throughout the simulation. Sutherland's law is used for the viscosity (μ), and the polynomial equation presented in Equation (19) is used for thermal conductivity (κ).

$$\kappa = 1.5207 \cdot 10^{-11} \times T^3 - 4.8574 \cdot 10^{-8} \times T^2 + 1.0184 \cdot 10^{-4} \times T - 3.9333 \cdot 10^{-4}.$$
(19)

3. Results and Discussions

3.1. Validation Case 1: Double Ramp. The experiment on the hypersonic flow over a double ramp [51], which was conducted in the TH2 shock wave tunnel of the RWTH Aachen University, is selected as the first benchmark problem for the validation of the numerical methods used in this study. The case of a double ramp with a sharp leading edge is considered. The cross-section geometry and flow conditions are presented in Figure 3 and Table 1. Three cases of wall temperature (T_w) conditions 300, 600, and 760 K are considered. The values of the Tu_∞ and viscosity ratio (μ_t/μ) are set to 0.9% and 0.01, respectively, which are identical to those used in the analysis performed by Krause [43].

The computational domain, grid system, and boundary conditions are shown in Figure 4. The end of the first ramp is set as the origin of the x-axis, and the leading edge is set as the origin of the y-axis. A grid convergence test is carried out for the case of $T_w = 300 \text{ K}$ using the γ -Re_{θ t} model. The information regarding the finest, fine, medium, and coarse grids used in the grid test is listed in Table 2. Based on the results for each grid, a comparison of the pressure coefficient $(C_{\rm p})$ and Stanton number (St) distributions along the surface is shown in Figure 5. The differences in the results with respect to the grid resolution are barely evident in the $C_{\rm p}$ distribution. However, the coarse grid exhibits a noticeable difference in the St distribution around the maximum value $(x \approx 0.03 \text{ m})$ over the second ramp. The medium grid exhibits slight differences in the results compared to the fine and finest grids at the downstream region of the second ramp. A fine grid level that does not exhibit grid dependency for both the C_p and St distributions is finally selected.

The analysis was performed using the laminar, $k-\omega$ SST, γ -Re_{θ t}, Krause, and blended models. In the case where $T_w =$ 300 K, a comparison of the C_p and St distributions obtained using the models is shown in Figure 6. Regarding the results of the laminar simulation, laminar separation occurs near the ramp corner, and compared to the experimental data, the separation point over the first ramp is found to be upstream. This indicates that the size of the separation bubble is overestimated. Additionally, lower values of the St are predicted in the separated flow region as a recirculation region is formed. Compared to the experimental data and the results of other models, lower values of St are also observed over most of



FIGURE 5: (a) C_p and (b) St distributions along the surface for several grid resolutions.



FIGURE 6: Comparison of the (a) $C_{\rm p}$ and (b) St distributions for the $T_{\rm w}$ = 300 K case.

the second ramp after the reattachment. Unlike the results in other analyses, the results of the k- ω SST model show that a high St value corresponding to the degree of the turbulence boundary layer appears even in the first ramp. This indicates that the turbulence boundary layer is being simulated. No separation bubble occurs at the ramp corner, and the maximum value of the St on the second ramp reaches a level that is similar to that of the experimental data. However, it is observed that the location where the maximum value is attained lies upstream of that observed in the experimental results. In the case of the analysis results of the three transition models (γ -Re_{θ t}, Krause, and blended models), the boundary layer on the first ramp maintains a laminar state, and the separation occurs at a point that is similar to that in the experimental data. Accordingly, the size of the separation bubble appears to be similarly predicted. The contour of the turbulent kinetic energy (TKE) in the region just downstream of the ramp corner is shown in Figure 7(a). The occurrence of the transition after the ramp corner and the increase in the TKE in the transitional region can be identified. The results from the γ -Re_{θ t} model and the blended model exhibit the fastest and slowest increase in the TKE, respectively. A comparison of the boundary layer profiles at the point where x = 0.04 m is shown in Figure 7(b). The results of the laminar analysis correspond to the thinnest velocity boundary layer and thermal boundary layer



FIGURE 7: (a) Contour of TKE and (b) boundary layer profiles.



FIGURE 8: Comparison of St distributions when (a) $T_w = 600$ K and (b) $T_w = 760$ K.

thicknesses and the lowest temperature gradient at the wall (heat transfer). Contrarily, the results of the k- ω SST model exhibit the thickest boundary layer thickness, which corresponds to the typical feature of the fully turbulent boundary layer. The velocity and temperature profiles and the boundary layer thickness exhibit only slight differences among the results from the three transition models. This is attributed to the value of v_t being determined differently because of the differences in the TKEs predicted by the transition model. It is noteworthy that the transition models yield results that are similar to those obtained using the k- ω SST model in terms of the near-wall behavior of velocity and temperature. Meanwhile, the boundary

layer thickness remains similar to that observed in the case of laminar analysis. This indicates that the results from the transition models possess intermediate characteristics of the laminar and fully turbulent states at this location. Based on the models, the gradient, maximum value, and location of the St along the second ramp are predicted differently. The results of the γ -Re_{θ t} and Krause models show similar levels for the maximum values of the St, and these values appear to be higher compared to those of experimental data, as shown in Figure 6(b). The blended model predicts the maximum value of the St as similar to that of the *k*- ω SST model and the experimental data. The tendency of change in the value of St in the second ramp

	3	00 K	6	00 K	760 K		
	Separation point (m)	Reattachment point (m)	Separation point (m)	Reattachment point (m)	Separation point (m)	Reattachment point (m)	
γ -Re _{θt}	-0.02472	0.01343	-0.02971	0.01709	-0.03249	0.01868	
Krause	-0.02713	0.01484	-0.03394	0.01868	-0.037	0.02034	
Blended	-0.0284	0.01632	-0.03545	0.0195	-0.0386	0.02207	

TABLE 3: Separation onset points and reattachment points of transition models.



FIGURE 9: Configuration and cross-sectional geometry of the SWL scramjet intake.

and the maximum location also agrees relatively well with the experimental data.

The results for the cases of $T_w = 600$ and 760 K are depicted in Figure 8. As the measurement data for the $C_{\rm p}$ distribution are not provided in the works of Neuenhahn and Olivier [51], only the results for the St are compared. According to the linear stability theory (LST), the boundary layer transition tends to delay as the wall temperature increases at hypersonic Mach numbers [59]. Therefore, similar to the 300 K case, the laminar state of the boundary layer over the first ramp and the formation of laminar separation bubbles in the vicinity of the ramp corner can also be expected for the 600 and 760 K cases. The results of the three transition models indicate that the laminar boundary layer is along the first ramp, and the separation point and size of the separation bubble are similar to those of the experiment. From the results shown in Figures 6–8, for all the wall temperatures (T_w) considered in this study, the three transition models seem to accurately predict essential features, such as the laminar state of the boundary layer along the first ramp, formation and size of the laminar separation bubble at the ramp corner, and transition of the boundary layer on the second ramp. Regarding the streamwise extent of the separation bubble, the γ -Re_{θ t} model yields the smallest bubble among the three transition models, whereas the blended model yields the largest bubble. The sizes of the separation bubbles increase with the increase in wall temperature, as shown in Table 3.

In the case of higher-wall temperature conditions $(T_w = 600 \text{ and } 760 \text{ K})$, the experimental data are available only for specific streamwise extents near the ramp corner (see Figure 8). The available data and the results of all three transition models show that the tendency of increase in the

TABLE 4: Flow conditions for the SWL scramjet intake.

T ₀ (K)	<i>p</i> ₀ (Pa)	T_{∞} (K)	$T_{\rm w}$ (K)	M_{∞}	$\operatorname{Re}_{x,\infty}(1/m)$
1520	750	125	300, 600, 800	7.7	4.1×10^6

value of St in the region immediately after the reattachment to the second ramp is similar to that of the experimental data. However, because of the unavailability of experimental data, including the maximum value and the location of St, there are limitations in the complete comparison and analysis of the prediction capability of the three transition models throughout the transitional region. Consequently, by comparing the results shown in Figures 6 and 8, it is difficult to verify whether the transition models can accurately predict the dependence of the transition on the wall temperature in a similar tendency to that of actual flow.

Through the analysis of the double ramp case, the k- ω SST model predicts the boundary layer over most of the first ramp as a turbulent state. As a result, no separation bubble is formed at the ramp corner. On the other hand, the three models that consider the transition predict the boundary layer as a laminar state over the first ramp, and a separation bubble is formed in the vicinity of the ramp corner. There are differences in the separation and reattachment points on the first and second ramps, respectively, depending on the transition model. After the reattachment, the increasing tendency and the maximum value of St in the transitional region also exhibit some differences depending on the models. Such discrepancies are attributed to the differences in the shear layer profiles of the region near and downstream of the separation bubble because of the differences in the prediction of the transition onset location and the extent



FIGURE 10: Computational domain, grid system, and boundary conditions.

TABLE 5. One information of the grid dependency test for the 500 betainjet induce case.								
Grid type	Number of cells	Number of grids along the streamwise direction	Number of grids along the transverse direction	Max <i>y</i> +	Average <i>y</i> +			
Finest	178,000	891 (41 + 171 + 201 + 481)	201	2.51	0.326			
Fine	120,000	751 (41 + 141 + 181 + 391)	161	5.62	0.82			
Medium	100,800	631 (41 + 131 + 161 + 301)	161	5.63	0.858			
Coarse	70,000	501(41 + 111 + 141 + 211)	141	8.4	2.27			

TABLE 5: Grid information of the grid dependency test for the SWL scramjet intake case

of the transitional region. Among the three transition models, the blended model seems to provide better prediction performance than the other models for the wallbounded shear flows at a hypersonic Mach number, including the separated flow region.

3.2. Validation Case 2: SWL Scramjet Intake. The experiment on the SWL scramjet intake was carried out in the TH2 wind tunnel at the RWTH Aachen University as part of the GRK 1095/1 project [1] along with the double ramp experiment presented in Section 3.1 [52]. The configuration and crosssection geometry are depicted in Figure 9, and the flow conditions are summarized in Table 4. The computational domain, the grid system, and the types of boundary conditions are shown in Figure 10. The analyses are carried out using the freestream conditions of $Tu_{co} = 0.9\%$ and $\mu_t/\mu =$ 0.01, following the study conducted by Krause [43]. The leading edge of the model is set as the origin of the coordinate system.

Four grids with different grid resolutions, i.e., finest, fine, medium, and coarse, are generated for the grid convergence test. The information about different grids is listed in Table 5. The grid convergence test is carried out using the γ -Re_{θ t} model for the $T_w = 300$ K case, and a comparison between the C_p distributions obtained along the first, second, and isolator ramps is depicted in Figure 11. In the isolator ramp region, the differences in the results, according to the grid resolution, are identifiable near the location of the maximum C_p . The difference between the results of the finest and fine grids is approximately 1.9% at the point where x = 0.453 m. Hereafter, the fine grid is chosen for the analyses in consideration of the number of cells and the grid dependency of the result.

The analyses were performed using the k- ω SST, γ -Re_{θ t}, Krause, and blended models. Figure 12 compares the results obtained for three wall temperature conditions. The geometry of the inlet part (first and second ramps) of the SWL



FIGURE 11: Results of the grid convergence test for the SWL scramjet intake.

intake engine is similar to that of the double ramp case presented in Section 3.2. Based on the turbulence/transition models and T_w , it can be observed that the location and size of the separation bubble exhibit characteristics and tendencies that are similar to those observed in the double ramp case. Among the transition models, the blended model and the γ -Re_{θ t} model predict the separation bubbles of the largest size and smallest size at the ramp corner, respectively. The results from the k- ω SST model still show no separation bubble near the ramp corner.

Fischer and Oliver [52] provide the pressure measurement data only along the inner walls of the isolator channel (isolator ramp and lip). Therefore, the results along the isolator ramp and lip walls are compared with the experimental data, as shown in Figures 13 and 14. The corresponding



FIGURE 12: Comparison of C_p distribution along the first, second, and isolator ramps.

contour of the C_p in the isolator channel is illustrated in Figure 15. From the comparison of the pressure along the isolator ramp (Figure 13), the noticeable difference in the results for the k- ω SST model and the other three transition models $(\gamma-\text{Re}_{\theta t}, \text{Krause, and blended models})$ can be identified. The difference appears to increase as the wall temperature increases. For the case where $T_w = 300$ K, all models seem to provide reasonable agreement with the experimental data in terms of the surface pressure. Along the isolator ramp, the pressure increases rapidly after passing the reattachment shock ($x \approx 0.42$ m), after which it decreases from approximately $x \approx 0.45$ m as the flow expands. The pressure increases again after passing the location of $x \approx 0.52 - 0.53$ m, where the shock wave that reflected from the lip side impinges (see Figures 13(a) and 15(a)). Near $x \approx 0.45$ m, where the pressure attains its maximum value, the results obtained using the k- ω SST model exhibit a different pressure variation compared to the other three models. The analysis of the results reveals that this is because the angle of the expansion wave formed at the corner of the isolator ramp (x = 0.3872 m) differs depending on the model used. This expansion wave impinges on the lip side, and it is subsequently reflected toward the isolator ramp. The oblique shock from the leading edge of the lip interacts differently at the isolator ramp, thereby creating different shock structures. The positions at which the shock wave reflects from the lip side and impinges on the isolator ramp side appear to be different in the results obtained using the k- ω SST and other transition models.

As shown in Figure 13(b), in the case where $T_w = 600$ K, it is evident that the three transition models, except the *k*- ω SST model, exhibit prediction results that are similar to those in the experimental data. Regarding the results of the *k*- ω SST model, as separation occurs near the end of the second ramp



FIGURE 13: Comparison of C_p distribution along the isolator ramp surface.

 $(x \approx 0.386 \text{ m})$, a separation-induced shock occurs instead of an expansion wave at the end of the second ramp (see Figure 15(b)). Therefore, as shown in Figure 15(b), the separation-induced shock results in the high-pressure region at the leading edge of the isolator ramp. Moreover, in the numerical study performed by Fischer and Oliver [52], the separation point demonstrated a tendency to move upstream with the increase in the wall temperature. The separationinduced shock wave, which is reflected from the lip side, impinges on the isolator ramp side, and it interacts with the oblique shock generated at the leading edge of the lip. This interaction makes the maximum pressure appear further upstream on the isolator ramp side compared to the other three models, resulting in a considerable difference from the experimental results. Similar to the case where $T_w =$ 300 K, the predicted location ($x \approx 0.51-0.53$ m) of the second impingement of the shock wave that is reflected from the lip side is also different for the k- ω SST model compared to the other three models. The results of the three transition models are slightly different.

In the case where $T_w = 800$ K, it is evident that the results exhibit higher pressure along the ramp side in the vicinity of the isolator channel's entrance compared to the experimental data ($x \approx 0.39-0.41$ m) (see Figure 13(c)). This is because all the models predict the occurrence of separation near the end of the second ramp. The pressure increases after the separation-induced shock wave, similar to the results of the $k-\omega$ SST model in the case where $T_w = 600$ K. Therefore, it can be conjectured that the simulated flow structure around the entrance of the isolator channel is considerably different from the actual flow in the experiment. The separationinduced shock is predicted in the analysis. Similar to the



FIGURE 14: Comparison of C_p distribution along the lip side surface.

two wall temperature cases mentioned above and compared to the k- ω SST model, it can be observed that the three transition models predict the pressure distribution similar to the experimental data. The $T_w = 800$ K case shows the lowest agreement with the experiment throughout the isolator ramp's interior area because of the difference in the prediction accuracy near the entrance.

In the case where $T_w = 300$ K, on the lip side (Figure 14(a)) and in the region where C_p decreases $(x \approx 0.405-0.465 \text{ m})$, the results of the $k-\omega$ SST model exhibit local increasing and decreasing behavior in the relatively upstream region $(x \approx 0.415 \text{ m})$ compared to the other three models. As mentioned earlier, this is attributed to the difference in the angle of the expansion wave generated from the isolator ramp side, as can be identified from the C_p contour shown in Figure 15(a). Downstream of the lip side, it can

be observed that the results of the three transition models indicate that the location where the pressure starts to increase $(x \approx 0.48 \text{ m})$ agrees well with the experimental data. This increase in pressure is a consequence of the impingement of the reattachment shock generated from the isolator ramp side. The SWBLI phenomenon is shown in Figure 14 around the location of the shock impingement ($x \approx 0.47-0.48 \text{ m}$). The region of the SWBLI and the resulting separation bubble can also be observed clearly in the contours of the Mach number shown in Figure 16. In the figure, this region is marked using a dashed circular line. For the k- ω SST model, it can be observed that the size and streamwise extent of the separation bubble resulting from the SWBLI are predicted differently compared to those of the other three models.

Similar to the case where $T_w = 300$ K, in the case where $T_w = 600$ K (Figures 14(b) and 15(b)), the three transition



FIGURE 15: $C_{\rm p}$ contours in the isolator channel obtained using different turbulence models.

models provide results that are similar to the experimental results downstream of the isolator. However, in the results of the $k-\omega$ SST model, the impingement of the separation-induced shock occurs at the end of the second ramp on the lip, which can be identified in Figures 14(b) and 15(b) ($x \approx 0.41-0.42$ m). The impingement results in a steep increase in pressure, starting from $x \approx 0.405$ m. The pressure decreases as the flow expands to the downstream region ($x \approx 0.42-0.455$ m). It increases again after passing the impinging shock wave, which is the reattachment shock that occurs upstream of the isolator ramp side. In the $k-\omega$ SST model, the location of the shock impingement is predicted to be further upstream compared to that of the three transition models and the experimental data.

In the case where $T_w = 800 \text{ K}$ (Figures 14(c) and 15(c)), all the models predict the separation-induced shock wave at the end of the second ramp. Therefore, a rapid increase in pressure appears from approximately $x \approx 0.405 \text{ m}$, where the separation-induced shock wave impinges on the lip side. Compared to experimental data, the analysis results reveal the shift of the shock impingement location upstream on the lip side. Specifically, the results of the k- ω SST model exhibit a significant difference.

The analysis of the SWL intake reveals that the characteristics and tendencies obtained for the inlet part (first and second ramps) could be identified as similar to those pertaining to the double ramp case discussed in the previous section. The results obtained using all models reveal that as the wall temperature increases, separation tends to occur near the end of the second ramp (expansion corner), which is upstream of the isolator ramp. Most of the differences in the results downstream of the expansion corner (isolator channel) are attributed to the flow structure formed through the expansion and shock waves, which are generated at the entrance region of the isolator and during the interaction of shock waves/reflected shock waves. Unlike the three transition models, the k- ω SST model predicts the separation even at low temperatures, and it exhibits the most significant differences from the experimental data. The differences in the results of the three transition models are not considered because these models provide results that are significantly close to those in the experimental data.

The above-described analyses reveal that the consideration of the laminar-turbulent transition is essential for the analysis of scramjet intake flows at hypersonic Mach numbers to obtain reasonably accurate results. Moreover, the observed dependence on the wall temperature and separation occurrence significantly affects the flow structure. Therefore, the models must demonstrate the appropriate predictive capability for practical application with sufficient reliability.

3.3. Application Example: SCP-01 Scramjet Forebody. In this section, based on the turbulence/transition models and from an engineering application perspective, the difference and uncertainty of the results are evaluated in terms of flow fields and air intake performances. To achieve this objective, the forebody of the scramjet combustion propulsion-01 (SCP-01) flight vehicle, which adopts a double ramp-type inlet that is similar to that in the cases discussed in the previous sec-



FIGURE 16: Comparison of Mach number contours in the entrance region of the isolator channel ($T_w = 300$ K).

tions, is considered a geometry for investigation. The SCP-01 is a concept scramjet flight vehicle [53] designed by the Agency for Defense Development (ADD) for basic research purposes. Its cross-sectional geometry is depicted in Figure 17.

The angles of the first, second, and isolator ramps are $\theta_1 = 3^\circ$, $\theta_2 = 12^\circ$, and $\theta_3 = 6^\circ$, respectively. The corresponding horizontal lengths are $l_1 = 601.94$ mm, $l_2 = 694.28$ mm, and $l_3 = 168.41$ mm. The length of the main section of the isolator (l_4) is 500.0 mm. The angle (θ_4) and horizontal length (l_5) of the expansion part at the end of the isolator are -2.5 and 150.0 mm, respectively. The height (h_1) of the flow channel formed by l_4 and l_7 , which are parallel to each other, is 40.0 mm, and the thickness (h_2) of the lip is 20.0 mm. The lengths of the outer and inner lips are $l_6 = 687.58$ mm and $l_7 = 801.0$ mm, respectively.

Around the design operating conditions of the SCP-01 were considered the flow conditions of interest, and the analyses were performed in the 20-30 km altitude range at intervals of 2.5 km. The freestream Mach numbers (M_{∞}) in the 5-6.5 range were considered at intervals of 0.5. The angle of attack was set to 0°. The atmospheric conditions [60] and unit Reynolds numbers according to the altitude and Mach number are listed in Table 6. The computational domain, grid system, and boundary conditions used in this study are shown in Figure 18. The wall boundaries were set to no-slip and adiabatic conditions. The total number of face cells was 139,800. Two values of Tu_{∞} were selected to consider the flow conditions in the ground and flight tests. In the ground test condition, $Tu_{\infty} = 0.9\%$ similar to previous sections, whereas $Tu_{co} = 0.1\%$ for the flight test condition in consideration of the atmospheric conditions at high altitudes. For both freestream turbulence intensity cases, μ_t/μ was set to 0.01. Because the correlations involved in the transition



FIGURE 17: Forebody geometry of the SCP-01 flight vehicle.

Altitude (km)	p_{∞} (Pa)	T_{∞} (K)	M_{∞}	Unit Re (1/m)	M_{∞}	Unit Re (1/m)
20	5520.05	21.6.62	5	9.22×10^{6}	6	1.1×10^{7}
20	5529.85	216.69	5.5	1.01×10^7	6.5	1.19×10^7
22.5	2524 (5	21 ((2)	5	6.23×10^{6}	6	7.48×10^6
22.5	3734.67	216.69	5.5	6.85×10^6	6.5	$8.1 imes 10^6$
25	0500.05	21 ((2)	5	4.2×10^6	6	5.05×10^6
25	2522.27	216.69	5.5	4.63×10^{6}	6.5	5.47×10^6
25.5	1602.02	224.10	5	$2.68 imes 10^6$	6	3.22×10^6
27.5	1682.92	224.19	5.5	2.95×10^6	6.5	3.49×10^6
20	1150.00	221 (1	5	1.76×10^{6}	6	2.12×10^6
30	1158.33	231.61	5.5	1.94×10^6	6.5	2.29×10^6

TABLE 6: Flow conditions according to altitudes and Mach numbers.



FIGURE 18: Computational domain, grid system, and boundary conditions for the forebody of the SCP-01 flight vehicle.

prediction using the Krause and blended models are defined as functions of Tu_{∞} , an attempt is made to identify the differences in the results based on the two different values of Tu_{∞} . Similar to the previous section, the analyses were performed using the *k*- ω SST and three transition models.

Apart from the analysis of RANS, an LST-based [61] analysis of the laminar boundary layer on the first ramp is performed to predict the possibility of the occurrence of the transition and transition location based on the e^N method [62, 63], which is a semiempirical method. To simplify the problem, a flow over a single wedge with the same angle as that of the first ramp is considered. The Mach number and the properties of the inviscid wedge flow after passing the single oblique shock wave are calculated using the oblique shock relation. The laminar mean flow data required for the stability analysis are obtained by calculating the numerical solution of the compressible boundary layer equation with the 4th-order accuracy using the conditions of inviscid flow along the wedge surface as the boundary layer edge conditions. The stability of the resulting mean flow data is analyzed using the LST code developed by Park and Park [64]. The amplification rate (growth rate) of the instability wave is obtained for various frequencies and various locations along the streamwise direction of each frequency. The N factor, which indicates



FIGURE 19: N factor curves over the first ramp for Mach numbers (a) 5 and (b) 6.5 at an altitude of 20 km.

the degree of overall amplification of the disturbance amplitude, is obtained by integrating the downstream growth rate from the neutral point of the lower branch. The *N* factor curves for various frequencies at the altitude of 20 km and at $M_{\infty} = 5$ and 6.5 are illustrated in Figure 19 as examples. The analysis of the Mach number, mode shape, and range of unstable frequencies indicates that the unstable mode found corresponds to Mack's 2nd mode [65].

The results depicted in Figures 19(a) and 19(b) reveal that the *N* factor reaches the value of 2.1 and 3.8, respectively, at $x \approx 0.6$ m, where the first ramp ends for the SCP-01 configuration. For the e^N method, $N \approx 5.5$ for Mack's 2^{nd} mode shows a good correlation with the transition onset locations of hypersonic boundary layers, measured from the wind tunnel experiments using the ground facilities [66-68]. In addition, it is well known that the transition onset in the ground test conditions using a quiet tunnel or in the flight test conditions corresponds to a higher N value than those obtained in conventional noisy tunnels, owing to the good flow quality of freestream vorticity, entropy, and low acoustic pressure levels [69]. If the transition caused by Mack's 2nd mode under flight conditions is assumed to be at the level of N = 7-8 or higher, it can be expected that the transition will not occur on the first ramp during the flight at an altitude of 20 km. Additionally, keeping in mind that the transition onset location moves downstream as the unit Reynolds number decreases, it can be predicted that the boundary layer transition will not occur on the first ramp at altitudes higher than 20 km based on the stability theory and the e^N method. From further analysis, the possibility of occurrence for the transition on the first ramp is identified for altitudes that are lower than 15 km because of the high unit Reynolds number.

Returning to the RANS simulation, analyses of all the flow conditions listed in Table 6 were performed using two freestream turbulence intensities (Tu_{∞}) and four models. Among the air intake performance parameters, the mass flow rate (per unit span) at the isolator exit, i.e., the entrance of the combustion chamber, is considered to quantify the differences in the results according to Tu_{∞} and the models. The



FIGURE 20: Variance of mass flow rate results with respect to various models and Tu_{∞} .

variance of the mass flow rate is calculated for each flow condition and plotted, as shown in Figure 20. A smaller value of the variance means that the differences in the resultant flow field and air intake performance, depending on the models or freestream turbulence intensities, are insignificant under the corresponding flow conditions. From the variance results shown in Figure 20, it can be observed that the resultant air intake performance is not influenced significantly by the turbulence/transition models or the applied turbulence intensity at altitudes of 20 and 22.5 km with Mach numbers of 6 and 6.5 at an altitude of 25 km. On the other hand, in the case of the altitudes of 27.5 and 30 km and the Mach numbers of 5 and 5.5 at an altitude of 25 km, considerable differences in air intake performance are identifiable.

The resulting mass flow rates at an altitude of 20 km, where the variance is significantly small for all Mach

Model			k-ω	k - ω SST		γ -Re _{θt}		Krause		Blended	
Tu_{∞}			0.1	0.9	0.1	0.9	0.1	0.9	0.1	0.9	
		5	25.02	25.01	25.27	25.27	26.13	25.28	25.79	25.47	
20 1	М	5.5	29.81	29.81	30.05	30.05	31.11	30.08	30.68	30.3	
20 KM	IVI _{co}	6	35.13	35.13	35.37	35.26	36.64	35.41	36.1	35.67	
		6.5	40.37	40.37	40.37	40.37	40.37	40.37	40.37	40.37	
25 km		5	11.32	11.32	11.56	11.56	11.89	11.56	6.86	11.6	
	М	5.5	13.5	13.5	13.75	13.76	14.17	13.75	8.23	13.8	
	IVI _{co}	6	15.9	15.9	16.19	16.19	16.68	16.19	16.68	16.25	
		6.5	18.41	18.41	18.41	18.41	18.41	18.41	18.41	18.41	

TABLE 7: Mass flow rates for altitudes of 20 km and 25 km.

*Unit: kg/m•s.



FIGURE 21: Mach number contours at 25 km and M_{∞} = 5.5.

numbers, and an altitude of 25 km, where the variance varies significantly according to the Mach number, are summarized in Table 7. In the case of the altitude of 25 km, the blended model at $M_{\infty} = 5$ and 5.5 with Tu_{∞} = 0.1% exhibits lower mass flow rates compared to the other models, resulting in high variance. For example, the contours of the Mach numbers are compared in Figure 21 based on the model and the value of Tu_{∞} for the flow condition of $M_{\infty} = 5.5$ at an altitude of 25 km, which exhibits the largest variance. As discussed in Sections 3.2 and 3.3, because the *k*- ω SST model yields the turbulent boundary layer on the first ramp, no separation bubble is formed at the ramp corner. In the other three models considering a transition, a laminar separation bubble is formed at the ramp corner because the laminar boundary layer is maintained over the first ramp.

The state of the boundary layer and the streamwise extent of the separated flow region over the first ramp are shown in Figure 22 in which the skin friction coefficients (C_f) along the first ramp are compared. In Figure 22, the lines with symbols indicate the results of the analysis when $Tu_{\infty} = 0.1\%$, and the lines without symbols indicate the results of the analysis when $Tu_{\infty} = 0.9\%$. For the γ -Re_{θ t} model, the size of the separated flow region does not depend on $\mathrm{Tu}_\infty.$ However, the Krause and blended models exhibit different predictions in the size of the separated flow region based on Tu_{∞} (Figures 21 and 22). For the case where $Tu_{\infty} = 0.9\%$, the three transition models provide separation bubbles of similar sizes. However, when $Tu_{\infty} = 0.1\%$, the blended model predicts the largest size of the separation bubble. Furthermore, for the results of the blended model where Tu_{∞} = 0.1%, the separated flow that occurs over the ramp corner does not reattach on the second ramp surface, and thus, a significantly long separation region is formed (see Figure 21(a)). Therefore, a significant portion of the inlet flow field becomes the



FIGURE 22: Skin friction coefficient along the first ramp.



FIGURE 23: Mach number contours at 27.5 km and $M_{\infty} = 5$.

separated flow region, resulting in considerable deterioration of the air intake performance. The same feature and characteristic are also observed in the results at M_{∞} = 5.

Figure 23 shows the results for Mach number 5 at an altitude of 27.5 km. When $Tu_{\infty} = 0.1\%$, the separation occurs at the first ramp, and the flow does not reattach on the second ramp for both the Krause and blended models. At an altitude of 27.5 km, the results with Mach numbers other than 5 exhibit the same features as observed when $M_{\infty} = 5$ and 5.5 at an altitude of 25 km. Consequently, the air intake performance is significantly degraded because of the massive sepa-

ration, seen in the results of the blended model. Figure 24 shows the Mach number contour for $M_{\infty} = 6$ at an altitude of 30 km. In the results of the Krause and blended models where Tu_{∞} = 0.1%, the air intake performance is significantly degraded as a result of the long separated flow region formed throughout the inlet region without reattachment at the second ramp. Similar characteristics are observed for other Mach numbers. In the case where $M_{\infty} = 6$ at an altitude of 30 km, the contours of intermittency from the results of the Krause and blended models are illustrated in Figure 25. The examination of intermittency confirms that the long



FIGURE 24: Mach number contours at 30 km and $M_{\infty} = 6$.



FIGURE 25: Contours of intermittency at 30 km, M_{∞} = 6, and Tu_{∞} = 0.1%.

separation region is formed because the flow does not reach the fully turbulent state until it reaches the entrance of the isolator channel when using both models.

The results obtained for the $Tu_{\infty} = 0.1\%$ case reveal that the air intake performance degrades owing to the occurrence of massive separations at higher altitudes, as observed using the Krause and blended models. These two models use Equations (13), (14), (17), and (18), which are correlations involving the prediction of transition, and they are functions of Tu_{∞} . The equations are plotted in Figure 26 (Equations (13) and (18) are the same). The analysis indicates that because Equation (13) is modeled to increase rapidly (see Figure 26(a)) and Equations (14) and (17) are modeled to have very small values (see Figure 26(b)) at low values of Tu_{∞} , the transition can be predicted to be delayed excessively so that the fully turbulent state is not reached even at the isolator channel. Upon further analysis, the transition is excessively delayed when $Tu_{\infty} = 0.1\%$ because very large values of the $Re_{\theta c}$ are achieved and F_{length} has very small or even slightly negative values at significantly low values of Tu_{∞} . Therefore, it seems that when similar analyses are performed for conditions involving significantly low values of Tu_{∞} , the results should be carefully interpreted considering the model reliability and a valid range of parameters or data on which the correlations are modeled. The excessive extrapolation of determining parameters, such as $Re_{\theta c}$ and F_{length} , can result in unrealistic predictions.

Based on the e^N method combined with the LST analysis, it is predicted that no boundary layer transition occurs at the first ramp of the forebody of the SCP-01 flight vehicle under



FIGURE 26: Modified correlations for (a) $\text{Re}_{\theta c}$ and (b) F_{length}

the flow conditions considered in this study and at altitudes higher than 20 km. Therefore, it can be assumed that for the $k-\omega$ SST model that yields a fully turbulent flow, the analysis of the high-altitude conditions can yield a larger difference in the actual flow and the resulting air intake performance. For the analysis of hypersonic intake flow under high-altitude conditions, the use of a model that can predict the transition is essential, and a careful analysis of the results, considering the reliability of the model, is necessary. Under all analysis conditions, the results of the γ -Re_{θ t} model exhibit the laminar boundary layer over the first ramp, and the transition occurs at the second ramp. As mentioned previously, the validity of the model is guaranteed only for twodimensional subsonic shear flows, and the limitations of the model in the hypersonic regime were identified [34]. Therefore, although an improvement of the results compared to the k- ω SST model can be expected, the accuracy of the analysis results cannot be easily assured. This is because this model is still not free from the issue of its reliability on the prediction of the hypersonic flow transition. The Krause and blended models considering the transition of the hypersonic boundary layer yield the results in which the laminar boundary layer is maintained over the first ramp for all analysis conditions. In the cases where $Tu_{\infty} = 0.9\%$, the differences in the results of the two models are not significantly large according to the flow conditions (altitude and Mach number). There is no noticeable difference in the flow field structure and air intake performance for the two models. However, in the cases where $Tu_{\infty} = 0.1\%$, the occurrence of the long region of separated flows is determined by the flow conditions and the models, and the separation results in considerable deterioration of the air intake performance. As discussed earlier, the flow field results and air intake performance can possess significant uncertainties based on the model and related correlations with parameters, such as Tu_{∞} . Therefore, further studies on the improvement of models and their assessment and validation are required to enhance the reliability of the model, especially for low-value Tu_∞ conditions.

4. Conclusions

This study assesses and compares the prediction accuracy of several turbulence/transition models by performing RANS simulation for hypersonic flows over the double ramp and within two scramjet intakes installed with double ramp-type inlets. The results obtained using one turbulence model (k- ω SST) and three transition models (γ -Re_{θ t}, Krause, and blended) are investigated.

In the double ramp case, the k- ω SST model could not capture the separation bubble formed in the vicinity of the ramp corner, whereas the transition models appropriately captured the formation and size of the separation bubble, which agrees well with the experimental data. In the SWL intake case, the transition models provided results in which the surface pressure distributions were in good agreement with the experimental data for low-wall temperature conditions. In terms of predictive performance, the transition models proved to be superior to the k- ω SST model for all the wall temperatures analyzed. Although the transition models still exhibited better prediction capability than the k- ω SST model, the accuracy of the results deteriorated as the wall temperature increased, even for the transition models. The separation that occurred near the second ramp owing to the change in the shock structures throughout the entrance region and the isolator channel was identified as the main cause of the discrepancies. The observations made from two benchmark cases indicated that the transition of the shear layer must be considered for RANS analysis of the hypersonic intake flow to achieve reliable results. Additionally, the accurate prediction of the separation point and the reflection of the dependence of transition and separation on the wall temperature is an important factor that determines the nature of all the results.

In the results for the forebody of the SCP-01 flight vehicle, a boundary layer transition over the first ramp was not expected according to the LST and the e^N method under the selected flow conditions. Intake flow analyses were performed for various altitude and Mach number conditions using the developed models. Under the moderate freestream turbulent intensity condition (Tu $_{\infty}$ = 0.9%), which represents the ground test conditions, there was no significant difference in the results obtained from the model in terms of the overall flow structure and air intake performance. In the case of the Krause and blended models, wherein $Tu_{\infty} = 0.1\%$ represents the flight test condition, the air intake performance deteriorated under certain flow conditions because of the occurrence of a long streamwise region of separation over the second ramp. This characteristic is dominant over a wide range of Mach numbers as the altitude (unit Reynolds number) increases or decreases. Consequently, the uncertainty of the results increases considerably based on the model, especially at higher altitudes. The issues of uncertainty and validity resulting from the parameters that alter the transition prediction by affecting the correlations are recognized and discussed.

In all the cases studied, it was predicted that the transition does not occur at the first ramp for the Krause and blended models. It was not determined whether the models can accurately predict the transition of the other flow conditions, when the transition occurs on the first ramp in actual flow, or the other geometries. As a result, further comprehensive validations and assessments of such situations are required. An appropriate model must be carefully selected for analyzing hypersonic flows. The model should consider factors, such as accuracy, uncertainty, and validity, in terms of its prediction capability of shear layer transition and separation.

Data Availability

The data that support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare there is no conflict of interest regarding the publication of this paper.

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