Fuzzy Adaptive Prescribed Performance Fault-Tolerant Control for HFVs with Fixed-Time Convergence Guarantee

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Received 23 March 2022; Revised 27 August 2022; Accepted 6 September 2022; Published 20 September 2022

A new fixed-time fuzzy adaptive fault-tolerant control methodology is proposed for the longitudinal dynamics of hypersonic flight vehicles (HFVs) in the presence of actuator faults, uncertain dynamics, and external disturbances. In contrast with the conventional fixed-time control schemes that typically contain the fractional powers of errors in their designs, this work develops a low-complexity control structure in the sense of removing the dependence on the need of aforementioned fractional power terms by means of prescribed performance control (PPC) method. Different from the most existing PPC approaches where the initial conditions of tracking errors are required to be known, the newly proposed prescribed performance function (PPF) can relax such restrictions through choosing properly small initial values of PPF. Fuzzy logic systems (FLSs) are employed to handle unknown dynamics, and minimal learning parameter (MLP) technique is incorporated into the design for the purpose of alleviating computation burden. Closed-loop stability is rigorously proved via Lyapunov stability theory, and simulation results are eventually given to validate the effectiveness of the proposed control strategy.

1. Introduction

Hypersonic flight vehicles have already attracted considerable attention due to their advantages of high flight speed, remarkable penetration ability, and cost-effectiveness [1–4]. Nevertheless, the controller design for HFVs remains an intractable issue due to their peculiar features. For example, the engine-airframe structure results in strong couplings between propulsive and aerodynamic forces, and there exist intricate flexible deformation due to the slender geometry of vehicle structure, which influences the aerodynamic characteristics prominently [5]. In addition, the fast time-varying flight environment and the unknown external disturbances lead to frequent parameter variations and model uncertainties, dramatically increasing the difficulty of controller design. To address these problems, many effective methods have been presented, including robust control [5–7], neural/fuzzy control [8–10], prescribed performance control [11, 12], and disturbance observer-based control [13]. Although these efforts solve the above-mentioned issues to some extent, these results rarely focus on the rate of convergence.

To be specific, only the exponential convergence of tracking error is guaranteed in the aforementioned work, which reveals the convergence time tends to be infinite. From a practical perspective, the rate of convergence is of great significance to the transient tracking performance [14]. Recently, the finite-time tracking control for HFVs is investigated in [14–17], which can make the tracking error converge into the predefined compact set within a finite time. Nevertheless, the convergence time, which is commonly achieved in [14–17], depends on the initial states of the system. It inevitably brings up a problem, that is, the convergence time cannot be accurately settled when the initial states of the system are unknown. To solve such problem, the fixed-time control [18–21] is proposed skillfully,
by which the tracking error can converge into a predefined impact set within a fixed time and the connection between the convergence time and initial states is eliminated.

However, there still exist some shortcomings with the conventional fixed-time tracking control scheme [18–21], where the inequation $\dot{V}(x) \leq -\tau V^2(x) - cV^4(x) + \kappa$ holds. On the one hand, the derivative of virtual and actual control laws will tend to infinite when the tracking error approaches to zero, giving rise to singularity issues in the control design [21]. On the other hand, when the system encounters the unknown external disturbances [22–26], the dynamic uncertainties [27–32], the system faults [33–36], or input nonlinearities [37], it is complicated to both ensure the fixed-time stability of the system and determine the convergence accuracy by selecting the design parameters. As is known to us, the actuator faults not only affect handling performance for HVFs, but even cause closed-loop instability [33–36]. The fault-tolerant control is an inevitable issue for HVFs, due to complex and variable flight environment that may lead to actuator faults, such as control effectiveness decline and drifting. Furthermore, it is well known that the transient and steady-state performances are important to the controller design for HVFs [38]. Nonetheless, the existing PPC for HVFs commonly fails to explicitly contain a convergence time $T$ in the performance function. Thus, it is urgent to develop a new low-complexity fixed-time fault-tolerant control (FTFTC) strategy for HVFs with the prescribed performance.

Motivated by these observations, we present a fixed-time adaptive fuzzy fault-tolerant control scheme for HVFs by utilizing a new prescribed performance function. The contributions mainly contain the follows:

(1) This paper presents a structurally inexpensive FTFTC framework for HVFs in the sense that no fractional powers are involved in the design. There is no fractional power of tracking error in the controller, and thus the singularity problem caused by the derivative of fraction term is removed

(2) By constructing the intermediate control laws and adaptive laws, the adverse impact of actuator faults of HVFs (e.g., loss of effectiveness and drift) is compensated effectively

The remainder of the work is organized as follows. The HVF dynamics and preliminaries are introduced in Section 2. In Section 3, the FLSs-approximator-based FTFTC is designed and the closed-loop stability is verified in Section 4. Section 5 provides simulations to demonstrate the effectiveness of the proposed methods and the work is concluded in Section 6.

A preprint has previously been published by Zehong Dong et al. [39].

2. Problem Formulation and Preliminaries

2.1. Hypersonic Flight Vehicle Dynamics. The longitudinal control-oriental model is originally developed by Parker et al. [40, 41], which can be formulated as

$$
\dot{V} = \frac{T \cos (\theta - y) - D}{m} - g \sin y, \dot{\theta} = V \sin y,
$$

(1)

$$
\dot{\theta} = \frac{T \sin (\theta - y) + L}{mV}, \dot{y} = Q,
$$

(2)

$$
Q = \frac{M + \psi_1 \dot{\eta}_1 + \psi_2 \dot{\eta}_2}{I_{yy}},
$$

(3)

$$
k_1 \dot{\eta}_1 = -2 \zeta_1 \omega_1 \eta_1 - \omega_1^2 \eta_1 + N_1 - \frac{\psi_1 M}{I_{yy}} - \psi_1 \psi_2 \dot{\eta}_2,
$$

(4)

$$
k_2 \dot{\eta}_2 = -2 \zeta_2 \omega_2 \eta_2 - \omega_2^2 \eta_2 + N_2 - \frac{\psi_2 M}{I_{yy}} - \psi_1 \psi_2 \dot{\eta}_1,
$$

(5)

where $T$, $D$, $L$, $M$, $N_1$, and $N_2$ are expressed as

$$
L = \tilde{q}SC_L(\alpha, \delta, \eta),
$$

$$
M = z_T T + \tilde{q}SC_M(\alpha, \delta, \eta),
$$

$$
T = \tilde{q} \cos \Theta \left[ C_{T,\alpha}(\alpha) \nu(\Phi) + C_T(\alpha) + C_{T,\alpha}^0 \eta \right],
$$

(6)

$$
N_i = \tilde{q} \left[ N_i^0 \alpha^2 + N_i^0 \alpha + N_i^0 \delta + N_i^0 + N_i^0 \eta \right], i = 1, 2,
$$

where $\alpha = \theta - y$, $\tilde{q}$, $S$, $z_T$, and $\tilde{c}$ denote dynamic press, reference area, thrust moment arm, and reference length. $\eta = [\eta_1, \dot{\eta}_1, \eta_2, \dot{\eta}_2]^T$ denotes the flexible modes. $\delta = [\nu(\delta_x), \delta_x]_x^T$, where the deflection of canard $\delta_x$ is set to be ganged with $\nu(\delta_x)$ and $\delta_c = k_{\delta c}/C_{L,0}$. This approach was originally proposed in [41] as a way to remove some non-minimum phase characteristics of the dynamics. Considering the actuator fault, the actual output signals of the fuel equivalence ratio and the elevator angular deflection are denoted as $\nu(\Phi)$ and $\nu(\delta_x)$, respectively. The aerodynamic model is obtained by curve fitting and can be expressed as

$$
C_D(\cdot) = C_D^0 \alpha^2 + C_D^0 \alpha + C_D^0 \nu(\delta_x) + C_D^0 \nu(\delta_x) + C_D^0 \delta_x,
$$

(7)

$$
C_M(\cdot) = C_M^0 \alpha^2 + C_M^0 \alpha + C_M^0 \nu(\delta_x) + C_M^0 \nu(\delta_x) + C_M^0 \delta_x,
$$

(8)

$$
C_L(\cdot) = C_L^0 \alpha + C_L^0 \nu(\delta_x) + C_L^0 \delta_x + C_L^0 \delta_x + C_L^0 \nu(\delta_x),
$$

(9)

$$
C_{T,\alpha}(\cdot) = C_{T,\alpha}^0 + C_{T,\alpha}^0 \alpha^2 + C_{T,\alpha}^0 \alpha + C_{T,\alpha}^0 \nu(\delta_x),
$$

(10)

$$
C_T(\cdot) = C_T^0 \alpha^3 + C_T^0 \alpha^2 + C_T^0 \alpha + C_T^0,
$$

(11)

$$
C_\Theta^0 = [C_{\Theta}^0, 0, C_{\Theta}^0, 0], j = T, M, L, D,
$$

(12)

$$
N_i^0 = [N_i^0, 0, N_i^0, 0], i = 1, 2,
$$

(13)

and more detailed definitions can be found in [40–41].
2.2. The Actuator-Fault Model. The actuator-fault model is developed by the following formula [34]:

\[
\begin{align*}
\dot{v}(\Phi) &= \omega_0 \Phi + \varepsilon_\Phi, \\
\dot{v}(\delta_\epsilon) &= \omega_0 \delta_\epsilon + \varepsilon_\delta_\epsilon,
\end{align*}
\]

(14)

where \(v(\Phi)\) and \(v(\delta_\epsilon)\) denote the actual output signals of the fuel equivalence ratio and the elevator angular deflection, respectively. \(\omega_0\) and \(\varepsilon_\epsilon\) represent the actual control effectiveness and drift distance, respectively.

**Assumption 1.** There exists an unknown positive constant \(\tilde{\varepsilon}_\epsilon\) such that \(|\varepsilon_\epsilon| \leq \tilde{\varepsilon}_\epsilon\), \(0 < \omega_0 \leq 1\).

**Remark 2.** Assumption 1 is commonly applied in FTC research to ensure the controllability of system when the actuator faults occur [34]. During flight, actuator fault inevitably occurs due to multiple factors such as aging and damage of components or screw shedding, which deteriorates the flight performance and even causes the serious flight accident in severe circumstance. Therefore, it is of great significance to consider the possible actuator faults when designing the control strategy. With different values of \(\omega_0\) and \(\varepsilon_\epsilon\), (14) can be divided into the following four cases:

1. \(\omega_0 = 1\) and \(\varepsilon_\epsilon = 0\), representing the fault-free case.
2. \(0 < \omega_0 \leq \omega_0 \leq 1\) and \(\varepsilon_\epsilon = 0\), where \(\omega_0\) and \(\varepsilon_\epsilon\) are unknown positive constants, denoting partial loss of effectiveness.
3. \(\omega_0 = 1\) and \(\varepsilon_\epsilon \neq 0\), indicating the bias fault.
4. \(0 < \omega_0 \leq \omega_0 \leq 1\) and \(\varepsilon_\epsilon \neq 0\), where \(\omega_0\) and \(\varepsilon_\epsilon\) are unknown positive constants, signifying that partial loss of effectiveness and bias fault occur at the same time.

2.3. Model Transformation and Decomposition. According to (1) and (7), \(V\) and \(h\) are mainly regulated by \(\Phi\) and \(\delta_\epsilon\), respectively. To facilitate the controller design, the HFVs dynamics are decomposed into velocity subsystem and altitude subsystem.

Considering the actuator-fault model (14) and inspired by [42, 43], the velocity subsystem is written as

\[
\dot{V} = f_\gamma + g_\gamma v(\Phi) + d_\gamma,
\]

(15)

where \(f_\gamma = -\bar{q}S/m(C_{\gamma\alpha} \beta^2 + C_{\gamma\alpha}^2 \alpha + C_{\gamma\alpha}^2 \alpha^2 + C_{\gamma\alpha}^2 \delta_\epsilon + C_{\gamma\alpha}^2 \delta_\epsilon^2 + C_{\gamma\alpha}^2 \eta) - g \sin \gamma + (\bar{q}S/m) \cos \alpha C_{\gamma\alpha}^2 \alpha^2 + C_{\gamma\alpha}^2 \alpha + C_{\gamma\alpha}^2 \alpha + C_{\gamma\alpha}^2 \eta), \ g_\gamma = (\bar{q}S/m) \cos \alpha(C_{\gamma\alpha}^2 \alpha + C_{\gamma\alpha}^2 \alpha^2 + C_{\gamma\alpha}^2 \alpha + C_{\gamma\alpha}^2 \eta). \)

\(f_\gamma\) and \(g_\gamma\) stand for unknown functions due to the time-varying aerodynamic parameters, and \(d_\gamma\) represents the external disturbance on velocity.

Considering the actuator-fault model (14) and taking the assumption \(\sin(\gamma) = \gamma, \cos(\gamma) = 1\), then the altitude subsystem can be considered as

\[
\begin{align*}
\dot{h} &= V\gamma + d_\gamma, \\
\dot{\gamma} &= f_\gamma + g_\gamma \theta + d_\gamma, \\
\dot{\theta} &= Q, \\
\dot{Q} &= f_Q + g_Q v(\delta_\epsilon) + d_Q.
\end{align*}
\]

(16)

Similar to velocity subsystem, the functions \(f_\gamma, g_\gamma, f_Q, g_Q\) and \(d_\gamma\) are unknown functions; \(d_\gamma\), \(d_\gamma\), and \(d_\gamma\) are the external disturbances of altitude subsystem. Along the standard ideas as [10–11], we assume there exist unknown positive functions \(g_{Vm}, g_{ym}, g_{Qm}\) such that \(0 < g_{Vm} \leq g_v, 0 < g_{ym} \leq g_y, 0 < g_{Qm} \leq g_Q\).

**Remark 3.** In practice, it is rather difficult to know the values of functions \(f_*^{}, g_*^{} \in \{V, \gamma, Q\} \) accurately. There are mainly the following reasons: On the one hand, the aerodynamic parameters are constantly changing with the flight environment (i.e., velocity, altitude, and attack of angle), where there inevitably exist measuring errors in the sensors of flight control system [2]. On the other hand, it is impossible to take all flight environment of HFVs into account in a wind tunnel so that we have to rely on curve fitting technology to build the aerodynamic model [7]. Consequently, an exact model for HFVs is difficult to be obtained and in order to facilitate the design of flight control system; we regard \(f_*^{}, g_*^{} \in \{V, \gamma, Q\} \) as unknown function and regard \(g_*^{}\) as unknown positive function.

**Assumption 4.** See [44]. The reference trajectory \(y_{ref}(t)\), together with its \(i\)-order derivative \(y_{ref}^{(i)}(t)\), is continuous and bounded \((i = 1, 2 \cdots n)\).

2.4. A New Fixed-Time Performance Function

**Definition 5.** See [44]. A smooth function \(\rho(t)\) is called fixed-time performance function (FTPF), if the following conditions are satisfied:

1. \(\rho(t) > 0\), i.e., \(\rho(t)\) is ensured to be a positive function.
2. \(\dot{\rho} \leq 0\), that is, \(\rho(t)\) is monotonically decreasing.
3. \(\lim_{t \to T^-} \rho(T) = \rho(T)\) and \(\rho(t) = \rho(T)\) for any \(t > T\), where \(\rho(T)\) and \(T\) denote an arbitrarily small positive constant and settling time, respectively.

According to Definition 5, we construct an FTPF in the form of

\[
\rho(t) = \begin{cases} 
\coth \left( \frac{\theta}{T - t} + r \right) - 1 + \rho(T), & 0 \leq t < T \\
\rho(T), & t \geq T.
\end{cases}
\]

(17)
Consider the following transformation:

$$q(t) = \frac{e(t)}{\rho(t)},$$  \hspace{1cm} (23)

where $e(t)$ represents an error function; the error transformation function is chosen as

$$z(q) = \frac{q}{1-q^2},$$  \hspace{1cm} (24)

and we abbreviate $q = q(t)$ there-in-after.

Remark 7. From (24), it can be observed that the inequality $-1 < q < 1$ holds if $z(q)$ is bounded. In view of (23), we conclude that $|e(t)| < |\rho(t)|$ holds as long as $|e(0)| < |\rho(0)|$. Choosing $\theta = 0.5$, $r = 0.1$, $T = 6$, and $\rho(T) = 1$, the convergence performance of the proposed FTFP is shown in Figure 1(b). In contrast to traditional PPC [11], where the prescribed performance function is in the form of $\rho(t) = \cosh (\theta t + r) - 1 + \rho_0$, the proposed control scheme explicitly contains a convergence time $T$ in the FTFP. By this means, we can easily preset the convergence time as needed.

2.5. Fuzzy Logic System. In the process of designing the flight controller, the fuzzy logic system is used to estimate the dynamics uncertainties of HFVs. Define a set of fuzzy IF-THEN rules, where the $l$th IF-THEN rule is written as follows [22, 23, 45, 46]:

$$\mathcal{A}^l: \text{If } x_1 \text{ is } F_1^l, \text{ and } \cdots \text{ and } x_n \text{ is } F_n^l, \text{ then } y \text{ is } B^l.$$  \hspace{1cm} (25)

where $x = [x_1, \cdots, x_n]^T \in \mathbb{R}^n$, and $y \in \mathbb{R}$ are the input and output of the FLSs, respectively, and $F_1^l, \cdots, F_n^l$ and $B^l$ are fuzzy sets in $\mathbb{R}$. Let $F(x)$ be a continuous function defined on a compact set $\Omega_w$. Then, for a given desired level of accuracy $\varepsilon > 0$, there exists a FLS $W^T S(x)$ such that $\sup_{x \in \Omega_w} |F(x) - W^T S(x)| \leq \varepsilon$, where $W = [w_1, \cdots, w_p]^T$ is the adaptive fuzzy parameter vector in a compact set $\Omega_w$, $p$ is the number of the fuzzy rules, and $S(x) = [S_1(x), \cdots, S_p(x)]^T$ is the fuzzy basis function vector with $S_l(x) = \sum_{j=1}^{m_l} \mu_{F_l^j}(x_j) / \sum_{j=1}^{m_l} \mu_{F_l^j}(x_j)$ where $\mu_{F_l^j}(x_j)$ is a fuzzy membership function of the variable $x_j$ in IF-THEN rule. Let $W^*$ be the optimal parameter vector, which is defined as

$$W^* = \arg \min_{W \in \Omega_w} \left\{ \sup_{x \in \Omega_w} |F(x) - W^T S(x)| \right\}.$$  \hspace{1cm} (26)

Then, we can further obtain

$$F(x) = W^*^T S(x) + \phi,$$  \hspace{1cm} (27)

where $\phi$ is the minimum fuzzy approximation error.

In order to reduce the computational burden, the functions $\xi_V = (\|W_V\|)^2/\gamma_{V_m}$, $\xi_h = (\|W_h\|)^2/\gamma_{V_m}$, $\xi_p = (\|W_p\|)^2/\gamma_{V_m}$...
Lemma 8. See [47]. The hyperbolic tangent function \( \tanh(\gamma_m) \) is continuous and differentiable; for each \( \gamma_m \in \mathbb{R} \) and \( \forall \mu > 0 \), it has

\[
0 \leq |e| - \zeta \tanh\left( \frac{\zeta}{\mu} \right) \leq 0.2785 \mu.
\]

Lemma 9. See [48]. For any positive constants \( \omega \) and \( \delta \), the following inequation holds

\[
0 \leq \omega^2 - \frac{\omega^2}{\sqrt{\omega^2 + \delta^2}} \leq \delta.
\]

The control objective of this article is to design an adaptive fuzzy controller such that

1. The velocity and altitude tracking errors are guaranteed to obey the prescribed performance boundaries at all times and finally converge into the predefined residual sets within preassigned time \( T \) even in the presence of actuator faults.
2. All signals of the closed-loop system remain bounded.

3. The FTFTC Design

Corresponding to the decomposition in Section 2.3, the control design is also decomposed into a velocity control design and an altitude control design.

3.1. Velocity Controller Design. We first define the velocity tracking error as

\[
e_V = V - V_{\text{ref}},
\]

where \( V_{\text{ref}} \) is the velocity reference trajectory. In view of (15), (23), and (24), the time derivative of \( e_V \) is

\[
\dot{e}_V = f_V + g_V v(\Phi) + d_V - \dot{V}_{\text{ref}} = \dot{\rho}(t)q + \dot{\zeta}_V \rho(t) \frac{\partial q}{\partial z}.
\]

Then, \( \dot{e}_V \) can be rewritten as

\[
\dot{e}_V = f_V + g_V v(\Phi) + d_V - \dot{V}_{\text{ref}} - \dot{\rho}(t)q
\]

\[\frac{\rho(t)(\partial q/\partial z)}{-\rho(t)(\partial q/\partial z)} = Y_V + E_V (f_V + g_V v(\Phi) + d_V - \dot{V}_{\text{ref}}), \]

with \( Y_V = -\dot{\rho}(t)qE_V, \ E_V = (1/\rho(t))(\partial q/\partial z) \).

It can be deduced from (24) that \( \partial q/\partial z = ((1-q^2)^2)/(1+q^2) \) \leq 1 when \(-1 < q < 1\), and noticing the fact \( g_V \geq g_{\text{vm}} > 0 \) leads to

\[
E_V g_V = \frac{g_V}{\rho(t)(\partial q/\partial z)} \leq \frac{g_{\text{vm}}}{\rho(0)} > 0.
\]

Choose the following quadratic function:

\[
L_V = \frac{1}{2} \varepsilon_V^2 + \frac{1}{2\mu_V} \tilde{g}_{\text{vm}} \tilde{e}_V^2,
\]

where \( \tilde{g}_{\text{vm}} = (g_{\text{vm}}/\rho(0)) \), \( \tilde{e}_V = \varepsilon_V - \tilde{\zeta}_V \), \( \tilde{\zeta}_V \) denotes the estimation of adaptive parameter \( \tilde{\zeta}_V \) and \( \mu_V \) is the positive user-defined parameter.

Utilizing (14) and (31), the time derivative of \( L_V \) is

\[
\dot{L}_V = \varepsilon_V (Y_V + E_V f_V + E_V g_V v(\Phi) + E_V g_V v(\Phi) + E_V d_V - E_V \dot{V}_{\text{ref}})
\]

\[\frac{1}{\mu_V} \tilde{g}_{\text{vm}} \tilde{e}_V \tilde{e}_V.
\]
Define the nonlinear function
\[ F_V = Y_V + E_V f_V + E_V d_V - E_V V_{ref} + \frac{1}{2} \bar{z}_V, \] (36)
where \( f_V \) and \( d_V \) are unknown due to the existence of unknown external disturbances and fast time-varying flight environment. An FLSs-approximator is constructed to estimate \( F_V \) as
\[ F_V = W^T V S_V(x_V) + \phi_V, \] (37)
where \( x_V = [V, V_{ref}, \dot{V}_{ref}]^T \). Substituting (36) and (37) into (35), we have
\[ \dot{L}_V = z_V \left( W^T V S_V + \phi_V + E_V g_V \omega_0 \Phi + E_V g_V \epsilon_0 \right) - \frac{1}{\mu_V} \dot{g}_V \tilde{\epsilon}_V \tilde{\epsilon}_V - \frac{1}{2} \bar{z}_V. \] (38)

According to Young’s inequation, we can further have
\[ z_V W^T V S_V \leq \frac{z_V^2 \| W_V \|^2 S^T V S_V}{4 \tau_V} + \tau_V, \] (39)
\[ z_V \phi_V \leq \frac{1}{2} \tilde{z}_V^2 + \frac{1}{2} \phi_V^2, \]
where \( \tau_V \) is a positive constant; then, we can rewrite (38) as
\[ \dot{L}_V \leq z_V \left( E_V g_V \omega_0 \Phi + E_V g_V \epsilon_0 \right) + \frac{z_V^2 \| W_V \|^2 S^T V S_V}{4 \tau_V} + \tau_V + \frac{1}{2} \phi_V^2 - \frac{1}{\mu_V} \dot{g}_V \tilde{\epsilon}_V \tilde{\epsilon}_V. \] (40)

According to Assumption 1, \( \omega_0 \) and \( \epsilon_0 \) are unknown. Therefore, we define the upper and lower bounds of fault parameters to achieve robustness, which are expressed as
\[ \omega_0 = \inf \left( E_V g_V \omega_0 \right), \quad \vartheta_V = \frac{1}{\omega_0}, \]
\[ \xi_V = \sup \left( E_V g_V \epsilon_0 \right). \] (41)

Consider the Lyapunov function candidate
\[ L_V = L_V + \frac{1}{2 l_V} \omega_0 \dot{\vartheta}_V \vartheta_V + \frac{1}{2 r_V} \tilde{\xi}_V^2, \] (42)
where \( l_V > 0 \) and \( r_V > 0 \) are the parameters to be designed and \( \vartheta_V = \vartheta_V - \vartheta_V \) and \( \tilde{\xi}_V = \xi_V - \tilde{\xi}_V \) represent estimation errors with \( \vartheta_V \) and \( \tilde{\xi}_V \) being the estimations of \( \vartheta_V \) as \( \xi_V \), respectively.

The time derivative of \( L_V \) gives
\[ \dot{L}_V = L_V - \frac{1}{l_V} \omega_0 \dot{\vartheta}_V \vartheta_V - \frac{1}{r_V} \tilde{\xi}_V \tilde{\xi}_V. \] (43)

Define \( \xi_V = \| W_V \|^2 / \dot{g}_V \) and choose the intermediate control law as
\[ \Phi = k_V z_V + \frac{z_V^2 \dot{\epsilon}_V S^T V S_V}{4 \tau_V} + \tilde{\xi}_V \tanh \left( \frac{z_V}{a_V} \right). \] (44)

It can be induced that
\[ \dot{L}_V \leq z_V E_V g_V \omega_0 \Phi + \frac{z_V^2 \dot{\epsilon}_V S^T V S_V}{4 \tau_V} + \tau_V + \frac{1}{2} \phi_V^2 - \frac{\omega_0}{\mu_V} \dot{g}_V \tilde{\epsilon}_V \tilde{\epsilon}_V \]
\[ + z_V \Phi - k_V z_V \frac{z_V^2 \dot{\epsilon}_V S^T V S_V}{4 \tau_V} \]
\[ + \xi_V \left( |z_V| - z_V \tanh \left( \frac{z_V}{a_V} \right) \right) - \frac{\dot{g}_V m b}{\mu_V} \tilde{\epsilon}_V \tilde{\epsilon}_V \]
\[ + \frac{1}{r_V} \tilde{\xi}_V \left( r_V z_V \tanh \left( \frac{z_V}{a_V} \right) - \tilde{\xi}_V \right). \] (45)

Choose the adaptive laws as follows:
\[ \tilde{\epsilon}_V = \frac{\mu_V z_V^2 S^T V S_V}{4 \tau_V} - r_V \tilde{\epsilon}_V, \] (46)
\[ \tilde{\xi}_V = r_V z_V \tanh \left( \frac{z_V}{a_V} \right) - b_V \tilde{\xi}_V, \] (47)
\[ \bar{\vartheta}_V = l_V z_V \Phi - \vartheta_V \bar{\vartheta}_V, \] (48)
where \( a_V > 0 \), \( b_V > 0 \), and \( c_V > 0 \) are the parameters to be designed.

Substituting (46)–(48) into (45) yields
\[ \dot{L}_V \leq z_V E_V g_V \omega_0 \Phi + \tau_V + \frac{1}{2} \phi_V^2 + \frac{\mu_V}{l_V} \dot{g}_V \tilde{\epsilon}_V \tilde{\epsilon}_V \]
\[ - \omega_0 \dot{\vartheta}_V z_V \Phi + \frac{\omega_0 c_V}{l_V} \dot{\vartheta}_V \vartheta_V + z_V \Phi - k_V z_V^2 \]
\[ + \xi_V \left( |z_V| - z_V \tanh \left( \frac{z_V}{a_V} \right) \right) + b_V r_V \tilde{\xi}_V \tilde{\xi}_V. \] (49)

Now, we design the actual control law as
\[ \Phi = - \frac{z_V^2 \dot{\epsilon}_V^2 \Phi^2}{\sqrt{z_V^2 \dot{\epsilon}_V^2 + \sigma_V^2}}, \] (50)
where \( \sigma_V > 0 \) is a predefined constant, which is designed to avoid the singularity issue. According to Young’s inequation, one has
\[ \tilde{\epsilon}_V \tilde{\epsilon}_V \leq \left( \epsilon_V - \tilde{\epsilon}_V \right) \tilde{\epsilon}_V \leq \frac{1}{2} \epsilon_V^2 + \frac{1}{2} \tilde{\epsilon}_V^2, \] (51)
\[ \tilde{\vartheta}_V \tilde{\vartheta}_V \leq \left( \vartheta_V - \tilde{\vartheta}_V \right) \tilde{\vartheta}_V \leq \frac{1}{2} \vartheta_V^2 + \frac{1}{2} \tilde{\vartheta}_V^2, \] (52)
Substituting (50)-(53) into (49) and applying Lemma 8 and Lemma 9, we can rewrite (49) as

\[
L_V \leq -k_v z_v^2 - \frac{1}{2\mu_V} \Gamma_v \theta_v^2 z_v^2 - \frac{1}{2\mu_V} \omega_{\theta_v} c_{\theta_v} \theta_v^2 - \frac{1}{2\mu_v} b_{v} \dot{z}_v^2 + \frac{1}{2\mu_V} \Gamma_v \theta_v^2 + \frac{1}{2\mu_v} b_{v} \dot{z}_v + \frac{1}{2} \phi_v^2 + \omega_{\theta_v} \theta_v + 0.2785 a_v z_v V.
\]

\[
V_h = \frac{z_h^2}{4r_h} + \tau_h.
\]

where \( \tau_h \) is a positive constant; noting that \( \gamma = z_h + \gamma \), we can further rewrite (61) as

\[
L_h = \frac{z_h^2}{4r_h} + \tau_h
\]

with the definition of \( L_h = \langle \| W_h \| \rangle / \langle V_m \rangle \). Choose the virtual control law and updating law as follows:

\[
\dot{\gamma}_h = -k_h \gamma_h - \frac{z_h Z_h S_h^2 S_h}{4r_h},
\]

\[
\dot{\gamma}_h = \frac{\mu_h z_h^2 S_h^2 S_h}{4r_h} - \Gamma_h \gamma_h.
\]

Substituting (64)–(65) into (63) yields

\[
L_h \leq -k_h \gamma_h^2 + \frac{1}{2} \mu_h \beta_h \gamma_h^2 + 1/2 \frac{1}{2} \mu_h V_m \dot{\gamma}_h + \frac{1}{2} \phi_h^2 + \tau_h.
\]

Step 11. Take the Lyapunov function candidate as

\[
L_\gamma = \frac{1}{2} \gamma_h^2 + \frac{1}{2} \mu_h \beta_h \gamma_h^2
\]

Similar to step 10, by defining \( \gamma_h = \langle \| W_h \| \rangle / \langle g_{ym} \rangle \), the time derivative of \( L_\gamma \) can be formulated as

\[
\dot{\gamma}_h = \dot{\gamma}_h + z_h \left( W_h^2 S_h + \phi_h + g_{ym} \right) - \frac{1}{2} \mu_h \beta_h \gamma_h^2 - E_h \beta_h^2 z_h \gamma_h - \frac{1}{2} \gamma_h^2
\]

with \( F_h = f_h + E_h \beta_h^2 z_h \gamma_h - (1/2) \gamma_h^2 \) being approximated by FLS \( W_h^2 S_h(x_h) + \phi_h \), where \( \chi_h = (d \chi_h / dh) \hat{h} + (d \chi_h / dh) \hat{h} \) and \( x_h = [h, \gamma_h, \hat{h} \gamma_h, \hat{h} \gamma_h] \).
Design the virtual control law and adaptive law as

\[
\chi_\theta = -k_\gamma z_\gamma - \frac{z_\gamma^T S_\gamma^T S_\gamma}{4T_\gamma},
\]

\[
\hat{\xi}_\gamma = \frac{\mu_\gamma z_\gamma^T S_\gamma}{4T_\gamma} - \Gamma_\gamma \hat{\xi}_\gamma.
\]  

(69)

(70)

Based on (66)–(70), we obtain

\[
\dot{L}_\gamma \leq -k_h z_h^2 - k_\gamma z_\gamma^2 + \frac{1}{\mu_\gamma} \Gamma_h V_m \hat{\xi}_h^T \hat{\xi}_h + \frac{1}{\mu_\gamma} \Gamma_\gamma g_{ym} \hat{\xi}_\gamma^T \hat{\xi}_\gamma + g_j z_\gamma z_\theta
\]

\[
+ \frac{1}{2} \phi_\delta^2 + \frac{1}{2} \phi_\tau^2 + r_\theta + r_\gamma.
\]

(71)

Step 12. Consider the following Lyapunov function candidate:

\[
L_\theta = L_\gamma + \frac{1}{2} \hat{\xi}_\theta^2 + \frac{1}{2\mu_\theta} \hat{\theta}_\theta^2.
\]  

(72)

Defining \( \xi_\theta = \|W_\theta\|^2 \), the time derivative of \( L_\theta \) gives

\[
\dot{L}_\theta = \dot{L}_\gamma + z_\theta (W_\theta^T S_\theta + \phi_\theta + Q) - \frac{1}{\mu_\theta} \hat{\xi}_\theta \dot{\xi}_\theta - g_j z_\gamma z_\theta - \frac{1}{2} z_\theta^2,
\]

(73)

with \( F_\theta = g_j z_\gamma - \hat{\chi}_\theta + (1/2) z_\theta \) being approximated by FLS \( W_\theta^T S_\theta(x_\theta) + \phi_\theta \), where \( \hat{\chi}_\theta = \sum_{\chi=0} (\partial \chi_\theta / \partial \chi) \hat{\chi} + \sum_{\chi=0} (\partial \chi_\theta / \partial \theta) \hat{\theta} + \sum_{i=0} (\partial \chi_\theta / \partial \theta_i) \hat{\theta}_i \) and \( x_\theta = [h, \gamma, \theta, h_{ref}, h_{ref}^{(2)}, h_{ref}^{(3)}, h_{ref}^{(4)}, e_h, e_\gamma, e_\theta]^T \).

Construct the virtual control law and adaptive law

\[
\chi_\theta = -k_\theta z_\theta - \frac{z_\theta^T S_\theta^T S_\theta}{4T_\theta},
\]

\[
\hat{\xi}_\theta = \frac{\mu_\theta z_\theta^T S_\theta^T S_\theta}{4T_\theta} - \Gamma_\theta \hat{\xi}_\theta.
\]  

(74)

(75)

From (72)–(75), one has

\[
\dot{L}_\theta \leq -k_h z_h^2 - k_\gamma z_\gamma^2 - k_\theta z_\theta^2 + \frac{1}{\mu_\theta} \Gamma_\theta V_m \hat{\xi}_\theta^T \hat{\xi}_\theta + \frac{1}{\mu_\gamma} \Gamma_\gamma g_{ym} \hat{\xi}_\gamma^T \hat{\xi}_\gamma + z_\theta z_\gamma
\]

\[
+ \frac{1}{2} \phi_\delta^2 + \frac{1}{2} \phi_\tau^2 + r_\theta + r_\gamma.
\]

(76)

Step 13. Choose the following Lyapunov function candidate:

\[
L_Q = L_\theta + \frac{1}{2} \hat{\xi}_Q^2 + \frac{1}{2\mu_Q} \hat{\theta}_Q^2.
\]  

(77)

Constructing \( \xi_Q = (\|W_Q\|^2 / g_{Qm}) \) and taking the derivative of \( L_Q \) yield

\[
\dot{L}_Q = \dot{L}_\theta + z_\theta (W_Q^T S_Q + \phi_Q + g_Q a_Q \delta_\epsilon + g_Q b_Q e_\theta)
\]

\[
- \frac{1}{\mu_Q} g_{Qm} \hat{\xi}_Q \hat{Q}_Q - z_\theta \delta_\epsilon - \frac{1}{2} \delta_\epsilon^2,
\]

(78)

with \( F_Q = f_Q + z_\theta + d_Q - \dot{\chi}_Q + (1/2) z_\theta \) being approximated by FLS \( W_Q^T S_Q(x_Q) + \phi_Q \), where \( \hat{\chi}_Q = \sum_{\chi=0} (\partial \chi_Q / \partial \chi) \hat{\chi} + \sum_{\chi=0} (\partial \chi_Q / \partial \theta) \hat{\theta} + \sum_{i=0} (\partial \chi_Q / \partial \theta_i) \hat{\theta}_i \) and \( x_Q = [h, \gamma, \theta, h_{ref}, h_{ref}^{(2)}, h_{ref}^{(3)}, h_{ref}^{(4)}, e_h, e_\gamma, e_\theta]^T \).

The upper and lower bounds of fault parameters are defined as

\[
\omega_\theta = \inf \left( g_\theta \omega_\theta \right), \quad \theta_Q = \frac{1}{g_\theta},
\]

\[
\xi_Q = \sup \left( g_\theta \xi_\theta \right).
\]  

(79)

Construct the Lyapunov function:

\[
L_Q = L_\theta + \frac{1}{2\mu_Q} \omega_\theta \xi_\theta^2 + \frac{1}{2\mu_Q} \xi_\theta^2
\]

(80)

where \( l_Q > 0 \) and \( r_Q > 0 \) are designed parameters and \( \delta_\theta = \theta_\theta - \hat{\theta}_\theta \) and \( \xi_\theta = \xi_\theta - \hat{\xi}_\theta \) represent estimation errors with \( \hat{\theta}_\theta \) and \( \hat{\xi}_\theta \) being the estimates of \( \theta_\theta \) and \( \xi_\theta \), respectively. Choose the intermediate control law and adaptive laws as follows:

\[
\delta_\epsilon = k_Q \delta_\epsilon + \frac{z_\theta}{4T_Q} S_\theta^T S_\theta + \xi_Q \tanh \left( \frac{z_\theta}{a_Q} \right),
\]

\[
\hat{\vartheta}_Q = \frac{\mu_\theta z_\theta^T S_\theta^T S_\theta}{4T_Q} - \Gamma_\theta \hat{\vartheta}_Q,
\]

\[
\hat{\xi}_Q = r_Q \delta_\epsilon \tan \left( \frac{z_\theta}{a_Q} \right) - b_Q \hat{\xi}_Q,
\]

\[
\hat{\theta}_Q = l_Q \delta_\epsilon - c_Q \hat{\theta}_Q,
\]

(81)

(82)

(83)

(84)

where \( k_Q, r_Q, a_Q, b_Q, \) and \( c_Q \) are designed positive parameters.

Finally, we choose the actual control law as

\[
\delta_\epsilon = -\frac{z_\theta^2 \delta_\epsilon^2}{\sqrt{z_\theta^2 \delta_\epsilon^2 + \sigma_\delta^2}},
\]

(85)

where \( \sigma_\delta > 0 \) is a predefined constant. Following similar analysis to velocity subsystem, we can further deduce that
The tracking errors $e_v$ and $e_h$ can converge into a pre-defined residual set within an user-defined time $T$.

(1) The overshoot and convergence rate are guaranteed by FTPF, and all signals of the closed-loop system are SGPFS.

**Proof.** Take the Lyapunov function candidate as

$$L = L_V + L_Q.$$  

Applying (54) and (86), the derivative of $L$ gives

$$
\dot{L}_Q \leq -k_h \delta^2 - k_v \gamma - k_o \delta^2 - k_2 \theta^2 - k_3 \xi^2 - k_4 \xi_2^2 - \frac{\Gamma_h V_m^2}{\mu_h} \xi_2^2 - \frac{\Gamma_y V_m^2}{\mu_y} \xi_2^2 - \frac{\omega_o \sigma Q}{2 \mu_Q} \xi_2^2 - \frac{b_Q}{2 \mu_Q} \xi_2^2 + \frac{b_Q}{2 \mu_Q} \xi_2^2 + \Delta V + \Delta \theta + \Delta Q + \omega_o \sigma Q + 0.2785 a_Q \xi_Q.
$$  

(86)

The whole FTFTC design for HFVs is shown in Figure 2.

**4. Closed-Loop Stability Analysis**

**Theorem 14.** Despite the occurrence of unknown actuator fault (14), consider the closed-loop system composed by (15) and (16); the virtual control laws (64), (69), and (74); the intermediate control laws (44) and (81); the actual control laws (50) and (85); and the parameter adaptation laws (46)-(48), (65), (70), (75), and (82)-(84). Let Assumptions 1–4 hold. By designing the parameters properly, it therefore holds the following.

The whole FTFTC design for HFVs is shown in Figure 2.
Integrating both sides of (89) yields

\[ L \leq L(0) + \frac{C}{k}. \]  

In accordance with (34) and (60), we have

\[ z_y \leq \sqrt{2L(0) + \frac{C}{k}} z_h \leq \sqrt{2L(0) + \frac{C}{k}}. \]
In view of (42), (60), (67), (72), (77), (80), (87), and (90), the estimation errors of the adaptive parameters will converge to the following compact sets:

\[
\begin{align*}
\dot{e}_V & \leq \sqrt{2\mu_V\left(L(0) + \frac{C}{K}\right)}, \\
\dot{e}_h & \leq \sqrt{2\mu_h\left(L(0) + \frac{C}{K}\right)}, \\
\dot{e}_\gamma & \leq \sqrt{2\mu_\gamma\left(L(0) + \frac{C}{K}\right)}, \\
\dot{e}_\theta & \leq \sqrt{2\mu_\theta\left(L(0) + \frac{C}{K}\right)}, \\
\dot{e}_Q & \leq \sqrt{2\mu_Q\left(L(0) + \frac{C}{K}\right)},
\end{align*}
\]

Therefore, the transformational errors $e_V$ and $e_h$ are bounded. By reviewing (17), we can conclude that the velocity and altitude tracking errors converge to a residual set within a fixed time $T$ and the prescribed performances are guaranteed. Besides, all signals of closed-loop system are SGPFS. According to the design of velocity controller (44), altitude controller (64), and the FTPF (17), the overshoots of velocity and altitude do not exceed their preset threshold. This completes the proof.

Remark 15. The existing fixed-time control strategies for HFVs [18–20] fail to take system transient and steady-state performances into account, and it is fairly complicated to make tracking error convergence into a pre-assigned compact set within the fixed time by selecting design parameters. It is worth noting that the fixed-time tracking control is achieved as long as the bounded condition is satisfied in the proposed design. Consequently, the complexity of the control structure is reduced and the initial states need not to be known accurately via the proposed control approach.

Remark 16. It is worth mentioning that the control performance is depended closely on the designed parameters of prescribed function. In (17), large initial errors are allowed by choosing a small enough $r$; larger $\theta$ and smaller $T$ will increase the convergence rate. However, too large $\theta$ or too small $T$ will give rise to actuator input saturations. In (44), (64), (69), and (81), designed parameters $k_V$, $a_V$, $k_h$, $a_h$, $k_\gamma$, $k_\theta$, $k_Q$, $a_Q$, and $\alpha_Q$ determine the convergence rate and convergence accuracy. In (46)–(48), (65), (70), and (82)–(84), designed parameters $\mu_V$, $\Gamma_V$, $r_V$, $b_V$, $k_h$, $a_h$, $k_\gamma$, $k_\theta$, $k_Q$, $a_Q$, $\alpha_Q$, $\Gamma_Q$, $r_Q$, $b_Q$, $k_Q$, $a_Q$, and $\alpha_Q$ effect convergence rate of the adaptive parameters. In the controller design, we need to design the parameters properly to improve the tracking performance and avoid the saturation phenomenon.

Remark 17. When the actuator failure occurs, the upper and lower bounds of fault parameters are estimated by
the adaptive laws (47), (48), (83), and (84), then the intermediate control laws (44) and (81) and the actual control laws (50) and (85) are executed. By this way, the flight control system is robust to actuator failure. Compared with the state-of-the-art FTC methods [30–33], the fixed-time stability is guaranteed, and the prescribed performance is ensured.

5. Simulations

In this section, simulation results are used to demonstrate the effectiveness and superiority of the proposed methodology. The model parameters of HFVs can be consulted from [40]. HFVs are expected to climb a maneuver from the initial trim conditions, depicted in Table 1, to the final values \( V = 8700 \text{ft/s} \) and \( h = 8800 \text{ft} \). The external disturbances in velocity subsystem and altitude subsystem are set as \( d_v = \sin (0.1\pi t) \text{ft/s} \), \( d_h = 0.001 \sin (0.01\pi t) \text{deg} \), \( d_Q = 0.01 \sin (0.01\pi t) \text{deg/s} \). The reference trajectories of velocity and altitude are generated via the following filters [11]:

\[
\begin{align*}
\frac{V_{\text{ref}}(s)}{V_c(s)} &= \frac{0.03^2}{s^2 + 2 \times 0.95 \times 0.03 \times s + 0.03^2}, \\
\frac{h_{\text{ref}}(s)}{h_c(s)} &= \frac{0.03^2}{s^2 + 2 \times 0.95 \times 0.03 \times s + 0.03^2}.
\end{align*}
\]  

(93)

where \( V_{\text{ref}}(s) \) and \( h_{\text{ref}}(s) \) represent the inputs of filters and \( V_c(s) \) and \( h_c(s) \) represent the outputs of filters, respectively. It is assumed that HFVs actuators failed at 100 s and the details of failure are formulated in the form of

\[
\begin{align*}
\nu(\Phi) &= 0.8\Phi - 0.1, \\
\nu(\delta_c) &= 0.8\delta_c + 0.0349.
\end{align*}
\]  

(94)
The FTPFs are selected as
\[
\rho_V(t) = \begin{cases} 
\coth \left( \frac{t}{50} + 0.4 \right) - 0.9, & 0 \leq t < 50 \\
0.1, & t \geq 50 
\end{cases}
\]
\[
\rho_h(t) = \begin{cases} 
\coth \left( \frac{t}{50} + 0.4 \right) - 0.5, & 0 \leq t < 50 \\
0.5, & t \geq 50 
\end{cases}
\]

The fuzzy rules in \( W^*_V S_V \) are listed as
\( \mathcal{R}^V \): If \( V \in F^i_V \), then \( y \in B^i \), where \( i = 1, 2, 3 \) and \( l = 1, 2, 3 \).

The fuzzy rules in \( W^*_h S_h \) are listed as
\( \mathcal{R}^h \): If \( h \in F^i_h \), and \( y \in F^j_y \), then \( y \in B^l \), where \( i = 1, 2, 3 \); \( j = 1, 2, 3 \); and \( l = 1, 2, \ldots, 9 \).

Then, the fuzzy rules in \( W^*_\theta S_\theta \) are listed as
\( \mathcal{R}^\theta \): If \( h \in F^i_h \), and \( y \in F^j_y \), and \( \theta \in F^k_\theta \), then \( y \in B^l \), where \( i = 1, 2, 3 \); \( j = 1, 2, 3 \); \( k = 1, 2, 3 \); and \( l = 1, 2, \ldots, 27 \).

The fuzzy rules in \( W^*_Q S_Q \) are listed as

\[\begin{array}{c|c|c}
\text{Control method} & \text{Performance index of } \Phi & \text{Performance index of } \delta_e \\
\hline
\text{FTFTC} & 99.4683 & 15.2615 \\
\text{CFTC} & 99.4574 & 15.2620 \\
\end{array}\]

Figure 8: The flexible states and control inputs via different methodologies.

Figure 9: The performance indexes via different methodologies.
In this example, two different initial states cases, i.e., case 1 \( (e_{Vr}(0) = e_{Qr}(0) = 0) \) and case 2 \( (e_{Vr}(0) = e_{Qr}(0) = 1) \), are considered, respectively.

The obtained simulation results, depicted in Figures 3–5, reveal that the proposed FTFTC has full capabilities to deal with the condition that aerodynamic parameters are perturbed and initial states are uncertain. It can be observed from Figures 3(a)–3(b) that the velocity and altitude tracking errors are guaranteed not to exceed the prescribed bounds. Furthermore, the velocity and altitude can rapidly track their reference trajectories even if the actuator fails.

\[ E = \int_0^t e^2 dt \]

The fuzzy membership function is given as follows:

\[
\begin{align*}
\mu_{f_1} &= \exp \left[-(V - 7500)^2/300\right], \\
\mu_{f_2} &= \exp \left[-(V - 8100)^2/300\right], \\
\mu_{f_3} &= \exp \left[-(V - 8700)^2/300\right],
\end{align*}
\]

\[ h(t) = \begin{cases} 1, & \text{if } t = 0 \\ 0, & \text{otherwise} \end{cases} \]

The controller parameters are selected as \( k_y = \mu_y = r_y = l_y = l_h = l_y = l_q = i_Q = 1, \quad g_{\nu_m} = 0.0001, \quad g_{\nu_m} = 1, \quad \tau_{\nu} = \Gamma_{\nu} = e_{\nu} = b_{\nu} = r_{\nu} = r_Q = q_{\nu} = q_{\nu} = q_{\nu} = \Gamma_{Q} - c_{Q} = b_{Q} = 0.1, \quad a_{\nu} = 0.2, \quad a_{Q} = 0.25, \quad \sigma_{\nu} = \sigma_{\nu} = (1/(t^2 + 0.01)), \quad \delta_h = 10, \quad \delta_v = 2, \quad \delta_{\nu} = 10, \quad \delta_q = 50. \]

Taking the engineering practice into account, the limitations of the actuators are set as \( \Phi \in [0, 1], \quad \delta_{\nu} \in [-20 \text{ deg}, 20 \text{ deg}] \) [40]. To prove that the proposed FTFTC is realizable and independent of the initial states, the first example is taken, while the superiority of the proposed FTFTC over the conventional fixed-time control (CFTC) in [49] is explained in the second example.

**Example 18.** In this example, simulations via the CFTC [49] and the proposed FTFTC are demonstrated, where the initial states are set as \( e_{Vr}(0) = e_{Qr}(0) = 1 \). In order to expound the advantages of tracking performances of the proposed FTFTC, the performance index of tracking error \( E_u = \int_0^t e^2 dt \)

\[ t \text{ is introduced, where } e \text{ denotes the tracking error. In order to compare the energy consumption between the FTFTC and CFTC, the performance index of control input } E_u = \int_0^t u^2 dt \text{ is defined where } u \text{ denotes the control input.} \]

Simulation results are depicted in Figures 6–9. Figures 6(a)–6(b) shows the velocity and attitude tracking performance, in which the velocity and altitude tracking errors are limited in the preset bounds by the proposed FTFTC and the proposed FTFTC can provide higher rate of convergence compared with the CFTC. Besides, the attitude angles and flexible states are shown in Figures 7 and 8(a), indicating that smaller oscillation amplitudes of attitude angles and flexible states are achieved in the presence of actuator failures by means of the proposed FTFTC. Figure 8(b) shows that the control inputs are smooth and within realistic limits by means of the proposed method. In addition, Figure 9 gives that less error energy is produced via the proposed FTFTC in contrast with the CFTC. Table 2 shows that the energy consumption of actuator with FTFTC is almost equal to that with CFTC. That is to say, the proposed FTFTC can achieve more accurate tracking.

**Abbreviations**

- \( V \): Velocity
- \( \theta \): Pitch angle

**6. Conclusions**

A novel fixed-time fuzzy adaptive fault-tolerant control methodology based on performance function is developed for hypersonic flight vehicles in this work. In contrast with the conventional fixed-time control, the proposed approach not only guarantees that the velocity and altitude tracking errors converge into a preassigned compact set, but also satisfies both the prescribed transient and steady performance. In addition, the proposed scheme can avoid the singularity problem caused by the differential of fractional order tracking error and remain valid in spite of actuator faults. Comparative simulation results confirm the validity and superiority of the presented control strategy. Note that the distributed adaptive containment fault-tolerant control of multi-HFVs formation is an important research region for the future [50–53]; thus, the extension of our control scheme to the case of multi-HFVs formation will be an interesting topic for further investigation.

**Example 19.** In this example, simulations via the CFTC [49] and the proposed FTFTC are demonstrated, where the initial states are set as \( e_{Vr}(0) = e_{Qr}(0) = 1 \). In order to expound the advantages of tracking performances of the proposed FTFTC, the performance index of tracking error \( E_u = \int_0^t e^2 dt \)
Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this article.

Acknowledgments

This study was supported by the National Natural Science Foundation of China (grant numbers 62103440 and 62003368) and the Youth Talent Promotion Project of Shaanxi Science and Technology Association (20220101).

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