Research Article

Variable Structure Control and Its Ground Experimental Test for the Space Station Robot

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Building a simulated weightless test system on the ground while making comprehensive comparisons of design controllers for a large and heavy multijointed space station robot is not an easy task. To save cost and improve the efficiency of the test, this paper develops a plan in which controllers undergo preliminary testing in a 6-DOF industrial robot. The key idea is gravity compensation included within the dynamic control algorithm of the robot to replace the function of the microgravity environment. It is generally difficult to build an accurate dynamic model for a serial-joint robot in a practical manner. Therefore, to guarantee the stability of the 6-DOF industrial robot in which the dynamic model is built inaccurately, we propose one of the simplest variable structure (VS) controllers, and the stability of the system is analyzed through the Lyapunov method. Last, experiments are carried out to provide preliminary comparisons among three potential algorithms for the space robot in a low-cost and efficient approach.

1. Introduction

Space robots have been playing an important role in orbital servicing missions, such as assisting the assembly of space stations, capturing or repairing faulty satellites, and cleaning orbital debris [1–5]. The primary characteristics of applied space equipment controllers are typically simple but reliable and effective. At present, there is a lack of confidence in launching equipment controlled by complex advanced control algorithms into space considering the risk and price of failure. Space-bound robots are used to serve the corresponding space station routines and tasks, after their performance and reliability in various types of working conditions are verified by trial and error, and under the simulated weightlessness or microgravity environment on the ground. Furthermore, without the weightlessness simulation system, the space robot designed for the space station may not even carry its own weight on the ground. To date, a variety of strategies have been published to compensate for the gravity of space robots for experimental tests on the ground. [6] emulates the zero-gravity environment for the space manipulator using an air-bearing platform, while the platform is mainly suitable for the planar mechanism whose motion is parallel to the air-bearing table [7]; A microgravity environment can also be created in a plane while in a free fall or a free-falling capsule at the microgravity center; however, the test time is too short, and available space for the robot is limited [8–10]; A microgravity test can be performed in the pool using neutral buoyancy. Nevertheless, the influence of fluid damping on test results cannot be negligible [11]; [12–15] emulate the process of capturing the space target by the free-floating robot mounted on satellites, using two industrial robots. However, the purpose is not to create the microgravity environment for the space robot but to generate the trajectory of the satellite; [16–19] design the suspension system to provide the zero gravity of the space robot for maintaining tension. However, it is difficult to remove the influences of the test results caused by the coupled vibration of the serial multijoint space robot and suspension system. Unfortunately, building a simulated weightless test system
for a large and heavy serial multijoint space station robot while choosing the most practical control algorithm from potential candidates is not a simple task and consumes a large amount of time, manpower, auxiliary facilities, and material resources [6–21].

The central difference between the controller designed for a space robot and that for an industrial robot on the ground is determined by considering gravity compensation or not. The gravity compensation included in the dynamic control algorithm of industrial robots can be regarded as a replacement for simulated weightless system functions. As a result, preliminary comparisons between various potential dynamic control algorithms on a 6-DOF industrial robot can be made to save cost and improve the efficiency of the test. This allows contrasting effects to be obvious and easily acquired, which provides prior knowledge and makes preliminary judgment regarding controllers designed for the space robot.

To achieve high precision and efficiency of robot tracking performance, it is necessary to introduce the robotic dynamics into the controller to reduce the effect of robotic nonlinear dynamic characteristics. The dynamics of serial-link robots have been well covered by almost all standard robotic textbooks [22–27]. Two main approaches are feedforward control and computed torque control [28], and both are based on robotic dynamics combined with one diagonal PD controller. Compared with computed torque control, the feedforward control is computationally less expensive at operating time and more easily realizes a high servo rate. This is all possible because the model-based dynamic compensation is “outside” the servo loop, and then, a fast inner servo loop is achieved. Furthermore, PD plus dynamic feedforward compensation in [22] is one of the simplest and most appealing dynamic controllers for robots [29]. It consists of one linear PD feedback and a nominal robot dynamic model computed as a function along the desired path only. Therefore, the advantage of the simple and efficient structure of this type of algorithm provides a reference to the controller design of the space robot.

Generally, the motion control of robots is not an easy task due to nonlinearity, strong coupling, uncertain modeling structure, or parameters. These factors may cause inaccurate joint motion and lead to poor robot performance. To solve these problems, various dynamic feedforward compensation plus PID-like or plus other single-loop feedback control algorithms and computed torque control-based methods have been proposed, including sliding-mode control (SMC) [30–36] or some other variable structure controllers [37–40], neural networks [41–46], fuzzy control [47, 48], and adaptive control [49, 50]. However, many of them are only tested in simulations or double-jointed robotic systems and may have more difficulties achieving the expected performance in the multijoint serial robotic system. One reason is that the current advanced control algorithms, such as neural networks, fuzzy control, and adaptive control, are characterized by complex structures and are time-consuming, and their computational time is possibly several times larger than the commercial robot servo period which is generally less than 10 ms. This may lead to poor real-time performance, let alone the current situation that most of those advanced control algorithms are mainly based on the computed torque control structure. Furthermore, reliable and practical controllers are always those simple ones in experience. At present, there is a lack of confidence to launch space robots controlled by those complex advanced control algorithms into space on account of the risk and price of failure. By comparison, a kind of variable structure controller is proposed in this paper to enhance the tracking performance of the robot under modeling uncertainty. It shows higher efficiency and a better control performance than [22] in the experiment because it combines the PD plus dynamic feedforward compensation and the Bang-Bang control to achieve compensation for the tracking error caused by the modeling uncertainty. One noticeable merit of the proposed method lies in the easier design and application in real robot systems than the controllers designed in [30–48] because of its simplicity in structure.

The main contributions of this paper are summarized as follows: (i) Since testing the different controllers and comparing them in a serial multijoint space station robot on the ground are difficult, this paper initially develops a strategy for controllers to be preliminarily tested in an industrial robot. The key idea is the gravity compensation included in the dynamic control algorithm of the robot to act in the function of a microgravity environment. It will be more efficient to obtain the superior controller from others; (ii) [22] points out that the dynamic model is always known to be inaccurate, and one reason is that it is severely challenging to acquire the structure of the friction model, let alone the corresponding parameters. To improve the control performance of the system, we propose the simplest variable structure control through the combination of the PD plus dynamic feedforward compensation and the Bang-Bang control together to compensate for tracking error caused by the modeling uncertainty, and the stability of the system is analyzed by Lyapunov theory. Then, we experiment to provide preliminary comparisons among several potential algorithms for the space robot in a low-cost and highly efficient way.

The rest of the paper is organized as follows. In Section 2, the dynamic parameters of the robot are identified by the least square method. In Section 3, to guarantee the stability of the system under the modeling uncertainty, a variable structure controller is designed based on the Lyapunov direct method. Then, we design the PD plus and the cascaded PD plus dynamic feedforward compensation to make an experimental comparison in Section 4. As a result, the cascaded PD plus dynamic feedforward compensation control more easily achieves the high-precision tracking performance than the proposed VS control or the PD plus, under the roughly built robotic dynamics. Finally, we present the conclusions in Section 5.

2. Identification of Dynamic Parameters

A typical 6-DOF serial industrial robot is shown in Figure 1.
The dynamic equation of the 6-link robot is written as
\[ \tau = M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + f_c \text{ sign}(\dot{q}) + f_j \dot{q}. \]  
(1)

where \( q \in \mathbb{R}^{6 \times 1} \) is the vector of joint displacements, \( \tau \in \mathbb{R}^{6 \times 1} \) is the vector of applied torque, \( M(q) \in \mathbb{R}^{6 \times 6} \) is the mass matrix of the manipulator, \( C(q, \dot{q}) \in \mathbb{R}^{6 \times 6} \) is the centrifugal/Coriolis force matrix, \( G(q) \in \mathbb{R}^{6 \times 1} \) is the vector of gravity terms, and \( f_c, f_j \in \mathbb{R}^{6 \times 6} \) are diagonal matrices that consist of the viscous and Coulomb friction parameters, respectively.

Then, in accordance with [51], (1) can be rewritten into the following linear form with \( n \times N_s = 6 \times 13 \) identifiable base parameters:
\[ \tau = Y(q, \dot{q}, \ddot{q})p, \]  
(2)

where
\[ p_{78 \times 1} = [p_1, \ldots, p_{6}]^T, \]
\[ p_i = [m_i, m_i, m_i, m_i, I_{xxi}, I_{xxi}, I_{xxi}, I_{xxi}, I_{xxi}, I_{xxi}, f_{ci}, f_{ci}]^T. \]

Obviously, \( 6 \times 13 \) unknown parameters constitute the dynamic parameters (3) of the robot. We generally reformulate (2) on account that not all parameters in \( p \) are independent. In this paper, the independent parameters are collected through QR decomposition.

We perform the robot under a random trajectory to obtain the multiple matrices \( Y(\cdot) \) which form the matrix
\[ [W]_{6M \times 78} = \begin{bmatrix} Y(q(t_1), \dot{q}(t_1), \ddot{q}(t_1)) \\ \vdots \\ Y(q(t_M), \dot{q}(t_M), \ddot{q}(t_M)) \end{bmatrix}. \]  
(4)

The number of matrices \( Y(\cdot) \) or the sample number is \( M = 10000 \). The sample period is 0.1 s. We apply QR decomposition (5) to find the independent base dynamic parameters, with the orthonormal matrix \( Q \in \mathbb{R}^{M \times nN_s} \) and upper triangular matrix \( R \):
\[ W = QR. \]  
(5)

Supposing that the main diagonal elements \( r_{ii} \) in the \( i \)th column of \( R \) are zero, the corresponding columns of \( R \) are collected in \( R_i \), while the rest constitute \( R_i \):
\[ R = [R_1, R_2]. \]  
(6)

Similarly, the corresponding columns of \( W \) are collected consistent with \( R \). According to (2), we have
\[ W_{\pi} = \begin{bmatrix} [W_1]_{6M \times 52} & [W_2]_{6M \times 26} \end{bmatrix} \begin{bmatrix} \pi_1 \\ \pi_2 \end{bmatrix} = T = \begin{bmatrix} \tau(1) \\ \vdots \\ \tau(M) \end{bmatrix}, \]  
(7)

with independent \( \pi_1 \) and dependent \( \pi_2 \) which are collected from \( p \) in the same way as \( W \).

According to (6) and (7), we have
\[ T = W_1 \pi \ast = W_1 (\pi_1 + k\pi_2), \]  
(8)

where \( k = R_2 R_1^{-1} \).

Then, the unknown dynamic parameters are estimated by
\[ [\pi^*]_{52 \times 1} = (W_1^T W_1)^{-1} W_1^T T. \]  
(9)

The excitation reference trajectories for every joint are applied with the finite sum of 5 harmonic sine and cosine functions. The joint position, velocity, and acceleration of the \( i \)th joint are
\[ q_i(t) = \sum_{l=1}^{5} \left[ a_i \frac{w_j}{w_j} \sin (w_j t) - \frac{b_i}{w_j} \cos (w_j t) + q_{i0} \right], \]
\[ \dot{q}_i(t) = \sum_{l=1}^{5} \left[ a_i \cos (w_j t) + b_i \sin (w_j t) \right], \]
\[ \ddot{q}_i(t) = w_j^2 \sum_{l=1}^{5} \left[ a_i \cos (w_j t) + b_i \sin (w_j t) \right], \]  
(10)

where the fundamental frequency is \( w_j = 0.05 \) and \( q_{i0} \) is the offset of the joint position of the trajectory. The parameters \( a_i \) and \( b_i \) are determined by trial and error or the following optimization process. The excitation trajectory \( q^*(t) \) is determined by the optimization issue of (11) which is directly equivalent to optimizing the condition number for less estimation error while having less complexity and
calculation time:
\[
q^*(t) = \arg \min (W_1)
\]
subject to
\[
-q_{\text{max}} \leq q_i(t) = \sum_{i=1}^{5} \frac{a_i}{w_j} \sin (w_j t) - \frac{b_i}{w_j} \cos (w_j t) \leq q_{\text{max}},
\]
\[
-v_{\text{max}} \leq \dot{q}_i(t) = \sum_{i=1}^{5} a_i \cos (w_j t) + b_i \sin (w_j t) \leq v_{\text{max}},
\]
\[
-a_{\text{max}} \leq \ddot{q}_i(t) = w_j \sum_{i=1}^{5} a_i \cos (w_j t) + b_i \sin (w_j t) \leq a_{\text{max}},
\]
\[
q_i(t_0) = q_i(0) = \sum_{i=1}^{5} \frac{b_i}{w_j} + q_{\mu} = 0,
\]
\[
\dot{q}_i(t_0) = \dot{q}_i(0) = \sum_{i=1}^{5} a_i = 0,
\]
\[
\ddot{q}_i(t_0) = \ddot{q}_i(0) = \sum_{i=1}^{5} w_j b_i = 0,
\]
where \(q_{\text{max}} = [q_{\text{max} 1}, \ldots, q_{\text{max} 6}]^T = [153, 24, 40, 153, 90, 180]^T /180\pi, \quad v_{\text{max}} = [v_{\text{max} 1}, \ldots, v_{\text{max} 6}]^T = [296, 240, 310, 355, 267, 500]^T /180\pi, \quad a_{\text{max}} = [a_{\text{max} 1}, \ldots, a_{\text{max} 6}]^T = [1800, 1500, 1800, 1800, 1300, 3000]^T /180\pi \) are the constraint vectors for position, velocity, and acceleration, respectively.

It is possible to use any optimization method to solve the above optimization problem. In this paper, the parameters \(a_i\) and \(b_i\) are solved by the interior-point method. The maximum number of iterations is 20000. The initial values of \(a_i\) and \(b_i\) are chosen as random numbers within \([-0.5, 0.5]\).

The dynamic model is often not known accurately. For example, the structure of the friction model is generally difficult to know, let alone the parameter values. Furthermore, it is unrealistic to have precise parameter values in the model at all times through applying the least square method [51, 52] in one time since the dynamic parameters always change as the robot moves. Therefore, the following controller is inevitably designed in accordance with the inaccurate dynamic parameters.

### 3. Controller Design

The robotic dynamic model has the following properties [23, 27]:

(P1) The matrix \(M(q)\) is positive definite symmetric and satisfies \(M_{ij} \leq x_i x_j M(q) x \leq M_{ii} x_i^2\) for positive constants \(M, \tilde{M} > 0\).

(P2) \(M(q) - 2C(q, \dot{q})\) is skew-symmetric.

(P3) The matrix \(C(q, \dot{q})\) satisfies \(\|C(q, \dot{q})\| \leq \mu_c\) for positive constant \(\mu_c > 0\).

(P4) The vector \(G(q)\) satisfies \(\|G(q)\| \leq \mu_g\) for positive constant \(\mu_g > 0\).

From (9), we can obtain \(\tilde{M}(q), \tilde{C}(q, \dot{q}), \tilde{G}(q), \tilde{f}_c,\) and \(\tilde{f}_v\), which represent the estimation of \(M(q), C(q, \dot{q}), G(q), f_c,\) and \(f_v\), respectively. Define \(\tilde{M}(q) = \tilde{M}(q_d) - M(q), \tilde{C}(q, \dot{q}) = \tilde{C}(q_d, \dot{q}_d) - C(q, \dot{q}), \tilde{G}(q) = G(q_d) - G(q),\) and the nonnegative definite diagonal gain matrices \(K_p, K_D,\) and \(Y = \text{diag} \left(r_1, \ldots, r_6\right); \quad \dot{q}_d = [q_d^1, \ldots, q_d^6]^T\) represents the desired trajectory for the robot joints.

#### Table 1: Controller parameters of each joint.

<table>
<thead>
<tr>
<th>Joint</th>
<th>(K_p : K_D)</th>
<th>(r_s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>962260; 80000</td>
<td>1000</td>
</tr>
<tr>
<td>2</td>
<td>886260; 209500</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>209500; 69000</td>
<td>100</td>
</tr>
<tr>
<td>4</td>
<td>1852500; 42000</td>
<td>1000</td>
</tr>
<tr>
<td>5</td>
<td>489500; 26700</td>
<td>1000</td>
</tr>
<tr>
<td>6</td>
<td>87000; 7500</td>
<td>7000</td>
</tr>
</tbody>
</table>

#### Figure 2: Control scheme of the robot.
Define $e = q_d - q$. We choose the Lyapunov function (19) by differentiating (18) as follows:

$$V = \frac{1}{2} \dot{e}^T M(q) \dot{e} + \frac{1}{2} \dot{e}^T K_p e,$$

which is continuous and nonnegative. According to the properties of the robotic dynamic model, we can obtain

$$\dot{V} = \dot{e}^T M(q) \dot{e} + \frac{1}{2} \dot{e}^T \dot{M}(q) \dot{e} + \dot{e}^T K_p e$$

$$= \dot{e}^T (M(q) \ddot{e} + C(q, \dot{q}) \dot{e} + K_p e)$$

$$= \dot{e}^T (M(q) \ddot{q}_d + C(q, \dot{q}) \dot{q}_d + K_p e + G(q) + f_c \text{sign}(\dot{q}) + f_v \dot{q} - \tau).$$

### Table 2: Estimated dynamic parameters of the robot.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Joint 1</th>
<th>Joint 2</th>
<th>Joint 3</th>
<th>Joint 4</th>
<th>Joint 5</th>
<th>Joint 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_i$ (kg)</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>$m_{xi}$ (kg·m)</td>
<td>0.000000</td>
<td>1467.621893</td>
<td>120.973665</td>
<td>0.877570</td>
<td>-5.497015</td>
<td>-3.474394</td>
</tr>
<tr>
<td>$m_{yi}$ (kg·m)</td>
<td>0.000000</td>
<td>-485.168899</td>
<td>512.922191</td>
<td>-4.105343</td>
<td>49.201786</td>
<td>9.947225</td>
</tr>
<tr>
<td>$m_{zi}$ (kg·m)</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
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<tr>
<td>$I_{xxi}$ (kg·m$^2$)</td>
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<td>$I_{xyi}$ (kg·m$^2$)</td>
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<td>-4.105343</td>
<td>49.201786</td>
<td>9.947225</td>
</tr>
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<td>$I_{zzi}$ (kg·m$^2$)</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
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<tr>
<td>$I_{ai}$ (kg·m$^2$)</td>
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<td>315.581754</td>
<td>123.918230</td>
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<td>$f_{ci}$ (Nm·s/rad)</td>
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<td>4450.476442</td>
<td>8730.326694</td>
<td>2888.554652</td>
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<td>$f_{vi}$ (Nm·s/rad)</td>
<td>7102.364470</td>
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<td>3909.460084</td>
<td>1322.947888</td>
<td>1322.947888</td>
</tr>
</tbody>
</table>

Figure 3: Tracking performance of each robotic joint controlled by variable structure control.
Figure 4: Tracking performance of each robotic joint controlled by PD plus.

Figure 5: Tracking error comparison between variable structure control and PD plus.
Then, we design the controller
\[ \tau = \tau_{\text{dynamic}} + \tau_{\text{PD}}, \]  
\[ \tau_{\text{dynamic}} = \tilde{M}(q_d)\ddot{q}_d + \tilde{C}(q_d, \dot{q}_d)\dot{q}_d + \tilde{G}(q_d) + \tilde{F}_f, \]  
\[ \tau_{\text{PD}} = K_P e + K_D \dot{e} + \gamma \sign(\dot{e}), \]  
where
\[ \tilde{\sigma} = \tilde{M}(q)\ddot{q}_d + \tilde{C}(q, \dot{q})\dot{q}_d + \tilde{G}(q) + \tilde{F}_f, \]  
\[ \tilde{F}_f = f_c \sign(q_d) - f_c \sign(q) + \tilde{f}_v \dot{q}_d - f_v \dot{q}. \]

Herein, we choose
\[ r_i > \|\tilde{\sigma}\|_{\text{max}}, \]  

to have
\[ \dot{V} < 0. \]  

For the case \( n = 6 \), the iterative Newton-Euler scheme is approximately 100 times more efficient than the Lagrangian approach. Consequently, we normally realize (21) by the iterative Newton-Euler dynamics shown as follows.

(a) Outward Iterations \( i : 0 \rightarrow 5 \)
The joint \( i + 1 \) rotational velocity is
\[ \omega_{i+1} = R_i^{i+1} \omega_i + \dot{q}_{i+1}^* \tilde{Z}_{i+1}, \]  
where \( \tilde{Z}_{i+1} \) represents the axis pointing along the \( i + 1 \) joint axis and \( R_i^{i+1} \) is the rotation matrix.
**Figure 7:** Tracking performance of each robotic joint controlled by cascaded PD plus.

**Figure 8:** Tracking error of each robotic joint controlled by cascaded PD plus.
The angular acceleration from one link to the next is described by
\[ \ddot{\theta}_{i+1} = R_{i+1}^{i+1} \dot{\theta}_i + R_{i+1}^{i+1} \Gamma_i \dot{\theta}_i + \ddot{\theta}_{i+1} \cdot Z_{i+1}. \] (28)

Noticeably, (27) and (28) are different from the iterations in the textbook [22], since the velocity in (27) and acceleration in (28) of each joint are the desired velocity and acceleration, respectively.

Then, we obtain the linear acceleration of each link-frame origin through
\[ \ddot{v}_{i+1} = R_{i+1}^{i+1} \left( \dot{v}_i \times P_{i+1}^{i} + v_i \times (\dot{P}_{i}^{i+1} + \dot{v}_i) \right) \] (29)
and obtain the linear acceleration of the center of mass of each link
\[ \ddot{v}_{Ci+1} = \dot{v}_{i+1} \times P_{Ci+1}^{i} + v_{i+1} \times (\dot{P}_{Ci}^{i+1} + \dot{v}_i) \] (30)
where \( C_i \) represents a frame attached to each link and has its origin located at the center of mass of the link with the same orientation as the link frame \( \{ i \} \); \( P_{Ci}^{i+1} \) locates the position of \( C_{i+1} \) relative to the \( i \) th joint.

Then, we can obtain the inertial force and torque acting at the center of mass of each link (31) and (32) through Newton-Euler equations as follows:
\[ F_{i+1} = m_{i+1} \ddot{v}_{Ci+1}, \] (31)
\[ N_{i+1} = I_{Ci+1}^{i+1} \ddot{\theta}_{i+1} + I_{Ci+1}^{i+1} \dot{v}_{Ci+1}, \] (32)
where \( I_{Ci+1}^{i+1} \) determines the mass moments of inertia relative to the center of mass in the \( i \) th link.

(b) Inward Iterations \( i : 6 \rightarrow 1 \)
The force exerted on link \( i \) by link \( i-1 \) is
\[ f_i = R_{i-1}^{i} f_{i-1} + F_i. \] (33)

12The torque exerted on link \( i \) by link \( i-1 \) is
\[ \tau_i = N_i + R_{i-1}^{i} n_{i-1} + P_i^{i-1} f_i + P_{i-1}^{i} f_{i-1} \] (34)

Finally, we obtain the required joint torques
\[ \tau_i = n_i^T \dot{Z}_i, \] (35)
which constitute \( \tau_{\text{dynamic}} = [\tau_1, \cdots, \tau_6]^T \) in (21).

Controller (20) guarantees the stability of the system in theory. Furthermore, if we let \( r_i = 0, i = 1, \cdots, 6 \), the controller is the feedforward nonlinear control shown in Figure 10.6 in [22]. Furthermore, if we correct \( \tau_{\text{PD}} \) (22) into the following cascaded PD controller
\[ \tau_{\text{PD}} = K_{p}^d \left( (K_p e + K_D \dot{e}) - \dot{\theta} \right) + K_{d}^d \frac{d}{dt} \left( (K_p e + K_D \dot{e}) - \dot{\theta} \right). \] (36)

The position control loop (outer loop) is to maintain the positional trajectory tracking. The error in the position control loop provides the desired velocity for the velocity loop (inner loop). We apply a PD controller \( (K_p e + K_D \dot{e}) \) on the basis of the error between the desired and actual positions to generate the desired speed of the motor so that the actual position tracks the desired position closely. The velocity loop uses the PD controller plus dynamic feedforward compensation to generate the desired torque for the robot joint (motor) so that the actual velocity tracks the desired velocity closely. We normally adjust the gains of the joint controller: \( K_p, K_D, K_p^+, \text{and} K_D^+ \) to change the behaviors of the robot subject to variation in the dynamic model caused by various poses and payloads, variation in friction with temperature and time, and some disturbance torques owing to complicated velocity and acceleration coupling.

By removing the gravity compensation item of the controller (20), we can obtain the practical controller for the on-orbit space robot:
\[ \tau = \hat{M}(q_d) \ddot{q}_d + \hat{C}(q_d, \dot{q}_d) \dot{q}_d + \hat{f}_c \text{ sign } (\dot{q}_d) + \hat{\tau}_c, \] (37)

4. Experiment
The proposed variable structure method is applied to the robot, and the controller parameters of each joint are listed in Table 1. The estimated dynamic parameters of the robot are shown in Table 2. The tracking performance of each robotic joint is shown in Figure 3.

When we let \( r_i = 0, i = 0, \cdots, 6 \), the proposed variable structure controller degenerates into the PD plus. The tracking performance of each robotic joint controlled by PD plus is shown in Figure 4. The tracking error comparison between VS control and PD plus is shown in Figure 5.

Figures 3 and 4 show that both outputs of the robot controlled by the proposed VS control and PD plus are able to track the desired trajectories. Figure 5 shows that the tracking error curves of the robot controlled by the proposed VS method and PD plus are close. Hence, the drawing of partial enlargement of the output of the 3rd joint and the output of the 6th joint of the robot controlled by variable structure control and PD plus are shown in Figure 6 to make a further comparison.

Figure 6 shows that the tracking performance of the 3rd joint of the robot controlled by the proposed VS control is better than that controlled by PD plus for less tracking error from 27200 ms to 27800 ms and from 24500 ms to 25500 ms, while there is no obvious difference between the two methods acting at the 6th joint. These findings are possibly because the proposed VS control combines the PD plus dynamic feedforward compensation and the Bang-Bang control together, and the Bang-Bang control item \( \gamma \text{ sign } (\dot{\theta}) \) can be used to suppress all matching uncertainties and unpredictable system dynamics. In this experiment, the
Bang-Bang control item can provide timely compensation for more tracking errors caused by the modeling uncertainty and consequently improves both the tracking accuracy and tracking speed of the robot.

Then, we apply cascaded PD plus dynamic feedforward compensation control to the robotic manipulator for comparison with the aforementioned two methods. The cascaded PD controller parameters of each joint are listed in Table 3.

The tracking performance of each robotic joint is shown in Figure 7. The tracking error of each joint controlled by cascaded PD plus is shown in Figure 8.

With less effort spent on the adjustment of controller parameters in the experiment, the cascaded PD plus easily guarantees the tracking error of each joint of the robot within ±8 × 10⁻³ rad which is shown in Figure 8. The reason is summarized as follows: As demanded by the outer loop, the inner loop is adopted to generate the desired torque for the robot joint so that the actual velocity tracks the desired velocity closely. The outer loop is used to determine the velocity of the joint that minimizes position error. By comparison, we have tested the proposed VS control and PD plus by trial and error in our experiment to guarantee the tracking error of each joint of the robot within ±2 × 10⁻² rad which is shown in Figure 5. By comparing Figures 8 and 5, we find that the cascaded PD plus dynamic feedforward compensation control more easily guarantees the high-precision tracking performance than the proposed VS control or the PD plus in the experiment, under the roughly built robotic dynamics. Actually, the PD plus typically represents a class of single-loop feedback control. Furthermore, this also implies that many SMC or other variable structure controls based on a single-loop feedback strategy may not more easily achieve better performance than the cascaded PD method. Based on this, we recommend the cascaded PID plus feedforward dynamic model for the calculation of the desired torque in the 6-DOF serial robot rather than the single-loop feedback strategy. The controller structure may be more important than the adjustment of the controller parameters in the robotic system.

The controller design or the choice of the controller from different candidates for the space robot remains prudent work. One purpose of the above analyses and experimental results is to provide a source of prior knowledge and practice experience for the controller design and the test of the space robot. Furthermore, by removing gravity compensation, we can design the controller of the space station robot in the same way as the above controllers of the industrial robot.

5. Conclusions

To suggest simple yet effective controllers for the large and heavy serial multijoint space station robot, we made comparisons among our proposed variable structure method, PD plus and the cascaded PD plus on the industrial robot in this paper. First, the dynamic parameters of the robot are identified by the least square method. Second, to guarantee the stability of the system under modeling uncertainty in theory, a variable structure controller is designed based on the Lyapunov direct method. It can be separated into two parts: the PD plus and the sign function. Third, experiments show that the cascaded PD plus dynamic feedforward compensation control more easily guarantees the high-precision tracking performance than the proposed variable structure control or the PD plus, under the roughly built robotic dynamics.

Data Availability

The experimental data used to support the findings in this paper can be available from the first author and corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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