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Research Article

A Novel Nonlinear Quadrotor Attitude Regulation with Disturbances and Preassigned Convergences

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This article develops a novel unknown system dynamic estimator-based funnel control scheme for nonlinear quadrotor attitude regulation with preassigned convergence subject to parametric uncertainties and external perturbations. An invariant manifold equipped with first-order filtering is established. To online identify the lump disturbances, an unknown system dynamic estimator is employed with a simple formula, which need a lower computation burden. Based on aforementioned estimator, a novel funnel control via utilizing funnel variable is investigated, where an exponentially decaying funnel function is preset with a prior preassigned convergence for regulation angle error. The angle tracking errors are proved to be ultimately uniformly bounded, and angle regulation error can evolve within the preset funnel boundary. Simulation results demonstrate the effectiveness of the developed control scheme.

1. Introduction

In recent years, quadrotors have been one of obbligato aircrafts in monitoring, disaster rescue, forest fire detection, power inspection, express transportation [1–5], and other fields due to their simple structure, low-budget manufacturability, and flexible maneuverability. However, it is not trivial to design a controller for attitude regulation featured with multiple input and multiple output and having nonlinear systems. Besides, parametric uncertainties and unknown external environmental disturbances are ubiquitous and unavoidable in practical scenarios, which the performance of quadrotors may be affected by aforementioned lumped disturbances. Therefore, effective and robust attitude regulation control for quadrotors in the presence of uncertainty and external disturbance is a meaning and challenging problem.

With the requirement of strong disturbance rejection as well as precise control performance, some attempts have been made by adding feedforward compensation based on the controller. To guarantee the improvement of feedforward performance, the disturbance observer and corresponding theory have been investigated for quadrotors in traditional techniques, such as sliding mode observers (SMO) [6–9], function approximators by neural networks (NN) [10–13], fuzzy logistic system (FLS) [14, 15], and extended state observers (ESO) [16, 17]. In [8], a first-order SMO equipped with high gain observer is designed to estimate unknown disturbance. Based on the estimation of high-order SMO, the composite control is structured in position and attitude loop with different controller in [9], respectively. In [11], a robust and adaptive controller-based NN is conducted for quadrotor by introducing the generalized regression neural network. To realize the finite-time convergence, an adaptive integral sliding mode control is proposed with a novel fully connected recurrent neural networks with finite time learning process in [13]. In [14], an adaptive backstepping control is proposed with the command filtering technique for quadrotor trajectory tracking, where FLS is employed to estimate the uncertainty dynamics in the quadrotor model. By incorporating FLS into control scheme, the adaptive quantized control is considered for trajectory tracking in position loop and attitude loop in [15]. By constructing ESO to estimate the uncertainties, a robust state feedback controller is designed in [16] for linear quadrotor model derived by the flatness theory. In [17], a robust backstepping sliding mode control is exploited in position loop, and ESO is applied in the attitude loop for quadrotor to derive desired trajectory. Although the above literature achieves satisfactory performance with prominent disturbance estimation, the chattering phenomenon may arise owing to design of SMO. The heavy computational burden that is caused by function approximation with adjusting parameters repeatedly in NN and FLS is not practical in practical engineering applications. In addition, many piratical problems in engineer usually focus not only on steady-state error but also on overshoot and convergence time, which is not taken into account in above-mentioned controller design.

Another significant issue related to controlling performance is the preassigned convergence of the tracking error. To tackle this issue, a funnel boundary is first established to regulate both transient and steady-state behavior in [18], which depict the convergence by predefine funnel function. By utilizing funnel controller (FC), the tracking error is converted to a funnel variable such that it can remain with the proposed funnel boundary. Compared to prescribed performance control (PPC) proposed in [19], it is flexible to design the corresponding controller based on funnel control without encountering nonsingularity problem, which will exist during presenting inverse transformation function in PPC. Thus, there has been some research focusing on funnel controller. In [20], a low-complexity FC tracking controller is developed for a class of system with more inputs than outputs. For arbitrary relative degree nonlinear system, the tracking controller is considered such that the evolvement of tracking error is within the range of predefined funnel [21]. By introducing high-gain observer to derive virtual output, a FC is proposed in [22] for nonlinear systems via transforming arbitrary relative degree system into relative degree one system. However, the FC in [21, 22] are only developed for certain system. For nonlinear system with disturbance, the FC-based backstepping control in [23] is explicated for quadrotors with improved funnel function, where SMO is employed to estimate the disturbance. In [24], by incorporating dynamic surface control with NN, an adaptive tracking controller is developed for hydraulic manipulators to ensure that tracking error evolves within the new funnel function. Although the tracking performance can be guaranteed with the funnel boundary by the robust control in [23–25], the high estimation of disturbance may affect the tracking performance.

The unknown system dynamic estimation (USDE) proposed by [26] is a novel estimator with simple formula and few parameter adjustment to handle the uncertainties and disturbance for nonlinear system, avoiding utilizing the function approximation by NN or FLS with inducing computational burden. Different from the ESO and other estimators, there is only one parameter tuning without intermediate variable during the USDE designer. Based on improved unknown disturbance estimator (MUDE), a motion control is considered in [27] for robotic systems to settle the lumped disturbance. In [28], sliding mode control is developed by employing USDE to compensate for disturbance, achieving fast convergence and strong robustness. However, due to quadrotor with coupling and multivariables, it is challenging to design USDE-based controller for quadrotor attitude regulation, unlike the above literature USDE aimed at system in a chain of integrators. In addition, the above controller cannot guarantee the preassigned convergence of tracking error, which may cause disaster practical with obstacle. Hence, it is imperative and urgent to investigate USDE-based funnel control to eliminate tracking errors of quadrotor attitude.

Inspired by the foregoing statements, a novel USDE-based tracking control with funnel control is investigated for quadrotor attitude regulation with total disturbances. First, the USDE is adopted for quadrotor for precise estimation of the uncertainty and external disturbance with less computational burden. Second, a novel funnel control involving the approximation by USDE is designed to such that the attitude regulation error remains the preset funnel function. The novelties of this work are twofold.

(i) Unlike the above stating SMO-based controller [6–9] and controller with function approximators [10–15], where chattering and heavy computational burden with adjusting parameters always arise, we adopt the unknown system dynamic estimator (USDE) with an invariant manifold for quadrotor attitude regulation to confirm the uncertainty in quadrotor model. In comparison with the ESO-based controller [16, 17], due to state in quadrotor calculated with simple filter, a perspicuous framework consisting of USDE with low computation is conducted

(ii) A novel USDE-based funnel control is proposed for quadrotor attitude regulation to enable preassigned convergence with existence of estimation error. Different from the conventional controllers [13, 29], where the tracking error of quadrotors is only guaranteed to be ultimately uniformly bounded (UUB), a FC is evolved with the estimation of USDE with novel funnel variable to guarantee preassigned convergences defined by funnel function, which is regarded as the prior criterion for transient-state and stead-state regulation errors

2. Problem Statements

Figure 1 describes the quadrotor attitude dynamics by establishing the body-fixed coordinate $\{\mathbf{B}\}$ and the inertial-fixed coordinate $\{\mathbf{E}\}$, and F_i denote the thrusts generated by the rotor actuations. Based on the previous work [30, 31], a model can be established when faced with external disturbances.

$$\begin{cases} \dot{\mathbf{\Theta}} = \mathbf{\Gamma}\boldsymbol{\omega}, \\ \mathbf{J}\dot{\boldsymbol{\omega}} = -\boldsymbol{\omega} \times \mathbf{J}\boldsymbol{\omega} + \mathbf{F} + \mathbf{d}, \end{cases} \tag{1}$$

where $\mathbf{\Theta} = [\phi, \theta, \psi]^T$ is the Euler angle in frame $\{\mathbf{E}\}$, consisting of roll angle ϕ, pitch angle θ, and yaw angle ψ. $\boldsymbol{\omega} = [\dot{\phi}, \dot{\theta}, \dot{\psi}]^T$, where $\dot{\phi}, \dot{\theta}, \dot{\psi}$ denotes the angular rate. In addition, matrix $\mathbf{\Gamma} = [1, \tan\theta \sin\phi, \tan\theta \cos\phi\,; 0, \cos\phi, -\sin\phi\,;$

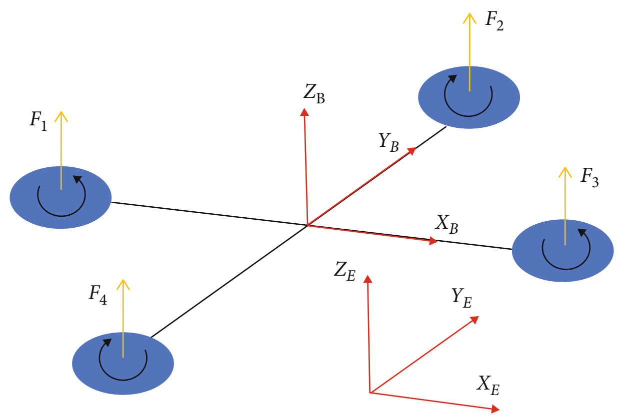

Figure 1: Elementary diagram of quadrotor.

$0, \sec\theta \sin\phi, \sec\theta \cos\phi]$ reveals the relationship between angle $\boldsymbol{\Theta}$ and angular velocity $\boldsymbol{\omega}$ in the attitude loop. $\mathbf{J} = \mathrm{diag}\,(J_\phi + J_\phi^*, J_\theta + J_\theta^*, J_\psi + J_\psi^*)$ stands for the positive diagonal matrix, where J_ϕ, J_θ, J_ψ are inertia nominal moments and $J_\phi^*, J_\theta^*, J_\psi^*$ are inertia moment uncertainties. The control input vector is defined as $\mathbf{F} = [F_\phi, F_\theta, F_\psi]^T$. And $\mathbf{d} = [d_\phi, d_\theta, d_\psi]^T$ could be used to represent the external disturbances faced by the attitude dynamics of the quadrotors.

Furtherly, it can be obtained along (1) with $\mathbf{J}_\Theta = \mathrm{diag}\,(J_\phi, J_\theta, J_\psi)$ and $\mathbf{J}_\Theta^* = \mathrm{diag}\,(J_\phi^*, J_\theta^*, J_\psi^*)$ representing uncertainty parameters as

$$\ddot{\boldsymbol{\Theta}} = -\boldsymbol{\Gamma}(\mathbf{J}_\Theta)^{-1}\boldsymbol{\omega} \times \mathbf{J}\boldsymbol{\omega} + \boldsymbol{\Gamma}(\mathbf{J}_\Theta)^{-1}(\mathbf{d} - \mathbf{J}_\Theta^*\dot{\boldsymbol{\omega}}) + \dot{\boldsymbol{\Gamma}}\boldsymbol{\omega} + (\mathbf{J}_\Theta)^{-1}\boldsymbol{\Gamma}\mathbf{F}. \tag{2}$$

Taking into account $\boldsymbol{\Delta} = -\boldsymbol{\Gamma}(\mathbf{J}_\Theta)^{-1}\boldsymbol{\omega} \times \mathbf{J}\boldsymbol{\omega} + \boldsymbol{\Gamma}(\mathbf{J}_\Theta)^{-1}(\mathbf{d} - \mathbf{J}_\Theta^*\dot{\boldsymbol{\omega}}) + \dot{\boldsymbol{\Gamma}}\boldsymbol{\omega}$ and $\mathbf{u} = (\mathbf{J}_\Theta)^{-1}\boldsymbol{\Gamma}\mathbf{F}$ and introducing new vector variable with $\boldsymbol{\Omega}_1 = \boldsymbol{\Theta} = [\Omega_{1,\phi}, \Omega_{1,\theta}, \Omega_{1,\psi}]^T$ and $\boldsymbol{\Omega}_2 = \boldsymbol{\Gamma}\boldsymbol{\omega} = [\Omega_{2,\phi}, \Omega_{2,\theta}, \Omega_{2,\psi}]^T$, it can be rewritten as a chain of integrators:

$$\begin{cases} \dot{\boldsymbol{\Omega}}_1 = \boldsymbol{\Omega}_2, \\ \dot{\boldsymbol{\Omega}}_2 = \mathbf{u} + \boldsymbol{\Delta}. \end{cases} \tag{3}$$

The aim of this paper is to design attitude regulation control for quadrotor based on unknown system dynamic estimator to deal with total disturbances in model, which can accurately bring the angles to converge preset attitude with preassigned convergence.

3. Main Results

3.1. Filter-Based Unknown System Dynamic Estimator Design. Noting that $\boldsymbol{\Delta}$ is unknown due to the lump disturbances, which is mainly based on the second equation of (3), USDE is designed to estimate $\boldsymbol{\Delta}$. In line with [26, 32], filtering operations are introduced firstly for the available states $\boldsymbol{\Omega}_2$ and $\mathbf{u}$.

$$\begin{cases} k\dot{\boldsymbol{\Omega}}_2^{\mathrm{f}} + \boldsymbol{\Omega}_2^{\mathrm{f}} = \boldsymbol{\Omega}_2,\ \boldsymbol{\Omega}_2^{\mathrm{f}}(0) = \mathbf{0} \in \mathbb{R}^3, \\ k\dot{\mathbf{u}}^{\mathrm{f}} + \mathbf{u}^{\mathrm{f}} = \mathbf{u},\ \mathbf{u}^{\mathrm{f}}(0) = \mathbf{0} \in \mathbb{R}^3, \end{cases} \tag{4}$$

where control inputs $\mathbf{u} = [u_\phi, u_\theta, u_\psi]^T$, $\boldsymbol{\Omega}_2^{\mathrm{f}}$, and $\mathbf{u}^{\mathrm{f}}$ represent the corresponding auxiliary filtering vector for $\boldsymbol{\Omega}_2$ and $\mathbf{u}$ and k is the adjustable filtering parameters.

Lemma 1 [33]. *Given the positive filtering parameter $k \in (0,+\infty)$, the auxiliary vector $\boldsymbol{\eta} = k^{-1}(\boldsymbol{\Omega}_2 - \boldsymbol{\Omega}_2^{\mathrm{f}}) - (\mathbf{u}^{\mathrm{f}} + \boldsymbol{\Delta})$ is bounded and the following condition holds:*

$$\lim_{k \to 0}\left\{\lim_{t \to \infty}\left[\frac{1}{k}\left(\boldsymbol{\Omega}_2 - \boldsymbol{\Omega}_2^{\mathrm{f}}\right) - \left(\mathbf{u}^{\mathrm{f}} + \boldsymbol{\Delta}\right)\right]\right\} = 0. \tag{5}$$

Notably, the auxiliary vector $\boldsymbol{\eta}$ denotes the invariant manifold that indicates a mapping between filtered vectors $\boldsymbol{\Omega}_2^{\mathrm{f}}$ and $\mathbf{u}^{\mathrm{f}}$ and lumped disturbances $\boldsymbol{\Delta}$. Therefore, the USDE can be developed based on filtered vectors as

$$\widehat{\boldsymbol{\Delta}} = \frac{1}{k}\left(\boldsymbol{\Omega}_2 - \boldsymbol{\Omega}_2^{\mathrm{f}}\right) - \mathbf{u}^{\mathrm{f}}, \tag{6}$$

where $\widehat{\boldsymbol{\Delta}} = [\widehat{\Delta}_\phi, \widehat{\Delta}_\theta, \widehat{\Delta}_\psi]^T$ is conducted to approximate lumped disturbances. To characterize the estimation, the following assumption is necessary.

Assumption 2 [34–36]. The time derivatives of unknown lump disturbances can be bounded with an unknown positive constant δ, i.e., $\|\dot{\boldsymbol{\Delta}}\| \le \delta$.

Theorem 3. *For angular velocity dynamics and estimation (6) with filtered vector in (4), the disturbance estimation errors $\tilde{\boldsymbol{\Delta}} = \boldsymbol{\Delta} - \widehat{\boldsymbol{\Delta}}$ can converge to the following neighborhood of origin:*

$$\|\tilde{\boldsymbol{\Delta}}\| \le \sqrt{\|\tilde{\boldsymbol{\Delta}}(0)\|^2 e^{-t/k} + k^2\delta^2}. \tag{7}$$

Proof. By adding the first-order filter $1/(ks+1)$ on the second equation of (3), it can be given with Laplace operator s in the form of

$$\frac{1}{ks+1}\dot{\boldsymbol{\Omega}}_2 = \frac{1}{ks+1}\mathbf{u} + \frac{1}{ks+1}\boldsymbol{\Delta}. \tag{8}$$

From the quadrotor model (1), its transformation (3), and the filtering manipulation (4), we can get

$$\dot{\boldsymbol{\Omega}}_2^{\mathrm{f}} = \frac{1}{k}\left(\boldsymbol{\Omega}_2 - \boldsymbol{\Omega}_2^{\mathrm{f}}\right) = \mathbf{u}^{\mathrm{f}} + \boldsymbol{\Delta}^{\mathrm{f}}, \tag{9}$$

with $\boldsymbol{\Delta}^{\mathrm{f}} = \boldsymbol{\Delta}/(ks+1)$, so that $\boldsymbol{\Delta}^{\mathrm{f}} = \widehat{\boldsymbol{\Delta}}$ can be deduced from (6).

Thus, the estimation errors can be expressed by

$$\tilde{\boldsymbol{\Delta}} = \boldsymbol{\Delta} - \boldsymbol{\Delta}^{\mathrm{f}} = \frac{ks}{ks+1}\boldsymbol{\Delta}, \tag{10}$$

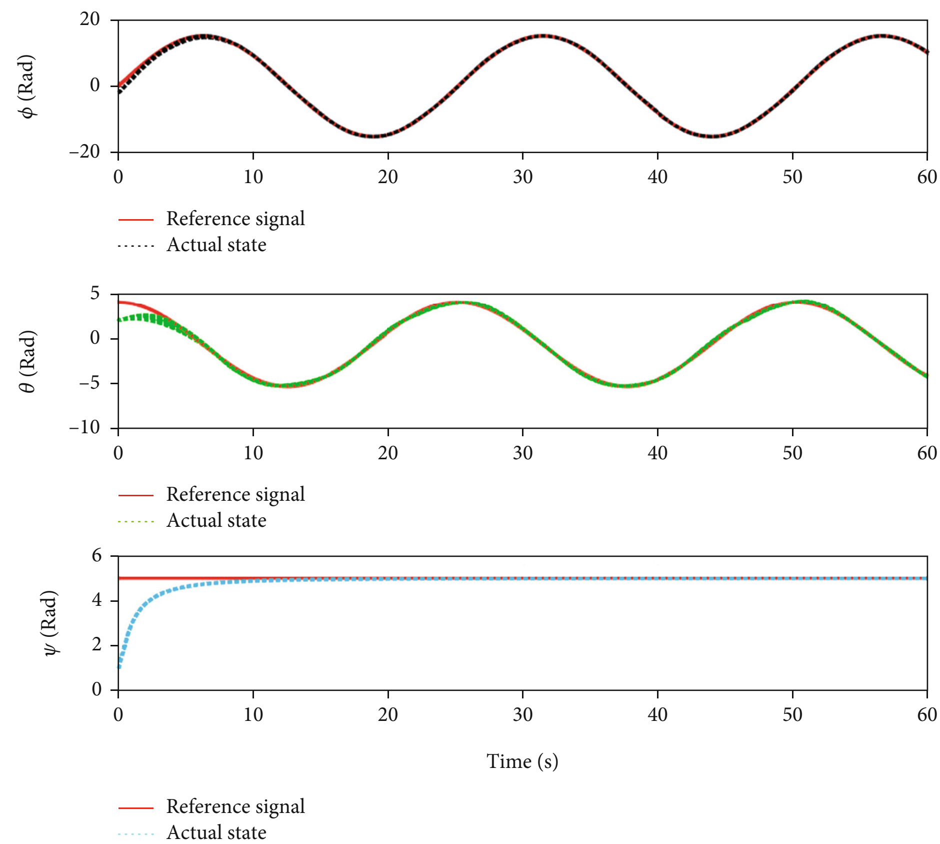

Figure 2: Response of attitude regulation.

and its derivative with respect to time will then be given along (1), (3), and (4) as

$$\begin{aligned}\dot{\tilde{\boldsymbol{\Delta}}} &= \dot{\boldsymbol{\Delta}} - \dot{\boldsymbol{\Delta}}^{\mathbf{f}} = \dot{\boldsymbol{\Delta}} - \frac{\dot{\boldsymbol{\Omega}}_2 - \dot{\boldsymbol{\Omega}}_2^{\mathbf{f}} + k\dot{\mathbf{u}}^{\mathbf{f}}}{k} \\ &= \dot{\boldsymbol{\Delta}} - \frac{\dot{\boldsymbol{\Omega}}_2 - \left(\boldsymbol{\Omega}_2 - \boldsymbol{\Omega}_2^{\mathbf{f}}\right)/k + k\dot{\mathbf{u}}^{\mathbf{f}}}{k} \\ &= \dot{\boldsymbol{\Delta}} - \frac{\mathbf{u} + \boldsymbol{\Delta} - \left(\boldsymbol{\Omega}_2 - \boldsymbol{\Omega}_2^{\mathbf{f}}\right)/k + k\dot{\mathbf{u}}^{\mathbf{f}}}{k} \\ &= \dot{\boldsymbol{\Delta}} - \frac{\boldsymbol{\Delta} - \boldsymbol{\Delta}^{\mathbf{f}}}{k}.\end{aligned} \quad (11)$$

Now, the estimation error $\tilde{\boldsymbol{\Delta}}$ is investigated by considering the following Lyapunov function:

$$V_1 = \frac{1}{2}\tilde{\boldsymbol{\Delta}}^{\mathbf{T}}\tilde{\boldsymbol{\Delta}}. \quad (12)$$

Based on (11), the time derivative of V_1 can be expressed as

$$\begin{aligned}\dot{V}_1 &= \tilde{\boldsymbol{\Delta}}^{\mathbf{T}}\dot{\tilde{\boldsymbol{\Delta}}} = \tilde{\boldsymbol{\Delta}}^{\mathbf{T}}\left(\dot{\boldsymbol{\Delta}} - \frac{\boldsymbol{\Delta} - \boldsymbol{\Delta}^{\mathbf{f}}}{k}\right) \\ &= \tilde{\boldsymbol{\Delta}}^{\mathbf{T}}\dot{\boldsymbol{\Delta}} - \frac{1}{k}\tilde{\boldsymbol{\Delta}}^{\mathbf{T}}\left(\boldsymbol{\Delta} - \boldsymbol{\Delta}^{\mathbf{f}}\right) \\ &= \tilde{\boldsymbol{\Delta}}^{\mathbf{T}}\dot{\boldsymbol{\Delta}} - \frac{1}{k}\tilde{\boldsymbol{\Delta}}^{\mathbf{T}}\tilde{\boldsymbol{\Delta}}.\end{aligned} \quad (13)$$

It can be easily derived from Young's inequality $ab \leq (a^2\beta/2) + (b^2/2\beta)$ for positive constant β that

$$\dot{V}_1 \leq \frac{1}{2k}\left\|\tilde{\boldsymbol{\Delta}}\right\|^2 + \frac{k\delta^2}{2} - \frac{1}{k}\tilde{\boldsymbol{\Delta}}^{\mathbf{T}}\tilde{\boldsymbol{\Delta}} = -\frac{1}{k}V_1 + \frac{k\delta^2}{2}. \quad (14)$$

One will have

$$V_1(t) \leq V_1(0)e^{-t/k} + \frac{k^2\delta^2}{2}. \quad (15)$$

Thus, we obtain the following conclusion about estimation errors:

$$\left\|\tilde{\boldsymbol{\Delta}}\right\| = \sqrt{2V_1(t)} \leq \sqrt{\left\|\tilde{\boldsymbol{\Delta}}(0)\right\|^2 e^{-t/k} + k^2\delta^2}. \quad (16)$$

It can be shown that, for the given filtering parameter $k \longrightarrow 0$, the estimation error $\tilde{\boldsymbol{\Delta}}$ is guaranteed to converge exponentially to the origin when time approximates to be infinite. □

3.2. Controller Design. During the controller design, $\mathbf{r} = [\phi_d, \theta_d, \psi_d]^T$ represents the given reference signal. Then, $\mathbf{e} = \boldsymbol{\Omega}_1 - \mathbf{r}$, where $\mathbf{e} = [e_\phi, e_\theta, e_\psi]^T$ is defined as the angle tracking error. In order to drive the tracking error within the preset funnel function $-\vartheta_i(t) < e_i(t) < \vartheta_i(t)$, the funnel boundary is described by the following function as [26]

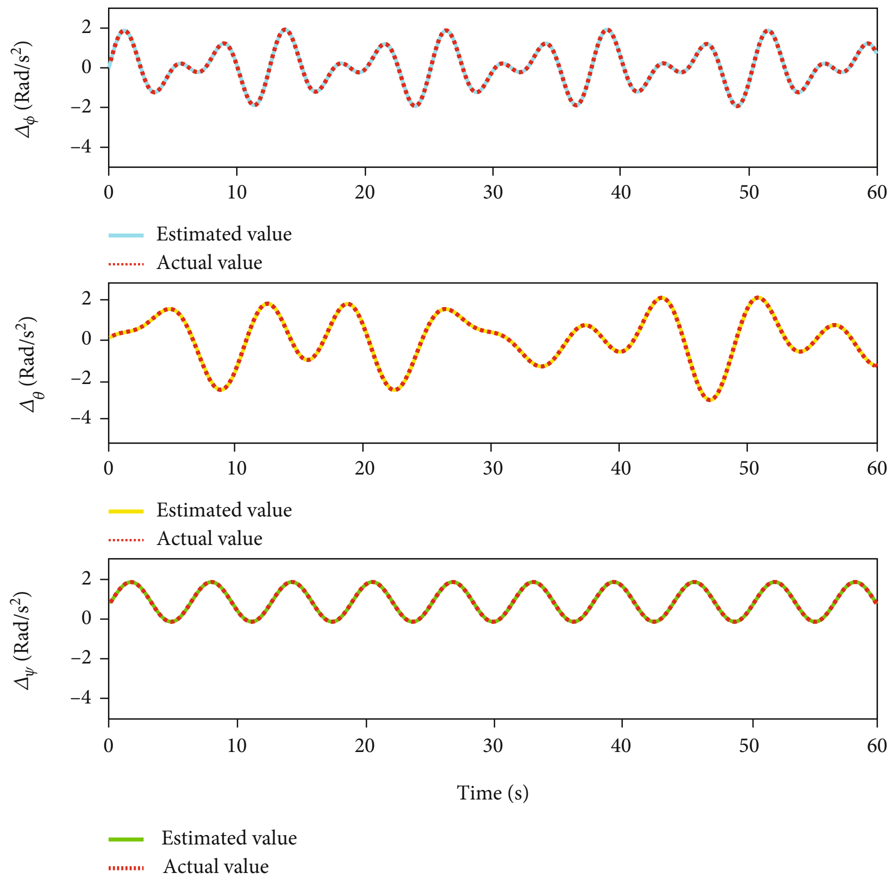

FIGURE 3: Estimation performances for total disturbances based on USDE.

$$\vartheta_i(t) = \alpha e^{-lt} + \gamma, \text{ for } i = \phi, \theta, \psi, \tag{17}$$

where parameters α, γ, l are selected to meet the conditions that $\alpha \geq \gamma > 0$ and $|e_i(0)| < \vartheta_i(0) = \alpha + \gamma$.

To fulfill preassigned convergence, we consider a novel funnel variable:

$$\zeta_i(t) = \frac{e_i(t)}{\sqrt{\vartheta_i(t) - |e_i(t)|}}, \text{ for } i = \phi, \theta, \psi, \tag{18}$$

with derivative of (18) yields

$$\dot{\zeta}_i = \frac{\dot{e}_i\left(\sqrt{\vartheta_i - |e_i|}\right) - (1/2)e_i\left(\dot{\vartheta}_i - |\dot{e}_i|\right)^{-1/2}(\vartheta_i - |e_i|)}{\vartheta_i - |e_i|}, \text{ for } i = \phi, \theta, \psi. \tag{19}$$

Furthermore, it could be redescribed as

$$\dot{\zeta}_i = \frac{1}{\sqrt{\vartheta_i - |e_i|}}\left(\dot{e}_i - \frac{(1/2)\left(\dot{\vartheta}_i - |\dot{e}_i|\right)e_i}{\vartheta_i - |e_i|}\right) = \chi_i(\dot{e}_i + \mu_i), \tag{20}$$

where $\chi_i = 1/\sqrt{\vartheta_i - |e_i|}$ and $\mu_i = -((\dot{\vartheta}_i - |\dot{e}_i|)e_i)/(2(\vartheta_i - |e_i|))$.

Thus, the virtual control input τ_i can be established by stabilizing the funnel variable $\zeta_i(t)$:

$$\tau_i = -k_{\Theta,i}\chi_i^{-1}\zeta_i + \dot{r}_i - \mu_i, \text{ for } i = \phi, \theta, \psi, \tag{21}$$

where the subsystem gain $k_{\Theta,i} > 0$ can be adjusted to stabilize system. Naturally, we can get angular rate errors:

$$\mathbf{z} = \mathbf{\Omega_2} - \boldsymbol{\tau}, \tag{22}$$

where $\mathbf{z} = [z_\phi, z_\theta, z_\psi]^T$ and $\boldsymbol{\tau} = [\tau_\phi, \tau_\theta, \tau_\psi]^T$.

Based on the dynamical model (3), calculating the differential of angular rate errors yields

$$\dot{\mathbf{z}} = \mathbf{u} + \boldsymbol{\Delta} - \dot{\boldsymbol{\tau}}. \tag{23}$$

Noting that $\boldsymbol{\Delta}$ is unavailable, the control input $\mathbf{u}$ can be constructed via substituting the disturbance estimation (6) as follows:

$$u_i = -k_{\omega,i}z_i + \dot{\tau}_i - \widehat{\Delta}_i \text{ for } i = \phi, \theta, \psi, \tag{24}$$

where the controller gain $k_{\omega,i} > 0$ can be regulated.

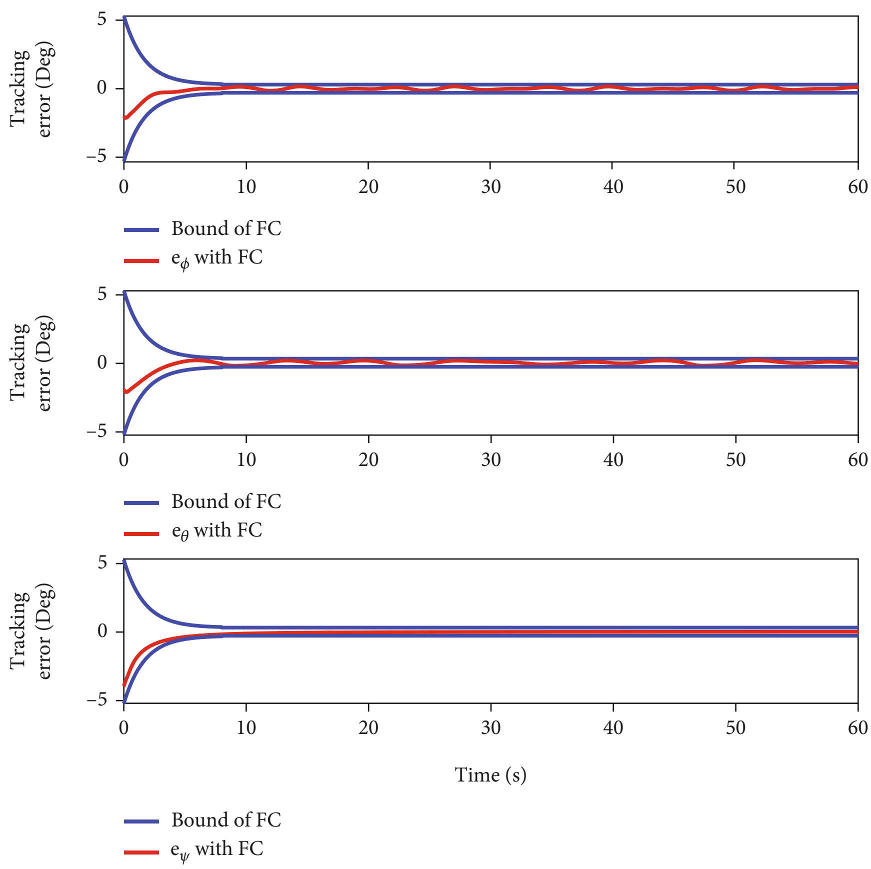

FIGURE 4: The angle tracking errors under the FC.

4. Stability Analysis

To depict the controller property, we first give the statements on funnel-based angle errors and angle rate error. The derivate of funnel vector can be rewritten with (21) as

$$\dot{\zeta}_i = -k_{\Theta,i}\zeta_i + \chi_i z_i, \text{ for } i = \phi, \theta, \psi. \tag{25}$$

Likewise, one can take derivation of angular rate errors along (24) that

$$\dot{z}_i = -k_{\omega,i} z_i + \tilde{\Delta}_i, \text{ for } i = \phi, \theta, \psi. \tag{26}$$

Theorem 4. *For the system (3), the proposed novel control laws (21) and (24) for angle and angle rate with estimation from USDE (6) can drive all the regulation error into the neighborhood of origin. Moreover, the angle errors $e_i(t)$ can be regulated with preassigned convergence via preset funnel function provided that the premier angle errors exist in interval $e_i(0) \in (-\vartheta_i(0), \vartheta_i(0))$ with positive variable $\vartheta_i(0)(i = \phi, \theta, \psi)$ and corresponding suitable controller gains $\mathbf{k}_\Theta$ and $\mathbf{k}_\omega$.*

Proof. Taking into account the whole attitude kinetics, we can construct the Lyapunov function:

$$V = \frac{1}{2}\sum_{i=\phi,\theta,\psi}\left(\zeta_i^2 + z_i^2\right). \tag{27}$$

Based on error dynamics (25) and (26), the derivation of (27) can be taken as

$$\begin{aligned}\dot{V} &= \sum_{i=\phi,\theta,\psi}\left(\zeta_i\dot{\zeta}_i + z_i\dot{z}_i\right)\\ &= \sum_{i=\phi,\theta,\psi}\left(\zeta_i(-k_{\Theta,i}\zeta_i + \chi_i z_i) + z_i\left(-k_{\omega,i}z_i + \tilde{\Delta}_i\right)\right)\\ &= \sum_{i=\phi,\theta,\psi}\left(-k_{\Theta,i}\zeta_i^2 - k_{\omega,i}z_i^2 + \zeta_i\chi_i z_i + z_i\tilde{\Delta}_i\right).\end{aligned} \tag{28}$$

Then, the second term and last term can be relaxed by Young's inequality:

$$\begin{aligned}\sum_{i=\phi,\theta,\psi}|\zeta_i\chi_i z_i| &\le \frac{1}{2}\lambda_{\max}(\boldsymbol{\chi})\left(\|\boldsymbol{\zeta}\|^2 + \|\mathbf{z}\|^2\right),\\ \sum_{i=\phi,\theta,\psi}\left|z_i\tilde{\Delta}_i\right| &\le \frac{1}{2}\|\mathbf{z}\|^2 + \frac{1}{2}\left\|\tilde{\boldsymbol{\Delta}}\right\|^2,\end{aligned} \tag{29}$$

where $\boldsymbol{\zeta} = [\zeta_\phi, \zeta_\theta, \zeta_\psi]^T$, $\boldsymbol{\chi} = \text{diag}\,(\chi_\phi, \chi_\theta, \chi_\psi)$, and its maximum eigenvalue is expressed as $\lambda_{\max}(\cdot)$.

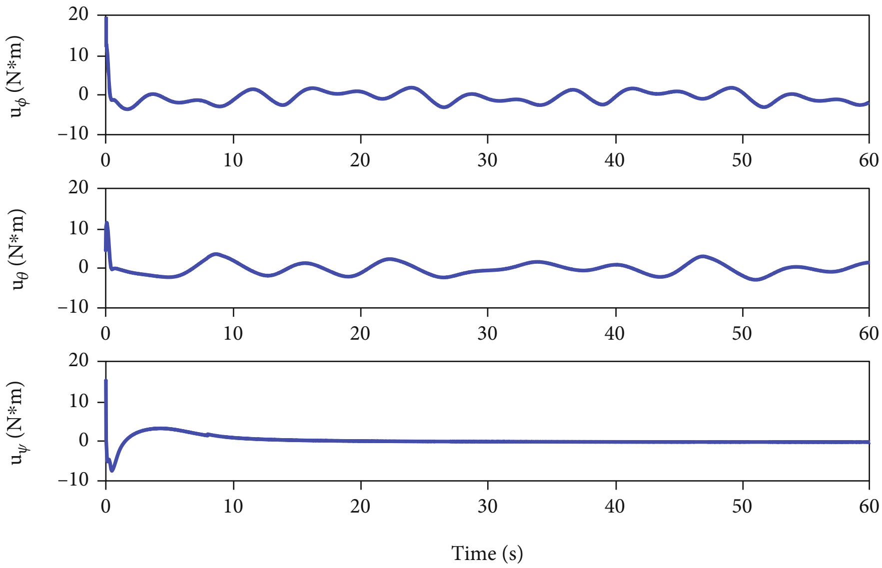

Figure 5: Control inputs.

Therefore, we can restate (28) with simple operation as

$$\begin{aligned}\dot{V} \le & -\left(\lambda_{\min}(\mathbf{k}_\Theta) - \frac{1}{2}\lambda_{\max}(\boldsymbol{\chi})\right)\|\boldsymbol{\zeta}\|^2 \\ & - \left(\lambda_{\min}(\mathbf{k}_\omega) - \frac{1}{2}\lambda_{\max}(\boldsymbol{\chi}) - \frac{1}{2}\right)\|\mathbf{z}\|^2 + \frac{1}{2}\left\|\tilde{\boldsymbol{\Delta}}\right\|^2,\end{aligned} \tag{30}$$

where $\mathbf{k}_\Theta = \text{diag}\,(k_{\Theta,\phi}, k_{\Theta,\theta}, k_{\Theta,\psi})$, $\mathbf{k}_\omega = \text{diag}\,(k_{\omega,\phi}, k_{\omega,\theta}, k_{\omega,\psi})$, and $\lambda_{\min}(\cdot)$ is the minimum value of the corresponding matrix.

Based on Theorem 3 results, one has

$$\lim_{t\longrightarrow\infty}\left\|\tilde{\boldsymbol{\Delta}}\right\| \le k\delta. \tag{31}$$

We can express $\dot{V}$ as

$$\begin{aligned}\dot{V} \le & -\left(-\frac{1}{2}\lambda_{\max}(\boldsymbol{\chi}) + \lambda_{\min}(\mathbf{k}_\Theta)\right)\|\boldsymbol{\zeta}\|^2 \\ & - \left(-\frac{1}{2}\lambda_{\max}(\boldsymbol{\chi}) + \lambda_{\min}(\mathbf{k}_\omega) - \frac{1}{2}\right)\|\mathbf{z}\|^2 + \frac{1}{2}k^2\delta^2,\end{aligned} \tag{32}$$

so that we can integrate (32) and derive

$$\dot{V} \le -\kappa V + \sigma, \tag{33}$$

where $\sigma = k^2\delta^2/2$ and $\kappa = \min\{2\kappa_1, 2\kappa_2,\} > 0$ with suitable controller gains satisfying

$$\begin{aligned}\kappa_1 &= -\frac{1}{2}\lambda_{\max}(\boldsymbol{\chi}) + \lambda_{\min}(\mathbf{k}_\Theta) > 0, \\ \kappa_2 &= -\frac{1}{2}\lambda_{\max}(\boldsymbol{\chi}) + \lambda_{\min}(\mathbf{k}_\omega) - \frac{1}{2} > 0.\end{aligned} \tag{34}$$

Table 1: Parameters of controllers.

Sections	Values
USDE	$k = 0.01$
Funnel control	$\alpha = 5, l = 0.6, \gamma = 0.3$
Control gains	$k_{\Theta,\phi} = 1, k_{\Theta,\theta} = 1, k_{\Theta,\psi} = 1, k_{\omega,\phi} = 4,$ $k_{\omega,\theta} = 4, k_{\omega,\psi} = 4$

Furthermore, by solving the inequality (33), it can be obtained as

$$0 \le V(t) \le \frac{\sigma}{\kappa}\left(1 - e^{-\kappa t}\right) + V(0)e^{-\kappa t}. \tag{35}$$

The boundedness of tracking errors $\boldsymbol{\zeta}$ and $\mathbf{z}$ can be guaranteed from (35). When the time tends to infinity, the upper boundedness can be indicated as

$$\begin{aligned}\|\boldsymbol{\zeta}\| &\le \sqrt{2\sigma/\kappa}, \\ \|\mathbf{z}\| &\le \sqrt{2\sigma/\kappa}.\end{aligned} \tag{36}$$

Then, for any positive constant ε, it can be further expressed as

$$\zeta_i^2 \le \frac{2\sigma}{\kappa} < \varepsilon, \text{for } i = \phi, \theta, \psi. \tag{37}$$

In particularly, we can obtain along (18) with $\zeta_i(t)$ as

$$\begin{aligned}\frac{e_i^2}{\sqrt{\vartheta_i(t) + |e_i(t)|}\sqrt{\vartheta_i(t) - |e_i(t)|}} &\le \frac{e_i^2}{\sqrt{\vartheta_i(t) - |e_i(t)|}\sqrt{\vartheta_i(t) - |e_i(t)|}} \\ &= \frac{e_i^2}{\vartheta_i(t) - |e_i(t)|} < \varepsilon.\end{aligned} \tag{38}$$

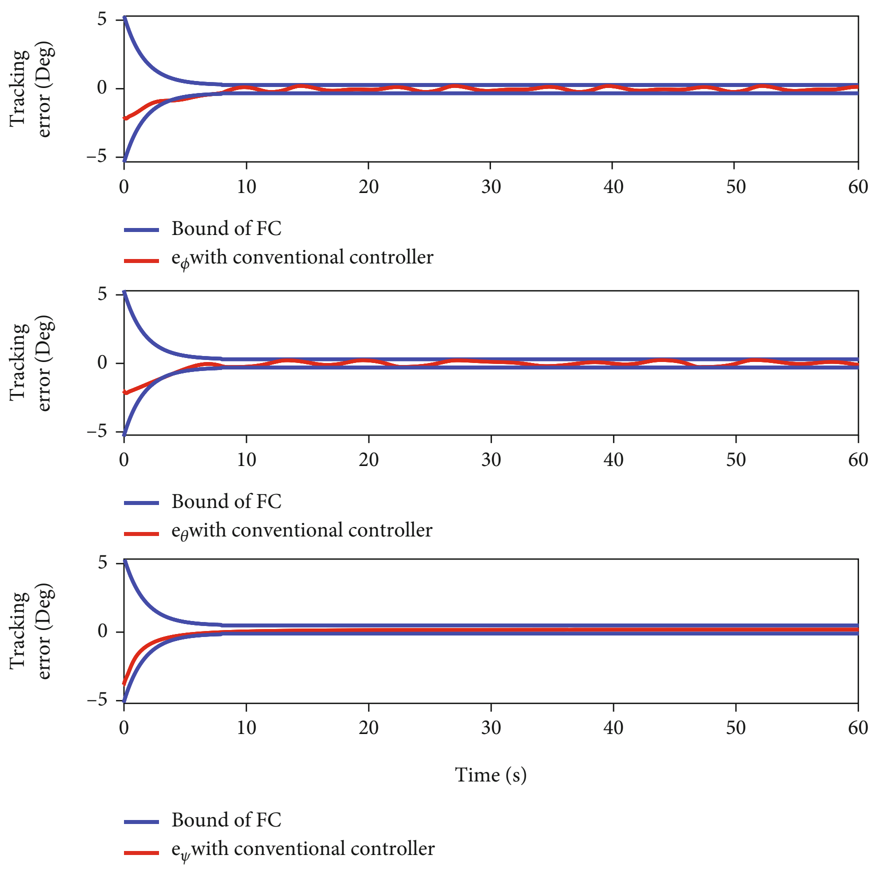

FIGURE 6: The angle tracking errors under conventional controller [37].

By simple arithmetic, angle error satisfies

$$e_i^2 \le \varepsilon\sqrt{\vartheta_i^2 - e_i^2} < \varepsilon\sqrt{\vartheta_i^2 - e_i^2 + \frac{e_i^2\vartheta_i^2}{\varepsilon}}. \tag{39}$$

It follows that

$$\left(e_i^2 + \varepsilon^2\right)e_i^2 < \left(e_i^2 + \varepsilon^2\right)\vartheta_i^2. \tag{40}$$

Thus, for arbitrarily $t \ge 0$, one has

$$-\vartheta_i(t) < e_i(t) < \vartheta_i(t) \text{ for } i = \phi, \theta, \psi. \tag{41}$$

□

5. Simulation Results

As is shown in Section 2, we set the parameters of quadrotor dynamics as follows: the known moment of inertia is given as $\operatorname{diag}(J_\phi, J_\theta, J_\psi) = \operatorname{diag}(0.16, 0.16, 0.32)$. To verify the robustness of controller, the uncertainties in inertia moment and external disturbance are set as $\operatorname{diag}(J_\phi^*, J_\theta^*, J_\psi^*) = \operatorname{diag}(0.012, 0.012, 0.012)$, $\Delta_\phi = 2(\sin(t) + \sin(1.5t))$, $\Delta_\theta = 2(\cos(t) + \sin(0.5t)\cos(0.8t))$, and $\Delta_\psi = 2\sin(t)$. The initial angles are set as $\Theta^0 = [-2, 3, 1]$, and the angle reference is $\phi_d = [15\sin(0.25t), 5\cos(0.25t), 5]^T$.

TABLE 2: Contrastive outcomes between suggested method and conventional controller [37].

Indices	Notation	Proposed method	Conventional controllers [37]
Convergence time	e_ϕ	4.536	10.240
	e_θ	3.021	10.238
	e_ψ	2.980	11.786
Tracking precision	e_ϕ	0.0022	0.1087
	e_θ	0.0261	0.1117
	e_ψ	0.0263	0.0781
Integrated absolute of control inputs	u_ϕ	35.8	29.86
	u_θ	38.95	29.73
	u_ψ	108.3	106.4

To display the effectiveness of our proposed controller scheme, the simulation results are listed in Figures 2–5 with controller parameters in Table 1. In Figure 2, it is shown that the attitude is governed by the proposed controller to regulate the reference attitude with total disturbances. Meanwhile, it shows that USDE can promptly observe and capture the unmeasurable perturbations with an improved accuracy in Figure 3. Figure 4 reveals that the tracking error evolves the funnel boundary defined funnel function and control inputs are displayed in Figure 5. To present the superiority of proposed controller, conventional control scheme [37] is employed to the nonlinear quadrotor attitude regulation for comparison. Although the attitude successfully tracks the reference attitude, a slower convergence rate and a worse tracking accuracy with control consumptions are derived in Figure 6. Furthermore, we compare the time for converging to steady state, tracking precision with index of standard derivation and control inputs in Table 2.

6. Conclusion

This article investigated a novel funnel control for quadrotor attitude regulation with uncertainty parameters and external disturbance by incorporating unknown system dynamic estimator to ensure tracking preassigned convergences. The USDE is designed to online estimate the lumped disturbances by establishing connection between environmental perturbations and filtered dynamics. Furthermore, the USDE-based novel funnel control is proposed to drive the angle regulation error within preset funnel boundary, where funnel function and corresponding variable are introduced to adjust the angle error. Eventually, the tracking error of angle and angle rate are UUB with the preassigned convergence. For quadrotor attitude regulation, the validity and availability of proposed control method are verified by simulations. Note that time-triggered and infinite convergence are investigated in this work and more event-trigger and appointed-time techniques based on funnel control can be considered as in [38, 39].

Data Availability

(1) The quantitative comparison between proposed method and conventional controller data used to support the findings of this study are included within the article. (2) The quadrotor model data used to support the findings of this study have been deposited in the (X.L. Shao, L.X. Xu, W.D. Zhang, Quantized control capable of appointed-time performances for quadrotor attitude tracking: experimental validation, IEEE Transactions on Industrial Electronics) repository (doi: 10.1109/TIE.2021.3079887). (3) The simulation code data used to support the findings of this study were made. Requests for access to these data can be made to the corresponding author (zbkz_gao@163.com or 1910480231@student.cumtb.edu.cn).

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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