Hindawi
International Journal of Aerospace Engineering
Volume 2022, Article ID 3590103, 9 pages
https://doi.org/10.1155/2022/3590103

Research Article

Surface Adjustment Method Based on Fuzzy Theory for Cable Net Structures under Multi-Uncertainties

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Received 27 April 2022; Revised 8 May 2022; Accepted 12 May 2022; Published 25 May 2022

Academic Editor: Fangzhou Fu

Because of manufacturing errors, measuring errors, and unpredictable service environment, the cable net structure to be further adjusted is in an uncertainty state. In this paper, the uncertain factors including elastic deformation, thermal deformation, and measurement uncertainties are considered as fuzzy variables, which are equivalent into fuzzy tensions to simplify calculation. A fuzzy force density method is developed for accuracy analysis of the cable net structures under multi-uncertainties, and an optimization model is developed for surface adjustment. The above method is applied to numerical model adjustment of circular truss cable net structure. The results show that the adjusted surface accuracy is significantly enhanced and its fuzziness is concentrated compared with the initial surface accuracy, which verify the validity of the proposed method.

1. Introduction

Cable net structures have been widely applied to space deployable reflector antennas such as the AstroMesh antenna, the TerreStar antenna, the antenna of JAXA Engineering Test Satellite, and the SkyTerra antenna [1–5]. High-accuracy surface is a prerequisite for ensuring the electrical performance of the antennas. However, limited by manufacturing and assembling technology, artificial surface adjustment is an essential and tiresome step to improve the surface accuracy, which has been revealed very sensitive to manufacturing errors and environmental changes [6–8].

There have been many researchers that have done a lot to improve the efficiency of adjustment strategies. Hiroaki and Natori [9] proposed a shape control method based on the concept of self-equilibrated stresses to improve the control efficient. Du et al. [10] presented a shape adjustment procedure based on optimization and then converted the procedure into a sequential quadratic programming problem to make it more easily. Niu et al. [11] established an optimal adjustment model that an influence coefficient matrix was treated as one target. The above adjustment methods regarding the current configuration of the cable net structure can be accurately obtained. Actually, because of the limitation of measurement accuracy and changeable environment, there must be some uncertain factors such as nodal positions, cable pretensions, material parameters, and environmental temperature [12, 13]. Under the influence of these uncertain factors, how does the surface accuracy of the cable net structure change and how to ensure the surface accuracy in a reliable range becomes particularly important.

Nowadays, the methods for dealing with uncertainty can be divided into three categories including the probability theory [14–16], the fuzzy algorithm [17], and the interval method [18]. The probability theory is a helpful tool in modeling situations where the primary source of uncertainty is randomness [19, 20]. But sometimes, we argue that uncertainty takes other forms; instead of asking whether something is true, we ask how much of it is true and how much a certain property is exhibited in a particular instance. In our previous work [21], an interval force density was proposed and an optimization adjustment model was established for the surface adjustment of cable net structures. However, it is found that only mean and marginal cases can be obtained by the interval method and probability distribution functions need to be further studied. Probability method needs to study the probability distribution of uncertain variables based on a large number of statistical data, while fuzzy algorithm can study uncertainty model by membership function which can be estimated by experience. In order to improve the efficiency of cable net structure adjustment considering uncertainty in engineering, it is necessary to study how to apply fuzzy algorithm to adjustment. Thus, this paper proposes a fuzzy force density method to deal with the surface adjustment problem of the cable net structures under multi-uncertainties. The paper has a guiding significance for the adjustment of cable net structure considering uncertainty in the case of few samples in engineering.

2. Brief Summary of the Fuzzy Set Theory

Define a fuzzy subset of U as function $A: U \longrightarrow [0, 1]$, that is, a characteristic function from U into interval $[0, 1]$. The value $A(u)$ is called the membership of point u in the fuzzy set U or the degree to which point u belongs to set A.

$$^{\lambda}A = \{u|A(u) \geq \lambda, u \in U\}, \tag{1}$$

where $^{\lambda}A$ is called the λ-cut of the fuzzy set A, $\lambda \in [0, 1]$.

If the lower bound $u^l(\lambda)$ and upper bound $u^u(\lambda)$ are given, the fuzzy number A can be obtained by summing all λ-cut sets as

$$^{\lambda}A = \left\{ \left[u^l(\lambda), u^u(\lambda)\right], \lambda \in [0, 1] \right\}. \tag{2}$$

Defining $\tilde{x}$ as a fuzzy variable and its fuzzy number as $A(\lambda)$, $\tilde{x}$ can be described by

$$\tilde{x} = \tilde{x}(\lambda, \delta) = x^c(\lambda) + x^r(\lambda)\delta, \tag{3}$$

where $x^c(\lambda)$ and $x^r(\lambda)$ are the midvalue and the amplitude of $\tilde{x}$, respectively, and where

$$\begin{aligned} x^c(\lambda) &= \frac{x^u(\lambda) + x^l(\lambda)}{2}, \\ x^r(\lambda) &= \frac{x^u(\lambda) - x^l(\lambda)}{2}. \end{aligned} \tag{4}$$

Then, the fuzzy variable $\tilde{x}$ can be described by the interval variables λ and δ, where $\lambda \in [0, 1]$ and $\delta \in [-1, 1]$. When the cut level λ is given, the fuzzy variable $\tilde{x}$ becomes an interval variable. Therefore, the operation of the fuzzy variable $\tilde{x}(\lambda, \delta)$ can be discretized into the operation of interval variables.

3. Mathematical Models for Fuzzy Cable Net Structures

Cable net structures inevitably suffer from multiple sources of uncertainty in the process of manufacture, assembly, and on-orbit service. Limited by our ability to get information, parameters of the structures like nodal positions, cable pretensions, material parameters, and environmental temperatures must be uncertain. Thus, we use some fuzzy variables to describe these uncertainties. In order to reveal the influence of these uncertainties on the surface accuracy of the cable net structures, the mathematic models are firstly established based on the force density method and the fuzzy theory in this section.

3.1. Equivalent Fuzzy Cable Tensions for a Cable Net Structure under Multi-Uncertainties. For the cable net structure whose geometric forms are given, the sources of uncertainty can be divided into three categories: elastic deformation uncertainty, thermal deformation uncertainty, and measurement uncertainty [21], among which the uncertainties which would cause cable tension changes can be equivalent into a total fuzzy tension to simplify calculation.

According to Hooke's Law, elastic property of a cable can be expressed by the following equation.

$$\varepsilon = \frac{F}{EA} = \frac{L - L_0}{L_0}, \tag{5}$$

where ε is the cable strain, F is the cable tension, E is the elastic modulus, A is the cross-sectional area, L is the stretched length, and L_0 is the unstretched length.

3.1.1. Uncertainty of Elastic Deformation. Because of multi-uncertainties, the axial tension, the elastic modulus, and the cross-sectional area are fuzzy variables. Thus, the above equation can be rewritten as

$$\tilde{\varepsilon} = \frac{F + \Delta\tilde{F}_E(\lambda, \delta)}{\tilde{E}(\lambda, \delta)\tilde{A}(\lambda, \delta)} = \frac{L - \tilde{L}_0(\lambda, \delta)}{\tilde{L}_0(\lambda, \delta)}, \tag{6}$$

where $\tilde{\varepsilon}$ is the fuzzy strain; $\tilde{E}$ is the fuzzy elastic modulus; $\tilde{A}$ is the fuzzy cross-sectional area; $\tilde{L}_0$ is the fuzzy unstretched length; $\Delta\tilde{F}_E(\lambda, \delta)$ is the fuzziness of the cable tension caused by uncertainty of elastic deformation; see below.

$$\Delta\tilde{F}_E(\lambda, \delta) = \frac{L - \tilde{L}_0(\lambda, \delta)}{\tilde{L}_0(\lambda, \delta)} \tilde{E}(\lambda, \delta)\tilde{A}(\lambda, \delta) - F. \tag{7}$$

According to the fuzzy set theory, the above equation can be rewritten as

$$\begin{aligned} \Delta\tilde{F}_E(\lambda, \delta) = {} & \frac{L - [l_0^c(\lambda) + l_0^r(\lambda)\delta]}{l_0^c(\lambda) + l_0^r(\lambda)\delta} [E^c(\lambda) + E^r(\lambda)\delta] \\ & \cdot [(A^c(\lambda) + A^r(\lambda)\delta)] - F, \end{aligned} \tag{8}$$

where l_0^c and l_0^r are the midvalue and the amplitude of the unstretched length; E^c and E^r are the midvalue and the amplitude of the elastic modulus; A^c and A^r are the midvalue and the amplitude of the cross-sectional area.

3.1.2. Uncertainty of Thermal Deformation. According to the thermoelasticity theory, the thermal strain ε_T of a cable is

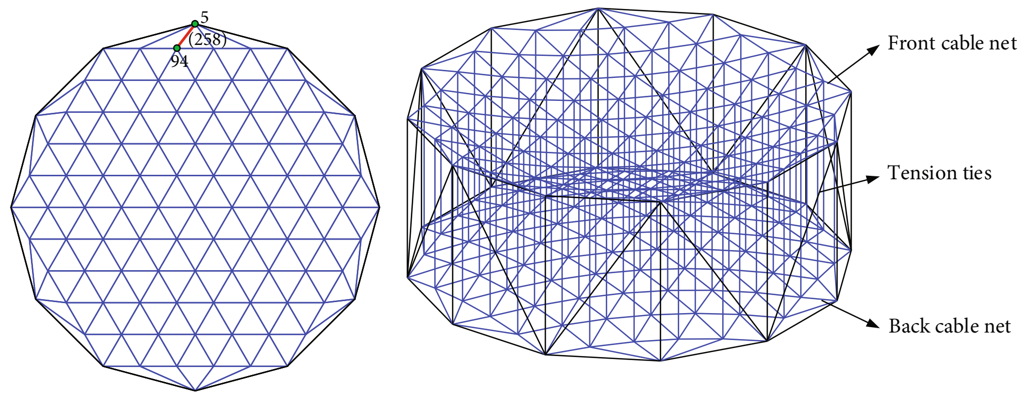


FIGURE 1: Circular truss cable net structure: (a) top view; (b) side view.

TABLE 1: Geometric parameters of the cable net structure.

Items	Value
Diameter of aperture	7 m
Focal lengths of front and back cable net	4 m
Piecewise number	5
Cable radius	1 mm
Elastic modulus of cables	20 GPa
Coefficient of thermal expansion	2×10^{-7}

directly proportional to the temperature difference ΔT, specifically as follows.

$$\varepsilon_T = \alpha \Delta T, \tag{9}$$

where α is the coefficient of thermal expansion.

The tension uncertainty caused by thermal deformation can be equivalent to

$$\Delta \tilde{F}_T = EA\alpha\Delta T. \tag{10}$$

Considering the uncertainties of the temperature difference, the coefficient of thermal expansion, the elastic modulus, and the cross-sectional area, the uncertainty of the tension caused by the thermal deformation can be obtained as

$$\Delta \tilde{F}_{\Delta T}(\lambda, \delta) = \tilde{E}(\lambda, \delta)\tilde{A}(\lambda, \delta)\tilde{\alpha}(\lambda, \delta)\Delta \tilde{T}(\lambda, \delta), \tag{11}$$

where $\Delta \tilde{F}_{\Delta T}$ is the fuzziness of the cable tension caused by the uncertainty of the temperature difference; $\tilde{\alpha}$ is the fuzzy coefficient of thermal expansion; $\Delta \tilde{T}$ is the fuzzy temperature difference.

According to the fuzzy set theory, the above equation can be rewritten as

$$\begin{aligned}\Delta \tilde{F}_{\Delta T}(\lambda, \delta) = {} & [E^c(\lambda) + E^r(\lambda)\delta][A^c(\lambda) + A^r(\lambda)\delta] \\ & \cdot [\alpha^c(\lambda) + \alpha^r(\lambda)\delta][\Delta T^c(\lambda) + \Delta T^r(\lambda)\delta],\end{aligned} \tag{12}$$

where α^c and α^r are the midvalue and the amplitude of the coefficient of thermal expansion, respectively; ΔT^c and ΔT^r are the midvalue and the amplitude of the temperature difference.

3.1.3. Uncertainty of Tension Measurement. There will be some uncertainties when applying and measuring cable tensions. Defining the fuzzy tension caused by manufacture and measure as $\Delta \tilde{F}_M$, the following equation can be obtained.

$$\Delta \tilde{F}_M(\lambda, \delta) = \Delta F^c_M(\lambda) + \Delta F^r_M(\lambda)\delta, \tag{13}$$

where ΔF^c_M and ΔF^r_M are the midvalue and the amplitude of the fuzzy tension caused by manufacture and measure.

3.1.4. Equivalent Fuzzy Tension. Combining Equations (8), (12), and (13), the equivalent fuzzy tension can be obtained as

$$\tilde{F}_{\text{total}}(\lambda, \delta) = F + \Delta \tilde{F}_E(\lambda, \delta) + \Delta \tilde{F}_T(\lambda, \delta) + \Delta \tilde{F}_M(\lambda, \delta), \tag{14}$$

where $F\sim_{\text{total}}$ is the equivalent fuzzy tension.

3.2. Surface Accuracy Analysis for a Fuzzy Cable Net Structure. For an arbitrary node j connected by some cables, the force balance equations can be obtained as

$$\begin{cases} \sum_{j\in S_j}^{S_j} \tilde{F}_{ij} \dfrac{\tilde{x}_j - \tilde{x}_i}{\tilde{L}_{ij}} = 0, \\ \sum_{j\in S_j}^{S_j} \tilde{F}_{ij} \dfrac{\tilde{y}_j - \tilde{y}_i}{\tilde{L}_{ij}} = 0, \\ \sum_{j\in S_j}^{S_j} \tilde{F}_{ij} \dfrac{\tilde{z}_j - \tilde{z}_i}{\tilde{L}_{ij}} = 0, \end{cases} \tag{15}$$

where $\tilde{F}_{ij}$ denotes the fuzzy tension of the cable connected to nodes i and j; $(\tilde{x}_i, \tilde{y}_i, \tilde{z}_i)$ is the fuzzy coordinates of node i; S_j

Table 2: Fuzzy parameters of the cable net structure.

Item	L_0(mm)		T(°C)		E(Pa)		A(mm^2)		F(N)		α(10^{-7}/°C)	
	a_L	σ_L	a_T	σ_T	a_E	σ_E	a_A	σ_A	a_F	σ_F	a_α	σ_α
Value	L_0	0.1%	0	10	2×10^{10}	1%	π	0.01	F	0.1	2	0.01

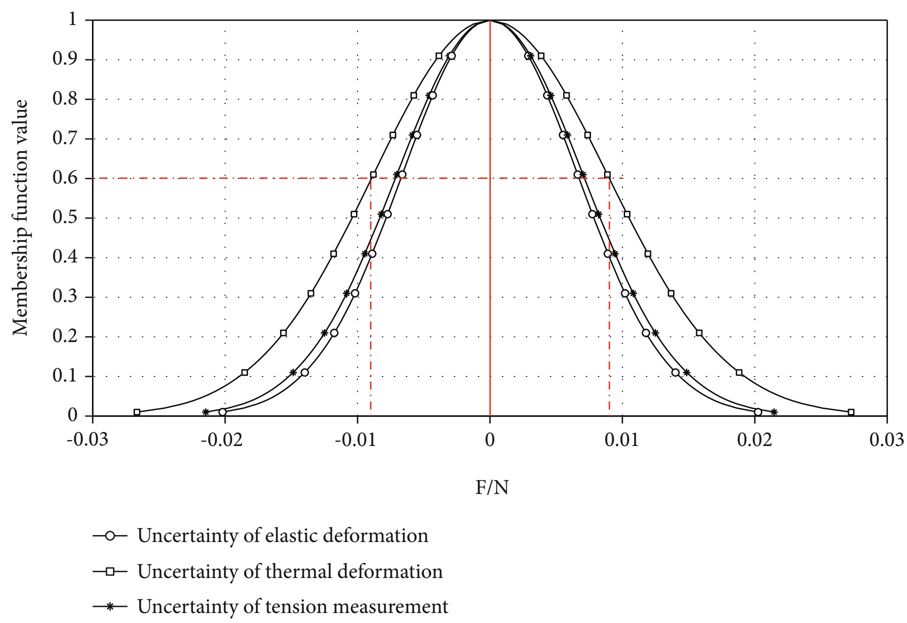


Figure 2: The distribution of the membership functions for three equivalent fuzzy tensions.

is the set of all cables connected to node j; $\tilde{L}_{ij}$ is the fuzzy length of the cable connected to nodes i and j.

According to Section 3.1, the fuzzy tension $\tilde{F}_{ij}$ can be obtained by Equation (14).

$$\tilde{F}_{ij}(\lambda, \delta) = F_{ij} + \Delta\tilde{F}_{ij,E}(\lambda, \delta) + \Delta\tilde{F}_{ij,T}(\lambda, \delta) + \Delta\tilde{F}_{ij,M}(\lambda, \delta). \tag{16}$$

In order to consider the effects of geometric and tension uncertainties simultaneously, a fuzzy force density is introduced as follows.

$$\tilde{q}_{ij}(\lambda, \delta) = \frac{\tilde{F}_{ij}(\lambda, \delta)}{\tilde{L}_{ij}(\lambda, \delta)}. \tag{17}$$

Referring to the force density method [8], the static equilibrium equations can be obtained as follows.

$$\begin{cases} \mathbf{C}_s^{\mathrm{T}}\tilde{\mathbf{Q}}\mathbf{C}_s\tilde{\boldsymbol{x}}(\lambda, \delta) = \mathbf{0}, \\ \mathbf{C}_s^{\mathrm{T}}\tilde{\mathbf{Q}}\mathbf{C}_s\tilde{\boldsymbol{y}}(\lambda, \delta) = \mathbf{0}, \\ \mathbf{C}_s^{\mathrm{T}}\tilde{\mathbf{Q}}\mathbf{C}_s\tilde{\boldsymbol{z}}(\lambda, \delta) = \mathbf{0}, \end{cases} \tag{18}$$

where $\tilde{\mathbf{Q}}$ is a diagonal matrix containing fuzzy force densities; $\mathbf{C}_s$ is the incidence matrix of the cable net structure; $\tilde{\boldsymbol{x}}(\lambda, \delta)$ is a column vector of x-coordinates; $\tilde{\boldsymbol{y}}(\lambda, \delta)$ is a column vector of y-coordinates; $\tilde{\boldsymbol{z}}(\lambda, \delta)$ is a column vector of z-coordinates. If some nodal coordinates are given, e.g., these nodes are attached to a foundation, $\mathbf{C}_s$ can be partitioned as

$$\mathbf{C}_s = \left[\mathbf{C}_u\ \mathbf{C}_f\right], \tag{19}$$

where the restrained nodes have been put at the end of the numbering sequence. Equation (18) can be rewritten as

$$\begin{aligned} \tilde{\boldsymbol{x}}_u\left(\lambda_{x_u}, \delta\right) &= -\left(\mathbf{C}_u^{\mathrm{T}}\tilde{\mathbf{Q}}\mathbf{C}_u\right)^{-1}\mathbf{C}_u^{\mathrm{T}}\tilde{\mathbf{Q}}\mathbf{C}_f\tilde{\boldsymbol{x}}_f(\lambda, \delta), \\ \tilde{\boldsymbol{y}}_u\left(\lambda_{y_u}, \delta\right) &= -\left(\mathbf{C}_u^{\mathrm{T}}\tilde{\mathbf{Q}}\mathbf{C}_u\right)^{-1}\mathbf{C}_u^{\mathrm{T}}\tilde{\mathbf{Q}}\mathbf{C}_f\tilde{\boldsymbol{y}}_f(\lambda, \delta), \\ \tilde{\boldsymbol{z}}_u\left(\lambda_{z_u}, \delta\right) &= -\left(\mathbf{C}_u^{\mathrm{T}}\tilde{\mathbf{Q}}\mathbf{C}_u\right)^{-1}\mathbf{C}_u^{\mathrm{T}}\tilde{\mathbf{Q}}\mathbf{C}_f\tilde{\boldsymbol{z}}_f(\lambda, \delta), \end{aligned} \tag{20}$$

where $\tilde{\boldsymbol{x}}_u$, $\tilde{\boldsymbol{y}}_u$, and $\tilde{\boldsymbol{z}}_u$ are the column vectors of unknown x-, y-, and z-coordinates; $\tilde{\boldsymbol{x}}_f$, $\tilde{\boldsymbol{y}}_f$, and $\tilde{\boldsymbol{z}}_f$ are the column vectors of the given x-, y-, and z-coordinates, respectively.

Taking the ideal coordinates $\{\boldsymbol{x}_0, \boldsymbol{y}_0, \boldsymbol{z}_0\}$ as a reference, the fuzzy root-mean-square error (RMS), which can be used to evaluate the structure accuracy, can be obtained as follows.

$$\tilde{w}_{\mathrm{rms}}(\lambda, \delta) = \sqrt{\frac{\left(\|\tilde{\boldsymbol{x}}_u - \boldsymbol{x}_0\|_2^2 + \|\tilde{\boldsymbol{y}}_u - \boldsymbol{y}_0\|_2^2 + \|\tilde{\boldsymbol{z}}_u - \boldsymbol{z}_0\|_2^2\right)}{N_u}}, \tag{21}$$

where N_u is the number of the nodes with unknown coordinates.

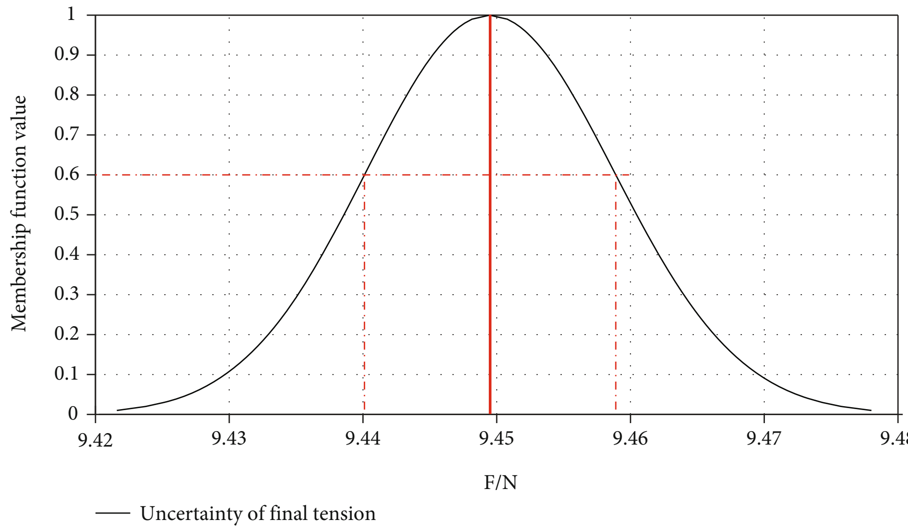


Figure 3: The distribution of the membership functions for the total equivalent fuzzy tension.

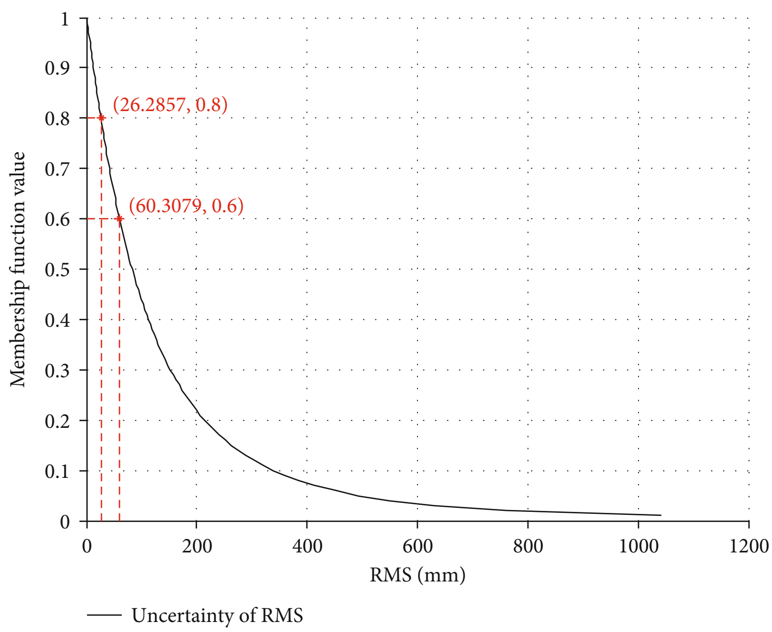


Figure 4: Distribution of the membership function for RMS.

4. Optimization Model for Adjustment of a Cable Net Structure under Multi-Uncertainties

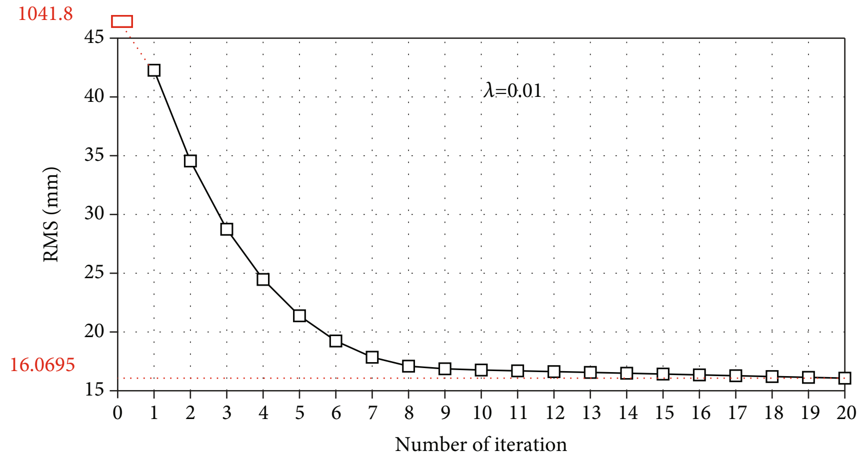
The force density of an adjustable cable connected to nodes i and j can be modified as

$$\tilde{q}_{ij} = \frac{\tilde{F}_{ij}}{\tilde{L}_{ij} - a_{ij}}, \tag{22}$$

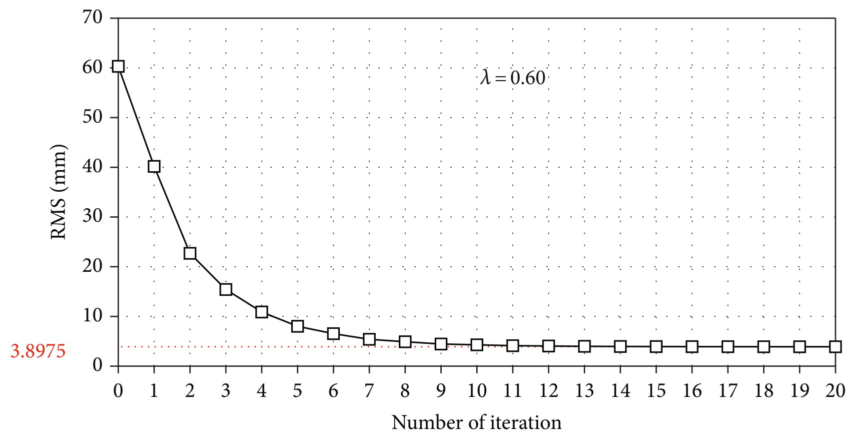
where a_{ij} is the adjustment amount of the cable.

Substituting Equation (22) into Equation (20), the cable net structure after adjustment can be obtained and the surface accuracy can be then calculated by Equation (21). Base on this, we establish the following optimization model, which can be solved by the advance and retreat algorithm [21], for the adjustment of the cable net structure under multi-uncertainties:

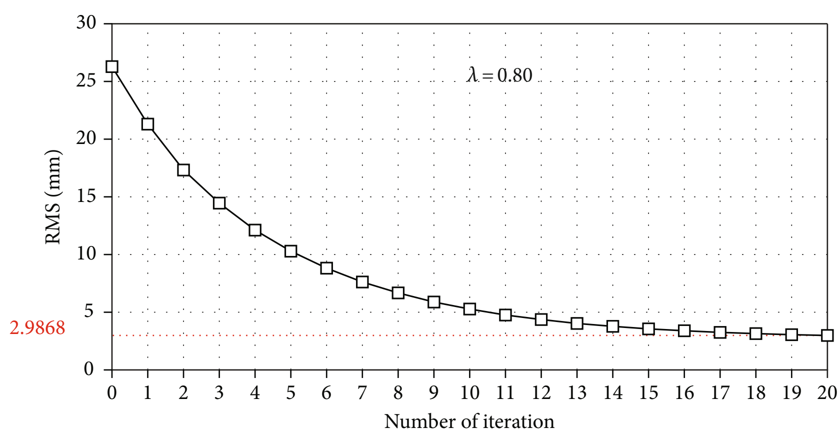
$$\begin{aligned} \text{Find} \quad & \{a_{ij}\} \\ \min \quad & \text{mean}(\tilde{w}_{\text{rms}}) \\ \text{s.t.} \quad & \text{Equation (20)} \\ & g_1^{ij} = a_0 - |a_{ij}| \le 0 \\ & g_2^{ij} = \frac{\tilde{F}_{ij}}{\tilde{L}_{ij} - a_{ij}} > 0 \end{aligned} \tag{23}$$



(a)

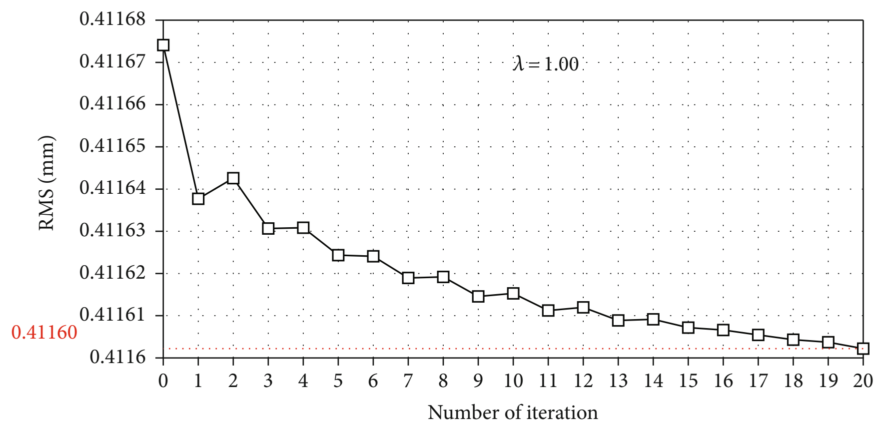


(b)



(c)

FIGURE 5: Continued.



(d)

Figure 5: RMS iterative process under different cut levels: (a) $\lambda = 0.01$; (b) $\lambda = 0.60$; (c) $\lambda = 0.80$; (d) $\lambda = 1.00$.

where $\{a_{ij}\}$ is the set of adjustment amount containing all adjustable cables; $\mathrm{mean}(\tilde{w}_{\mathrm{rms}})$ is the mean value of the fuzzy RMS; a_0 is the minimum adjustment length; g_1^{ij} denotes an inequality constraint that the a_{ij} cannot be smaller than the minimum adjustment amount a_0 which is dependent on the engineering practice; and g_2^{ij} denotes an inequality constraint that the force density of the cables must be positive due to the fact that the cable has no compressive rigidity.

The adjustment progress of the cable net structure under multi-uncertainties is as follows.

Step 1. Uniformly discrete the horizontal cut set λ into λ^i ($i = 1, 2, 3, \cdots$). When $k = 0$ and horizontal cut set is equal to λ^i, solve initial node coordinates and initial cable tensions by membership function of fuzzy variables.

Step 2. Calculate adjustment amount $\{a_{ij}\}^{(k)}$ by Equation (23).

Step 3. Update fuzzy force density matrix $\tilde{\mathbf{Q}}^{(k)}$ by Equation (22). Update node coordinates and cable tensions by Equation (20). Calculate $\tilde{w}_{\mathrm{rms}}^{(k)}$ by Equation (21).

Step 4. When $\tilde{w}_{\mathrm{rms}}^{(k)} \le \varepsilon_{\mathrm{ideal}}$ ($\varepsilon_{\mathrm{ideal}}$ is a given ideal RMS), turn to Step 5; else, let $k = k + 1$ and turn to Step 2.

Step 5. Record the optimal objective function value and design variable value under level cut set λ^i. When $\lambda^i = 1$, turn to Step 6; else, let $i = i + 1$; turn to Step 1.

Step 6. The distribution of membership function of the adjusted RMS is obtained by curve fitting.

5. Numerical Example

Take a cable net structure which has been applied to the hoop truss reflectors as an example to illustrate the proposed method. As shown in Figure 1, the cable net structure is composed of a front cable net, a back cable net, and tension ties, among which the front cable net is usually used to support the wire mesh to reflect electromagnetic wave and tension ties are adjustable cables. The geometrical parameters and the material parameters of the cable net structure are shown in Table 1, and the uncertain parameters are shown in Table 2. In this example, the membership functions of the fuzzy variables obey a normal distribution of which the function can be written as

$$\mu(x) = \exp\left\{-\left(\frac{x-a}{\sigma}\right)^2\right\} \sigma > 0, \quad x \in R, \tag{24}$$

where a and σ are the mean and the standard deviation of the fuzzy variable, respectively.

5.1. Equivalent Fuzzy Cable Tensions. When the λ-level cut value is specified as 0.01, 0.02 ... 0.99, and 1.0, respectively, for the fuzzy variables, the interval value of the equivalent fuzzy cable tension $\tilde{F}_{ij}(\lambda, \delta)$ corresponding to the λ-level cut set can be obtained by Equation (16). Then, the discrete intervals can be connected and fitted to obtain the membership distribution curve of the equivalent fuzzy cable tension $\tilde{F}_{ij}$. Taking cable 258 which is connected to nodes 49 and 5 as shown Figure 1 as an example, the cores of the cable length and the cable tension are 0.6132 m and 9.4495 N, respectively. The distributions of the membership functions for the cable tensions caused by elastic deformation uncertainty ($k = 1$), thermal deformation uncertainty ($k = 2$), and tension measurement uncertainty ($k = 3$) can be obtained by Equations (13), (18), and (19) and drawn in Figure 2.

It can be seen from the figure that the fuzziness of the three equivalent tensions $\tilde{F}_{ij}^k \in [-0.01, 0.01]$N ($i = 49$, $j = 5$, $k = 1, 2, 3$) when the cut level $\lambda > = 0.6$, that is to say the membership degrees of the equivalent fuzzy tensions are greater than 0.6 when $|\tilde{F}_{ij}^k| < = 0.01$ ($i = 49$, $j = 5$, $k = 1, 2, 3$). By Equation (23), the distribution of the membership function for the total equivalent fuzzy tension can be obtained and shown as Figure 3, from which it can be seen that $\tilde{F}_{ij} \in$

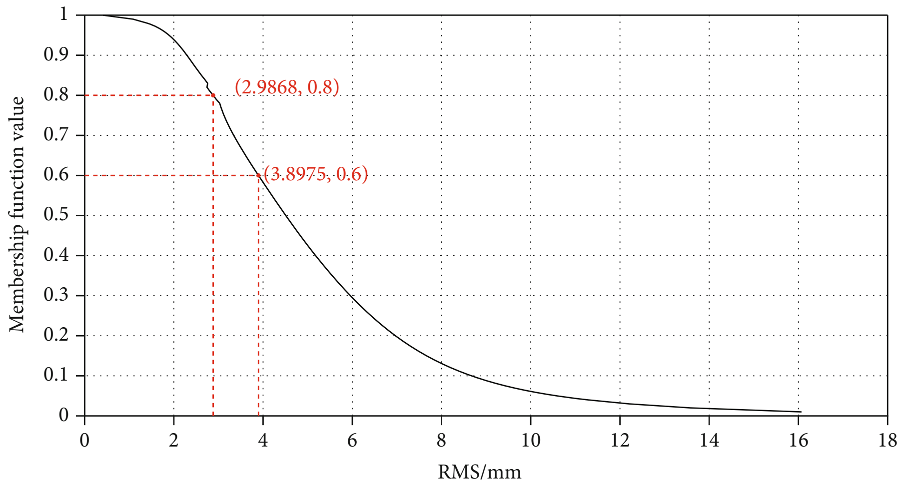


Figure 6: Distribution of RMS membership function after adjustment.

$[9.44, 9.46]$ when the cut level is greater than 0.6 and the curve obeys the normal distribution with 9.4495 N mean and 0.0131 N standard deviation.

5.2. Surface Accuracy Analysis. Due to the measurement error, the nodal positions are also of uncertainty. It is assumed that the membership function for the nodal coordinates obeys the normal distribution with zero mean and 1 mm standard deviation. According to Equation (23), the membership function for RMS can be obtained as shown in Figure 4. The curve is an addition of multiple Gauss curves but not a single normal distribution curve. It can be seen from the figure that RMS < 60.3079 mm when the degree of the membership is greater than 0.6 and RMS < 26.2857 mm when the degree of the membership is greater than 0.8. The fuzziness of the surface accuracy is dispersed, and further adjustment should be carried out to increase the surface accuracy.

5.3. Adjusting the Cable Net Structure. To calculate the adjustment amount of the tension ties, the advance and retreat algorithm is used to solve the optimization model introduced in Section 4, and the RMS value changes in the interactive process are drawn in Figure 5 with the level cut set specified as 0.01, 0.6, 0.8, and 1.0, respectively, for example. From the iterative curves, it can be seen that the convergence speeds are fast in the first 10 steps and the RMS are rapidly decreased, but after that the convergence speed slows down; repetitive adjustment work is needed. What is more, it can be seen from Figure 5, the closer the λ-level cut value is to 1, the smaller the deviation of the RMS is. When $\lambda = 1.00$ with a small initial RMS, the coupling effect of front nodes is significant. It may occur that adjusting the tension tie can make the front nodes directly connected to it closer to the ideal position, but the nodes around tensioning tie are affected to deviate from the ideal position, thus making the adjusted RMS larger. This phenomenon makes the adjustment efficiency of advance and retreat algorithm reduced when RMS is small.

The results after 20 iterations are chosen to be the adjustment amount. After adjusting, the RMSs at different cut levels are drawn in Figure 6. It can be seen that RMS $<$ 3.8975 mm when the degree of the membership is greater than 0.6, and RMS < 2.9868 mm when the degree of the membership is greater than 0.8. Compared with the initial surface accuracy, the fuzziness of the surface accuracy is concentrated and the surface accuracy is increased.

6. Conclusion

We have developed a surface adjustment method for the cable net structures under multi-uncertainties including elastic deformation uncertainty, thermal deformation uncertainty, and measurement uncertainty.

The main contributions of this paper are presented as follows. (1) The uncertain variables are considered as the fuzzy values, membership functions, and λ-level cut sets are introduced to describe the fuzzy values. (2) The elastic deformation uncertainty, thermal deformation uncertainty, and tension measurement uncertainty are equivalent into a total equivalent fuzzy tension to simplify calculation. (3) The force density method is applied to modeling static equilibrium equations for cable net structures with fuzzy parameters. (4) An optimization model for the adjustment of cable net structures under multi-uncertainties is established.

According to the numerical example, the following conclusions can be summarized. (1) Using the advance and retreat algorithm to solve the optimization model can achieve fast convergence. (2) The adjustment efficiency of advance and retreat algorithm will be reduced when the initial RMS is small. (3) This method can be used to obtain the uncertainty distribution of the surface accuracy of cable net structure without sample only by engineering experience. (4) The fuzziness of the surface accuracy can be concentrated effectively by the proposed method.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

Acknowledgments

The authors are very grateful to the references for their previous research works. This project is supported by the National Natural Science Foundation of China (Nos. 51775403 and 51905401).

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