Research Article

Investigation on Viscoelastic Poisson's Ratio of Composite Materials considering the Effects of Dewetting

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A direct numerical method is introduced herein to investigate time-dependent Poisson's ratio of solid propellant based on a representative volume element (RVE) model. Time-dependent longitudinal and transverse strains are considered in the calculation of time-dependent Poisson's ratio under the relaxation test. The molecular dynamics (MD) packing algorithm is used to generate the high area fraction RVE model of solid propellants consisting of ammonium perchlorate (AP) particles whose radius follows lognormal distribution. In order to simulate the dewetting response of the interface between particles and matrix, the PPR model is modified and utilized during the analysis. Time-dependent Poisson's ratio is measured under different cohesive parameters, loading conditions (loading temperature, loading rate, and fixed strain), and area fraction. Numerical results reveal that time-dependent Poisson's ratio can be nonmonotonic or monotonic according to the different cohesive parameters. A concept of critical cohesive parameters is proposed to judge whether the monotonic property of time-dependent Poisson's ratio appears or not. According to the numerical analysis, the cohesive contact and the shrinkage of the bulk element are two main factors which will control the change of monotonic property. All time-dependent Poisson's ratios will increase at the beginning of the relaxation stage because the effects of cohesive contact can be ignored compared with the large shrinkage of the bulk element. However, with the increased shrinkage of the bulk element, the increased cohesive contact will defend further shrinkage at the same time. Although the shrink of the bulk element never changes its direction, the ratio of the transverse strain to longitudinal strain may decrease or keep increasing in this stage. When transverse and longitudinal strains stop to change, all time-dependent Poisson's ratios will achieve their equilibrium values.

1. Introduction

Solid propellant acts as the power source of the solid rocket motor which is one of the most complicated parts of a missile system. Structure integrity as well as the physical response of different designs will be evaluated by the numerical simulation during the design stage to reduce the cost of production. Poisson's ratio of solid propellant is one of the main input parameters which will strongly affect the structural response, especially during the ignition period [1–5].

Traditionally, Poisson's ratio is assumed to be a constant (e.g., 0.4995) for simplifying the experimental task, but the real value varies depending on the chemical design of the solid propellant. Actually, Poisson's ratio is a function of time that depends on the time regime chosen to elicit it in a viscoelastic material as pointed by Tschoegl et al. [6].

Series experimental and numerical approaches are proposed in the determination of time-dependent Poisson's ratio including direct and indirect methods. For the direct method, a constant uniaxial deformation is imposed on the

specimen; then, the lateral contraction is measured as a function of time. The ratio is then calculated using the appropriate relation which will be introduced in Section 2. Kugler et al. [7] utilized a novel optoelectronic system by which stress-relaxation data were employed to investigate time-dependent aspects of Poisson's ratio in a series of filled elastomers. All curves showed the expected increase in the lateral contraction ratio with time but did not reach either the glassy or the equilibrium region. In addition, only three significant figures for Poisson's ratio can be achieved in their work. Pandini and Pegoretti [8] measured the longitudinal and transverse deformations of the dumb-bell specimens which are made of crosslinked epoxy resins simultaneously by means of longitudinal and transversal extensometers, clipped on the specimens. Results show that Poisson's ratio is seen to increase with temperature and time, and an overall decreasing trend is found with the strain rate. Further, Poisson's ratio increases with the longitudinal deformation. In Lu et al. [9], time-dependent specimen deformation is monitored by the image moiré method. Monotone nondecreasing Poisson's ratio is found in their test. However, their research concluded that standard measurement accuracy is totally inadequate to allow conversion of these properties to bulk-related time-dependent behavior. Pan et al. [10] employed the digital image correlation method to investigate time-dependent Poisson's ratio at room temperature. Poisson's ratio is not a constant value but shows an increasing trend to the incompressible value of 0.5 as time increases based on the experimental data. Relative studies are accomplished by Cui et al. [11]; Poisson's ratio increases with temperature and longitudinal strain and decreases with preload and storage time, while amplitude Poisson's ratio increases with preload and decreases with longitudinal strain and storage time. Miller [12] determined elastic Poisson's ratio of two types of solid propellants with the three-dimensional digital image correlation method. Statistical analysis reveals that neither the temperature nor the strain rate had a significant effect on Poisson's. In addition, longitudinal strain-dependent elastic Poisson's ratio of solid propellant was investigated in Shekhar and Sahasrabudhe's work [13]. Apart from monotone nondecreasing Poisson's ratio found in the above direct and some indirect [14, 15] experimental tests, different tendencies were pointed out with the indirect numerical method in which Poisson's ratio is always calculated from two other time-dependent material functions. Lakes and Wineman [16] thought that viscoelastic Poisson's ratio need not increase with time, and it need not be monotonic with time. A designed microstructure is selected to prove their deduction. Grassial et al. [17] found that in the region of α -relaxation, Poisson's ratio which is analytically evaluated from the bulk and shear responses can be a non-monotonic function of time, with a weak minimum at short times, when the shear response is broader than bulk response. While the shear and bulk responses share similar timescales and relaxation time distributions, a monotonically increasing function of time appears. Although experimental data from literature they claimed is used in their paper, the method did not satisfy the demands that highly accurate measurements are made on the same specimen, at

TABLE 1: Statistical parameters of the RVE model.

Af (%)	$\bar{r}$ (μm)	σ^2	N	$\bar{r}'$ (μm)	μ'
60	50	0.25	95	48.62	3.85
50	50	0.25	79	48.91	3.86
40	50	0.25	63	49.11	3.86

Note: N represents the number of the particles, $\bar{r}'$ is the real average radius of the RVE model, and μ' is the real standard deviation of the radius' natural logarithm. In order to satisfy the designed Af, the average of the radius and standard deviation of the radius' natural logarithm needs to make compromises.

the same time, and under the same conditions of the experimental environment proposed by Tschoegl et al. [6]. Similarly, Charpin and Sanahuja [18] reviewed several definitions of viscoelastic Poisson's ratio by various scientific communities. While considering a composite material made up of elastic inclusions embedded into a Maxwell nonaging viscoelastic matrix, depending on the volume fraction, effective Poisson's ratios which are calculated by using the Mori-Tanaka scheme [19] can be either monotonic (increasing or decreasing) or nonmonotonic.

According to the above literature review, nonmonotonic time-dependent Poisson's ratio seems to appear under hypothetical conditions by the indirect numerical method. In our opinion, the nonmonotonic property of Poisson's ratio may be very hard to appear in the direct experimental test. According to the definition of time-dependent Poisson's ratio under relaxation tests as will be introduced in Section 2, the determination of the longitudinal and transverse strains is under the macroscopic scale, the scale of specimens. In this condition, the longitudinal strain is considered to keep unchanged, and the increased transverse strain will be recorded; thus, time-dependent Poisson's ratio will never decrease. However, the longitudinal strain is uneven in the body of the specimen; the constant longitudinal strain used in the definition will deduce incorrect Poisson's ratio. This is the key reason why nonmonotonic property of Poisson's ratio appears under the indirect method among the literature.

In order to reduce the high cost and avoid complicated factors in direct and indirect experiments, the numerical investigation method is utilized here. In this paper, a direct numerical method which determines time-dependent Poisson's ratio on the representative volume element (RVE) model of solid propellant is proposed under a relaxation test. A famous cohesive zone model (PPR) is modified and utilized to describe the dewetting phenomenon in solid propellants. Unlike the existing direct method, the time-dependent longitudinal strain is considered during the analysis. The effects of cohesive parameters of the debonding interface between particles and matrix, loading conditions, and area fractions are analyzed. Both nonmonotonic and monotonic properties of Poisson's ratio are captured during the analysis. The origin of nonmonotonic and monotonic properties is discussed according to the numerical analysis results. At the end of this paper, the numerical results are validated by two designed tests.

FIGURE 1: Distribution of the radius (left) and the packing model in the RVE (right) with different A_f .

2. Definition of Time-Dependent Poisson's Ratio in the Viscoelastic Domain

In the viscoelastic domain, the correspondence principle is applied to the moduli but not to Poisson's ratio. Tschoegl et al. [6] give the detailed definition of time-dependent Poisson's ratio in the relaxation test by the Laplace transform method which follows:

$$\nu(t) = \frac{-\varepsilon_y(t)}{\varepsilon_x^0}, \quad (1)$$

where ε_x^0 is the step longitudinal strain and $\varepsilon_y(t)$ is the transverse strain. A similar definition is given by Lakes and Wineman [16] through the bulk and stretch moduli for Poisson's ratio in relaxation. The definition is valid

TABLE 2: Parameters in the Prony series for relaxation modulus master curve of solid propellant.

i	0	1	2	3	4	5
τ_i^E (s)	--	0.0064	0.0953	1.4291	21.4364	321.5465
E_i (MPa)	31.59	18.67	5.50	2.63	1.90	1.33

everywhere in the material; however, it is hard to obtain Poisson's ratio from the an isolate point in reality.

For the numerical analysis by using the RVE model, the determination of the average strain seems easy than the bulk and stretch moduli according to the standard protocol Tschoegl et al. [6] proposed. Considering that the longitudinal strain is uneven on the surface of the RVE model during the relaxation stage, Poisson's ratio will be calculated by the mean longitudinal and transverse strain:

$$\nu(t) = \frac{-\bar{\varepsilon}_y(t)}{\bar{\varepsilon}_x(t)}, \quad (2)$$

in which $\bar{\varepsilon}_y(t)$ and $\bar{\varepsilon}_x(t)$ refer to the mean transverse and longitudinal strains and are calculated as follows:

$$\bar{\varepsilon}_{ij} = \int_S \varepsilon_{ij} dS = \sum_{p=1}^{N_{tri}} \bar{\varepsilon}_{ij}^{tri} \frac{S_p^{tri}}{S_{rve}} + \sum_{q=1}^{N_{quad}} \bar{\varepsilon}_{ij}^{quad} \frac{S_q^{quad}}{S_{rve}}. \quad (3)$$

Here, N_{tri} (N_{quad}) represents the number of the triangular (quadrangular) elements. $\bar{\varepsilon}_{ij}^{tri}$ ($\bar{\varepsilon}_{ij}^{quad}$) is the mean strain of the triangular (quadrangular) element. S_p^{tri} (S_q^{quad}) and S_{rve} are the area of the p th (q th) triangular (quadrangular) element and the total area of the RVE model, respectively.

In order to research the properties of time-dependent Poisson's ratio, we define ν_0 as instantaneous Poisson's ratio, ν_∞ as equilibrium Poisson's ratio, and $\nu_A = \nu_\infty - \nu_0$ as amplitude Poisson's ratio.

3. Preparations of the RVE Model

In order to significantly reduce the cost of expensive experimental tests during the design and material development process, homogenization methods have been extensively utilized to estimate the effective properties of composite materials. Homogenization methods like the self-consistent model [20], the Mori-Tanaka model [19, 21], and the Halpin-Tsai equation [22] can predict the effective mechanical properties rapidly. However, while considering the dewetting phenomenon in composites, those analytical models are limited to capturing the real strain and stress distribution which is of great importance during the analysis. RVE is widely used to predict the effective properties of the composites accompanied with the FE analysis, especially for solid propellants [23–27]. In the published literature, there are different definitions of it. The main point of the definition can be concluded that the RVE model should represent the properties of the microstructure, like Poisson's ratio, while its dimensions are smaller than the macroscopic dimensions. A series of literature focus on

FIGURE 2: Normal traction-separation relationship of the PPR model in pure mode I [40].

FIGURE 3: Difference of the control parameters between Ref. [38, 39] and this paper.

how to determine the size of the RVE model, like image processing approaches [28], experimental-image processing methods [29], statistical-numerical methods [30], and analytical approaches [31]. In addition, researchers found that the periodic boundary conditions could represent a better prediction on the effective properties than Dirichlet and Neumann boundary conditions [32].

In order to work within the framework of the micro-mechanical methods, it is required to have the RVE model of the solid propellant. As we know, different model parameters as well as loading conditions will affect the RVE size during the analysis which will introduce another effect indeed. Considering this reason, the same RVE size will be used when we investigate the influence of the model parameters and loading conditions. Through analyzing lots of simulation results under the different loading conditions and cohesive parameters with various RVE sizes, results show that while RVE size is larger than $1000 \mu\text{m}$, effective Poisson's ratio will be almost the same; thus, $1000 \mu\text{m}$ is chosen to be the RVE size in this paper. The detailed analysis will be ignored here considering the huge amount of space.

FIGURE 4: Response under different critical displacement Δ_n^c , $\sigma_{\max} = 0.5$ MPa, $m = 0.1672$, $\alpha = 3.10$ [40].

Solid propellants have complex microstructures and are mainly composed of polymeric binders, crystal oxidizers, and fuel particles. As we know, some particles like aluminum (Al) are unsuitable for direct use in numerical analysis. In order to simplify the analysis, only one main kind of particle, ammonium perchlorate (AP), is considered here. The small particles as well as the binder will be considered the matrix in the RVE model. The Mori-Tanaka method can be utilized to obtain the mechanical properties of the new matrix.

The actual experimental results reveal that the radius of AP follows lognormal distribution $\ln(r) \rightarrow N(\mu, \sigma^2)$, μ and σ that are, respectively, the mean and standard deviation of the radius' natural logarithm. In addition, the average radius equals to $\bar{r}$. In order to compare the effect of area fraction (Af), three different Af (60%, 50%, and 40%) are designed in this paper. The corresponding statistical parameters are listed in Table 1.

Considering the high area (volume) fraction of the solid propellant, the common RVE generation method like the Monte Carlo method may be hard to converge [33]. In this paper, the molecular dynamics (MD) [34] method is utilized to generate a random ensemble of particles. As shown in Figure 1, the distribution of the radius and the packing model in the RVE model are plotted with the RVE size $1000 \mu\text{m}$. In order to apply the periodic boundary condition (PBC) more easily, the periodic geometry of the RVE model is generated. Several methods about how to apply PBC can be referred to [35, 36]; there is no need to spend space on it here.

4. Mechanical Modeling of the Components and the Interface

4.1. Constitutive Modeling of the Matrix and Particles. Some literature used the elastic matrix of the solid propellant in the micromechanical analysis. However, this simplification may cause inaccurate results. Linear viscoelastic material is always

FIGURE 5: Response under different cohesive strength $\sigma_{\max}$, $\Delta_n^c = 2.5$ mm, $m = 0.1672$, $\alpha = 3.10$ [40].

FIGURE 6: Response under different transition parameter m , $\sigma_{\max} = 0.5$ MPa, $\Delta_n^c = 2.5$ mm, $\alpha = 3.10$ [40].

FIGURE 7: Response under different shape parameter α for pure mode I [40].

FIGURE 8: The mesh of the RVE model with Af equal to 60%.

utilized to model the mechanical properties of the matrix. The relaxation model of the matrix is expressed by the Prony series as

$$E(t) = E_0 - \sum_{i=1}^{N_E} E_i \left(1 - e^{-t/\tau_i^E}\right), \quad (4)$$

where E_0 is the initial modulus, N_E is the term number of the Prony series, t represents the loading time, and τ_i^E and E_i are material coefficients. While N_E equals to 5, the other model parameters are listed in Table 2.

A matrix is a simple thermorheological material, and its material properties always satisfy the time-temperature equivalence principle. According to the WLF equation [37]

$$\log a_T = \frac{-C_1(T - T_s)}{C_2 + T - T_s}, \quad (5)$$

where a_T is the shift factor, T is the current temperature, T_s is the reference temperature, and C_1 and C_2 are material constants. During the numerical analysis, $C_1 = 6.01$ and $C_2 = 178.17$, with the reference temperature equal to 20°C used to model the mechanical property of the matrix. In addition, Poisson's ratio of 0.4995 is chosen to model the incompressibility of the matrix. The particles are considered isotropic elastic material in this paper with the elastic moduli 32.45 MPa and Poisson's ratio 0.14.

4.2. PPR Cohesive Zone Model. The debonding phenomenon is common between the particles and matrix for the solid propellant. Cohesive zone models are very useful in dealing with this nonlinear fracture process. The PPR model which is proposed by Park et al. [38, 39] to demonstrate the consistency of the constitutive relationship under mixed-mode conditions is chosen to simulate the fracture process of the debonding interface between matrix and particles.

TABLE 3: Common parameters of PPR model and loading conditions.

Parameters of PPR model	Loading parameters
$\sigma = \sigma_{\max} = \tau_{\max} = 0.2 \text{ MPa or } 1.0 \text{ MPa}$	$\epsilon_x^0 = 10\%$
$\Delta = \Delta_n^c = \Delta_t^c = 120 \text{ mm}$	$\nu = 500 \text{ min/min}$
$k = m = n = 0.2$	$T = 20^\circ\text{C}$
$\alpha = \beta = 3.10$	

The potential of the model is expressed by

$$\begin{aligned} \Psi(\Delta_n, \Delta_t) = & \min(\phi_n, \phi_t) + \left[\Gamma_n \left(1 - \frac{\Delta_n}{\Delta_n^c}\right)^\alpha \left(\frac{m}{\alpha} + \frac{\Delta_n}{\Delta_n^c}\right)^m + \langle \phi_n - \phi_t \rangle \right] \\ & \times \left[\Gamma_t \left(1 - \frac{|\Delta_t|}{\Delta_t^c}\right)^\beta \left(\frac{n}{\beta} + \frac{|\Delta_t|}{\Delta_t^c}\right)^n + \langle \phi_t - \phi_n \rangle \right]. \end{aligned} \quad (6)$$

Here, ϕ_n and ϕ_t represent the normal and tangential cohesive energy, respectively. Shape parameters α ($\alpha > 1$) and β ($\beta > 1$) are introduced to control the whole shape of the traction-separation curve. Δ_n^c and Δ_t^c are the critical displacement (or final crack opening width) of the normal and tangential separation Δ_n and Δ_t (see Figure 2). Energy constants Γ_n and Γ_t are character parameters which are related to cohesive energy, shape parameters, and transition parameter (m and n). The detailed information including unloading/reloading and contact condition of the model can be found in Ref. [38, 39].

The normal traction of the PPR model can be calculated by the derivatives of the potential with respect to the normal separation

$$\begin{aligned} T_n(\Delta_n, \Delta_t) = & \frac{\Gamma_n}{\Delta_n^c} \left[m \left(1 - \frac{\Delta_n}{\Delta_n^c}\right)^\alpha \left(\frac{m}{\alpha} + \frac{\Delta_n}{\Delta_n^c}\right)^{m-1} - \alpha \left(1 - \frac{\Delta_n}{\Delta_n^c}\right)^{\alpha-1} \left(\frac{m}{\alpha} + \frac{\Delta_n}{\Delta_n^c}\right)^m \right] \\ & \times \left[\Gamma_t \left(1 - \frac{|\Delta_t|}{\Delta_t^c}\right)^\beta \left(\frac{n}{\beta} + \frac{|\Delta_t|}{\Delta_t^c}\right)^n + \langle \phi_t - \phi_n \rangle \right]. \end{aligned} \quad (7)$$

FIGURE 9: The curves of the (a) longitudinal strain, (b) transverse strain, and (c) Poisson's ratio versus time under different cohesive strengths.

According to Park and Paulino [39], the normal critical displacement Δ_n^c has the form

$$\Delta_n^c = \frac{\phi_n}{\sigma_{\max}} \alpha \lambda_n (1 - \lambda_n)^{\alpha-1} \left(\frac{\alpha}{m} + 1 \right) \left(\frac{\alpha}{m} \lambda_n + 1 \right)^{m-1}. \quad (8)$$

The transition parameter m is expressed as

$$m = \frac{\alpha(\alpha-1)\lambda_n^2}{(1-\alpha\lambda_n^2)}; \quad (9)$$

here, initial slope indicator λ_n is expressed as $\lambda_n = \Delta_n^0 / \Delta_n^c$. In addition, the normal cohesive energy is the area surrounded by the curve and abscissa in Figure 2:

$$\phi_n = \int_0^{\Delta_n^c} T_n(\Delta_n, 0) d\Delta_n. \quad (10)$$

According to the above analysis, there are four control parameters (cohesive energy ϕ_n , cohesive strength $\sigma_{\max}$, shape parameter α , and initial slope indicator λ_n) utilized to form the normal traction accompanied with the normal separation Δ_n as done by Park et al. However, the changed

TABLE 4: Characteristic parameters of Poisson's ratio under different cohesive parameters.

Cohesive parameters		ν_0	ν_∞	ν_A
σ (MPa)	0.08	0.3284	0.3893	0.0609
	0.1	0.3302	0.3936	0.0634
	0.2	0.3437	0.4022	0.0585
	0.4	0.3603	0.3963	0.0360
	0.6	0.3709	0.3883	0.0174
	1.0	0.3863	0.3765	-0.0098
Δ (mm)	40	0.3503	0.3911	0.0408
	80	0.3479	0.4000	0.0521
	100	0.3446	0.4005	0.0559
	120	0.3437	0.4022	0.0585
	160	0.3394	0.4012	0.0618
	200	0.3373	0.3994	0.0621
k	0.01	0.3535	0.3811	0.0276
	0.05	0.3521	0.3966	0.0445
	0.1	0.3487	0.4005	0.0518
	0.2	0.3437	0.4022	0.0585
	0.5	0.3360	0.3993	0.0633
	0.7	0.3324	0.3958	0.0634

cohesive strength $\sigma_{\max}$ will have a direct effect on the cohesive energy ϕ_n . The four control parameters are not suitable for analyzing the model properties.

Combining Equation (8), the cohesive energy ϕ_n can be formed by cohesive strength $\sigma_{\max}$ and normal critical displacement Δ_n^c directly:

$$\phi_n = \frac{\sigma_{\max} \Delta_n^c}{\alpha \lambda_n (1 - \lambda_n)^{\alpha-1} (\alpha/m + 1) (\alpha/m \lambda_n + 1)^{m-1}}. \quad (11)$$

However, there are five unknown parameters still existing in the above equation. Considering the monotonous relationship between the initial slope indicator λ_n and transition parameter m , λ_n can be rewritten as

$$\lambda_n = \sqrt{\frac{m}{\alpha(m + \alpha - 1)}}. \quad (12)$$

Thus, in pure mode I, the normal traction can be fully determined by the four independent control parameters including critical displacement Δ_n^c , cohesive strength $\sigma_{\max}$, shape parameter α , and transition parameter m as illustrated in Figure 3. Figures 4–6 show the response of the traction under various control parameters Δ_n^c , $\sigma_{\max}$, and m in pure mode I. Obviously, the changes are monotonous; the three model parameters can be utilized further for complex loading conditions. Take the response under different transition parameter m for instance; the increased parameter m will delay the appearance of Δ_n^0 while the critical displacement Δ_n^c and cohesive strength $\sigma_{\max}$ keep unchanged. It is worth

mentioning that the modified PPR model is introduced first in the authors' another work [40].

Shape parameter α and β just control the concavity and convexity of the traction-separation curve. If $1 < \alpha$, $\beta < 2$, the shape of the traction-separation curve shows the concave shape, while $\alpha, \beta > 2$, the relation is the convex shape. For the condition that $\alpha, \beta \approx 2$, the softening phase of the curve closes to the straight line as illustrated in Figure 7.

4.3. Finite Element Modeling. The finite element method is used to simulate the response of the RVE model which is meshed by 3-node triangular bulk elements. The plane stress condition is employed for the computational simulation. The 4-node zero-thickness cohesive elements are inserted between the particles and matrix. As shown in Figure 8, the finite element mesh consists of 11607 nodes and 22810 elements for the RVE model. Besides the PBC, the corresponding loading temperature is also applied during the analysis.

5. Effects of Cohesive Parameters

To account for the effect of dewetting, in this section, three kinds of cohesive parameters are considered here, i.e., the cohesive strength, critical displacement, and transition parameter. The detailed basic model parameters of the interface and loading parameters are listed in Table 3. The relaxation test as introduced in Section 2 is utilized here. The data of the results are collected at the beginning of the relaxation period. The RVE model in Figure 1(a) is utilized in this part.

FIGURE 10: The curves of the (a) longitudinal strain, (b) transverse strain, and (c) Poisson's ratio versus time under different critical displacements.

In order to investigate the effects of the cohesive strength, six sets of the parameters are used here. The simulation results of the average strain, average stress, and Poisson's ratio are shown in Figure 9. The average longitudinal strain increased with time because of the slow response of the middle part of the specimen. During this stage, the average transverse strain increased. At the end of the period, the average longitudinal and transverse strains never changed. In addition, the average longitudinal and transverse strains increase with the cohesive strength from Figures 9(a) and 9(b). According to the definition, the corresponding average Poisson's ratio can be

obtained by the average strain. In Figure 9(c), the initial Poisson's ratio increases with the increased cohesive strength while the amplitude of Poisson's ratio decreases. There are two main tendencies of Poisson's ratio here which depend on the magnitude of the cohesive strength. While the cohesive strength is smaller than 0.2 MPa, Poisson's ratio increases first and keeps unchanged at the end of the test. The corresponding equilibrium Poisson's ratio increases with the cohesive strength. The detailed statistic data is listed in Table 4.

But while the cohesive strength is larger than 0.4 MPa, Poisson's ratio increases at the beginning, then decreases

FIGURE 11: The curves of the (a) longitudinal strain, (b) transverse strain, and (c) Poisson's ratio versus time under different transition parameters.

and keeps unchanged at the end. In this condition, the corresponding equilibrium Poisson's ratio decreases with the cohesive strength.

For the effect of the critical displacement on Poisson's ratio, six different critical displacements are chosen in the analysis. As can be seen from Figure 10, the average longitudinal and transverse strains decrease with the critical displacement and still increase with the relaxation time and reach balance at the end of the test. On the one hand,

in Figure 10(c), the initial Poisson's ratio decreases with the increased critical displacement while the amplitude of Poisson's ratio increases as listed in Table 4. On the other hand, Poisson's ratio shows the property of monotone nondecreasing while critical displacement is larger than 120 mm, and equilibrium Poisson's ratio decreases with the critical displacement in this condition. However, Poisson's ratio increases first then decreases to a constant and keeps unchanged at the end of the test while critical

FIGURE 12: The curves of the (a) longitudinal strain, (b) transverse strain, and (c) Poisson's ratio versus time under different fixed strains when cohesive strength equals to 0.2 MPa.

displacement is small than 100 mm. In addition, equilibrium Poisson's ratio increases with the increased critical displacement.

A similar response of the average longitudinal and transverse strains as well as Poisson's ratio under different transition parameter k can be found in Figure 11 and Table 4. For the changed cohesive parameters including cohesive strength, critical displacement, and transition parameter, there is a transition value (or section) that controls the tendency of the curve of Poisson's ratio. The two sides of this critical value will result in different responses.

In order to distinguish the two different tendencies, we call the phenomenon of decreased Poisson's ratio as Poisson's ratio mutation.

6. Effects of Loading Conditions

In this section, the effects of loading conditions will be discussed. The fixed strain, loading rates, and loading temperature are three main parameters when designing the relaxation test. Considering Poisson's ratio mutation, two kinds of cohesive strength (0.2 MPa and 1.0 MPa) are chosen

FIGURE 13: The curves of the (a) longitudinal strain, (b) transverse strain, and (c) Poisson's ratio versus time under different fixed strains when cohesive strength equals to 1.0 MPa.

in the analysis; the other model parameters are listed in Table 3.

As illustrated in Figures 12 and 13, the longitudinal and transverse strains increase with the fixed strain. For Poisson's ratio, the changed fixed strain will not change the tendency of the curve of Poisson's ratio. That means Poisson's ratio mutation is independent of the fixed strain. In addition, initial and equilibrium Poisson's ratios increase with the fixed strain while the amplitude one decreases without Poisson's ratio mutation ($\sigma = 0.2$ MPa). While Poisson's ratio mutation happened ($\sigma = 1.0$ MPa), initial Poisson's ratio increases with the fixed strain while the amplitude and equilibrium Poisson's ratio decrease.

For the changed loading rates, the longitudinal and transverse strains decrease with the fixed strain at first and keep the same after a while as can be seen in Figure 14 ($\sigma = 0.2$ MPa) and Figure 15 ($\sigma = 1.0$ MPa). Decreased initial Poisson's ratio, increased amplitude Poisson's ratio, and the same equilibrium Poisson's ratio happened when the loading rate increases. The corresponding parameters about Poisson's ratio are listed in Table 5. Obviously, the loading rates will not change the tendency of Poisson's ratio.

As we can see in Figure 16 ($\sigma = 0.2$ MPa) and Figure 17 ($\sigma = 1.0$ MPa), the increased longitudinal and transverse strains happened with the increased temperature. At the end of the test, the equilibrium longitudinal and transverse

FIGURE 14: The curves of the (a) longitudinal strain, (b) transverse strain, and (c) Poisson's ratio versus time under different loading rates when cohesive strength equals to 0.2 MPa.

strains are the same for different loading temperatures. Increased initial Poisson's ratio, decreased amplitude Poisson's ratio, and the same equilibrium (see Table 5) are introduced by the increased loading temperature. Obviously, the tendency will not change with the loading temperature. In addition, Poisson's ratio will spend a longer time to achieve the balance under the lower temperature.

7. Effects of Area Fraction

Area fraction is another significant parameter during the design phase. For common Poisson's ratio without mutation as illustrated in Figure 18(a), the initial Poisson's ratio decreases from 0.4025 to 0.3437 and the equilibrium one decreases from 0.4278 to 0.4022 when area fraction

FIGURE 15: The curves of the (a) longitudinal strain, (b) transverse strain, and (c) Poisson's ratio versus time under different loading rates when cohesive strength equals to 1.0 MPa.

changes from 40% to 60%. At the same time, amplitude Poisson's ratio increases from 0.0253 to 0.0585. Considering Poisson's ratio mutation while cohesive strength equals to 1.0 MPa in Figure 18(b), there is a similar tendency of initial, equilibrium, and amplitude Poisson's ratio from Table 6. Obviously, the increased area fraction will help Poisson's ratio to be close to the particle's Poisson's ratio 0.14.

8. Discussion

Table 7 shows the variation tendency of characteristic Poisson's ratio and its curve under different conditions. Generally speaking, Poisson's ratio will increase first and keep unchanged at the end of the relaxation stage. However, Poisson's ratio mutation will happen by changing the value of the cohesive parameter (cohesive strength, critical

TABLE 5: Characteristic parameters of Poisson's ratio under different loading conditions.

Loading parameters	σ (MPa)	ν_0	ν_∞	ν_A
Fixed strain (%)	3	0.3267	0.4008	0.0741
	5	0.3319	0.4010	0.0691
	10	0.3437	0.4022	0.0585
	20	0.3631	0.4037	0.0406
	25	0.3696	0.4060	0.0364
	3	0.3779	0.3892	0.0113
	5	0.3822	0.3858	0.0036
	10	0.3863	0.3765	-0.0098
	20	0.3939	0.3632	-0.0307
	25	0.3983	0.3575	-0.0408
Loading rate (%)	50	0.3572	0.4022	0.0450
	100	0.3533	0.4022	0.0489
	300	0.3455	0.4022	0.0567
	500	0.3437	0.4022	0.0585
	1000	0.3406	0.4022	0.0616
	50	0.3987	0.3765	-0.0222
	100	0.3960	0.3765	-0.0195
	300	0.3881	0.3765	-0.0116
	500	0.3863	0.3765	-0.0098
	1000	0.3824	0.3765	-0.0059
Temperature (°C)	-20	0.3364	0.4022	0.0658
	0	0.3384	0.4022	0.0638
	20	0.3437	0.4022	0.0585
	40	0.3515	0.4022	0.0507
	70	0.3620	0.4022	0.0402
	-20	0.3769	0.3765	-0.0004
	0	0.3779	0.3765	-0.0014
	20	0.3863	0.3765	-0.0098
	40	0.3959	0.3765	-0.0194
	70	0.4012	0.3765	-0.0247

displacement, and transition parameter) under a certain loading condition.

There is a critical value (or section) of the cohesive parameters (cohesive strength, critical displacement, and transition parameter) which can control the mutation of Poisson's ratio existing under a certain loading condition. Take the critical cohesive strength σ^t for instance; when $\sigma < \sigma^t$, the cohesive contact is relatively small; the shrinkage of the bulk element plays a dominant part. By increasing the cohesive strength, the shrinkage of the bulk element will increase as well as Poisson's ratio. In addition, the change of the initial Poisson's ratio can be explained easily by this reason, because the cohesive traction can be ignored at the beginning of the relaxation stage. However, while $\sigma > \sigma^t$, the cohesive contact is in the leading status, with the increased cohesive strength, the increased cohesive contact (see Figure 19), and decreased Poisson's ratio

appearing. However, Poisson's ratio mutation will not happen or vanish under various loading conditions under certain cohesive parameters. It reveals that Poisson's ratio mutation is controlled only by the cohesive parameters.

Another interesting phenomenon about equilibrium Poisson's ratio will support the above conclusion. Equilibrium Poisson's under different cohesive parameters and loading conditions will change in the opposite direction. The same reason can be used for this phenomenon. Obviously, the changed loading temperature and loading rate will not change the longitudinal strain in analysis; equilibrium Poisson's ratio will keep unchanged, while the fixed strain has a similar effect on equilibrium Poisson's ratio. For the changed Af, considering that Poisson's ratio will be close to Poisson's ratio of particles with the increased Af, equilibrium Poisson's ratio decreases.

FIGURE 16: The curves of (a) longitudinal strain, (b) transverse strain, and (c) Poisson's ratio versus time under different loading temperatures when cohesive strength equals to 0.2 MPa.

Before Poisson's ratio achieve the balanced value, there will be a period from 100s to 3000s, which is full of intense oscillation exit. During this stage, strong interactions happened between the shrinkage of bulk elements and the cohesive contact in this stage. When the two effects achieve a balance, Poisson's ratio will keep unchanged.

Note that the same equilibrium Poisson's ratio appears under the different loading rates and loading temperatures, while the increased one was captured in some literature [8, 11]. The phenomenon may be caused by the fixed cohesive parameters used under the differ-

ent loading rates and temperatures which should be changed according to some related works [41, 42]. Further researches will take the viscoelastic cohesive model into consideration.

9. Verification

In this section, two examples are designed to confirm the numerical results of the above analysis. The cohesive parameters and loading conditions are all from Table 3. The above simulation results are all obtained from the same direction, and the longitudinal direction parallels

FIGURE 17: The curves of (a) longitudinal strain, (b) transverse strain, and (c) Poisson's ratio versus time under different loading temperatures when cohesive strength equals to 1.0 MPa.

the horizontal direction. Considering the isotropic material we used for the solid propellant, Poisson's ratio from other directions should be the same.

For Poisson's ratio with or without mutation (see Figures 20(a) and 20(b)), the curves of Poisson's ratio from the two directions agree with each other. As shown in Figure 20(c), the relative error between Poisson's ratios is all within 1.5%. In other words, the direction has no effect on the determination of Poisson's ratio.

Mirkhalaf et al. [43] utilized the standard deviation of the deformation behavior to ensure that changing the realization, for the same sample size and same inclusion area

fraction, does not remarkably change the deformation behavior. In their work, 10% is considered for the variation of the first Piola-Kirchhof stress tensor. Here, a similar criterion about the standard deviation of Poisson's ratio is given as follows:

$$S_I(P) = \sqrt{\frac{1}{N_P - 1} \sum_{i=1}^{N_P} (I^i(P) - \overline{I(P)})^2} \leq 0.01 \times \overline{I(P)}, \quad (13)$$

where $S_I(P)$ is the standard deviation, N_P is the number of

FIGURE 18: The curves of Poisson's ratio versus time under different area fractions when cohesive strength equals to (a) 0.2 MPa and (b) 1.0 MPa.

TABLE 6: Characteristic parameters of Poisson's ratio under different area fractions.

σ (MPa)	A_f (%)	ν_0	ν_∞	ν_A
0.2	40	0.4025	0.4278	0.0253
	50	0.3806	0.4105	0.0299
	60	0.3437	0.4022	0.0585
1.0	40	0.4214	0.4162	-0.0052
	50	0.4037	0.3895	-0.0142
	60	0.3863	0.3765	-0.0098

TABLE 7: Variation tendency of characteristic Poisson's ratio and its curve under different conditions.

Conditions		ν_0	ν_∞	ν_A	Curve tendency
σ	$\sigma > \sigma^t$	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow \downarrow \rightarrow$
	$\sigma < \sigma^t$	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow \rightarrow$
Δ	$\Delta > \Delta^t$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow \rightarrow$
	$\Delta < \Delta^t$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow \downarrow \rightarrow$
k	$k > k^t$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow \rightarrow$
	$k < k^t$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow \downarrow \rightarrow$
ε_x^0	$\sigma > \sigma^t$	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow \downarrow \rightarrow$
	$\sigma < \sigma^t$	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow \rightarrow$
ν	$\sigma > \sigma^t$	$\downarrow$	$\rightarrow$	$\uparrow$	$\uparrow \downarrow \rightarrow$
	$\sigma < \sigma^t$	$\downarrow$	$\rightarrow$	$\uparrow$	$\uparrow \rightarrow$
T	$\sigma > \sigma^t$	$\uparrow$	$\rightarrow$	$\downarrow$	$\uparrow \downarrow \rightarrow$
	$\sigma < \sigma^t$	$\uparrow$	$\rightarrow$	$\downarrow$	$\uparrow \rightarrow$
Af	$\sigma > \sigma^t$	$\downarrow$	$\downarrow$	$\leftrightarrow$	$\uparrow \downarrow \rightarrow$
	$\sigma < \sigma^t$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow \rightarrow$

Note: $\uparrow$ and $\downarrow$ represent increasing and decreasing trends, respectively. $\rightarrow$ means keep unchanged with time.

FIGURE 19: Schematic of the effect of cohesive contact on the transverse strain.

the samples, $I^i(P)$ is the i th Poisson's ratio, and $\overline{I(P)}$ is the mean value which is expressed as

$$\overline{I(P)} = \sum_{i=1}^{N_p} \frac{I^i(P)}{N_p}. \quad (14)$$

Another two morphologies with Af of 60% and RVE size of $1000\mu\text{m}$ are shown in Figure 21. For the different morphologies, the distribution of the particle radius is the same. Only the position of the particles

in the RVE model changed among the three morphologies. Figure 22 shows the curves of Poisson's ratio versus the relaxation time of the three different morphologies with and without mutation. In order to check whether the RVE model used satisfies the criterion or not, five time points, 0.0001 s, 0.01 s, 1 s, 100 s, and 10000 s, are selected in this paper. As we can see from Table 8, the percent of standard deviation/average is all smaller than 1% which means that there is no remarkable change while the realization changes. The simulation result obtained from the proposed RVE model is persuasive.

FIGURE 20: The curves of Poisson's ratio versus time under different loading directions when cohesive strength equals to (a) 0.2 MPa and (b) 1.0 MPa. (c) The relative error between Poisson's ratio under different loading directions.

FIGURE 21: Another two realizations for A_f equals to 60% and RVE size equals to $1000 \mu\text{m}$ (realization in Figure 1(a) is marked as (c)).

FIGURE 22: The curves of Poisson's ratio versus time from different realizations when cohesive strength equals to (a) 0.2 MPa and (b) 1.0 MPa.

TABLE 8: The percent of (standard deviation/average) for Poisson's ratio at five time points.

Time (s)	0.0001	0.01	1	100	10000
Standard deviation/average (Figure 22(a))	0.61%	0.72%	0.15%	0.24%	0.07%
Standard deviation/average (Figure 22(b))	0.21%	0.10%	0.29%	0.49%	0.84%

10. Conclusion

In this paper, a fully numerical framework for the determination of time-dependent Poisson's ratio for solid propellants has been proposed. The definition of time-dependent Poisson's ratio, which is deduced under the relaxation test by the Laplace transform method, is introduced first. Considering the uneven distribution of the strain on the surface of the RVE model, mean longitudinal and transverse strains are utilized in the calculation. Fixed lognormal distribution parameters of particle size as well as the RVE size are used to generate the packing model for solid propellants by means of the MD method. During the analysis, the binder and small particles are treated as the matrix. The viscoelastic matrix, elastic particles, and PPR cohesive zone model are the main mechanical units in the analysis.

By changing the cohesive parameters, loading conditions, and Af during the analysis, some interesting findings are summarized. In order to explain Poisson's ratio mutation, the critical value (or section) of the cohesive parameters is proposed in this paper. The cohesive contact will change its leading role at the two sides of the critical value. Poisson's ratio mutation will be introduced when cohesive contact occupies the dominant position. While the shrinkage of the bulk element is dominant, Poisson's ratio mutation will disappear.

Two examples are designed to validate the simulation results. The first one is utilized to check the direction independence of Poisson's ratio. It was proved that Poisson's

ratio is almost the same for the two directions of the RVE model with or without mutation. By changing the realization of the RVE model, Poisson's ratio is checked by the standard deviation. It was found that the percent of standard deviation/average is all smaller than 1%. The size of the RVE model utilized in the analysis can be accepted.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflict of interest.

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