In this regard, mission autonomy becomes one of the main topics to provide operational safety and reliability for missions under uncertain and highly complex operational environments. From the flight-control point of view, there exists much effort to adopt the advanced methods rather than the conventional linear feedback-control design. It is well known that the linear feedback control can still provide an excellent control solution and effectively expand its applicable operational flight envelope (OFE) using the gain-scheduling strategy. However, it may fail to obtain strong robustness to model uncertainties and unknown external disturbances, which will be highly demanded in future missions. Furthermore, the gain-scheduling strategy typically requires exhaustive design workloads to extend its applicable area up to OFE [1].

As an alternative, various kinds of model-based nonlinear control strategies such as nonlinear dynamic inversion (NDI), backstepping control (BSC), and sliding-mode control (SMC) have been developed and efficiently implemented in the rotorcraft’s configuration in the last few decades [2–5]. A common obstacle to the model-based control design approach is that the method relies on the accuracy of the model dynamics. Among the control strategies mentioned above, NDI is applicable only to the feedback linearizable systems [6]. However, to the authors’ best knowledge, NDI has never been applied directly to the high-fidelity rotorcraft model without resorting to model simplification [2]. On the other hand, BSC and SMC are more flexible than NDI in that their controls can be designed without resorting to full feedback linearization. However, BSC and SMC may still suffer from instability and performance degradation due to unmodeled dynamics and external disturbances.
A good way for handling such a problem is to use various modular or integrated online adaptive schemes to estimate unknown parameters or external disturbances [7–12]. Among them, least-square (LS), tuning-function (TF), and immersion-and-invariance (I&I) methods are reported to be well suited to the BSC framework [7]. The LS-based control design is an indirect adaptive method that reduces trajectory-tracking errors by recursively estimating model parameters. It has several advantages related to the independent designs of the modular estimator. However, since LS adopts the certainty equivalence principle, it is difficult to prove the stability of the closed-loop system.

On the other hand, the TF method is a direct adaptive scheme, in which the adaptive update laws are designed in an integrated manner [9, 13]. The main advantage of the TF method is that the adaptive laws are designed to guarantee the stability of the closed-loop system. However, in a case where relatively large gains are used to enhance the control robustness, undesirable behavior might occur in the trajectory-tracking solution [14].

The I&I approach differs from the TF method in that it adopts an additional nonlinear function to assign stabilized estimation error dynamics [15]. Therefore, the I&I is capable of designing a modular estimator, and the associated controller can also be designed without resorting to the certainty equivalence principle [16]. The main advantage of the I&I method is that the designed I&I estimator guarantees asymptotic convergence of the estimation error. In addition, such a design process allows analyzing the stability of the closed-loop system.

Given the dynamics of rotorcraft and its flight characteristics, it is very difficult to consider all unknown parameters in each element of the mathematical model. Also, adaptive estimators may show poor performance in the presence of combined multiple uncertainties. Therefore, it may not be efficient to estimate specific parameters using the adaptive approach mentioned above. In addition, the effects of external disturbances such as wind gusts and wind shears should be considered during autonomous flight. In general, reducing such undesirable effects is one of the most difficult tasks in rotorcrafts' control problems. An efficient way to deal with both model uncertainties and external disturbances is to design a disturbance observer (DOB) that has been extensively studied and applied to UAVs [17–21]. In these cases, both model uncertainties and external disturbances are expressed in dynamics as a bounded lumped disturbance.

Previous studies have been conducted to effectively cope with the rotorcraft's underactuated system while extending controllers' robustness against existing uncertainties [22, 23]. Both approaches are based on driving incremental dynamics out of the original dynamics, and the slack variable approach has been used to reduce design parameters and simplify the control structure. In Ref. [22], an adaptive incremental backstepping controller (AIBSC) is designed to estimate the uncertain control effectiveness matrix based on the LS method. While the simulation studies using the AIBSC have validated that the separately designed modular LS estimator can identify the unknown control effectiveness matrix, the main limitation of such an approach is that the stability of the closed-loop system can only be guaranteed under the certainty equivalence principle. In Ref. [23], an adaptive incremental backstepping sliding mode controller (AIBS-SMC) is designed to compensate for mismatched uncertainties in the control effectiveness matrix. In the research, the AIBS-SMC proved its excellent trajectory-tracking performance and the stability of the closed-loop system. However, the suggested adaptive component's performance is limited by the overestimation problem due to the monotonically increasing switching control input. The suggested solution is to cease the calculation of the adaptive component when the certain limit is reached. However, such a method is limited in its performance since the method can cause the steady-state error. At last, previous studies have dealt only with constant uncertainties while, realistically, uncertainties are generally completely unknown and time-varying in nature.

In this regard, this paper focuses on developing an easy-to-tune and effective method to compensate for completely unknown, time-varying disturbances while guaranteeing closed-loop stability. Based on the previous studies mentioned above, this paper adopts the I&I method to design a modular nonlinear DOB that deviates from the certainty equivalence principle. As in the conventional I&I method, the observer state is represented as the sum of a dynamic update law and a continuous nonlinear design function. Here, a very simple form of the design function is present to minimize the tuning parameters of the controller. Even so, the resulting disturbance estimation error dynamics can be stabilized while showing the straightforward relationship between the estimation errors and observer gains. The I&I-based DOB is then combined with an adaptive backstepping control (ABSC). To effectively reduce design parameters and simplify the design process, a slack-variable approach [22–25] is adopted to deal with the underactuated system rather than the traditional hierarchical approach [26] or block-BSC approach [27]. The combined ABSC structure with the I&I-based DOB can effectively decouple the adaptive components and the slack variable update laws. The stability of the composite controller is then proved by Lyapunov's stability theorem under both time-invariant and time-varying disturbances. In this process, a relation between the control gains that guarantees the convergence of the proposed composite controller is revealed. The usefulness of the proposed controller is demonstrated through comparative simulations with the tuning function adaptive backstepping control (TF-ABSC) and the standard BSC. The simulations are conducted using the helicopter trim, linearization, and simulation (HETLAS) program, which has been developed for rotorcrafts' fly-by-wire flight control systems [28, 29].

This paper is structured as follows: the rotorcraft dynamics suitable for trajectory-tracking control are derived from Euler's equation in Section 2. The TF-ABSC and the I&I-ABSC designs and their stability analysis are addressed in Section 3. Section 4 presents a series of simulation results with discussions. Finally, conclusions are drawn in Section 5.
2. Rotorcraft Model Description

The rotorcraft dynamics can be expressed by the following compact form of the Euler equations and the kinematic relations.

\[ \dot{\mathbf{v}} = \frac{\mathbf{f}}{m} - \omega \times \mathbf{v}, \]

\[ \dot{\omega} = \mathbf{J}^{-1}(\mathbf{m} - \omega \times \mathbf{I} \omega), \quad \mathbf{v} = \mathbf{C} \mathbf{r}, \quad \omega = \mathbf{L} \dot{\phi}, \]

\[ \mathbf{v} = \begin{pmatrix} u \\ v \\ w \end{pmatrix}, \quad \mathbf{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad \dot{\phi} = \begin{pmatrix} \phi \\ \theta \\ \psi \end{pmatrix}, \]

where \( m, v, \omega, f, m, r, \) and \( \phi \) denote the mass, linear and angular velocities, moment of inertia matrix, external forces and moments, position, and Euler angles, respectively. The transformation matrices, \( \mathbf{C} \) and \( \mathbf{L} \), are represented using the definition of the trigonometric functions like \( \cos \alpha = \cos \phi \) and \( \sin \alpha = \sin \theta \) for \( \alpha = \phi, \theta, \psi \) as

\[
\begin{pmatrix}
\cos \phi \cos \psi & \sin \phi \cos \psi & -\sin \psi \\
\sin \phi \sin \psi & \cos \phi \sin \psi & \cos \psi \\
\cos \phi \cos \psi & \sin \phi \cos \psi & -\sin \psi
\end{pmatrix}
\]

\[
\begin{pmatrix}
1 & 0 & -\sin \phi \\
0 & \cos \phi & \sin \phi \\
0 & \sin \phi & \cos \phi
\end{pmatrix}
\]

The desired trajectories are typically prescribed by the position and heading angle of the aircraft. Therefore, the dynamics for these trajectory states are straightforwardly used in the control design. Using the kinematics in Equation (1), the second-order dynamics for \( (r, \phi) \) are derived as

\[
\dot{\mathbf{r}} = \mathbf{C}^{-1} \left\{ \frac{\mathbf{f}}{m} - \left( \mathbf{L} \dot{\phi} \right) \times \left( \mathbf{C} \mathbf{r} \right) \right\}, \tag{4}
\]

\[
\dot{\phi} = \mathbf{L}^{-1} \left\{ \mathbf{m} - \left( \mathbf{L} \dot{\phi} \right) \times \left( \mathbf{I} \mathbf{L} \dot{\phi} \right) \right\} - \mathbf{L}^{-1} \dot{\mathbf{L}} \dot{\phi}.
\]

The flap and inflow dynamics of the main rotor are omitted in the motion equation since the rotor trim solutions are used considering the separated time scales between the rigid body and the rotor dynamics [30]. In the paper, the Bo-105 helicopter dynamics implemented in HETLAS are used for the control design and validation [31]. Considering the component-based modeling technique adopted in HETLAS, the external forces and moments can be represented by the sum of contributions made by the main rotor (mr), tail rotor (tr), fuselage (fus), stabilizer (stb), and gravity (grav) as shown in the following equations.

\[
f = f_{mr} + f_{tr} + f_{fus} + f_{stb} + f_{grav},
\]

\[
m = m_{mr} + m_{tr} + m_{fus} + m_{stb} + m_{grav}.
\]

The conventional helicopter typically uses four primary controls \( u_p = (\delta_{\phi}, \delta_{cl}, \delta_{ls}, \delta_{tr})^T \) consisting of the main rotor collective \( (\delta_{\phi}) \), the lateral cyclic \( (\delta_{cl}) \), the longitudinal cyclic \( (\delta_{ls}) \), and the tail rotor collective \( (\delta_{tr}) \) to generate the required control forces and moments, which are directly affected by the flap, lead-lag, and feathering motions of rotors. As in the typical high-fidelity rotorcraft model, HETLAS also applies the blade element method (BEM) to integrate nonlinear air loads over the blades. Therefore, \( u_p \) is implicitly included in the rotor dynamics \( (f_{mr}, f_{tr}, m_{mr}, \) and \( m_{tr} \) rather than explicitly described in Equations (4) and (5). The detailed information of the rotor dynamics can be found in Ref. [30].

As a result, the nominal rotorcraft dynamics in Equation (4) can be expressed by the following general form for \( \mathbf{x} = (x, y, z, \phi, \theta, \psi)^T \):

\[
\dot{\mathbf{x}} = \mathbf{f}_n(\mathbf{x}, \dot{\mathbf{x}}, u_p).
\]

To additionally express the mismatching between the physical system and the nominal model, the lumped disturbance vector \( \mathbf{d} \) is added to the dynamic model as follows.

\[
\dot{\mathbf{x}} = \mathbf{f}_n + \Delta \mathbf{f}_n + \mathbf{d}_{\text{external}} = \mathbf{f}_n + \mathbf{d}, \quad \mathbf{d} = \Delta \mathbf{f}_n + \mathbf{d}_{\text{external}} = \begin{pmatrix} \mathbf{d}_f \\ \mathbf{d}_m \end{pmatrix}, \tag{7}
\]

where \( \mathbf{d}_f \) and \( \mathbf{d}_m \) represent vectors composing linear and angular lumped disturbances acting on the aircraft, respectively; \( \Delta \mathbf{f}_n \) represents unmodeled dynamics and parameter perturbation; and \( \mathbf{d}_{\text{external}} \) represents external disturbances. To be specific, the mismatch between the physical system and the nominal model in the force \( (\Delta \mathbf{f}) \), moment \( (\Delta \mathbf{m}) \), mass \( (\Delta \mathbf{m}) \), moment of inertia matrix \( (\Delta \mathbf{I}) \), etc. in Equations (4) and (5) can cause \( \Delta \mathbf{f}_n \). Also, wind gust typically causes the acceleration increment to the aircraft [32]. Thus, the lumped disturbance \( \mathbf{d} \) can be expressed as the sum of \( \Delta \mathbf{f}_n \) and \( \mathbf{d}_{\text{external}} \) [33].

Nonlinear dynamics represented by Equation (7) generally yield difficulties in the control design process since the system is nonlinear in the control and underactuated. Various methods have been developed to derive the approximated control affine form required for the nonlinear control design [34–36]. In this paper, the state-dependent-coefficient (SDC) factorization method proposed by Yang et al. [36] for the state-dependent-Riccati-equation (SDRE) control design is adopted. The method approximates the control effective matrix by the control Jacobian computed with the temporal states. By applying this concept, Equation (7) can be simply transformed into the desired affine form as
\[ \dot{x} = f_u(x, \dot{x}, u_p) + G_n(x, \dot{x}, u_p)u_p + d, \]  

where

\[ f_u = f_n - G_n u_p, \quad G_n = \frac{\partial f_n(x, \dot{x}, u_p)}{\partial u_p} \in \mathbb{R}^{6 \times 4}. \]  

Therefore, the system dynamics for the trajectory-tracking control can be rewritten as

\[ \dot{x} = f_u + G_n u_p + d, \tag{10} \]

\[ y = (x, y, z, \phi, \theta, \psi)^T. \]

The system dimension in Equation (10) is twelve, and there exist four unobservable internal dynamics for \((\phi, \theta, \psi, \dot{\phi})\) because the primary control \(u_p\) explicitly appeared from the second derivative of the system output with the relative degree \(\rho = 8\). It is well known that the internal attitude dynamics are generally unstable over the most rotorcraft OFE. As it was mentioned in Introduction, this paper adopts the slack-variable method to handle the nonsquare matrix \(G_n\) to preserve useful nonlinearities and to reduce the design parameters of the controller at the same time. Padhi et al. [25] introduced a slack variable vector and a slack variable matrix to make the control-effective matrix invertible. The proposed method has been successfully implemented in the SMC design for underactuated systems [24, 36, 37]. After applying the slack-variable approach, Equation (10) can be transformed into the following square dynamics.

\[ \dot{x} = f_u + G u + v + d, \tag{11} \]

where

\[ G = (G_n G_z), \quad u = \begin{pmatrix} u_p \\ u_z \end{pmatrix}, \quad u_z = \begin{pmatrix} u_5 \\ u_6 \end{pmatrix}. \tag{12} \]

Here, \(u_p \in \mathbb{R}^2\) is the slack control, \(u_z\) is the augmented control, and the slack variable matrix \(G_z \in \mathbb{R}^{6 \times 2}\) is selected to make \(G \in \mathbb{R}^{6 \times 6}\) invertible. The matrix \(G_z\) is simply defined by \(G_z = (0_{2 \times 3}, I_{2 \times 2}, 0_{2 \times 1})^T\), and the following control derivative matrix \(G\) is always nonsingular when the rotorcraft has enough control margins for each primary control. Since the main objective of the control problem is to track the desired position and heading angle, the primary control \(u_p\) is assigned to the associated axis. Therefore, the slack control vector \(u_z\) does not affect the closed-loop system in this structure. Even so, \(v\) will be designed using an adaptive update law as it was proposed by Kim et al. [24].

By observation, one can notice that the nominal system in Equation (7) has remained throughout the transformation process from Equation (8) to Equation (11). In summary, the above process can be expressed for later use as

\[ \dot{x} = f_u + d = f_u + Gu + v + d. \tag{13} \]

In order to deal with the underactuated system, the parameter \(v\) is added and subtracted from the dynamics as shown in Equation (11). As a result, the term itself has no effect on the nominal dynamics of the system, whereas the term \(d\) represents any deviation from the nominal model. This will be covered in depth in the control design sections.

3. Adaptive Backstepping Control Design

3.1. Tuning Function Approach. This subsection defines the ABSC structure, and the adaptive update law for the lumped disturbance is designed using the TF approach. The ABSC design starts by defining tracking errors with the continuously differentiable desired trajectory \(x_d\) and the virtual control \(a \in \mathbb{R}^6\) as

\[ z_1 = x - x_d, \tag{14} \]

\[ z_2 = \ddot{x} - \dot{a}. \tag{15} \]

The TF method is used to estimate the unknown slack variables and disturbances. The associated estimation errors can be represented by

\[ \dot{v} = v - \ddot{v}, \]

\[ \dot{d} = d - \ddot{d}. \tag{15} \]

Then, the following relations can be obtained.

\[ \dot{z}_1 = \ddot{x} - \ddot{x}_d = z_2 + a - \ddot{x}_d, \]

\[ \dot{z}_2 = \dddot{x} - \dot{a} = f_u + Gu + v + d - \dddot{a}, \tag{16} \]

Using the control Lyapunov function (CLF) defined with positive definite weight matrices \((Q, \Gamma, \nu) \geq 0\) as shown in Equation (17), the ABSC structure and update laws for estimates \((\dot{v}, \dot{d})\) can be derived to meet the Lyapunov stability criteria of Equation (18).

\[ V = V_{z_1} + V_{z_2} + V_v + V_d \]

\[ = \frac{1}{2} z_1^T Q^{-1} z_1 + \frac{1}{2} z_2^T z_2 + \frac{1}{2} v^T \Gamma_v^{-1} v + \frac{1}{2} d^T \Gamma_d^{-1} d, \tag{17} \]

\[ \dot{V} = \dot{V}_{z_1} + \dot{V}_{z_2} + \dot{V}_v + \dot{V}_d \]

\[ = z_1^T Q^{-1} \dot{z}_1 + z_2^T z_2 + \dot{v}^T \Gamma_v^{-1} \dot{v} + d^T \Gamma_d^{-1} \dot{d} \leq 0. \tag{18} \]

Substituting Equation (16) into Equation (18), Equation (19) is obtained.

\[ \dot{V} = z_1^T Q^{-1} (\alpha - \ddot{x}_d) + z_1^T \left( Q^{-1} \dot{z}_1 + f_u + Gu + v + d - \ddot{a} \right) \]

\[ + \dot{v}^T \left( z_2 + \Gamma_v^{-1} \dot{v} \right) + d^T \left( z_2 + \Gamma_d^{-1} \dot{d} \right). \tag{19} \]
Therefore, the TF-ABSC can be derived as shown in Equation (20), where $K_1 > 0$ and $K_2 > 0$ are positive diagonal gain matrices.

$$
\alpha = -QK_1z_1 + \hat{x}_d,
\quad u = -G^{-1}\left(K_2z_2 + Q^{-1}z_1 + f_a + \hat{v} + \hat{d} - \hat{\alpha}\right). \tag{20}
$$

Regarding the adaptive update laws, the slack variable approach typically assumes slow variation of $v$. Using this assumption ($\dot{v} = 0$), the derivations of the required adaptive laws and update dynamics can be finalized as the following equation:

$$
\begin{align*}
\dot{\hat{v}} &= \Gamma_{\nu}z_2, \\
\dot{\hat{d}} &= \Gamma_{\nu}d_2.
\end{align*}
\tag{21}
$$

By substituting Equations (20) and (21) to Equation (19), Equation (19) becomes

$$
\begin{align*}
\dot{\hat{d}} &= -\sum_{i=1}^{n_0} k_{1i}^2 z_1^2 - \sum_{i=1}^{n_0} k_{2i}^2 z_2^2 + d^T \Gamma_{\nu} \hat{d} = -\sum_{i=1}^{n_0} k_{1i}^2 z_1^2 - \sum_{i=1}^{n_0} k_{2i}^2 z_2^2 + \sum_{i=1}^{n_0} \frac{1}{2\gamma_i} \left(d_i - \hat{d}_i\right)^2 + \sum_{i=1}^{n_0} \frac{1}{2\gamma_i} \hat{d}_i^2 \tag{22}
\end{align*}
$$

Equation (22) states that the TF-ABSC meets input-to-state stability (ISS) with respect to $d$ and $\hat{d}$. Therefore, it is not only the system that converges to a bounded region, but also, the radius of convergence in the trajectory-tracking solution can arbitrarily be reduced by increasing $\gamma_i$. However, the boundedness of both $d$ and $\hat{d}$ must be guaranteed. If the lumped disturbance is time constant ($\hat{d} = 0$), the global asymptotic convergence can be guaranteed by the TF approach.

As shown in Equation (20), the feedback control is affected by the sum of the estimated slack variables and disturbances. Substituting Equation (21) to Equation (20) yields

$$
\begin{align*}
\dot{u} &= -G^{-1}\left(K_2z_2 + Q^{-1}z_1 + f_a + \left(\Gamma_{\nu} + \Gamma_d\right)\right)z_2 - \hat{\alpha}.
\end{align*}
\tag{23}
$$

Equation (23) shows that the TF method adopts integral feedback of the trajectory-tracking error $z_2$. From Equation (23), one can observe that two adaptive update laws are excited by the same input $z_2$. Therefore, the total uncertainty acting in each corresponding axis is compensated by the sum of two estimated values ($\hat{v} + \hat{d}$). As a result, estimation results for $\hat{d}_a$ and $\hat{d}_d$ may show poor accuracy, since slack variables are assigned to the roll and pitch axes in this controller. Even so, when the disturbed system is controllable, the performance and the stability for the TF-ABSC can be guaranteed due to the stability proof in Equation (22).

The controller gain matrices used throughout the derivation are defined with positive diagonal matrices and can be represented by

$$
\begin{align*}
Q &= \text{diag}\left(q_i\right)_{i=1}^{\infty}, \\
\Gamma_{\nu} &= \text{diag}\left(\gamma_i\right)_{i=1}^{\infty}, \\
K_1 &= \text{diag}\left(k_{1i}\right)_{i=1}^{\infty}, \\
K_2 &= \text{diag}\left(k_{2i}\right)_{i=1}^{\infty}, \\
K_d &= \text{diag}\left(k_{2i}\right)_{i=1}^{\infty}.
\end{align*}
\tag{24}
$$

3.2 Immersion-and-Invariance Approach. Details on the I&I using the definition of the invariant manifold can be found in Ref. [15, 38]. Here, a nonlinear DOB based on the I&I method is proposed to estimate the disturbance $d$ expressed in Equation (7). The design of the I&I-based DOB starts by representing the observer state $\hat{d}$ as the sum of the dynamic update law $\xi$ and a continuous nonlinear shaping function $\beta$ as

$$
\hat{d} = \xi + \beta(x, \dot{x}, u_p).
\tag{25}
$$

Then, the estimation error (off-the-manifold coordinate) is defined by

$$
\hat{\xi} = \xi + \beta(x, \dot{x}), \quad \hat{d} = d - (\xi + \beta(x, \dot{x})).
\tag{26}
$$

The estimation error dynamics is derived by taking the time derivative of Equation (26) as

$$
\begin{align*}
\dot{\hat{\xi}} &= \frac{\partial \beta}{\partial x} \dot{x} + \frac{\partial \beta}{\partial \dot{x}} \ddot{x} + \frac{\partial \beta}{\partial u_p} \left(f_p(x, \dot{x}, u_p) + d\right) \\
\dot{\hat{d}} &= \frac{\partial \beta}{\partial x} \dot{x} + \frac{\partial \beta}{\partial \dot{x}} \ddot{x} + \frac{\partial \beta}{\partial u_p} \left(f_p(x, \dot{x}, u_p) + \hat{d} + d\right).
\end{align*}
\tag{27}
$$

In Equation (27), $f_p(x, \dot{x}, u_p)$ is the forces and moments calculated through the stabilized ABSC controller. Assuming that there exists a full-information control law $u_p$ that satisfies the closed-loop stability, $\hat{\xi}$ and $\hat{d}$ can be selected to render Equation (27) stable. This paper proposes the following I&I-based DOB structure:

$$
\begin{align*}
\dot{\xi} &= -\frac{\partial \beta}{\partial x} x - \frac{\partial \beta}{\partial \dot{x}} \dot{x} + f_p(x, \dot{x}, u_p) + \hat{d}, \\
\beta &= \Gamma_{\nu} \dot{x},
\end{align*}
\tag{28}
$$

where $\Gamma_{\nu} > 0$ is a positive diagonal matrix with observer gains. By substituting Equation (28) into Equation (27), the following estimation error dynamics can be obtained.

$$
\dot{\hat{d}} = \dot{d} - \Gamma_{\nu} \hat{d}.
\tag{29}
$$

To analyze the performance of the DOB in Equation (26), the following Lyapunov-like function and its time derivative is considered.
Increasing the observer gain, the estimation error decreases where $k$ is the minimum observer gain. Therefore, if $\|\mathbf{d}\|_{\infty}$ is finite, the estimation converges to a bounded region, and by increasing the observer gain, the estimation error decreases since the region of convergence is bounded by $\|\mathbf{d}\|_{\infty}/y_{\min} > \|\mathbf{d}\|$. Also, when $\mathbf{d}$ is time invariant ($\mathbf{d} = \mathbf{0}$), the estimation can converge to its real value with the convergence rate depending on the values of $\mathbf{d}$.

The shaping function proposed in Equation (28) does not use any other terms except for $\hat{x}$. Substituting the $x$-related term in the shaping function may increase the rate of convergence depending on the desired trajectory, but it may be difficult to choose appropriate observer gains, and the computational workload will be increased.

Now, the I&I-ABSC can be designed using the defined modular I&I-based DOB. With reference to the CLF shown in Equation (17), $W$ in Equation (30) is substituted to the CLF instead of $V$.

\[
W = \frac{1}{2} \mathbf{d}^T \mathbf{d},
\]

\[
\dot{W} = \mathbf{d}^T \dot{\mathbf{d}} + \mathbf{d}^T \dot{\mathbf{d}} - \mathbf{d}^T \Gamma_d \dot{\mathbf{d}} \leq \|\mathbf{d}\|_{\infty} - y_{\min} \|\hat{\mathbf{d}}\|.
\]

\[
\text{where } y_{\min} \text{ is the minimum observer gain. Therefore, if } \|\mathbf{d}\|_{\infty} \text{ is finite, the estimation converges to a bounded region, and by increasing the observer gain, the estimation error decreases since the region of convergence is bounded by } \|\mathbf{d}\|_{\infty}/y_{\min} > \|\mathbf{d}\|. \text{ Also, when } \mathbf{d} \text{ is time invariant (} \mathbf{d} = \mathbf{0} \text{), the estimation can converge to its real value with the convergence rate depending on the values of } \Gamma_d.
\]

The I&I-ABSC structure can be summarized as

\[
\alpha = -QK_\nu \mathbf{z}_1 + \hat{x}_d,
\]

\[
u = -G^{-1}
\left(K_\nu \mathbf{z}_2 + Q^{-1} \mathbf{z}_1 + f_\nu + \hat{\nu} + \mathbf{d} - \mathbf{\hat{a}}\right),
\]

\[
\hat{\mathbf{d}} = \Gamma_\nu \mathbf{z}_2,
\]

\[
\mathbf{d} = \xi + \beta.
\]

At this point, it is worth mentioning that the proposed I&I-ABSC differs from the TF-ABSC only in its use of I&I-based DOB for estimating $\mathbf{d}$ instead of the TF update law. Thus, both methods are applicable for the same ABSC structure (or the same control gains) regardless of their different approach for handling the disturbances. Note that the same gain matrices in Equation (24) are adopted in the I&I-ABSC. For analyzing the stability of the I&I-ABSC, substituting Equations (34) into Equation (33) yields

\[
\dot{V} = -z_1^T K \mathbf{z}_1 - z_2^T K_\nu \mathbf{z}_2 + \mathbf{d}^T \mathbf{d} - \mathbf{d}^T \Gamma_d \mathbf{d}
\]

\[
= \sum_{i=1}^{n_c} k_{ij} z_{ij}^2 - \sum_{i=1}^{n_c} \lambda_{ii} z_{ij}^2 + \sum_{i=1}^{n_c} d_{ii} z_{ij}^2 - \frac{1}{2} \sum_{i=1}^{n_c} y_{ij} d_{ij}^2
\]

\[
= \sum_{i=1}^{n_c} k_{ij} z_{ij}^2 - \sum_{i=1}^{n_c} \lambda_{ii} z_{ij}^2 + \sum_{i=1}^{n_c} d_{ii} z_{ij}^2 - \frac{1}{2} \sum_{i=1}^{n_c} y_{ii} d_{ij}^2
\]

\[
= \sum_{i=1}^{n_c} \left( z_{ij} - y_{ij} d_{ij} \right)^2 + \sum_{j=1}^{n_c} \frac{d_{ij}^2}{2y_{ij}} \leq \sum_{i=1}^{n_c} \left( k_{ij} \right)^2 z_{ij}^2 + \sum_{j=1}^{n_c} \frac{d_{ij}^2}{2y_{ij}}.
\]

\[
\sum_{i=1}^{n_c} \left( k_{ij} \right)^2 z_{ij}^2 + \sum_{j=1}^{n_c} \frac{d_{ij}^2}{2y_{ij}}.
\]

Figure 1: Pirouette maneuver.
Equation (35) proves the ISS of the I&I-ABSC when \( \gamma_i > 1/2 k_{2,i} \). As a result, not only does the system converge to a bounded region, but the radius of convergence in the trajectory-tracking solution can be reduced arbitrarily by increasing \( \gamma_i \). Unlike Equation (22), however, the ISS of the I&I-ABSC only requires boundedness of \( \dot{d} \). When the uncertainties are time-invariant, Equation (35) becomes

\[
\dot{V} = - \sum_{i=1}^{i=6} k_{1,i} \dot{x}_{i,1,i}^2 - \sum_{i=1}^{i=6} k_{2,i} \left( z_{2,i} - \frac{\dot{d}_i}{2k_{2,i}} \right)^2 - \sum_{i=1}^{i=6} \gamma_i \frac{1}{4k_{2,i}} \dot{d}_i^2. 
\]

With \( \gamma_i \geq 1/4k_{2,i} \), the global stability can be guaranteed.

The I&I-ABSC design ensures global asymptotic stability for time-invariant disturbances while also improving stability criteria by requiring less stringent conditions on the lumped disturbances when compared to the TF-ABSC.

Moreover, unlike TF-ABSC, it is expected that the proposed I&I-ABSC can accurately estimate \( \ddot{d} \) regardless of the estimated \( \dot{\gamma} \) since \( \ddot{d} \) is calculated through available state values instead of \( z_2 \). This is possible since the I&I-based DOB only estimates the deviation from the nominal model as in Equation (28) and the term \( \dot{\gamma} \) does not affect the nominal model as demonstrated in Equation (13).

The performance of the I&I-ABSC (34) will be compared to the TF-ABSC (20)–(21) and the standard BSC in the simulation section. In this paper, the BSC without the adaptation law for \( \ddot{d} \) is simply defined as the following equation.

\[
\dot{\alpha} = -Q K z_1 + \dot{x}_d,
\]

\[
u = -G^{-1} (K z_2 + Q^{-1} z_1 + f_a + \dot{\gamma} - \dot{\omega}),
\]

\[
\dot{\gamma} = \Gamma_\gamma z_2.
\]

4. Simulation Results and Discussion

In this section, simulation results are presented. All simulations are conducted in Intel Fortran software, with the help of Intel Core i7-6700K CPU (4.0 GHz) with 32 GB of RAM. Also, the control sampling rate is set to 0.01 s.

To validate both steady and transient performances of each controller, the pirouette maneuver and the helical turn
The pirouette maneuver is initiated from a stabilized hover over a point above 10 ft from the ground on the circumference of a 100 ft radius circle with the heading angle pointed at the center of the circle during the maneuver. The maneuver is depicted in Figure 1. Each simulation starts at the initial hover point and ends at the same point after the steady turn maneuver around the circle with the lateral velocity of 8 knots for 45 seconds. The final hovering maneuver is then applied for 10 seconds at the final point to see if the rotorcraft can stabilize after the maneuver.

The helical turn maneuver is initiated from a 60-knot level flight state and returned to the same flight condition.
after 900 degrees of heading change with the attitude increase of 400 ft for 60 seconds. The maneuver is depicted in Figure 2.

4.1. Case A: Performance under External Disturbances—Pirouette. To compare the performances of the BSC (37), TF-ABSC (20)-(21), and I&I-ABSC (34) under large time-varying disturbances, Equation (39) is applied to the nominal plant from 5 to 45 seconds of the maneuver, allowing 3 cycles of sinusoidal disturbances to be applied during the steady-turn maneuver. The control gains in Equation (38) are uniformly applied to two adaptive controllers, whereas $K_1$, $K_2$, $Q$, and $\Gamma_\psi$ in Equation (38) are applied to the BSC.

$$K_1 = \{k_{1,i}\}_{i=1}^{i=6} = 0.75, K_2 = \{k_{2,i}\}_{i=1}^{i=6} = 2.50, Q = \{q_{i}\}_{i=1}^{i=6} = 1.50,$$

$$\Gamma_\psi = \{\gamma_{i}\}_{i=1}^{i=6} = 5.00, \Gamma_\psi = \{0, 0, 0, 1, 1, 0\},$$

(38)

\begin{table}[h]
\centering
\caption{Case A: root-mean-squared tracking errors.}
\begin{tabular}{lccc}
\hline
Tracking state & BSC & TF-ABSC & I&I-ABSC \\
\hline
$x$ (m) & 16.7489 & 10.0987 & 1.8242 \\
$y$ (m) & 16.6437 & 10.2394 & 1.9580 \\
$h$ (m) & 16.6497 & 10.0378 & 1.6223 \\
$\psi$ (deg) & 769.8182 & 460.4518 & 74.7361 \\
$x$ (m/s) & 7.7459 & 5.0238 & 0.8362 \\
$y$ (m/s) & 7.7923 & 5.1048 & 0.9200 \\
$h$ (m/s) & 7.7652 & 5.0106 & 0.8531 \\
$\psi$ (deg/s) & 363.5249 & 229.5596 & 39.2243 \\
\hline
\end{tabular}
\end{table}
discernible differences in their control inputs. On the other hand, for the BSC, the tail rotor collective reaches its limit (20 deg) around 37 s due to the excessive path deviation in the yaw axis. As shown in Figures 4 and 5, the trajectory-tracking performance can be improved by adopting adaptive approaches. Especially when comparing the tracking errors shown in Figure 6 (ε subscript denotes error in the
corresponding state), one can notice that the I&I-ABSC shows superior tracking performance among the three controllers. Quantitative values of tracking errors can be compared with the root-mean-squared errors in Table 1.

The main difference between the TF-ABSC and the I&I-ABSC lies in the disturbance estimation results shown in Figures 7 and 8. The estimation accuracy of the I&I-ABSC is higher compared to that of the TF-ABSC over the entire flight time when the same gains are applied. Furthermore, the proposed design method using the I&I-based DOB effectively estimates the disturbance acting on the pitch and roll axes, while the estimation of the TF-ABSC on the pitch and roll axes is inaccurate.

However, since the primary control $u_p$ is assigned to the position and heading angle, the compensation for external disturbances for the attitude states does not affect the calculation of $u_p$. This is due to the structure of the slack variable matrix suggested in Section 2. A different approach of slack variables can be found in Ref. [24] where the matrix is designed using the reduced row echelon form and the associated slack variables become effective. In this case, the gains for generating the desired values of $\varphi$ and $\theta$ should be carefully selected, and the slack variables must be generated adequately. However, if accurate autonomous trajectory tracking is required in harsh environments, it is difficult to find gains that satisfy stability and desired trajectory conditions at the same time. Therefore, the I&I-ABSC combined with the slack variable approach can greatly reduce the design workloads while still providing accurate tracking results.

### 4.2. Case B: Performance under Both Parametric Uncertainties and External Disturbances—Helical Turn

In this subsection, simulations are conducted to verify the performance of the I&I-ABSC under both model uncertainties and external disturbances. The inertial and mass properties of a rotorcraft can be frequently changed for several reasons. Underestimating both properties in the controller can lead to serious performance degradation. Furthermore, underestimating the rotor force necessitates more control effort to compensate for the degradation. In practice, this can happen to varying degrees due to a mismatch between the real physical system and the nominal model. The disturbances and uncertainties shown in Table 2 are applied during the helical turn maneuver to validate the performance of the I&I-ABSC under harsh conditions. The same controller gains in Equation (38) are used except for $\Gamma_d = 3.0$.

The results of the helical turn simulation confirm that complex maneuvers can be completed successfully with fixed gains using the proposed controller. According to Figure 9, the I&I-ABSC has the most stabilized controls of all. On the other hand, oscillations in the controls of the BSC and

<table>
<thead>
<tr>
<th>Uncertainty</th>
<th>Property</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>External disturbance</td>
<td>$d_{f,\text{external}} = 2.0 \times \sin \left(T\right) \text{ m/s}^2$</td>
<td>10–50 s</td>
</tr>
<tr>
<td></td>
<td>$d_{m,\text{external}} = 11.4592 \times \sin \left(T\right) \text{ deg/s}^2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$T = 3 \times 2\pi t/40 \text{ s}$</td>
<td></td>
</tr>
<tr>
<td>Mass &amp; moment of inertia uncertainty</td>
<td>50% less than the plant model</td>
<td>10–60 s</td>
</tr>
<tr>
<td>Rotor force uncertainty</td>
<td>20% degraded</td>
<td>0–60 s</td>
</tr>
</tbody>
</table>

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<tr>
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</tr>
<tr>
<td>Rotor force uncertainty</td>
<td>20% degraded</td>
<td>0–60 s</td>
</tr>
</tbody>
</table>

![Figure 9: Case B: calculated controls of BSC, TF-ABSC, and I&I-ABSC.](image-url)
the TF-ABSC are frequently observed. Dramatic difference between controllers can be found in Figures 10 and 11. In particular, the TF-ABSC and BSC shows oscillatory behaviors in their velocity states. Overall, the I&I-ABSC exhibits minimal tracking errors, as shown in Figure 12 and Table 3. Again, this is due to the I&I-based DOB's ability to estimate time-varying disturbances quickly enough, as shown in the I&I estimation results in Figure 13 whereas the TF-ABSC shows poor estimation results in Figure 14. Although the I&I-ABSC cannot estimate exact parameters, the I&I-DOB observes and compensates for the overall effect of the mismatch. As a result, even when the mismatch is severe, the controller can effectively maintain its performance. These simulation results showed that the I&I-ABSC
works well for both model uncertainties and external disturbances.

5. Conclusions

In this paper, the I&I-ABSC has been developed for the rotorcraft’s trajectory-tracking control problem. The rotorcraft’s dynamics were transformed into the position and attitude states to match the system states with the prescribed desired trajectory. Instead of adopting a hierarchical approach or block backstepping approach, the slack variable approach is adopted to cope with the rotorcraft’s underactuated system and to simplify the design process. To effectively compensate for the completely unknown lumped disturbance, the I&I-based DOB has been developed under the assumption that the time derivative of the disturbance is bounded. To minimize the design parameters, the observer structure is kept simple. The performance of the I&I-based DOB has been theoretically investigated using the Lyapunov-like function. Then, the I&I-based DOB is

**Figure 12:** Case B: position and velocity tracking errors of BSC, TF-ABSC, and I&I-ABSC.

**Table 3:** Case B: root-mean-squared tracking errors.

<table>
<thead>
<tr>
<th>Tracking state</th>
<th>Control strategy</th>
<th>BSC</th>
<th>TF-ABSC</th>
<th>I&amp;I-ABSC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$ (m)</td>
<td></td>
<td>19.1013</td>
<td>14.4671</td>
<td>12.7250</td>
</tr>
<tr>
<td>$y$ (m)</td>
<td></td>
<td>19.5573</td>
<td>12.1945</td>
<td>10.0530</td>
</tr>
<tr>
<td>$h$ (m)</td>
<td></td>
<td>37.8101</td>
<td>9.4125</td>
<td>3.1919</td>
</tr>
<tr>
<td>$\psi$ (deg)</td>
<td></td>
<td>1.1596e+03</td>
<td>475.6811</td>
<td>170.0442</td>
</tr>
<tr>
<td>$\dot{x}$ (m/s)</td>
<td></td>
<td>8.3542</td>
<td>5.1023</td>
<td>3.2202</td>
</tr>
<tr>
<td>$\dot{y}$ (m/s)</td>
<td></td>
<td>8.6369</td>
<td>5.3703</td>
<td>3.4476</td>
</tr>
<tr>
<td>$\dot{h}$ (m/s)</td>
<td></td>
<td>9.5125</td>
<td>4.6023</td>
<td>1.6852</td>
</tr>
<tr>
<td>$\dot{\psi}$ (deg/s)</td>
<td></td>
<td>530.3183</td>
<td>249.6678</td>
<td>88.9577</td>
</tr>
</tbody>
</table>
combined into the ABSC structure. The stability analysis of the I&I-ABSC’s closed-loop system demonstrates ISS for the time-varying disturbance and shows that Lyapunov’s global stability condition is satisfied when the disturbance is time invariant.

Finally, a series of simulations were conducted using the pirouette and helical turn maneuvers as the desired trajectories and the high-fidelity Bo-105 model as the system plant to validate the performance of the proposed controller. To prove the effectiveness of the proposed controller, external
disturbances, parameter changes, and modelling errors were considered during simulations. As a result, it has been shown that the I&I-ABSC guarantees superior trajectory tracking and estimation performance compared to the standard BSC and the TF-ABSC under complex disturbances.

**Data Availability**

The simulation program data used to support the findings of this study have not been made available to protect its intellectual property rights.

**Conflicts of Interest**

The authors declared that there is no conflict of interest regarding the publication of this paper.

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