Hindawi
International Journal of Aerospace Engineering
Volume 2022, Article ID 8650592, 17 pages
https://doi.org/10.1155/2022/8650592

Research Article

Adaptive Super-Twisting Cooperative Guidance Law with Fixed-Time Convergence

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Received 4 July 2022; Revised 7 August 2022; Accepted 8 August 2022; Published 27 August 2022

Academic Editor: Xingling Shao

For the guidance problem of multiple missiles cooperatively intercepting a maneuvering target, fixed-time cooperative guidance law is proposed with impact angle constraints. First, fixed-time consensus, integral sliding mode technique, and adaptive super-twisting algorithm are implemented to construct cooperative guidance law along the line-of-sight (LOS) direction. Second, based on fixed-time convergence observer and terminal sliding mode control, one fixed-time guidance law along the normal direction of LOS is designed to ensure that the missiles attack the target with expected angles. Finally, simulation results are carried out to demonstrate the effectiveness of the proposed fixed-time cooperative guidance law.

1. Introduction

As the development of missile defense system in recent years, the traditional one-to-one engagement has been difficult to complete the interception missions [1]. In such a circumstance, a concept on multimissile cooperation, which means that multiple missiles attack a single target or multiple targets cooperatively [2], is proposed. Multimissile cooperation is considered as an efficient measure to improve the interception probability of missiles, which can complete the interception missions that the single missile cannot accomplish [3].

In recent years, cooperative control protocols, which consist of leaderless consensus [4, 5], leader-following consensus [5–7], containment control protocols [8, 9], and cooperative surrounding control [10, 11], have been more investigated, while cooperative guidance, which can be considered one special cooperative control, has also been made great progress. It is known that cooperative guidance includes time-cooperative guidance and space-cooperative guidance. Time-cooperative guidance means that all missiles achieve consistency of the time-to-go through information sharing [12], which is beneficial for improving missiles' combat effectiveness and survivability. The simultaneous attack of multiple missiles can be achieved by two methods. The first method is called impact time control guidance (ITCG) [13], which requires all missiles to attack the target within a predetermined common impact time. An ITCG law for simultaneous attack is carried out in [14], in which the limit of seeker look angle is considered as an important factor in the design of guidance law. Based on the sliding mode (SM) control, a nonsingular guidance law with impact time constraint is presented in [15], in which the impact time is prescribed and cannot be adaptively adjusted. However, it is difficult to determine an appropriate impact time in advance for the different initial conditions of missiles. To solve this problem, impact time coordination guidance is studied as the second method, which requires every missile communicates with other missiles to reach a consensus of impact time [16]. In [16], two kinds of sliding-mode guidance laws are developed for intercepting various target motions, including maneuvering and nonmaneuvering targets. In the certain particular combat scenario, the constraints of impact angles should also be considered to enhance missiles' destruction effect [17]. In order to satisfy the impact angle constraints, a guidance law based on SM is carried out in [18], which is designed with a line-of-sight rate shaping process. The shaping process results in a tuning parameter for creating a line-of-sight rate profile that guarantees the final time and flight path angle requirements will be met. The cooperative guidance laws designed in [19] are capable of satisfying impact angle constraints, in which distributed cooperative guidance laws consist of a local control term to achieve zero miss distance and the desired impact angle. However, the cooperative laws utilize the time-to-go derived from the linearized engagement dynamics, that is, these cooperative guidance laws can only deal with low-speed targets. This point limits the applications in practical engineering where the targets are highly maneuvering.

The rapid convergence is an important requirement for missiles, quadrotors, and other flight vehicles, because the final guidance time is short and about a few seconds [20]. Therefore, variable high-performance control strategies, including funnel control [21, 22], prescribed performance control [23, 24], and finite/fixed-time sliding mode techniques [25–31], have been applied to flight control systems. Compared with the traditional asymptotic stability, finite-time control can obtain faster convergence rate [25, 32]. This can be achieved by a specific kind of guidance law based on terminal sliding mode (TSM) control. Based on time-varying TSM, a new guidance law for intercepting a maneuvering target is presented in [26], in which the distributed protocol with accurate time-to-go ensures the simultaneous attack task can be solved within finite time. A finite-time SM guidance law with impact angle constraints is presented in [27], which ensures that the LOS angular rate will converge to zero before the final time of the guidance process. In [28], Song et al. propose a finite-time adaptive super-twisting (ST) cooperative guidance law with impact angle constraints, which extends the adaptive ST cooperative guidance law to the three-dimensional guidance system. In order to simultaneously attack a target with impact angle constraints, a novel distributed cooperative guidance law for multiple missiles under directed communication topologies is carried out in [29]. However, these guidance laws in [25–29, 32] are studied on the finite-time control theory. The convergence time of finite-time cooperative guidance law is related to the initial conditions of the system. As an extension of the finite-time control, a concept on fixed-time control [33], which means that the convergence time is independent of the initial states of the system and can be given in advance by setting parameters, is proposed. A fixed-time cooperative guidance law for attacking stationary targets is carried out in [34], in which an adaptive law is designed for disposing of the uncertainty. In order to attack a maneuvering target, a fixed-time cooperative guidance law is presented with constraints including the impact time and the LOS angle [30]. The designed fixed-time cooperative guidance law in [31] can satisfy the impact angle constraints, which regards the interference in the direction of LOS as zero.

In the process of guidance law design, the external disturbances caused by the target acceleration are an important problem about which one should concern. An adaptive sliding mode guidance law without information about the bounds of disturbances is proposed in [35], which is discontinuous for the use of sign function and leads to chattering problem. In [36], a cooperative guidance law with intercepting strong maneuvering target is studied, which needs to know the upper bound of the accelerations of the missiles and the target. However, the information of accelerations is difficult to obtain in actual flight. Based on the leader-follower control scheme, a fixed-time cooperative guidance law with impact angle constraints is proposed in [37], in which followers are required to communicate with leader. In order to save communication resources, a fixed-time cooperative guidance law with intercepting maneuvering target is studied in [38], in which the upper bound of the acceleration in the direction of LOS is applied directly in the guidance law.

Motivated by the above discussions, this paper will pay more attention to the design of cooperative guidance law with impact angle constraints based on fixed-time stability control. An adaptive ST cooperative guidance law with fixed-time convergence is presented for intercepting a single maneuvering target. The main contributions of this paper can be concluded as follows.

(1) Compared with Refs. [30, 31, 34], a new fixed-time consensus protocol based on the integral sliding mode and adaptive ST algorithm, i.e., the acceleration command on the LOS direction, is designed to guarantee that the time-to-go for each missile achieves consensus within fixed time. The stability of the system is demonstrated by Lyapunov stability theory, and the convergence time is independent of the initial conditions of the system

(2) Different from Refs. [34–36, 38], the continuous control input is achieved by introducing adaptive ST algorithm without any knowledge of external disturbances. The problem of overestimation or underestimation of switching gain is solved effectively

(3) Compared with Refs. [30, 37], a novel fixed-time guidance law, i.e., the acceleration command on the normal direction of the LOS, is developed to ensure that the LOS angles of all missiles can converge to the expected LOS angles rapidly. It can guarantee the system stabilization is independent on initial conditions without singularity

The rest of this paper is arranged as follows. Section 2 states problem formulations and related preliminaries. The main results are developed in Section 3, adaptive ST fixed-time cooperative guidance laws are presented, and the proof of the corresponding stability is given in detail. In Section 4, numerical simulations are provided to demonstrate the effectiveness of the proposed cooperative guidance laws. Conclusions are finally drawn in Section 5.

2. Problem Formation and Preliminaries

In this section, a brief description about the planar guidance geometry of multiple missiles and target is provided. Then, the algebraic graph and fixed-time control theory are formulated in details.

2.1. Problem Statement. For simplicity, these assumptions are made: (1) the pursuer and target are considered as point

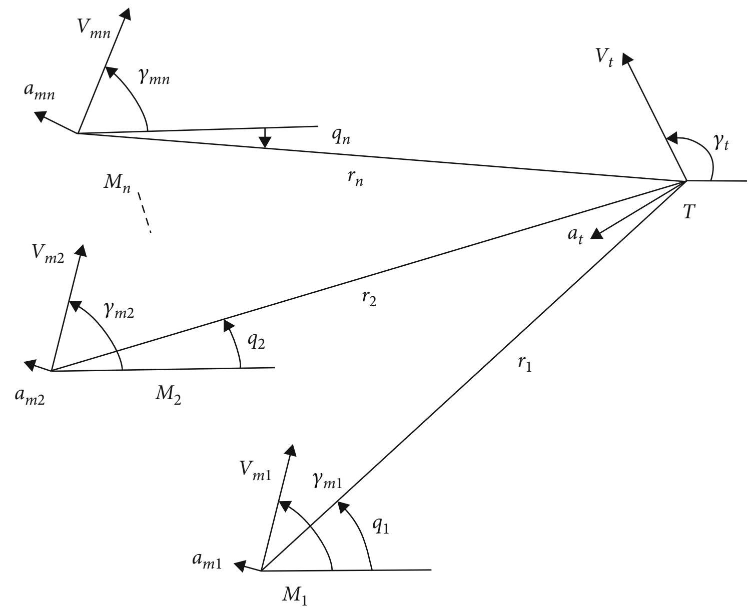


Figure 1: Homing engagement geometry.

masses; (2) seeker dynamics and the autopilot of the pursuer are fast enough to be neglected; (3) the speeds of the missile and target are constant. The 2D homing guidance geometry is shown in Figure 1, where the relevant explanations are provided below.

The corresponding relative kinematic equations can be described as follows

$$\begin{cases} \dot{r}_i = V_t \cos(\gamma_t - q_i) - V_{mi} \cos(\gamma_{mi} - q_i), \\ r_i \dot{q}_i = V_t \sin(\gamma_t - q_i) - V_{mi} \sin(\gamma_{mi} - q_i), \\ \dot{\gamma}_{mi} = \dfrac{a_{mi}}{V_{mi}}, \\ \dot{\gamma}_t = \dfrac{a_t}{V_t}, \end{cases} \tag{1}$$

where V_{mi} and V_t denote velocity vectors of missiles and target, $i = 1, \cdots n$; r_i is the relative distance between the missile i and the target. The flight path angles and LOS angle are denoted by γ_{mi}, γ_t and q_i, respectively. The normal acceleration of the missile and the target is denoted by a_{mi} and a_t.

The derivatives of the first two expressions of equation (1) can be obtained as

$$\begin{cases} \ddot{r}_i = r_i \dot{q}_i^2 + u_{ri} - \omega_{ri}, \\ \ddot{q}_i = -\dfrac{2\dot{r}_i}{r_i}\dot{q}_i - \dfrac{u_{qi}}{r_i} + \dfrac{\omega_{qi}}{r_i}, \end{cases} \tag{2}$$

where u_{ri}, ω_{ri} are the acceleration components of the missile and the target in the LOS direction, and u_{qi}, ω_{qi} are the acceleration components of the missile and the target in the normal direction of the LOS, respectively. We can get

$$\begin{aligned} u_{ri} &= a_{mi} \sin(\gamma_{mi} - q_i), \omega_{ri} = a_t \sin(\gamma_t - q_i), \\ u_{qi} &= a_{mi} \cos(\gamma_{mi} - q_i), \omega_{qi} = a_t \cos(\gamma_t - q_i). \end{aligned} \tag{3}$$

Define the state variables as $x_{1i} = r_i$, $x_{2i} = \dot{r}_i$, $x_{3i} = q_i - q_{di} = q_{ei}$, and $x_{4i} = \dot{q}_i = \dot{q}_{ei}$, where q_{di} is the terminal expected LOS angle. Therefore, the cooperative guidance model can be described as

$$\begin{cases} \dot{x}_{1i} = x_{2i}, \\ \dot{x}_{2i} = x_{1i} x_{4i}^2 + u_{ri} - \omega_{ri}, \\ \dot{x}_{3i} = x_{4i}, \\ \dot{x}_{4i} = -\dfrac{2x_{2i}x_{4i}}{x_{1i}} - \dfrac{u_{qi}}{x_{1i}} + \dfrac{\omega_{qi}}{x_{1i}}. \end{cases} \tag{4}$$

We introduce a new variable t_{goi} which denotes the time-to-go for the missile i. The change of the velocity $\dot{r}$ is relatively small, and the t_{goi} can be estimated by

$$t_{goi} = -\frac{r_i}{\dot{r}_i} = -\frac{x_{1i}}{x_{2i}}. \tag{5}$$

Taking the derivative of equation (6) yields

$$\dot{t}_{goi} = -1 + \frac{x_{1i}^2 x_{4i}^2}{x_{2i}^2} + \frac{x_{1i}}{x_{2i}^2} u_{ri} - \frac{x_{1i}}{x_{2i}^2} \omega_{ri}. \tag{6}$$

Define the new state variables as

$$\tilde{u}_{ri} = \frac{x_{1i}^2 x_{4i}^2}{x_{2i}^2} + \frac{x_{1i}}{x_{2i}^2} u_{ri}, d_{ri} = -\frac{x_{1i}}{x_{2i}^2} \omega_{ri}, d_{qi} = \frac{\omega_{qi}}{x_{1i}}, \tag{7}$$

where d_{ri}, d_{qi} are denoted as the external disturbance caused by the acceleration of the target, substituting (7) and (8) into (5), we obtain the new dynamic equations as follows

$$\begin{cases} \dot{t}_{fi} = \dot{t}_{goi} + 1 = \tilde{u}_{ri} + d_{ri}, \\ \dot{x}_{3i} = x_{4i}, \\ \dot{x}_{4i} = -\dfrac{2x_{2i}x_{4i}}{x_{1i}} - \dfrac{u_{qi}}{x_{1i}} + d_{qi}. \end{cases} \tag{8}$$

Remark 1. In view of equation (9), for multiple missiles intercepting a maneuvering target cooperatively, the design of the cooperative guidance law can be divided into two stages. The first stage is to design the control input $\tilde{u}_{ri}$ of the LOS direction, which guarantees that the interception time for all missiles reach the consistency within a fixed time. The second stage is to design the control input u_{qi} on the normal direction of LOS, which guarantees that the LOS angular velocity approaches zero and the LOS angle converges to the expected value.

Assumption 2. The unknown and bounded external disturbances are, respectively, denoted by d_{ri}, d_{qi}, which is caused by the acceleration of the target, $\dot{d}_{ri}$ and $\dot{d}_{qi}$ satisfy $|\dot{d}_{ri}| \le D_{r2}$ and $|\dot{d}_{qi}| \le D_{q2}$, where D_{r2} and D_{q2} are unknown positive constants.

Remark 3. Since the target maneuvers can be considered as generation of autopilot with first-order dynamics, the accelerations of the target and first-order derivatives of the target are bounded. So, $\dot{d}_{ri}$ and $\dot{d}_{qi}$ are also bounded. Therefore, in the process of interception engagement, external disturbances constraints are reasonable for Assumption 2.

2.2. Algebraic Graph. The missile can be regarded as an agent in the process of cooperative engagement. Graph theory is an important tool to study multiagent system, which describes the communication between all missiles by means of topology diagram.

The communication network between missiles is described by an undirected graph$G = \{V, E, A\}$, where $V = \{v_i, i = 1, 2, \cdots, n\}$ represents a set of n agents, an edge set $E \subseteq V \times V$ represents communication channel between two agents, and a matrix $A = [a_{ij}] \in R^{n\times n}$ represents the weighted adjacency matrix. If node v_i and v_j can communicate, then, $a_{ij} = a_{ji} = 1$, otherwise, $a_{ij} = a_{ji} = 0$. The neighbor of the node v_i is represented by the set $N_i = \{j : (v_i, v_j) \in E\}$, the degree of the node v_i is defined asdeg $(v_i) = d_i = \sum_{j=1}^{n} a_{ij} = \sum_{j\in N_i} a_{ij}$, and the degree matrix of the graph G is expressed as$D = \operatorname{diag}\{d_i, i = 1, 2, \cdots, n\}$. The Laplacian matrix corresponding to the undirected graph G is defined as $L = D - A = [l_{ij}]$, where$j = i$, $l_{ii} = \sum_{k=1}^{n} a_{ik}$,, otherwise, $l_{ij} = -a_{ij}$.

In addition, a graph is connected if there is an information exchange between any two nodes.

Lemma 4 (see [39]). *It supposes that $L \in R^{N\times N}$ is the Laplacian matrix of the graph G, then, the following conclusions are established:*

(1) The matrix L is semipositive definite with only one zero eigenvalue, and the other eigenvalues are positive real numbers if and only if the graph is unconnected

(2) If the graph G is an undirected graph, the following equation holds:

$$x^T L x = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\left(x_j - x_i\right)^2. \tag{9}$$

Lemma 5 (see [39]). *The second small eigenvalue of the Laplacian matrix L corresponding to an undirected graph G is defined as the algebraic connectivity of the graph G. If the undirected graph is connected, the following inequality is satisfied:*

$$\lambda_2(L) = \min_{\|x\|\neq 0, 1_N^T x = 0} \frac{x^T L x}{\|x\|^2} > 0. \tag{10}$$

Therefore, if $\sum_{i=1}^{N} x_i = 0$, the following inequality is satisfied:

$$x^T L x \ge \lambda_2(L) x^T x. \tag{11}$$

2.3. Fixed-Time Stability. Consider a nonlinear system

$$\dot{x}(t) = f(x(t)), x(0) = x_0, \tag{12}$$

with $x \in R^n$, and $f : R^n \longrightarrow R^n$ is an autonomous function such that $f(0) = 0$ $(x = 0)$ is an equilibrium point of nonlinear system (13).

Definition 6 (see [40]). The equilibrium point of system (13) is globally finite-time stable if it is globally asymptotically stable and any solution $\xi(t, x_0)$ of this system converge to the origin within finite time, $\forall t \ge T(x_0)$: $\xi(t, x_0) = 0$ where $T : R^n \longrightarrow R_+ \cup \{0\}$ and $R_+ = \{l : l \in R, l > 0\}$.

Definition 7 (see [33]). The equilibrium point of system (13) is fixed-time stable if the settling time is bounded and independent of initial conditions, i.e., $\exists T_{\max} > 0 : \forall x_0 \in R^n$ and $T(x_0) \le T_{\max}$.

Lemma 8 (see [33]). *For the system (13), if there exists Lyapunov function $V(x)$, scalars $\alpha, \beta, p, q \in R_+$ $(0 < p < 1, q > 1)$ such that $\dot{V}(x) \le -\alpha V(x)^q - \beta V(x)^p$ holds, the trajectory of this system is practical fixed-time stable. The bounded time required to reach the residual set is estimated by*

$$T(x_0) \le \frac{1}{[\beta(1-p)]} + \frac{1}{[\alpha(q-1)]}. \tag{13}$$

Lemma 9 (see [41]). *For $x_i \in R$ $(i = 1, 2, \cdots, n)$, $0 < \nu_1 \le 1$ and $\nu_2 > 1$, the following inequalities are fulfilled.*

$$\left(\sum_{i=1}^{n}|x_i|\right)^{\nu_1} \leq \sum_{i=1}^{n}|x_i|^{\nu_1} \quad and \quad \left(\sum_{i=1}^{n}|x_i|\right)^{\nu_2} \leq n^{\nu_2-1}\sum_{i=1}^{n}|x_i|^{\nu_2}. \tag{14}$$

3. Cooperative Guidance Scheme

In this section, one distributed fixed-time protocol, integral sliding mode, and adaptive ST technique are developed, and then, the fixed-time cooperative guidance laws on the LOS direction are proposed in order to control the time-to-go and achieve the cooperative interception. In the normal of the LOS direction, fixed-time nonsingular TSM technique is developed, while the fixed-time guidance laws are proposed to guarantee the LOS angular rates converge to zero within a fixed time. The main results are provided as follows.

3.1. Guidance Law in the LOS Direction. In this subsection, a new fixed-time consensus protocol based on adaptive ST algorithm and ISM is designed to guarantee that all missiles' time-to-go reach the agreement in fixed time. The key point of the proposed consensus protocol is continuous and requires on information on the upper bounds of the external disturbances and their derivatives.

Lemma 10. *Consider the following first-order multiagent system*

$$\dot{x}_i(t) = u_i(t). \tag{15}$$

If the distributed protocol is designed as follows:

$$\begin{aligned} u_i(t) &= \sum_{j=1}^{N}\phi_{ij}(x_j - x_i), \\ \phi_{ij}(x_j - x_i) &= a_{ij}\left[sig^{\alpha_1}(x_j - x_i) + sig^{\beta_1}(x_j - x_i)\right], \end{aligned} \tag{16}$$

where $0 < \alpha_1 < 1$, $\beta_1 > 1$, $x_i(t)$ and $u_i(t)$ represent the state and control input of the agent i, then, the state of the system (16) will reach consistency within a fixed time, and the upper bound of convergence time can be obtained as

$$T_1 \leq T^1_{\max} = \frac{1}{\bar{\alpha}(1-p)} + \frac{1}{\bar{\beta}(q-1)}, \tag{17}$$

where $\bar{\alpha} = 2^{2p-1}\lambda_2^p(L_{\alpha_1})$, $\bar{\beta} = 2^{2q-1}\lambda_2^q(L_{\beta_1})$, $p = \alpha_1 + 1/2$, $q = \beta_1 + 1/2$.

Proof. Define a new variable $\bar{x}(t) = 1/N\sum_{j=1}^{N}x_j(t)$, $e_i(t) = x_i(t) - \bar{x}(t)$, then, we can get

$$\phi_{ij}(x_j - x_i) = -\phi_{ji}(x_i - x_j),\ e_j(t) - e_i(t) = x_j(t) - x_i(t). \tag{18}$$

Taking the time of the derivative of $e_i(t)$ as

$$\begin{aligned} \dot{e}_i(t) &= \dot{x}_i(t) - \dot{\bar{x}}(t) = u_i(t) - \frac{1}{N}\sum_{j=1}^{N}u_j(t) = \sum_{j=1}^{N}\phi_{ij}(x_j - x_i) \\ &\quad - \frac{1}{N}\sum_{j=1}^{N}\sum_{i=1}^{N}\phi_{ji}(x_i - x_j) = \sum_{j=1}^{N}\phi_{ij}(x_j - x_i). \end{aligned} \tag{19}$$

Consider a Lyapunov function as $V(e) = 1/2\sum_{i=1}^{N}e_i^2(t)$ and the time derivative of $V(e)$ can be expressed as

$$\begin{aligned} \dot{V}(e) &= \sum_{i=1}^{N}e_i(t)\dot{e}_i(t) = \sum_{i=1}^{N}e_i(t)\sum_{j=1}^{N}\phi_{ij}(x_j - x_i) \\ &= \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}e_i(t)\phi_{ij}(x_j - x_i) + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}e_j(t)\phi_{ji}(x_i - x_j) \\ &= -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}(e_j - e_i)\phi_{ij}(e_j - e_i) \\ &= -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}(e_j - e_i)a_{ij}\left[sig^{\alpha_1}(e_j - e_i) + sig^{\beta_1}(e_j - e_i)\right] \\ &\leq -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\left(|e_j - e_i|^{\alpha_1+1} + |e_j - e_i|^{\beta_1+1}\right). \end{aligned} \tag{20}$$

According to Lemma 9, it can be got that

$$\begin{aligned} \sum_{i=1}^{N}a_{ij}|e_j - e_i|^{\alpha_1+1} &= \sum_{i=1}^{N}\left(a_{ij}^{2/\alpha_1+1}(e_j - e_i)^2\right)^{\alpha_1+1/2} \\ &\geq \left(\sum_{i=1}^{N}\left(a_{ij}^{2/\alpha_1+1}(e_j - e_i)^2\right)\right)^{\alpha_1+1/2}, \\ \sum_{i=1}^{N}a_{ij}|e_j - e_i|^{\beta_1+1} &= \sum_{i=1}^{N}\left(a_{ij}^{2/\beta_1+1}(e_j - e_i)^2\right)^{\beta_1+1/2} \\ &\geq N^{1-\beta_1/2}\left(\sum_{i=1}^{N}\left(a_{ij}^{2/\beta_1+1}(e_j - e_i)^2\right)\right)^{\beta_1+1/2}. \end{aligned} \tag{21}$$

Substituting (22) into (21) yields

$$\begin{aligned} \dot{V}(e) &\leq -\frac{1}{2}\left(\sum_{i=1,j=1}^{N}\left(a_{ij}^{2/\alpha_1+1}(e_j - e_i)^2\right)\right)^{\alpha_1+1/2} \\ &\quad - \frac{1}{2}N^{1-\beta_1/2}\left(\sum_{i=1,j=1}^{N}\left(a_{ij}^{2/\beta_1+1}(e_j - e_i)^2\right)\right)^{\beta_1+1/2} \end{aligned} \tag{22}$$

According to Lemma 4 and Lemma 5, we have

$$\sum_{i=1,j=1}^{N} a_{ij}^{2/\alpha_1+1}\left(e_j - e_i\right)^2 = 2e(t)^T L_{\alpha_1} e(t) \geq 2\lambda_2\left(L_{\alpha_1}\right)e(t)^T e(t) = 2\lambda_2\left(L_{\alpha_1}\right)\sum_{i=1}^{N} e_i^2(t) = 4\lambda_2\left(L_{\alpha_1}\right)V(e),$$

$$\sum_{i=1,j=1}^{N} a_{ij}^{2/\beta_1+1}\left(e_j - e_i\right)^2 = 2e(t)^T L_{\beta_1} e(t) \geq 2\lambda_2\left(L_{\beta_1}\right)e(t)^T e(t) = 2\lambda_2\left(L_{\beta_1}\right)\sum_{i=1}^{N} e_i^2(t) = 4\lambda_2\left(L_{\beta_1}\right)V(e), \tag{23}$$

where $L_{\alpha_1}, L_{\beta_1}$, respectively, represent the corresponding Laplacian matrix of the undirected graph $G(A^{2/(\alpha_1+1)})$, $G(A^{2/(\beta_1+1)})$, moreover, it can be obtained

$$\dot{V}(e) \leq -\frac{1}{2}\left(4\lambda_2\left(L_{\alpha_1}\right)V(e)\right)^{\alpha_1+1/2} - \frac{1}{2}N^{1-\beta_1/2} \cdot \left(4\lambda_2\left(L_{\beta_1}\right)V(e)\right)^{\beta_1+1/2} = -2^{\alpha_1}\lambda_2^{\alpha_1+1/2}\left(L_{\alpha_i}\right)V^{\alpha_1+1/2}(e) - 2^{\beta_1}N^{1-\beta_1/2}\lambda_2^{\beta_1+1/2}\left(L_{\beta_i}\right)V^{\beta_1+1/2}(e) = -\bar{\alpha}V^p - -\bar{\beta}V^q, \tag{24}$$

where $\bar{\alpha} = 2^{2p-1}\lambda_2^p(L_{\alpha_1}), \bar{\beta} = 2^{2q-1}\lambda_2^q(L_{\beta_1}), p = \alpha_1 + 1/2, q = \beta_1 + 1/2$.

According to Lemma 8, the state error $e_i(t)$ tends to zero within a fixed time, that is, the state $x_i(t)$ of multiagent system tends to be consistent within a fixed time, and the upper bound of convergence time is equation (18). □

Remark 11. In view of equation (17), the designed fixed-time consensus protocol of each agent relies on its states and those from the neighbors. Therefore, all agents exchange information in the process of control, and the proposed approach is a dynamic strategy.

In view of equation (9), the guidance model in the LOS direction can be considered as

$$\dot{t}_{fi} = \tilde{u}_{ri} + d_{ri}. \tag{25}$$

The objective is to design the consistency protocol $\tilde{u}_{ri}$ of the LOS direction based on the fixed-time consensus and the adaptive super-twisting algorithm, which can guarantee the state variables t_{fi} to achieve consistency within a fixed time.

In the literature [28], Song et al. proposed a cooperative guidance law for multiple missiles with finite-time convergence. In this paper, the relevant conclusions are improved to apply to the fixed-time convergence in the planar cooperative guidance systems.

Based on the fixed-time consensus dynamics (17), an integral sliding mode (ISM) surface can be designed as

$$s_{ri} = t_{fi} - t_{fi}(0) - \int_0^t u_{ri}^{nom} d\tau, \tag{26}$$

where $t_{fi}(0)$ is the initial value of t_{fi} and

$$u_{ri}^{nom} = \sum_{j=1}^{n} a_{ij}\left[sig^{\alpha_1}\left(t_{fj} - t_{fi}\right) + sig^{\beta_1}\left(t_{fj} - t_{fi}\right)\right]. \tag{27}$$

u_{ri}^{nom} is viewed as the nominal consensus protocol.

Based on the ISM (27) and adaptive ST algorithm, a continuous fixed-time consensus protocol is proposed as follows:

$$\tilde{u}_{ri} = u_{ri}^{nom} - l_1\varphi_1(s_{ri}) - \int_0^t l_2\varphi_2(s_{ri})d\tau,$$
$$\varphi_1(s_{ri}) = |s_{ri}|^{1/2}\boldsymbol{sign}(s_{ri}) + |s_{ri}|^{3/2}\boldsymbol{sign}(s_{ri}),$$
$$\varphi_2(s_{ri}) = \varphi_1'(s_{ri})\varphi_1(s_{ri}) = \frac{1}{2}\boldsymbol{sign}(s_{ri}) + 2s_{ri} + \frac{3}{2}s_{ri}^2\boldsymbol{sign}(s_{ri}), \tag{28}$$

where $\varphi_1'(s_{ri}) = 1/2|s_{ri}|^{-1/2} + 3/2|s_{ri}|^{1/2}$, and l_1, l_2 are the guidance law adaptive gains. Their adaptive law is given as [28]:

$$\dot{l}_1 = \begin{cases} \omega_1\left(\sqrt{\dfrac{\gamma_1}{2}}\right), s_{ri} \neq 0, \\ 0, s_{ri} = 0, \end{cases} \tag{29}$$

where $\omega_1, \omega_2, \gamma_1, \gamma_2 > 0$ are design gain parameters, $l_2 = 2l_1 a + b + 4a^2$.

Theorem 12. *Assume the undirected graph of system (26) is connected. If ISM occurs, the system (26) can achieve stability within a fixed time. The ISM can be achieved in finite time by the consensus protocol (29).*

Proof. The proof is carried out in two steps.

Taking the derivative of s_{ri} with respect to time and applying the designed fixed-time consistency protocol (29) yield

$$\dot{s}_{ri} = \dot{t}_{fi} - u_{ri}^{nom} = -l_1\varphi_1(s_{ri}) - \int_0^t l_2\varphi_2(s_{ri})dt + d_{ri}. \tag{30}$$

Let $z = -\int_0^t l_2\varphi_2(s_{ri})dt + d_{ri}, \xi = [\xi_1, \xi_2]^T = [\varphi_1(s_{ri}), z]^T$. According to Assumption 2, it can be obtained

$$\left|\dot{d}_{ri}\right| \leq D_{r2} \leq D_{r2} + 4D_{r2}|s_{ri}| + 3D_{r2}|s_{ri}|^2. \tag{31}$$

Therefore, the inequality (32) can be rewritten as $\dot{d}_{ri} = \chi_i \varphi_2(s_{ri})$ for some constant $|\chi_i| \leq 2D_{r2}$.

Now taking the first derivative of ξ, we get

$$\dot{\xi} = \begin{bmatrix} \varphi_1'(s_{ri})(-l_1\varphi_1(s_{ri}) + z) \\ -l_2\varphi_2(s_{ri}) + \chi_i\varphi_2(s_{ri}) \end{bmatrix} = \varphi_1'(s_{ri}) \begin{bmatrix} -l_1\varphi_1(s_{ri}) + z \\ -l_2\varphi_1(s_{ri}) + \chi_i\varphi_1(s_{ri}) \end{bmatrix} = \varphi_1'(s_{ri})H\xi, \tag{32}$$

where

$$H = \begin{bmatrix} -l_1 & 1 \\ -l_2 + \chi_i & 0 \end{bmatrix} \tag{33}$$

For stability analysis of system (26), we choose the following Lyapunov function as

$$V_1 = \xi^T P\xi + \frac{1}{2\gamma_1}(l_1 - l_1^*)^2 + \frac{1}{2\gamma_2}(l_2 - l_2^*)^2, \tag{34}$$

where l_1^*, l_2^* are the upper bound of l_1, l_2, and

$$P = \begin{bmatrix} b + 4a^2 & -2a \\ -2a & 1 \end{bmatrix}. \tag{35}$$

P is a symmetric positive definite matrix.

Define Lyapunov function $V_2 = \xi^T P\xi$ and take the first time derivative of V_2 is given by

$$\begin{aligned} \dot{V}_2 &= \dot{\xi}^T P\xi + \xi^T P\dot{\xi} = \left(\varphi_1'(s_{ri})H\xi\right)^T P\xi + \xi^T P\varphi_1'(s_{ri})H\xi \\ &= \varphi_1'(s_{ri})\xi^T H^T P\xi + \varphi_1'(s_{ri})\xi^T PH\xi \\ &= \varphi_1'(s_{ri})\xi^T\left(H^T P + PH\right)\xi = -\varphi_1'(s_{ri})\xi^T Q\xi, \end{aligned} \tag{36}$$

where

$$\begin{aligned} Q &= -\left(H^T P + PH\right) \\ &= \begin{bmatrix} 2l_1\left(b + 4a^2\right) - 4a(l_2 - \chi_i) & -2l_1 a + l_2 - \chi_i - b - 4a^2 \\ -2l_1 a + l_2 - \chi_i - b - 4a^2 & 4a \end{bmatrix}. \end{aligned} \tag{37}$$

.

It follows from the Schur complement that the matrix Q is positive definite if the following condition is satisfied

$$l_2 = 2l_1 a + b + 4a^2, l_1 > \frac{\chi_i^2/4a + 4a\left(b + 4a^2 - \chi_i\right)}{2b}. \tag{38}$$

Since V_2 is a quadratic positive definite function, it can be obtained

$$\lambda_{\min}(P)\|\xi\|^2 \le \xi^T P\xi \le \lambda_{\max}(P)\|\xi\|^2, \tag{39}$$

where $\lambda_{\min}(P), \lambda_{\max}(P)$ are the minimum and maximum eigenvalue of the matrix P, $\|\xi\|^2 = \xi_1^2 + \xi_2^2 = |s_{ri}| + |s_{ri}|^3 + |s_{ri}|^2 + z^2$, it can imply that

$$|s_{ri}|^{1/2} \le \|\xi\| \le \frac{V_2^{1/2}}{\lambda_{\min}^{1/2}}(p). \tag{40}$$

In view of equations (36), (39), and (40), we can get

$$\begin{aligned} \dot{V}_2 &= -\varphi_1'(s_{ri})\xi^T Q\xi \le -\lambda_{\min}(Q)\varphi_1'(s_{ri})\|\xi\|^2 \\ &= -\lambda_{\min}(Q)\|\xi\|^2\left(\frac{1}{2}|s_{ri}|^{-1/2} + \frac{3}{2}|s_{ri}|^{1/2}\right) \le -\frac{\lambda_{\min}(Q)}{2|s_{ri}|^{1/2}}\|\xi\|^2 \\ &\quad - \frac{3\lambda_{\min}(Q)|s_{ri}|^{1/2}}{2}\|\xi\|^2 \le -\frac{\lambda_{\min}(Q)\lambda_{\min}^{1/2}(p)}{2\lambda_{\max}(P)}V_2^{1/2} \\ &= -\eta_1 V_2^{1/2}, \end{aligned} \tag{41}$$

where $\eta_1 = \lambda_{\min}(Q)\lambda_{\min}^{1/2}(p)/2\lambda_{\max}(P)$.

Now, in view of equations (34) and (41), based on Lemma 9, the derivative of V_1 can be given by

$$\begin{aligned} \dot{V}_1 &\le -\eta_1 V_2^{1/2} + \frac{1}{\gamma_1}(l_1 - l_1^*)\dot{l}_1 + \frac{1}{\gamma_2}(l_2 - l_2^*)\dot{l}_2 \\ &\le -\eta_1 V_2^{1/2} - \frac{\omega_1}{\sqrt{2\gamma_1}}|l_1 - l_1^*| - \frac{\omega_2}{\sqrt{2\gamma_2}}|l_2 - l_2^*| \\ &\quad + \frac{\omega_1}{\sqrt{2\gamma_1}}|l_1 - l_1^*| + \frac{\omega_2}{\sqrt{2\gamma_2}}|l_2 - l_2^*| + \frac{1}{\gamma_1}(l_1 - l_1^*)\dot{l}_1 \\ &\quad + \frac{1}{\gamma_2}(l_2 - l_2^*)\dot{l}_2 \le -\min\{\eta_1, \omega_1, \omega_2\}V_1^{1/2} + \sigma. \end{aligned} \tag{42}$$

Applying the adaptive law (30), we suppose that $l_1^* > l_1, l_2^* > l_2$ are bounded. Therefore $\sigma = -\omega_1/\sqrt{2\gamma_1}(l_1 - l_1^*) - \omega_2/\sqrt{2\gamma_2}(l_2 - l_2^*) + 1/\gamma_1(l_1 - l_1^*)\dot{l}_1 + 1/\gamma_2(l_2 - l_2^*)\dot{l}_2$. To satisfy the finite-time convergence, we let $\sigma = 0$. Thus, the adaptive law can be designed as

$$\dot{l}_1 = \omega_1\left(\sqrt{\frac{\gamma_1}{2}}\right), \dot{l}_2 = \omega_2\left(\sqrt{\frac{\gamma_2}{2}}\right). \tag{43}$$

Based on Lemma 8, we can get V_1 converge to zero in finite time. Therefore, it can be obtained that $\xi = [\varphi_1(s_{ri}), z]^T$ and s_{ri} converge to zero in finite time.

If ISM occurs, according to the ISM (27), we can get $s_{ri} = 0$. Therefore, it can imply that

$$\dot{t}_{fi} = u_{ri}^{nom} = \sum_{j=1}^{n} a_{ij}\left[sig\left(t_{fj} - t_{fi}\right)^{\alpha_1} + sig\left(t_{fj} - t_{fi}\right)^{\beta_1}\right]. \tag{44}$$

According to Lemma 10, t_{fi} can reach the agreement in fixed time. The proof is completed. □

Remark 13. In Theorem 12, the first item of the consistency protocol is used to adjust the attack time of the missile so that it is consistent within a fixed time, and the second and third items use the super-twisting adaptive algorithm to compensate the influence of target maneuvering on the system in the LOS direction. The distributed communication mode is adopted in this paper, which not only saves communication resources but also improves the survivability of missile group effectively.

3.2. Guidance Law in the Normal of the LOS Direction. In this subsection, novel nonsingular fixed-time TSM guidance law on the normal direction of the LOS is derived to ensure the fixed-time convergence of the LOS angular rates and LOS angles. The proof of the stability is given in details.

In view of Eq. (9), the guidance model in the normal of the LOS direction is obtained as

$$\begin{cases} \dot{x}_{3i} = x_{4i}, \\ \dot{x}_{4i} = -\dfrac{2x_{2i}x_{4i}}{x_{1i}} - \dfrac{u_{qi}}{x_{1i}} + \dfrac{\omega_{qi}}{x_{1i}}. \end{cases} \tag{45}$$

The unknown target acceleration is a crucial factor for the design of the guidance law. A larger interference upper bound is often selected in order to ensure the robustness of the designed guidance law in the real flight, but this will cause chattering phenomenon. To address this issue, a fixed-time disturbance observer (FxTDO) is proposed and the estimated value of the interference is applied to the design of guidance law. According to the literature [42], the observer form is as follows

$$\begin{cases} \dot{z}_{1i} = k_1 \varphi_{1i}\left(\dfrac{x_{4i} - z_{1i}}{\delta}\right) + z_{2i} - \dfrac{2x_{2i}x_{4i}}{x_{1i}} - \dfrac{u_{qi}}{x_{1i}}, \\ \dot{z}_{2i} = \dfrac{k_2}{\delta}\varphi_{2i}\left(\dfrac{x_{4i} - z_{1i}}{\delta}\right), \end{cases} \tag{46}$$

where $\varphi_{1i}, \varphi_{2i}$ are the correction term, and there are

$$\begin{aligned} \varphi_{1i}(x) &= sig^{\alpha'}(x) + sig^{\beta'}(x), \\ \varphi_{2i}(x) &= sig^{2\alpha'-1}(x) + sig^{2\beta'-1}(x). \end{aligned} \tag{47}$$

z_{1i}, z_{2i} estimate the value of x_{4i}, d_{qi}, respectively, where $k_1, k_2 > 0$, $k_1 > 2\sqrt{k_2}$, $0 < \delta < 1$, $0.5 < \alpha' < 1$, and $1 < \beta' < 1.5$.

Consider a second order system of the form (45) and define the following sliding surface

$$s_{qi} = x_{4i} + 2\beta\sqrt{\left|\arctan\left(x_{3i}\right)\right|}\left(1 + x_{3i}^2\right)\operatorname{sgn}\left(x_{3i}\right). \tag{48}$$

Once a sliding motion is established on the surface, the dynamics of the variable x_{3i} are governed by

$$x_{4i} = \dot{x}_{3i} = -2\beta\sqrt{\left|\arctan\left(x_{3i}\right)\right|}\left(1 + x_{3i}^2\right)\operatorname{sgn}\left(x_{3i}\right). \tag{49}$$

In the case $x_{3i} > 0$, it holds

$$\frac{dx_{3i}}{\sqrt{\arctan\left(x_{3i}\right)}\left(1 + x_{3i}^2\right)} = -2\beta dt. \tag{50}$$

Solving equation (50), we can get $\sqrt{\arctan\left(x_{3i}(t)\right)} - \sqrt{\arctan\left(x_{3i}(0)\right)} = -\beta t$; it provides a settling time independently of the initial condition, given by

$$T_s\left(x_{3i}(t)\right) = \frac{1}{\beta}\sqrt{\arctan\left(x_{3i}(0)\right)} \le \frac{1}{\beta}\sqrt{\frac{\pi}{2}}. \tag{51}$$

Analogous considerations can be derived in the converse case $x_{3i} < 0$.

Theorem 14. *If the FxTDO is devised by (46), and the sliding-mode surface is proposed by (48), in order to guarantee the LOS angle error x_{3i} and the LOS angular velocity x_{4i} of multi-missiles are fixed-time convergent, the nonsingular terminal sliding mode guidance law can be designed as*

$$u_{qi} = \begin{cases} x_{1i}\left[-\dfrac{2x_{2i}x_{4i}}{x_{1i}} + \beta\dfrac{1 + 4x_{3i}\left|\arctan\left(x_{3i}\right)\right|\operatorname{sign}\left(x_{3i}\right)}{\sqrt{\left|\arctan\left(x_{3i}\right)\right|}}x_{4i} + l_3 sig^{\alpha_2}s_{qi} + l_4 sig^{\beta_2}s_{qi} + z_{2i}\operatorname{sign}\left(s_{qi}\right)\right] & s_{qi} = 0 \text{ or } s_{qi} \neq 0, \left|x_{3i}\right| \ge \eta, \\ x_{1i}\left(-\dfrac{2x_{2i}x_{4i}}{x_{1i}} - \left|z_{2i}\right|\operatorname{sign}\left(x_{4i}\right)\right) & s_{qi} \neq 0, \left|x_{3i}\right| < \eta. \end{cases} \tag{52}$$

Proof. If TSM occurs, it can be obtained that $s_{qi} = 0$ is fulfilled. Based on the previous analysis, LOS angles and angular rates can be stabilized to the equilibrium points within fixed time.

If $s_{qi} \neq 0$ and $|x_i| \ge \eta$ are satisfied, taking the derivative of s_{qi} with respect to time, combine with the guidance law (52), we get

$$\begin{aligned} \dot{s}_{qi} &= \dot{x}_{4i} + \beta\frac{1 + 4x_{3i}\left|\arctan\left(x_{3i}\right)\right|\operatorname{sign}\left(x_{3i}\right)}{\sqrt{\left|\arctan\left(x_{3i}\right)\right|}}x_{4i} \\ &= -\frac{2x_{2i}x_{4i}}{x_{1i}} - \frac{u_{qi}}{x_{1i}} + d_{qi} + \beta\frac{1 + 4x_{3i}\left|\arctan\left(x_{3i}\right)\right|\operatorname{sign}\left(x_{3i}\right)}{\sqrt{\left|\arctan\left(x_{3i}\right)\right|}}x_{4i} \\ &= -l_3 sig^{\alpha_2}s_{qi} - l_4 sig^{\beta_2}s_{qi} + d_{qi} - z_{2i}\operatorname{sign}\left(s_{qi}\right). \end{aligned} \tag{53}$$

Table 1: Initial conditions for three missiles.

Missile	Initial heading angle (°)	Initial distance (m)	Initial LOS angle (rad)	Desired LOS angle (°)
1	30	30067	-0.0666	-15
2	20	29069	0.0689	0
3	40	30265	0.1326	20

Select the following Lyapunov function as

$$V_3 = \frac{1}{2} s_{qi}^2. \tag{54}$$

Then, differentiating V_2 with respect to time yields

$$\begin{aligned}\dot{V}_3 &= s_{qi}\dot{s}_{qi} = s_{qi}\left(-l_3 sig^{\alpha_2} s_{qi} - l_4 sig^{\beta_2} s_{qi} + d_{qi} - z_{2i} \operatorname{sign}\left(s_{qi}\right)\right) \\ &\le -l_3\left|s_{qi}\right|^{\alpha_2+1} - l_4\left|s_{qi}\right|^{\beta_2+1} + \left(d_{qi} - z_{2i}\right)\left|s_{qi}\right|.\end{aligned} \tag{55}$$

According to the definition of the FxTDO in (38), there exists a bounded positive constant t^* as

$$d_{qi} - z_{2i} = 0, t > t^*. \tag{56}$$

Furthermore, the following result can be provided

$$\begin{aligned}\dot{V}_3 &\le -l_3\left|s_{qi}\right|^{\alpha_2+1} - l_4\left|s_{qi}\right|^{\beta_2+1} = -\sqrt{2}^{\alpha_2+1} l_3 V_3^{\alpha_2+1/2} \\ &- \sqrt{2}^{\beta_2+1} l_4 V_3^{\beta_2+1/2} = -\bar{l}_3 V_3^{p'} - \bar{l}_4 V_3^{q'},\end{aligned} \tag{57}$$

where $\bar{l}_3 = 2^{2p'-1} l_3, \bar{l}_4 = 2^{2q'-1} l_4, p' = \alpha_2 + 1/2$ and $q' = \beta_2 + 1/2$. According to Lemma 8, TSM variable can converge to the original within fixed time. That is to say, guidance system is practically fixed-time stable.

If $s_{qi} \neq 0$ and $|x_i| < \eta$ are fulfilled, it has

$$\begin{aligned}x_{4i}\dot{x}_{4i} &= x_{4i}\left(-\frac{2x_{2i}x_{4i}}{x_{1i}} - \frac{u_{qi}}{x_{1i}} + d_{qi}\right) = x_{4i}\left(\operatorname{sign}\left(x_{4i}\right)|z_{2i}| + d_{qi}\right) \\ &= |x_{4i}z_{2i}| + x_{4i}d_{qi} \ge 0.\end{aligned} \tag{58}$$

As a consequence, by the comparison principle, one has $|x_{4i}| \ge |x_{4i}(0)|$. Furthermore, the following result can be provided

$$|\dot{x}_{3i}| \ge |x_{4i}(0)| \ge 2\beta\sqrt{|\arctan\left(x_{3i}(0)\right)|}\left(1 + x_{3i}^2(0)\right). \tag{59}$$

The integral solution of equation (59) can be obtained

$$\begin{cases} x_{3i}(t) \ge x_{3i}(0) + 2\beta\chi t & \dot{x}_{3i} \ge 0, \\ x_{3i}(t) \le x_{3i}(0) - 2\beta\chi t & \dot{x}_{3i} < 0. \end{cases} \tag{60}$$

where $\chi = \sqrt{|\arctan\left(x_{3i}(0)\right)|}\left(1 + x_{3i}^2(0)\right)$. It can be seen that in the process of sliding mode variable approaching zero, the state variable x_{3i} will be far away from the region $|x_{3i}| < \eta$ and enter the region $|x_{3i}| \ge \eta$.

Through the analysis of the above three cases, it can be inferred that LOS angles and angular rates will reach stability with a fixed time. □

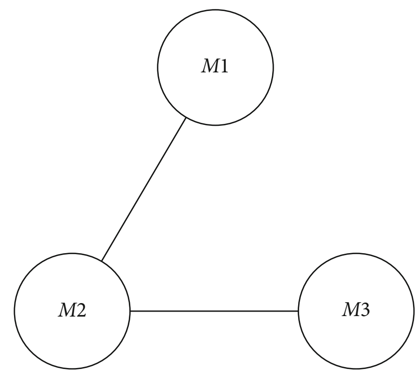


Figure 2: Communication topology.

Remark 15. In Theorem 14, the proposed guidance law with impact angle constraints is nonsingular. The second order system (45) can be reached to the sliding surface under the action of nonsingular terminal sliding mode guidance law in (52). Once the sliding surface is attained, the states of system (45) can reach the origin within a fixed time.

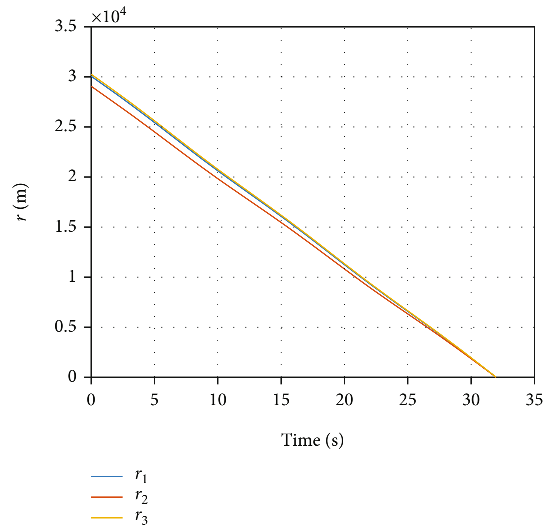
4. Simulation Results

In order to verify the effectiveness of the designed fixed-time cooperative guidance law with angle of attack constraint, this section considers the case where three missiles attack a maneuvering target at the same time. The velocity of the missiles is$V_M = 1800$ m/s. The maximum value of the available missile acceleration is assumed to be $80g$, where g is the acceleration of gravity ($g = 9.8\text{m/s}^2$). The velocity of the target is $V_T = 400\text{m/s}$, the initial flight-path is 120°. The initial conditions of missiles and targets as well as the expected sight angle of each missile are shown in Table 1.

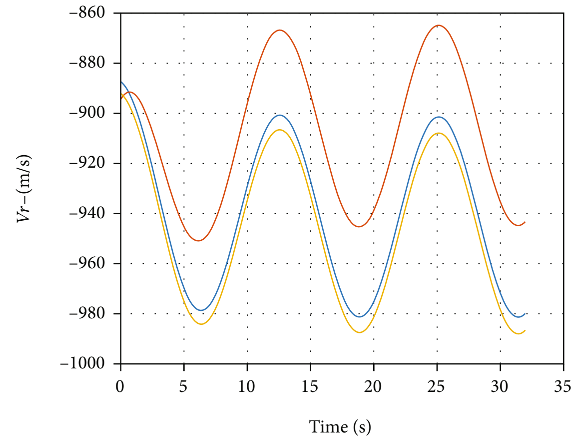
The communication topology of the multimissile team is illustrated in Figure 2, whose adjacency matrix A is describe by

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}. \tag{61}$$

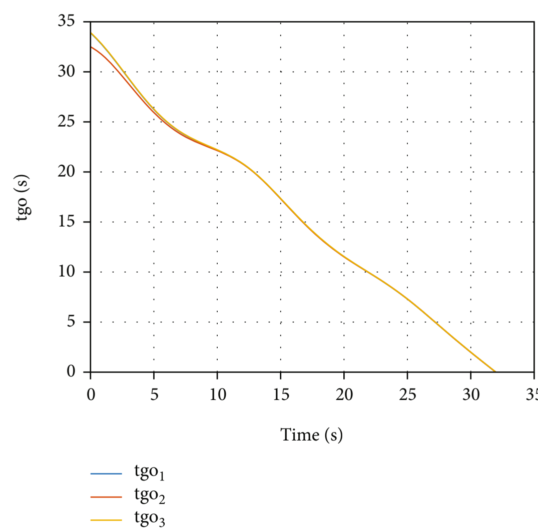
The parameters of fixed-time consistency protocol and adaptive ST algorithm are selected as $\omega_1 = 0.2, \gamma_1 = 1, a =$



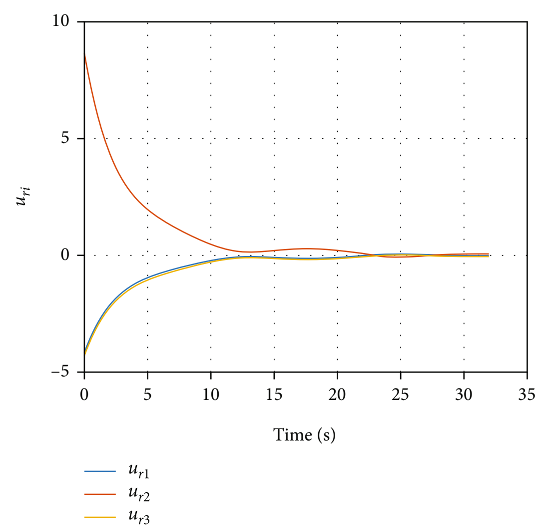
(a) Missile-target relative distance



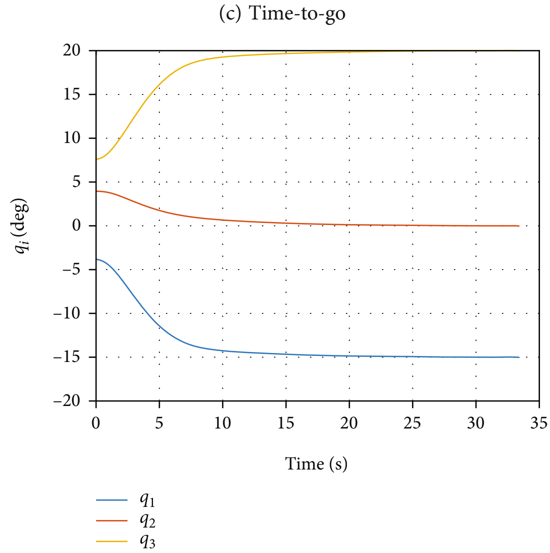
(b) Missile-target relative speed



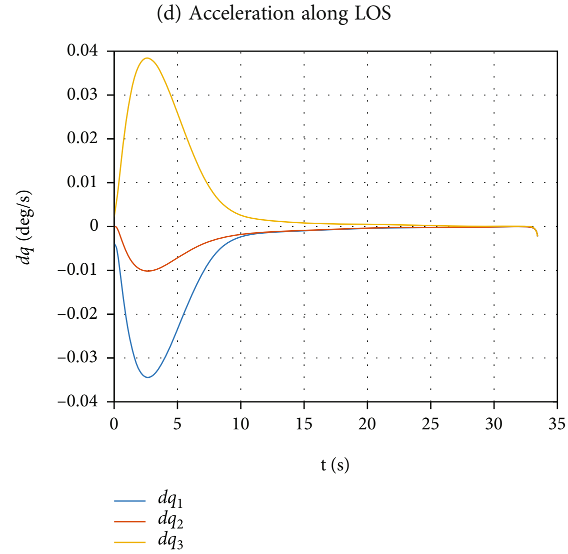
(c) Time-to-go



(d) Acceleration along LOS

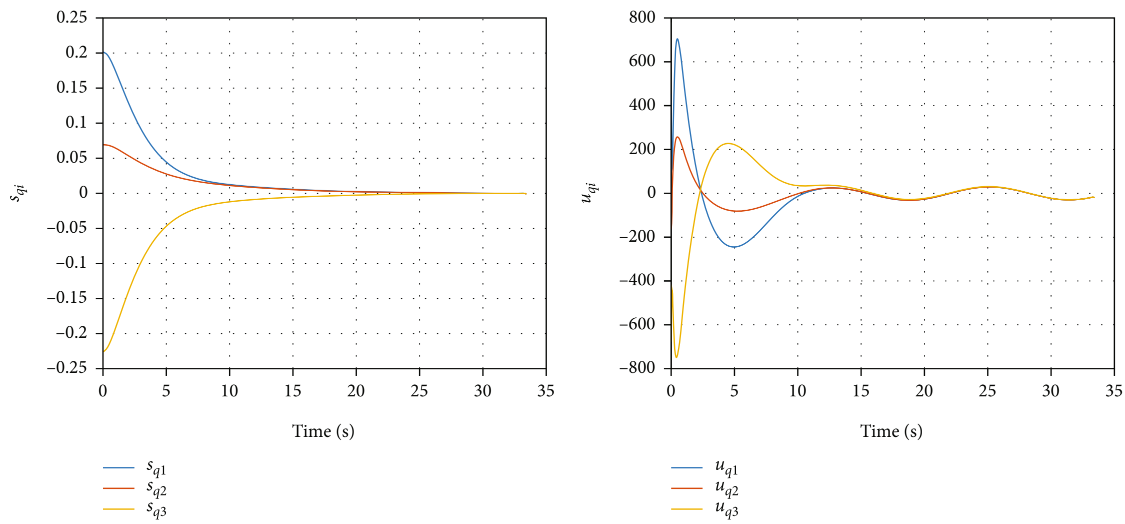


(e) LOS angle



(f) LOS angular velocity

FIGURE 3: Continued.



(g) Sliding mode surface normal to LOS

(h) Acceleration normal to LOS

FIGURE 3: Response curves under FxTCG [31] for Case 1.

$0.1, b = 1, \alpha_1 = 7/9$, and $\beta_1 = 11/9$. For A fixed-time disturbance observer, the parameters are set to be $k_1 = 3.0, k_2 = 1.5, \delta = 0.02, \alpha = 0.8$, and $\beta = 1.2$. The parameters of the guidance law in the normal direction of LOS are given by $l_3 = 0.4, l_4 = 1.4, \alpha_2 = 7/9$ and $\beta_2 = 11/9$.

To verify the robustness of the presented fixed-time cooperative guidance law (29) and (52), the two different cases for target maneuver are selected as follows:

Case 1. $a_{tr} = 20 \sin (0.5t) \mathrm{m/s^2}$ and $a_{tq} = 30 \cos (0.5t) \mathrm{m/s^2}$.

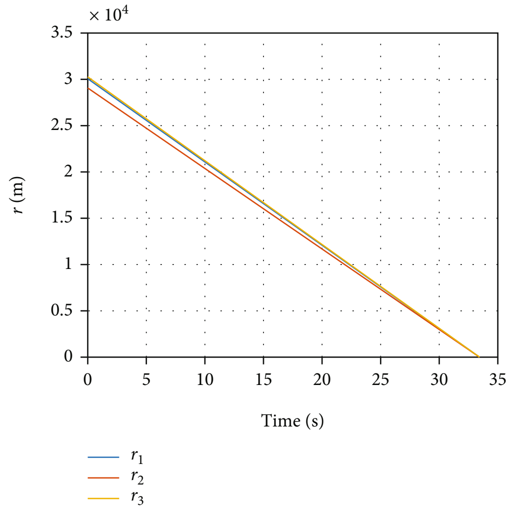
Case 2. $a_{tr} = 10 \mathrm{m/s^2}$ and $a_{tq} = 20 \mathrm{m/s^2}$.

First, simulation comparisons with recent research [31] are carried out to validate the advantages of the proposed scheme under Case 1, and the relevant results are given in Figures 3(a)–3(h) and Figures 4(a)–4(i), respectively. The same initial conditions are selected in Table 1. The curves of missile-target relative distance r are described in Figures 3(a) and 4(a), and it can be seen that all the missiles can be intercept maneuvering target. Due to a_{tr}, the relative speeds $\dot{r}_i$ of the FxTCG [31] have been changing in Figure 3(b), but $\dot{r}_i$ remains stable after transient convergence in Figure 4(b) under the proposed scheme. Compared with the gradients of time-to-go t_{goi} in Figure 4(c), they have been changing in Figure 3(c). The convergent rates are relatively slow under the FxTCG for fixed-time consensus command u_{ri} in Figures 3(d) and 4(e). From NFTSM surface s_{qi}, LOS angular rate $\dot{q}_i$, and angle q_i, it can be obtained that the settling time is shorter by the proposed guidance law than those under the FxTCG [31]. Compared with the proposed algorithm, guidance commands $u_{qi}$of the FxTCG [31] require more parameters and are more complex, and they have been depicted in Figures 4(i) and 3(h). Moreover, the miss distances are 3.3052 m, 0.7090 m, and 11.6712 m under the FxTCG, and therefore, it can be seen from Table 2 that the proposed scheme can greatly reduce the miss distances.

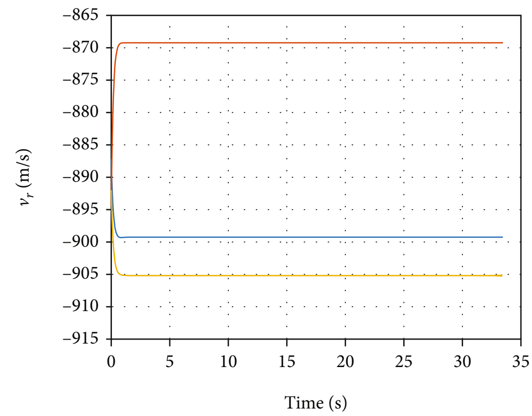
In two cases, the identical initial guidance conditions are set for target maneuver. The simulation results of Case 1 and Case 2 are shown, respectively, in Figures 4(a)–4(i) and Figures 5(a)–5(i). For two cases, the simulation curves including the missile-target relative distance r, relative speed $\dot{r}$, time-to-go t_{go}, integral sliding mode surface along LOS s_{ri}, fixed-time consensus command on the LOS direction u_{ri}, NFTSM guidance commands normal to LOS u_{qi}, NFTSM surface normal to LOS s_{qi}, LOS angular rate $\dot{q}_i$, and LOS angle q_i. Table 2 denotes the miss distances and interception times for all missiles under the Case 1 and Case 2.

With the implementation of the proposed fixed-time consistency protocol, it can be clearly observed that three missiles are both intercept the maneuvering target simultaneously under two cases from Figures 4(a) and 5(a). The relative distances reach zero after the missiles impact the target. From Figures 4(b) and 5(b), it can be seen that the relative speeds between missiles and target under two cases are both negative, which guarantees that missile-target relative distances tend to zero. In addition, the rates of change of relative velocities are relatively larger under Case 2 because the fixed maneuver is larger than sinusoidal maneuver in the beginning of the guidance progress. From Figures 4(c) and 5(c), the fact is seen that three missiles have different time-to-gos at initial position, then, the time-to-go of missiles reaches consensus after 2.181 s under the action of the consensus protocol in (29). Figures 4(d) and 5(d) indicate that the integral sliding mode converges to zero with a fixed time even while encountering target maneuvers for Case 1 and Case 2. However, the value of integral sliding mode is larger under Case 2, because the fixed maneuver is larger than sinusoidal maneuver in the beginning of the guidance progress.

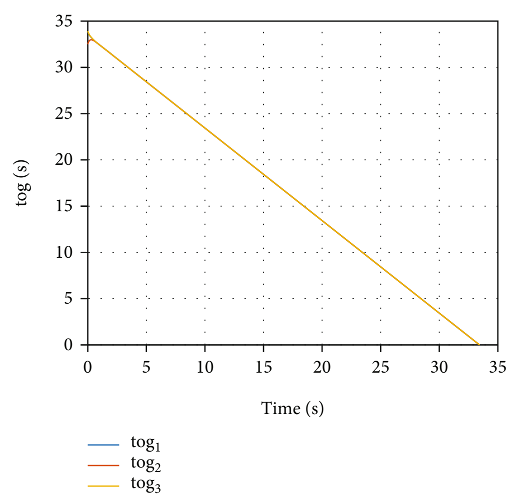
As shown in Figures 4(e) and 5(e), for the three missiles under two cases, all control commands on the LOS direction



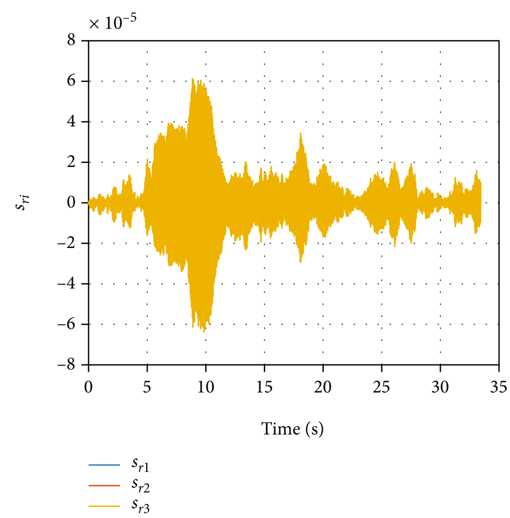
(a) Missile-target relative distance



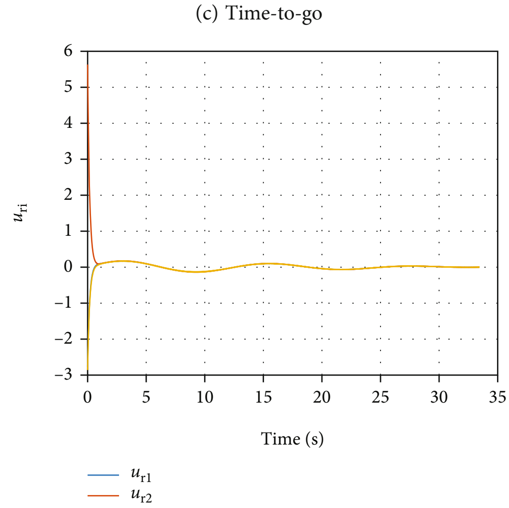
(b) Missile-target relative speed



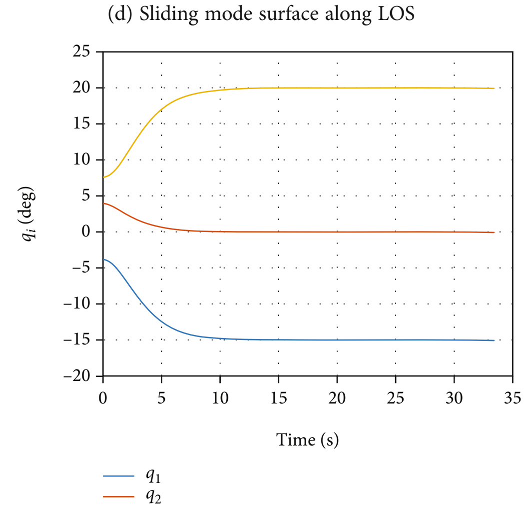
(c) Time-to-go



(d) Sliding mode surface along LOS



(e) Acceleration along LOS



(f) LOS angle

Figure 4: Continued.

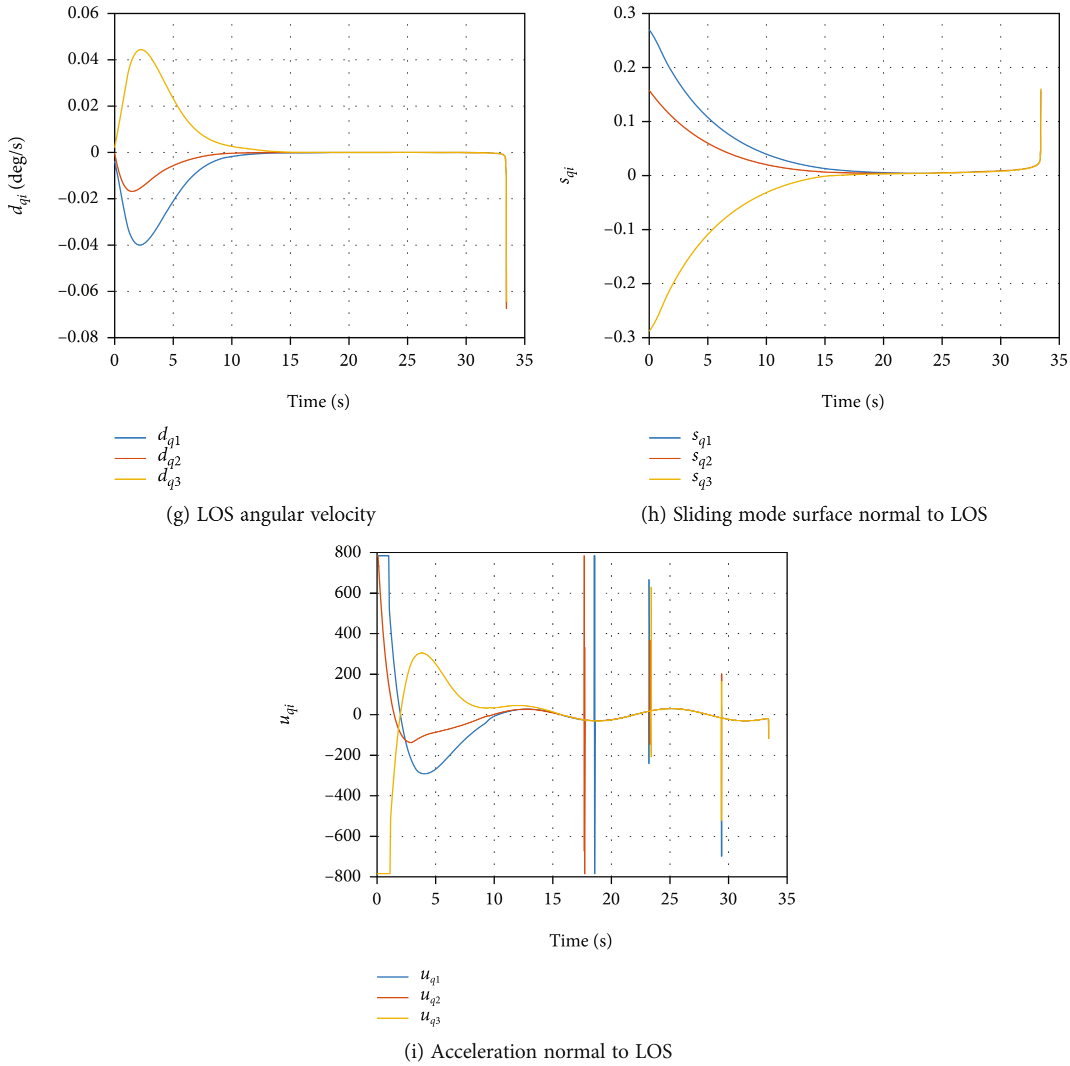
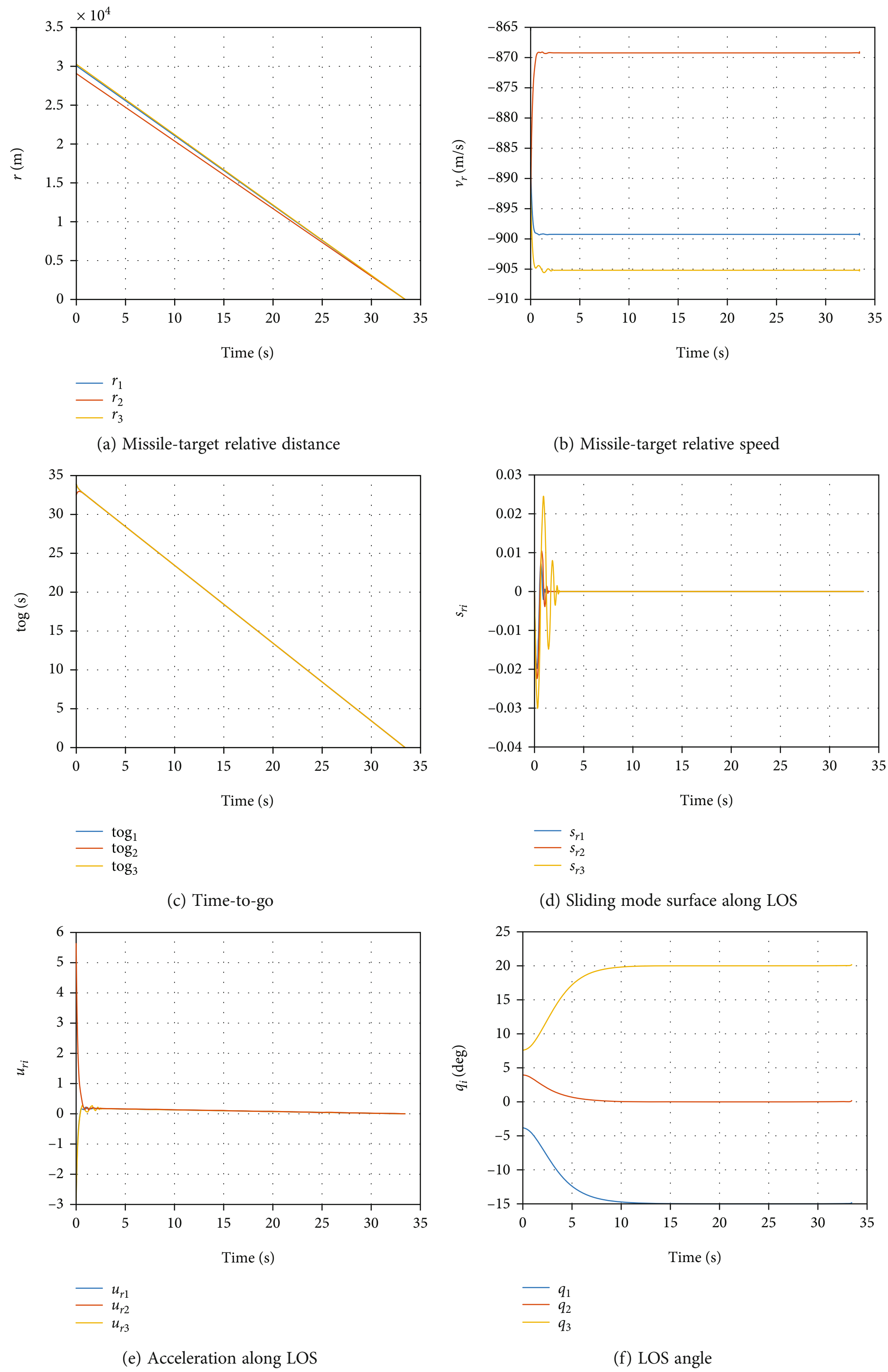


Figure 4: Response curves under the proposed algorithm for Case 1.

Table 2: Miss distances and interception times.

	Case 1		Case 2	
Missile	Miss distance (m)	Interception time (s)	Miss distance (m)	Interception time (s)
1	0.4671	33.438	0.4671	33.438
2	0.4520	33.438	0.4516	33.438
3	0.4692	33.438	0.4703	33.438

are within the reasonable bounds, and there are much larger values in the initial phase of the guidance process. The larger control inputs can make the time-to-go reach consensus in fixed time, and the control inputs decrease to reach the stability along with the time-to-go reaching consensus. Figure 5(e) indicates that the control commands on the LOS direction are more smoothly under Case 2 because the target maneuver is the constant. From Figures 4(f)–4(g) and Figures 5(f)–5(g), we can see that the LOS angles can reach the desired values, and the LOS angular velocities can also tend to zero smoothly in fixed-time, respectively. Figures 4(h) and 5(h) indicate that the nonsingular fixed time terminal sliding mode surface converges to a small neighborhood of zero in fixed-time for the three missiles. It can be seen from Figures 4(i) and 5(i) that all acceleration commands on the normal direction of the LOS for the three missiles are also within the reasonable bounds. There are much larger values in the initial phase of the guidance process for the larger acceleration commands can make the LOS angular rates preferably converge to zero. Then, the large acceleration commands decrease correspondingly as the LOS angular rates come close to zero. As the terminal sliding mode is obtained, the commands in the normal direction of LOS increase due to the conversion of the



(a) Missile-target relative distance

(b) Missile-target relative speed

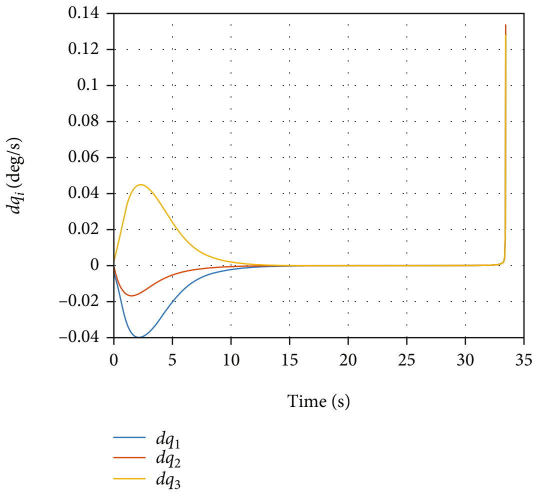
(c) Time-to-go

(d) Sliding mode surface along LOS

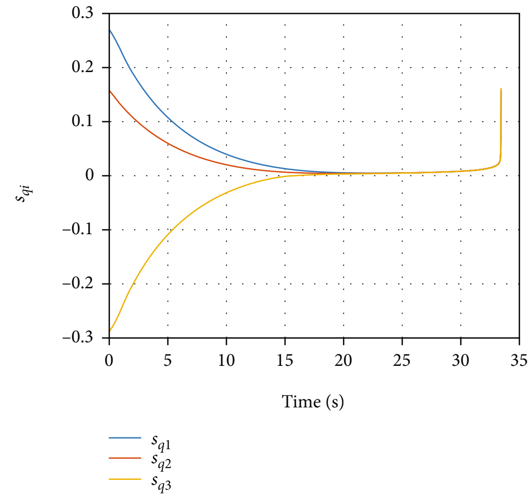
(e) Acceleration along LOS

(f) LOS angle

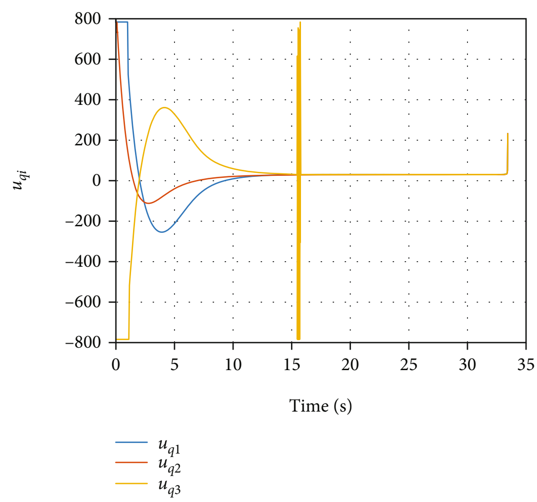
Figure 5: Continued.



(g) LOS angular velocity



(h) Sliding mode surface normal to LOS



(i) Acceleration normal to LOS

Figure 5: Response curves for Case 2.

guidance laws and are satisfied with saturation. In a word, from Figures 4(a)–4(i) and Figures 5(a)–5(i), it can be seen that the proposed cooperative guidance law can achieve good guidance performance.

From Table 2, it can be noted that the proposed cooperative guidance law can ensure all three missiles have much smaller miss distances and the same interception time under the Case 1 and Case 2. Thus, the designed cooperative guidance law can guarantee that three missiles intercept the maneuvering target simultaneously and successfully. Therefore, it can be concluded that the multiple missiles can achieve good guidance performance for intercepting the targets with different maneuvering cases from expected impact angle cooperatively.

5. Conclusion

In this paper, a novel cooperative guidance law with impact angle constraints is presented. It is continuous and requires no information on target maneuvers. Considering time-to-go as the agreement variable, the problem of multimissile cooperative guidance was regarded as the fixed-time consensus problem of multiagent system. The design of cooperative guidance law is divided into two stages. On the one hand, a new continuous fixed-time consensus protocol is designed to guarantee that all missiles simultaneously reach the maneuvering target in fixed-time. On the other hand, a nonsingular fixed-time terminal sliding mode control is developed to ensure that the LOS angular rate and the LOS angle converge to the equilibrium points in fixed-time. Finally, simulation results are presented to illustrate the effectiveness of the proposed cooperative guidance law. In the further, more multimissile cooperative guidance laws with directed topologies will be considered in the three-dimensional space.

Data Availability

Data are available on request from the authors. The data that support the findings of this study are available from the corresponding author, [Jiwei Gao], upon reasonable request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work is supported in part by the National Natural Science Foundation of China under Grant 62073124 and the Aeronautical Science Foundation of China under Grant 20180142001.

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