Research Article

Influence of Three-Body Gravitational Perturbation for Drag-Free Spacecraft

Tao Jia,¹ Decheng Jiang,¹ Chenglong Li,¹ Bo Ru,¹ Xiaobin Lian^{1b},² and Xusheng Zhu^{1b}

¹AVIC Chengdu Aircraft Industrial (Group) Co. Ltd., Chengdu 610092, China

²Shanghai Institute of Satellite Engineering, Shanghai 201109, China

Correspondence should be addressed to Xusheng Zhu; zhuxusheng@buaa.edu.cn

Received 8 October 2021; Revised 6 January 2022; Accepted 13 January 2022; Published 11 February 2022

Academic Editor: Chen Pengyun

Copyright © 2022 Tao Jia et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

This paper mainly studies the motion and control of the drag-free spacecraft system for a high-precision mission. In high-precision missions, the influence of three-body gravitational perturbation on a drag-free spacecraft system is still lack of relevant research. To this end, we first established a relative motion model of the test mass and the spacecraft based on multibody dynamics, and the equations of the relative motion of the test mass and the spacecraft cavity were obtained through coordinate transformation. The difference between the sun orbit and the Earth orbit is comparatively analyzed by a numerical solution. The influence of gravitational forces from the Earth, the moon, and the sun on the relative motion is studied in detail. Secondly, a sliding mode controller is designed for the high-precision mission. Finally, the performance of the controller is analyzed from the time domain and the frequency domain. Numerical simulation results show that the controller can meet the requirements of the time domain and the frequency domain.

1. Introduction

The concept of the drag-free spacecraft was first proposed by Pugh in 1959 [1]. It is a method that seals the test mass in a spacecraft cavity, which shields nongravitational interference such as atmospheric resistance and solar radiation pressure, then continuously controls the spacecraft by a microthruster to implement the feed-forward compensation of interference, so that the spacecraft runs in purely gravitational orbit. Because of the high accuracy and high stability of drag-free spacecrafts, they are considered to be used for space science mission.

At present, there are many achievements in the research on the dynamics model of the drag-free spacecraft, but the related research mainly focuses on different mission backgrounds. Lange et al. [2–4] has made great contributions to the development of drag-free spacecrafts. In [2], the motion of the spherical test mass relative to the spacecraft body in low Earth orbit was researched; the relative motion equations of the spacecraft body and the test mass can be obtained by the vector difference between the motion equa-

tions. Reference [4] studies the interference cancellation of spin spacecrafts. Wiegand et al. [5] analyzed the stability of the test mass's motion by using Mathieu differential equations and verified the applicability of the method by numerical simulation. Theil et al. [6–8] established a drag-free spacecraft dynamic model from the standpoint of the drag-free control. Reference [6] studied the relative linearization dynamics of the test mass and the spacecraft. By considering the uncertainty of the system, the model can be used to solve a type of drag-free spacecraft mission with nonspherical test mass. Canuto et al. [9–12] studied the dynamics of GOCE spacecrafts in the accelerometer mode based on the embedded model theory. In the LISA Pathfinder mission, the accelerometer mode and displacement mode are adopted simultaneously. Fichter et al. [13–15] and Wu and Fertin [16, 17] studied the dynamics of drag-free spacecrafts with 15 degrees of freedom, respectively.

The interference of drag-free spacecraft is mainly divided into three sources [18]: (1) The interference acts on the spacecraft body, such as sunlight pressure and atmospheric resistance. (2) The interference acts on the test mass,

such as residual gas damping, thermal radiation pressure, and electrostatic force. (3) The coupling interference between the spacecraft body and the test mass, such as gravity gradient from the sun/moon and capacitance gradient. In addition, interference will also be introduced in the TM release process. Lian et al. [19] analyzed the impact of the release process of the test mass and gave a capture controller design method.

Drag-free control is one of the core technologies of drag-free spacecraft. Chapman et al. [20] designed a PID controller for the Spacecraft Test of the Equivalence Principle (STEP) mission and designed the rotation and translation control laws as multiple single-input single-output (SISO) loops. Shi [21] designed a hybrid H_2/H_∞ controller for a drag-free spacecraft in displacement mode and verified the controller's excellent robustness through simulation. Francesco [22] used the quantitative feedback theory to solve the problem of drag-free and attitude control of LISA spacecrafts in his dissertation. Li [23] introduced a nonsmooth optimization tool to handle the parameters of the embedded model controller in his research. Lian et al. [24] proposed a frequency separation control for drag-free spacecraft with frequency domain constraints.

However, the above research mainly focuses on drag-free spacecraft in low Earth orbit, but it is less research on orbit around the sun/high Earth orbit. In addition, current research pays more attention to the influence of nonconservative forces, while less consideration is given to the influence of the three-body gravitational perturbation such as the sun and the moon. In this paper, the kinetic characteristic and high-precision control of the drag-free spacecraft are studied. The main contributions include the following:

- (1) The difference between the relative motion of the drag-free spacecraft on the orbit around the sun and the orbit around the Earth is comparatively studied
- (2) Considering that there are few studies on the influence characteristics of the sun-moon gravitational perturbation for the drag-free spacecraft of the Earth orbit, this paper analyses in detail the influence of the Earth-sun-moon and designs the controller to overcome the Earth-sun-moon gravitational perturbation

Based on the above analysis, the research on the relative motion of traditional drag-free spacecrafts is mainly focused on low Earth orbit and Solar orbit, while the research on the relative motion of high Earth orbit for gravitational wave detection is lacking. In this paper, the relative motion and control of a drag-free spacecraft system under the gravitational fields of the Earth, moon, and sun are studied with the background of gravitational wave detection in the geocentric very high orbit.

2. The Relative Motion Equation

2.1. Reference Coordinate System. The two test masses are assumed to be mass points; the attitude motion of test masses is not considered in this paper accordingly. To clarify

the problems clearly, some reference frames are presented as follows.

- (1) Earth-centered inertial (ECI): the origin is located at the Earth's mass center E , the x -axis points to the mean equinox J2000, the z -axis is along the direction the Earth's mass center to the Earth's north pole, and the y -axis forms a right-handed triad as shown in Figure 1
- (2) Spacecraft-centered orbit (SCO): the origin is located at the spacecraft's mass center O , the z -axis is along the direction the Earth's mass center to the spacecraft's mass center, the x -axis is perpendicular to the z -axis in the orbit plane and coincides with the direction of the velocity, and the y -axis forms a right-handed triad
- (3) The spacecraft cavity coordinate system: the origin is the cavity's center O_i , the x_i -axis is the sensitive axis along the line of sight, the z_i -axis is the vertical sensitive axis in the orbital plane, and the y_i -axis is the normal direction of the orbital plane

2.2. The Relative Motion Dynamics. Merely considering the effect of gravity of the Earth on the motion of the spacecraft, the motion equation of the spacecraft mass center and the test masses in the ECI coordinate system is as follows:

$$\ddot{\mathbf{r}} = -\frac{\mu_e}{r^3} \mathbf{r}, \quad (1)$$

where μ_e is a standard gravitational parameter and $\mathbf{r}$ is the position vectors from the Earth's mass center to the spacecraft mass center.

If the influence of lunar gravity is also considered, the equation of motion of the spacecraft body is

$$\ddot{\mathbf{r}} = -\frac{\mu_e}{r^3} \mathbf{r} - \frac{\mu_{mo}}{d_{mo-e}^3} \mathbf{d}_{mo-e} - \frac{\mu_{mo}}{d_{mo-sc}^3} \mathbf{d}_{mo-sc}, \quad (2)$$

where μ_{mo} is a standard gravitational parameter of the moon. $\mathbf{d}_{mo-e}$ and $\mathbf{d}_{mo-sc}$ are the position vectors from the moon's mass center to the Earth's mass center and the spacecraft mass center, respectively.

Similarly, the motion equation of the test mass 1 (TM1) and the test mass 2 (TM2) in the ECI coordinate system are

$$\ddot{\mathbf{r}}_1 = -\frac{\mu_e}{r_1^3} \mathbf{r}_1 - \frac{\mu_{mo}}{d_{mo-e}^3} \mathbf{d}_{mo-e} - \frac{\mu_{mo}}{d_{mo-tm1}^3} \mathbf{d}_{mo-tm1}, \quad (3)$$

$$\ddot{\mathbf{r}}_2 = -\frac{\mu_e}{r_2^3} \mathbf{r}_2 - \frac{\mu_{mo}}{d_{mo-e}^3} \mathbf{d}_{mo-e} - \frac{\mu_{mo}}{d_{mo-tm2}^3} \mathbf{d}_{mo-tm2}, \quad (4)$$

where $\mathbf{r}_1$ and $\mathbf{r}_2$ are the position vectors from the Earth's mass center to TM1 and TM2, respectively. $\mathbf{d}_{mo-tmi}$ is the position vectors from the moon's mass center to the i th test mass.

FIGURE 1: The relative motion between the test masses and the drag-free spacecraft.

The position vectors of TM1 and TM2 relative to spacecraft's mass center O are

$$\rho_3 = \mathbf{r}_1 - \mathbf{r}, \quad \rho_4 = \mathbf{r}_2 - \mathbf{r}. \quad (5)$$

As can be seen from Figure 1, the position vector of the test masses relative to the mass center of the spacecraft cavity can be expressed as

$$\rho_5 = \rho_3 - \rho_1, \quad \rho_6 = \rho_4 - \rho_2. \quad (6)$$

Differentiating Equation (5), substituting Equations (3) and (4) into it, and considering the influence of solar gravitation lead to

$$\begin{cases} \ddot{\rho}_3 = -\mu_e \left(\frac{\mathbf{r}_1}{r_1^3} - \frac{\mathbf{r}}{r^3} \right) - \mu_{mo} \left(\frac{\mathbf{d}_{mo-tm1}}{d_{mo-tm1}^3} - \frac{\mathbf{d}_{mo-sc}}{d_{mo-sc}^3} \right) - \mu_s \left(\frac{\mathbf{d}_{s-tm1}}{d_{s-tm1}^3} - \frac{\mathbf{d}_{s-sc}}{d_{s-sc}^3} \right), \\ \ddot{\rho}_4 = -\mu_e \left(\frac{\mathbf{r}_2}{r_2^3} - \frac{\mathbf{r}}{r^3} \right) - \mu_{mo} \left(\frac{\mathbf{d}_{mo-tm2}}{d_{mo-tm2}^3} - \frac{\mathbf{d}_{mo-sc}}{d_{mo-sc}^3} \right) - \mu_s \left(\frac{\mathbf{d}_{s-tm2}}{d_{s-tm2}^3} - \frac{\mathbf{d}_{s-sc}}{d_{s-sc}^3} \right), \end{cases} \quad (7)$$

where μ_s is a standard gravitational parameter of the sun. $\mathbf{d}_{s-sc}$ and $\mathbf{d}_{s-tmi}$ are the position vectors from the Solar's mass center to the spacecraft's mass center and the i th test mass, respectively.

According to Equation (7), the influence of the moon and the sun for the relative motion are

$$\mathbf{a}_{moon} = -\mu_{mo} \left(\frac{\mathbf{d}_{mo-tmi}}{d_{mo-tmi}^3} - \frac{\mathbf{d}_{mo-sc}}{d_{mo-sc}^3} \right), \quad \mathbf{a}_{sun} = -\mu_s \left(\frac{\mathbf{d}_{s-tmi}}{d_{s-tmi}^3} - \frac{\mathbf{d}_{s-sc}}{d_{s-sc}^3} \right). \quad (8)$$

TABLE 1: The parameters of the sun orbit and Earth orbit [25].

Orbit parameters	Sun orbit	Earth orbit
Semimajor axis a (km)	149597870.700	99995.5723
Eccentricity e	0.009648	0.00043
Orbit inclination i (°)	0.9529	74.5362
RAAN Ω (°)	348.16	211.6003
Argument of perigee w (°)	270	346.5528
True anomaly f (°)	180	61.3296

In SCO, we have

$$\begin{aligned} \mathbf{r} &= [00r]^T, \quad \rho_1 = [x_{s1} \ y_{s1} \ z_{s1}]^T, \quad \rho_2 = [x_{s2} \ y_{s2} \ z_{s2}]^T, \\ \mathbf{r}_1 &= [x_{s1} \ y_{s1} \ r + z_{s1}]^T, \quad \mathbf{r}_2 = [x_{s2} \ y_{s2} \ r + z_{s2}]^T, \end{aligned} \quad (9)$$

where $r = (1 - e^2)/(1 + e \cos v)$, a is the spacecraft's semimajor axes, e is an eccentricity of the orbit, v is a true anomaly. ρ_1 and ρ_2 are the position coordinates of TM1 and TM2 in SCO, respectively.

The equation of motion for test masses relative to the spacecraft's mass center in SCO can be expressed as

$$\begin{aligned} \frac{\delta^2 \rho_i}{\delta t^2} + 2\bar{\omega}_{SCO}^{ECI} \times \frac{\delta \rho_i}{\delta t} + \bar{\omega}_{SCO}^{ECI} \times (\bar{\omega}_{SCO}^{ECI} \times \rho_i) + \dot{\bar{\omega}}_{SCO}^{ECI} \times \rho_i \\ = -\mu_e \left(\frac{\mathbf{r}_i}{r_i^3} - \frac{\mathbf{r}}{r^3} \right) - \mu_{mo} \left(\frac{\mathbf{d}_{mo-tmi}}{d_{mo-tmi}^3} - \frac{\mathbf{d}_{mo-sc}}{d_{mo-sc}^3} \right) \\ - \mu_s \left(\frac{\mathbf{d}_{s-tmi}}{d_{s-tmi}^3} - \frac{\mathbf{d}_{s-sc}}{d_{s-sc}^3} \right), \end{aligned} \quad (10)$$

where $r_i = \sqrt{x_{si}^2 + y_{si}^2 + (r + z_{si})^2}$, $i = 1, 2$. $\dot{\omega}_{SCO}^{ECI}$ and $\ddot{\omega}_{SCO}^{ECI}$ are the angular velocity vector and the angular acceleration vector of the SCO with respect to the ECI, respectively.

And,

$$\dot{\omega}_{SCO}^{ECI} = [0 \quad \omega \quad 0]^T, \ddot{\omega}_{SCO}^{ECI} = [0 \quad \dot{\omega} \quad 0]^T, \quad (11)$$

where $\omega = \sqrt{(\mu_e(1 + e \cos f))/r^3}$, $\dot{\omega} = -(2\mu_e e \sin f)/r^3$.

Substituting Equations (9) and (11) into Equation (10) leads to

$$\begin{cases} \ddot{x}_{si} + 2n\dot{z}_{si} - n^2x_{si} + \dot{n}z_{si} = -\frac{\mu_e}{r_i^3}x_{si} - \frac{\mu_s}{d_{s-tmi}^3}(x_{si} - S_x) - \frac{\mu_s}{d_{s-sc}^3}S_x - \frac{\mu_{mo}}{d_{mo-tmi}^3}(x_{si} - M_{ox}) - \frac{\mu_{mo}}{d_{mo-sc}^3}M_{ox}, \\ \ddot{y}_{si} = -\frac{\mu_e}{r_i^3}y_{si} - \frac{\mu_s}{d_{s-tmi}^3}(y_{si} - S_y) - \frac{\mu_s}{d_{s-sc}^3}S_y - \frac{\mu_{mo}}{d_{mo-tmi}^3}(y_{si} - M_{oy}) - \frac{\mu_{mo}}{d_{mo-sc}^3}M_{oy}, \\ \ddot{z}_{si} - 2n\dot{x}_{si} - n^2z_{si} - \dot{n}x_{si} = -\frac{\mu_e}{r_i^3}(r + z_{si}) - \frac{\mu_s}{d_{s-tmi}^3}(z_{si} - S_z) + \frac{\mu_s}{d_{s-sc}^3}(r - S_z) - \frac{\mu_{mo}}{d_{mo-tmi}^3}(z_{si} - M_{oz}) + \frac{\mu_{mo}}{d_{mo-sc}^3}(r - M_{oz}), \end{cases} \quad (12)$$

where $d_{s-tmi} = \sqrt{(S_x - x_{si})^2 + (S_y - y_{si})^2 + (S_z - z_{si})^2}$, $i = 1, 2$. $[S_x, S_y, S_z]^T$ and $[M_{ox}, M_{oy}, M_{oz}]^T$ are the position coordinates of sun and moon in SCO, respectively. Equation (12) is the equation of motion for the i th test mass relative to the spacecraft's center of mass under the action of the Earth-moon-sun gravitation system.

The relative motion equation of the test mass and the cavity center in the spacecraft cavity coordinate system is as follows:

$$\begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} = \begin{bmatrix} \cos \theta_y & 0 & -\sin \theta_y \\ 0 & 1 & 0 \\ \sin \theta_y & 0 & \cos \theta_y \end{bmatrix} \begin{bmatrix} x_{si} - O_{ix} \\ y_{si} - O_{iy} \\ z_{si} - O_{iz} \end{bmatrix}, \quad (13)$$

where θ_y is the rotation angle of the SCO around the y -axis to the spacecraft cavity coordinate system, and $O_i = [O_{ix}, O_{iy}, O_{iz}]^T$ is the position coordinates from the spacecraft's mass center to the cavity's center.

3. The Relative Motion Analysis

In order to comparative study the relative motion between the spacecraft cavity and test masses in the sun orbit and the Earth orbit, the two orbit parameters are given here, as shown in Table 1.

Since the two test masses' analysis methods are similar, only the relative motion relation of TM1 is considered here. Let the coordinates of the cavity center relative to the center of the spacecraft's mass in the spacecraft orbital coordinate system be (0.3 m, 0, 0). Assuming that the initial motion state of TM1 in the cavity coordinate system is (0, 0, 0, 0, 0, 0), the motion of the TM1 under sun orbit and Earth orbit relative to the center of the cavity is shown in Figure 2. The simulation starts on January 1, 2034.

As can be seen from Figure 2(a), the maximum displacement of the TM1 relative to the center of the cavity in the

sensitive axis in sun orbit within a year is about 2.9 mm, and the relative motion in the nonsensitive axis is about 5 mm. In Figure 2(b), the relative motion displacement of the sensitive axis in Earth orbit is about 500 mm (Assuming the cavity size is large enough), while the relative motion displacement of the non-sensitive axis is about 900 mm. According to Figure 2, the test mass in the Earth orbit is significantly affected by the gravitational gradient of celestial bodies, while the motion in the sun orbit is relatively small. Therefore, for gravitational wave detection spacecrafts in Earth orbit, it is necessary to focus on the analysis of the effect of celestial gravity gradient on relative motion, so as to avoid the collision between test mass and cavity.

This study focuses on gravitational wave detectors in the Earth orbit, which are mainly subject to the gravity gradient of the Earth, moon and sun. The influence of the above three celestial bodies on the displacement and acceleration of the test mass relative to the center of the cavity are analyzed in detail, as shown in Figures 3–5.

It can be seen from Figure 3(a) that the relative displacement between the test mass and the center of the cavity in the sensitive axis caused by the gravitational gradient of the Earth-sun system is greater than other systems. The relative displacement caused by the gravitational gradient of the Earth-moon-sun system is smaller than that of the Earth-sun system, which may be due to the damping effect of the moon's gravity. Figure 3(b) shows the relative acceleration effects of the Earth, moon, and sun in the sensitive axis. It can be seen from the figure that the gravity gradient of the Earth is the main influencing factor of the relative motion in the sensitive axis, and its magnitude is 10^{-11} m/s^2 . The gravitational gradient of the sun fluctuates greatly, which makes the displacement of relative motion change greatly. The influence magnitude of the moon's gravitational gradient in the sensitive axis is 10^{-14} m/s^2 . Figure 5 shows the relative motion effects of the gravitational gradient of the Earth-moon-sun system on the direction of the vertically sensitive axis in the orbital plane. As can be seen from Figure 5, the

(a) The relative displacement in the sun orbit

(b) The relative displacement in the Earth orbit

FIGURE 2: The relative displacement of the sun orbit and Earth orbit.

influence of each system on the relative displacement is similar to the sensitive axis.

In Figure 4(a), the relative displacement caused by the Earth-moon-sun system in the normal direction is greater than that caused by other systems, while the relative displacement of the Earth-moon system in the normal direc-

tion is greater than that caused by the Earth-sun system. Figure 4(b) shows that the relative acceleration caused by the Earth's gravitational gradient in the normal direction is 0, which is because the normal direction has no weight in the initial state. And the normal direction is decoupled from the orbital plane, so the relative acceleration in the normal

FIGURE 3: The effect of the Earth-moon-sun system on the relative motion in the sensitive axis.

direction is 0. In the normal direction, the influence magnitude of the moon's gravitational gradient is $10^{-15} m/s^2$, while the sun is $10^{-16} m/s^2$. Therefore, the gravitational gradient of the moon dominates in the normal direction, and the influ-

ence of the gravitational gradient of the sun still needs to be considered.

A comparative analysis of the gravity gradient of the Earth system, the Earth-moon system, and the Earth-sun

FIGURE 4: The effect of the Earth-moon-sun system on the relative motion in the normal direction.

system shows that in gravitational wave detection spacecrafts of similar heights, the effects of the Earth, the moon, and the sun on the motion of the relative cavity center of the mass need to be considered simultaneously.

Therefore, the influence of the Earth, the moon, and the sun on the relative motion of the test mass and the cavity

center should be considered simultaneously in the gravitational wave detection spacecrafts at similar heights.

In the above analysis, it is assumed that the cavity size is large enough, while the actual cavity size is limited. Therefore, the movement of the test mass must be controlled to avoid collision with the cavity. Here, we study the

FIGURE 5: The effect of the Earth-moon-sun system on the relative motion in the vertically sensitive axis direction.

gravitational wave detector with two test masses, so the movement relationship between the two test masses and the cavity should be considered simultaneously when designing the control strategy. Figure 2(b) shows the relative

motion of TM1, and the relative motion of TM2 will be analyzed here. Let the position of the cavity center with TM2 in the SCO be $(-0.3 \text{ m}, 0, 0)$ and the initial motion state of TM2 in the cavity coordinate system be $(0, 0, 0, 0, 0, 0, 0)$. The

FIGURE 6: The movement of TM2 relative to the cavity center.

FIGURE 7: The structure of the control system.

TABLE 2: The control requirement.

Parameter	Requirement
x_1	$10^{-12} \text{ mHz}^{-1/2} @ 0.1 \text{ mHz}-1 \text{ Hz}$
y_1, z_1	$10^{-9} \text{ mHz}^{-1/2} @ 0.1 \text{ mHz}-1 \text{ Hz}$
a_{x1}	$10^{-15} \text{ ms}^{-2} \text{ Hz}^{-1/2} @ 0.1 \text{ mHz}-1 \text{ Hz}$
a_{y1}, a_{z1}	$10^{-13} \text{ ms}^{-2} \cdot \text{Hz}^{-1/2} @ 0.1 \text{ mHz}-1 \text{ Hz}$
Relative displacement deviation	5 mm

movement of TM2 relative to the cavity center is shown in Figure 6.

In Figures 2(b) and 6, the red line represents the relative motion on the sensitive axis, the blue dotted line represents the relative motion in the normal direction, and the green chain-dotted line represents the relative motion in the direction of the perpendicular sensitive axis in the orbital plane.

On the sensitive axis, the relative motion direction of TM1 and TM2 changes from negative to positive with time. On the vertical direction of the sensitive axis, the relative motion direction of TM1 changes from negative to positive with time, while TM2 changes from positive to negative with time.

4. Relative Motion Control

4.1. Sliding Mode Controller Design. In order to avoid the collision between the test mass and the cavity, the following two control strategies can be adopted:

- (i) Strategy 1 [24]: displacement mode+acceleration mode: the changes of TM1 and TM2 in the direction of the sensitive axis can first decompose the force to the coordinate axis and then offset by the micro thrusters. The change of nonsensitive axis direction can be controlled by the electrostatic controller alone

FIGURE 8: The control effect of relative motion in the time domain.

- (ii) Strategy 2: acceleration mode: the change of the sensitive axis and nonsensitive axis directions of TM1 and TM2 are all controlled by the electrostatic controller

In Strategy 1, displacement mode is adopted in the sensitive axis. Compared with Strategy 2, the control accuracy is higher, and the control force can avoid interference of the test mass in the direction of the sensitive axis. However, this method makes TM1 and TM2 mutually coupled, and the control force is relatively complex. In Strategy 2, each axis is controlled by electrostatic force, which can realize the decoupling of TM1 and TM2 controls, but the electrostatic force will introduce the negative stiffness effect. If the interference introduced in Strategy 2 is lower than the requirement, Strategy 2 can still be adopted.

The actuator of Strategy 2 is the capacitive plate, which directly acts on the test mass, so as to realize the decoupling of TM1 and TM2 controls and simplify the controller design process. Therefore, the sliding mode controller is designed for the relative motion control between the test mass and the cavity. Sliding mode control has the characteristics of fast response speed and insensitivity to parameter change and disturbance. The structure of the control system can be seen in Figure 7.

In the design of the controller, Equation (12) is first written as a form of state space, as follows:

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{f}(x_i, y_i, z_i, t) + \mathbf{B}\mathbf{u}, \\ \mathbf{y}(t) = \mathbf{C}\mathbf{x}(t), \end{cases} \quad (14)$$

where $\mathbf{x}(t) = [\dot{x}_i, \dot{y}_i, \dot{z}_i]^T$, $\mathbf{f}(x_i, y_i, z_i, t)$ is a nonlinear function, $\mathbf{B}$ and $\mathbf{C}$ are the coefficient matrix, and $\mathbf{u}$ is the control input.

We define the sliding mode function:

$$\mathbf{s}(t) = \mathbf{c}\mathbf{e}_\varepsilon + \dot{\mathbf{e}}_\varepsilon, \quad (15)$$

where $\mathbf{c} > 0$ and $\mathbf{e}_\varepsilon$ and $\dot{\mathbf{e}}_\varepsilon$ are error and error derivative, respectively.

Let

$$\mathbf{e}_\varepsilon(t) = \mathbf{x}_d - \mathbf{x}, \quad \dot{\mathbf{e}}_\varepsilon = \dot{\mathbf{x}}_d - \dot{\mathbf{x}}, \quad (16)$$

where $\mathbf{x}_d$ is an expected position.

In order to prove the stability of the controller, the Lyapunov function is defined as follows:

$$\mathbf{L}_y = \frac{1}{2} \mathbf{s}^2. \quad (17)$$

Differentiating Equations (15) and (17), we get

$$\dot{\mathbf{s}} = \mathbf{c}\dot{\mathbf{e}}_\varepsilon + \ddot{\mathbf{x}}_d - \mathbf{f}(x_i, y_i, z_i, t) - \mathbf{B}\mathbf{u}, \quad (18)$$

$$\dot{\mathbf{L}}_y = \mathbf{s}\dot{\mathbf{s}}. \quad (19)$$

FIGURE 9: The displacement accuracy and residual acceleration accuracy of the relative motion on x_1 .

We use the exponential reaching law; then, the control law is

$$\mathbf{u} = \frac{1}{\mathbf{B}} [\mathbf{c}\dot{\mathbf{e}}_e + \mathbf{x}_d - \mathbf{f}(x_i, y_i, z_i, t) + \mathbf{k}\mathbf{s} + \boldsymbol{\eta}\text{sat}(\mathbf{s})], \quad (20)$$

where

$$\text{sat}(\mathbf{s}) = \begin{cases} \text{sgn}(\mathbf{s}), & \mathbf{s} \geq \boldsymbol{\phi}, \\ \frac{\mathbf{s}}{\boldsymbol{\Gamma}}, & \mathbf{s} < \boldsymbol{\phi}. \end{cases} \quad (21)$$

Substituting Equations (18) and (20) in Equation (19) gives

$$\begin{aligned} \dot{\mathbf{L}}_y &= \mathbf{s}(\mathbf{c}\dot{\mathbf{e}}_e + \ddot{\mathbf{x}}_d - \mathbf{f}(x_i, y_i, z_i, t) - \mathbf{c}\dot{\mathbf{e}}_e - \ddot{\mathbf{x}}_d + \mathbf{f}(x_i, y_i, z_i, t) - \mathbf{k}\mathbf{s} - \boldsymbol{\eta}\text{sat}(\mathbf{s})) \\ &= -\mathbf{k}\mathbf{s}^2 - \boldsymbol{\eta}|\mathbf{s}| \leq 0. \end{aligned} \quad (22)$$

According to the LaSalle invariance principle, the closed-loop controller is asymptotically stable.

4.2. Performance Analysis of the Controller. The control analysis of TM1 is given, and the control analysis of TM2 is similar to that of TM1. This section studies the control of test mass relative to the cavity under the gravitational gradient of the Earth-moon-sun system when the geocentric

FIGURE 10: The displacement accuracy and residual acceleration accuracy of the relative motion on y_1 .

gravitational wave detector is operating in scientific mode. The control requirement can be given as shown in Table 2 [26].

In the scientific model, the initial state parameters of TM1 is (0, 0, 0, 0, 0, 0), and the starting time of simulation is January 1, 2034. With the controller designed by Equation (20), the control effect of the relative motion between TM1 and cavity in the time domain can be obtained, as shown in Figure 8.

By comparing Figures 2(b) and 8, it can be seen that the relative motion between the TM1 and the cavity after control is significantly reduced, and the relative displacement of three axes are in the magnitude of 10^{-9} m, 10^{-10} m, and 10^{-9} m,

respectively. These results meet the requirements of Table 2 and have the margins of several orders of magnitude. Therefore, the sliding mode controller can realize the high-precision control of drag-free spacecraft and has certain robustness.

In scientific mode, in order to avoid the interference of the controller on sensitive axis and nonsensitive axis to flood the gravitational wave detection signal, the control effect of the controller in the frequency domain is given here. Figures 9–11, respectively, show the displacement accuracy and residual acceleration accuracy of the relative motion in the frequency domain.

FIGURE 11: The displacement accuracy and residual acceleration accuracy of the relative motion on z_1 .

In Figures 9–11, Requirement denotes the control index from Table 2. The blue line represents the power spectral density (PSD) of relative displacement when the control force is applied in Figures 9(a)–11(a). It can be seen from Figure 9(a) that the accuracy of the relative displacement between the TM1 and the cavity does not meet the requirements in the detection frequency band when the control force of the sensitive axis is not applied. The slight constraint violation within 0.1 mHz–2 mHz may be caused by the negative stiffness of electrostatic force. In other frequency bands, the requirements are fully met. Considering that 0.1 mHz–2 mHz basically meets the requirements, this

violation can be tolerated. The figure in frequency band 0.025 Hz–1 Hz is not shown here, because the large interference is generally reflected in the low-frequency band. In Figure 9(b), the accuracy of residual acceleration in the direction of the sensitive axis is lower than required, and there is a certain margin. This result shows that the sliding mode controller not only achieves robustness in the time domain but also has a robustness to the frequency domain control results.

It can be seen from Figures 10 and 11 that the relative displacement accuracy and residual acceleration accuracy of the nonsensitive axis both meet the requirements and

have a certain margin. Therefore, the controller meets the requirements of the frequency domain. In addition, comparing the frequency domain control results of the relative acceleration on the three axes, it can be seen that the acceleration PSD margin of the sliding mode controller in the z-axis direction is greater than that of the other two axes, which is consistent with the position PSD results.

5. Conclusions

In this paper, the relative motion and control of a drag-free spacecraft system under the gravitational fields of the Earth, moon, and sun are studied for a high-precision mission. The relative motion between the test mass and the spacecraft cavity is analyzed by multibody dynamics, numerical solution, and coordinate transformation. Through the comparative study on the relative motion of the sun orbit and Earth orbit, it was found that under pure gravity, the relative motion of the LISA spacecraft on the sensitive axis was relatively small, and the collision between the test mass and the cavity on the sensitive axis would not occur. However, the relative motion of the Earth orbit drag-free spacecraft on the sensitive axis is relatively large, which will cause a collision if no control is applied. By decomposing the effects of different gravitational fields, the influence of the gravitational gradient of the Earth, moon, and sun in the Earth orbit is studied. It is found that the Earth has the greatest influence on relative motion in the orbital plane, while the moon has the greatest influence on relative motion in the normal direction. Finally, a sliding mode controller is designed to verify one of the control strategies in combination with the background of the high-precision mission. The simulation results show that the controller can meet the requirements of the time domain and frequency domain.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References

- [1] G. Pugh, "Proposal for a spacecraft test of the coriolis prediction of general relativity," in *Weapons Systems Evaluation Group Research Memorandum*, NASA, USA, 1959.
- [2] B. Lange, "The drag-free Satellite," *AIAA Journal*, vol. 2, no. 9, pp. 1590–1606, 1964.
- [3] B. Lange, *The Control and Use of Drag-Free Spacecrafts*, Stanford University, Stanford, 1964.
- [4] J. D. Powell, B. O. Lange, and P. Jhin, "Mass attraction reduction by integral control in spinning dragfree Satellites," *Journal of Spacecraft and Rockets*, vol. 9, no. 8, pp. 592–596, 1972.
- [5] M. Wiegand, S. Scheithauer, and S. Theil, "Step proof mass dynamics," *Acta Astronautica*, vol. 54, no. 9, pp. 631–638, 2004.
- [6] S. Theil, *Drag-Free Spacecraft Control. Lasers, Clocks and Drag-Free Control*, Springer, Berlin Heidelberg, 2008.
- [7] M. S. Guilherme, W. C. Leite Filho, and S. Theil, "Strategies for in-orbit calibration of drag-free control systems," *Aerospace Science and Technology*, vol. 12, no. 5, pp. 365–375, 2008.
- [8] L. Pettazzi, A. Lanzon, S. Theil, and A. E. Finzi, "Design of robust drag-free controllers with given structure," *Journal of Guidance, Control, and Dynamics*, vol. 32, no. 5, pp. 1609–1621, 2009.
- [9] E. Canuto, A. Molano, and L. Massotti, "Drag-free control of the GOCE satellite: noise and observer design," *IEEE Transactions on Control Systems Technology*, vol. 18, no. 2, pp. 501–509, 2010.
- [10] E. Canuto and L. Massotti, "All-propulsion design of the drag-free and attitude control of the European satellite GOCE," *Acta Astronautica*, vol. 64, no. 2-3, pp. 325–344, 2009.
- [11] E. Canuto, "Drag-free and attitude control for the GOCE satellite," *Automatica*, vol. 44, no. 7, pp. 1766–1780, 2008.
- [12] E. Canuto, C. Novara, D. Carlucci, C. P. Montenegro, and L. Massotti, *Spacecraft Dynamics and Control: The Embedded Model Control Approach*, Butterworth-Heinemann, 2018.
- [13] W. Fichter, P. Gath, S. Vitale, and D. Bortoluzzi, "LISA Pathfinder drag-free control and system implications," *Classical and Quantum Gravity*, vol. 22, no. 10, pp. S139–S148, 2005.
- [14] W. Fichter, A. Schleicher, S. Bennani, and S. Wu, "Closed loop performance and limitations of the LISA Pathfinder drag-free control system," in *AIAA Guidance, Navigation and Control Conference and Exhibit*, Hilton Head, South Carolina, 2007.
- [15] T. Ziegler and W. Fichter, "Test mass stiffness estimation for the LISA Pathfinder drag-free system," in *Aiaa Guidance, Navigation & Control Conference & Exhibit*, Hilton Head, South Carolina, 2011.
- [16] S. F. Wu and D. Fertin, "Spacecraft drag-free attitude control system design with quantitative feedback theory," *Acta Astronautica*, vol. 62, no. 12, pp. 668–682, 2008.
- [17] S. F. Wu and D. Fertin, *QFT based drag-free and attitude controller design/tuning*, European Space Agency Space Agency, 2005.
- [18] X. Lian, *Relative motion analysis and control strategy for test mass of drag-free spacecraft*, [Ph.D. thesis], Sun Yat-sen University, 2021.
- [19] X. Lian, J. Zhang, L. Chang, J. Song, and J. Sun, "Test mass capture for drag-free satellite based on RBF neural network adaptive sliding mode control," *Advances in Space Research*, vol. 69, no. 2, pp. 1205–1219, 2022.
- [20] P. Chapman, P. Zentgraf, and Y. Jafry, "Drag-free control design including attitude transition for the STEP mission," in *Spacecraft Guidance, Navigation and Control Systems*, European Space Agency-Publications-ESA SP, 2003.
- [21] L. Shi, *Dynamic Modeling and Control Research of Drag-Free Spacecraft*, Harbin Institute of Technology, Harbin, 2012.
- [22] L. Francesco, *Precise Control of LISA with Quantitative Feedback Theory*, Delft University of Technology, 2019.
- [23] H. Li, *Research of Drag-Free and Attitude Control System for the Next Generation Gravimetric Spacecraft*, Huazhong University of Science and Technology, Wuhan, 2017.
- [24] X. Lian, J. Zhang, L. Lu et al., "Frequency separation control for drag-free satellite with frequency-domain constraints," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 57, no. 6, pp. 4085–4096, 2021.

- [25] X. Lian, J. Zhang, J. Yang, Z. Lu, Y. Zhang, and Y. Song, "The determination for ideal release point of test masses in drag-free satellites for the detection of gravitational waves," *Advances in Space Research*, vol. 67, no. 2, pp. 824–833, 2021.
- [26] J. Luo, L. S. Chen, H. Z. Duan et al., "TianQin: a space-borne gravitational wave detector," *Classical and Quantum Gravity*, vol. 33, no. 3, pp. 035010–035032, 2016.