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Research Article

A Modified Robust Adaptive Fault Compensation Design for Spacecraft with Guaranteed Transient Performance

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A modified, robust adaptive fault compensation design is proposed for rigid spacecraft systems with uncertain actuator failures and unknown disturbances. The feedback linearization method is first introduced to linearize the nonlinear dynamics, and a model-reference adaptive controller is designed to suppress the unknown external disturbances and stabilize the linearized system. Then, a composite adaptive controller is developed by integrating multiple controllers designed for the corresponding actuator failure conditions, which can handle the essentially multiple uncertainties (failure time, values, type, and failure pattern) of actuator failures simultaneously. To further improve the transient performance problem in the failure compensation control, an $H\infty$ compensator is introduced as an additional item in the basic controller to attenuate the adverse effects on tracking performance caused by parameter estimation errors. From the theoretical analysis and simulation results, it is obvious that the designed scheme can not only guarantee the stability of the closed-loop system is stable and asymptotical tracking properties for a given reference signal but also greatly improve the transient performance of the spacecraft system during the process of failure compensation.

1. Introduction

Component (actuator or sensor) failures and external disturbances are common in performance-critical systems, which can lead to loss of performance and even cause catastrophic accidents. Hence, to maintain an acceptable level of performance and guarantee system stability in the event of uncertain component failures, remarkable progresses have been made in the area of fault-tolerant control (FTC) and disturbance suppression [1–7].

Reaction wheels are commonly used in spacecraft as the actuators, which may fail in the course of system operation. Precise attitude control in the case of disturbances and uncertain actuator failures have widely studied in the existing literature. Excellent overviews were provided by the survey papers [8, 9] to make FTC designs for spacecraft control system. In [10], a fault tolerant control scheme was proposed for spacecraft attitude stabilization by integrating learning observer and backstepping control design. A novel adaptive event-triggered controller was designed to handle disturbances, model uncertainties, actuator failures, and limited communication, simultaneously [11]. In this paper, only loss of actuator effectiveness fault was considered in the control system design. Adaptive observer-based fault-tolerant tracking control schemes were widely used to deal with the attitude tracking problem for spacecraft experience disturbances and actuator failures [12, 13]. Fault detection and identification based FTC scheme was designed for spacecraft control system subject to multiple actuator faults, parameter uncertainties, and external disturbances [14]. Nonlinear model predictive control approach was used to control the coupled translational-rotational motion of a spacecraft in the presence of one actuator failure [15]. In [16], a new adaptive attitude tracking control scheme was developed for a flexible spacecraft system subject to external disturbances and uncertain failures. The sliding mode control (SMC) technology is insensitive to some disturbances and uncertainties very much [17]. Numerous works related to SMC-based spacecraft FTC design were reported in [18] and the references therein.

For the recently advanced space missions, the system performance either in the stable state and the instantaneous one are equally important. Bad transient performance may exist although the steady performance can be obtained successfully finally, which are potentially dangerous to performance-critical systems. The problem of transient performance improvement has been widely researched based on several inspired control approaches, such as model reference adaptive control (MRAC) [19], H_∞ control [20], adaptive control [21, 22] and sliding mode control [23]. To improve the transient dynamics through modifying MRAC design, [24, 25] have used fuzzy logic and genetic algorithm, illustrated some simulations without analytical support to manifest the effectiveness of the designed scheme. Dynamic regressor extension and the technique of mixing parameter estimation were introduced by [26], which removed the assumption of some prior knowledge with high-frequency gain, whereas such a scheme may cause a complicated task for issues of practical interests. Considering that actuator saturation, together with the excessive tracking error of the closed-loop system's trajectory may also be led by large adaptive rates, with the exception of the plant's reference model, [27] introduced an auxiliary reference model with the characterization of the classical MRAC framework. However, the simulation results show that the larger values of the user-defined rate parameter will cause larger overshoots. Most of the aforementioned schemes deal only with system parameter uncertainties without considering the uncertainties of actuator failure.

Recently, more attention has been paid to the study of FTC designs with guaranteed transient performance [28]. Investigating the prescribe performance fault tolerance control for chaser spacecraft. The uncertainties of model, actuator failure, and external disturbances were summarized as lumped disturbances, which were estimated by a finite-time extended state observer. Based on the estimated information from the observer, an adaptive backstepping controller was designed to achieve the desired trajectory [29]. Addressed the problem of finite-time attitude-tracking control for a rigid spacecraft with inertial uncertainties, external disturbances, actuator saturations, and faults. A fast nonsingular terminal sliding mode manifold integrating with fuzzy approximation technique was constructed to develop an enhanced FTC scheme. It can guarantee the real finite-time stability instead of asymptotical stability. Similar to the study in [29, 30], we proposed a fault-tolerant nonsingular fixed-time control scheme based on neural networks for spacecraft maneuver mission, which can accelerate the convergence rate and improve control accuracy. A robust FTC algorithm was synthesized by employing a low-pass filter and an auxiliary dynamic system along with adaptive backstepping design, which achieved attitude tracking despite the presence of disturbances, actuator faults, and input saturation [31]. In [32], a fault-tolerant controller, based on dynamic surface design and nonlinear extended state observer, was developed for attitude tracking dynamics of the combined spacecraft in the presence of inertia uncertainty, actuator failure, and external disturbance. Such a scheme can drive the attitude tracking error to converge to one small neighborhood of zero. However, the uncertainties of both actuator failure and external disturbance were considered to be lumped disturbances by the previous literatures, and fuzzy logic system, neural networks, or observers were investigated to estimate the lumped disturbance. As [33] pointed out in most conditions, actuator failure and disturbance cannot be handled in the same way due to their different mechanisms.

As has been pointed out by [34], the parameter estimation error of the adaptive controller is one of the most significant factors that lead to the undesired transient. It is widely known that unanticipated actuator failures will bring about parametric uncertainties in the system. And the parameter estimation error is inevitable no matter which adaptive control scheme is adopted. Despite recent advances in transient performance improvement design for spacecraft fault-tolerant control system, it is still a challenging problem on how to guarantee transient performance for spacecraft system with uncertain actuator faults and external disturbances. Hence, it is an interesting and meaningful topic to investigate the problem of attitude tracking control with guaranteed transient and steady state performance for spacecraft subject to both actuator failures and external disturbances, which motivates the main results in this paper. The spacecraft attitude control problem under uncertain actuator failures and unknown disturbances is solved by proposed backstepping-based adaptive control scheme in literature [35]. On this basis, the problem of actuator failure compensation design for spacecraft attitude control system with guaranteed transient performance is further studied in this paper. The major contributions and excellence of our proposed methodology are as follows:

(i) Unlike the works which focus on the design of adaptive FTC for spacecraft systems with the weakness of a bad transient performance under the condition of unanticipated actuator failures, this paper further concerns the transient performance problem for the actuator failure compensation design. A performance index is adopted to assess the degree of the transient performance enhancement and characterize the weight of the designed H_∞ compensator in the modified MRAC system

(ii) Compared with the current MRAC based transient performance improvement schemes, this paper first couple the modified MRAC and direct adaptive FTC control techniques to increase the fault tolerance capability of the MRAC scheme

(iii) In contrast to the existing literatures regarding the uncertainties of both actuator failure and external disturbance as lumped disturbances, this paper solve the uncertain actuator failures and disturbances separately according to their different mechanisms

(iv) Detailed analysis of the performance of both transient and system steady state, and the valid proof according to the asymptotic output tracking as well

The remaining section is composed as follows. Section 2 formulates the control problems, describes some preliminaries on the feedback linearization theory, and presents two lemmas, which are important for the compensator design and performance analysis. Section 3 describes the modified

MRAC and the adaptive failure compensation design, as well as the closed-loop system performance analysis. Section 4 gives the simulation background and numerical simulation results.

2. Spacecraft Model and Problem Description

This chapter first introduces the rigid spacecraft system model and some basic concepts, then describes the actuator fault compensation of the spacecraft system.

2.1. Rigid Spacecraft Model. The mathematical model of a rigid spacecraft system is formulated by the following equations:

$$\begin{bmatrix} \dot{q}_0 \\ \dot{q} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -q^T \\ q_0 I + q^\times \end{bmatrix}\omega, \qquad J\dot{\omega} = -\omega^\times J\omega + Du(t) + d(t), \tag{1}$$

where q_0 and $q = [q_1, q_2, q_3]^T \in R^3$ denote the scalar and vector parts of the unit-quaternion, respectively. The quaternion also satisfies the constraint equation $q^T q + q_0^2 = 1$, $\omega \in R^3$ denotes the inertial angular velocity of the spacecraft expressed in the body frame, and the inertia matrix $J \in R^{(3\times 3)}$ is assumed to be known in this study. The notations $\zeta^\times$, $\forall \zeta = [\zeta_1, \zeta_2, \zeta_3]$, can be expressed as

$$\zeta^\times = \begin{bmatrix} 0 & -\zeta_3 & \zeta_2 \\ \zeta_3 & 0 & -\zeta_1 \\ -\zeta_2 & \zeta_1 & 0 \end{bmatrix}. \tag{2}$$

$u(t) \in R^4$ is the control input produced by reaction wheels. As the orientation matrix of the reaction wheel, $D \in R^{(3\times 4)}$ is available for a given spacecraft. In this research, we consider

$$D = \begin{bmatrix} -1 & 0 & 0 & \frac{1}{\sqrt{3}} \\ 0 & -1 & 0 & \frac{1}{\sqrt{3}} \\ 0 & 0 & -1 & \frac{1}{\sqrt{3}} \end{bmatrix}. \tag{3}$$

$d(t) = [d_1, d_2, d_3]^T \in R^3$ represents disturbance vector, which comes in many forms: gravity gradients, solar pressure, atmospheric drag, pressure forces, and so on. In practice, these forces are bounded. For the major topic of our interest, each component of $d(t)$ is modeled as

$$d_i(t) = c_i + \sum_{j=1}^{n_i} a_{ij} \sin \omega_{ij} t + \sum_{j=1}^{n_i} b_{ij} \cos \omega_{ij} t = \theta_{di}^{*T} \varpi_{di}(t), \tag{4}$$

where c_i, a_{ij}, and b_{ij} are unknown amplitudes, and ω_{ij} are known frequencies.

$$\theta_{di}^* = \left[c_{i0}, a_{i1}, \cdots, a_{i\,n_i}, b_{i1}, \cdots, b_{i\,n_i}\right]^T \in R^{2n_i+1}, \varpi_{di}(t) = \left[1, \sin \omega_{i_1} t, \cdots, \sin \omega_{i_{n_i}} t, \cos \omega_{i_1} t, \cdots, \cos \omega_{i_{n_i}} t\right]^T \in R^{2n_i+1}. \tag{5}$$

Define $x = [q^T, \omega^T]^T$ and $y = q$ as the state and output vector, the spacecraft attitude control system (1) is rewritten as

$$\dot{x} = f(x) + g(x)u_d(t) + g(x)d(t), \qquad y = h(x), \tag{6}$$

where

$$f(x) = \begin{bmatrix} 1/2\sqrt{1-(\|q\|)^2}\omega - 1/2\omega^\times q \\ -J^{-1}\omega^\times J\omega \end{bmatrix}, \qquad g(x) = [g_1, g_2, g_3] = \begin{bmatrix} 0 \\ J^{-1} \end{bmatrix}, \tag{7}$$

and

$$u_d = Du. \tag{8}$$

2.2. Control Problem Statement. This research mainly focuses on the issue of the spacecraft attitude control involves uncertain actuator failures. The classical actuator failures is expressed as

$$\bar{u}_j(t) = \bar{u}_{j0} + \sum_{i=1}^{q_j} \bar{u}_{ji} f_{ji}(t), \; t \geq t_j, \tag{9}$$

where $j \in \{1, 2, 3, 4\}$, $t_j > 0$, $\bar{u}_{j0}$, and $\bar{u}_{ji}$ represent unknown failure parameters. $f_{ji}(t)$, $i = 1, 2, \cdots, q_j$ are known. The Equation (9) can also be rewritten into the below-parameterized expression

$$\bar{u}_j(t) = \theta_j^T \varpi_j(t), \tag{10}$$

where $\theta_j = [\bar{u}_{j0}, \bar{u}_{j1}, \cdots, \bar{u}_{j\,q_j}]^T \in R^{q_j+1}$, $\varpi_j(t) = [1, f_{j1}(t), \cdots, f_{jq_j}(t)]^T \in R^{q_j+1}$. As specifically pointed out, the failure model (9) can describe stuck-in-place, complete failure, and oscillatory failure which usually occur in the spacecraft system.

In the system, if there is any uncertain actuator fault, then the input $u(t)$ applying to the system is

$$u(t) = (I - \sigma(t))v(t) + \sigma(t)\bar{u}(t), \tag{11}$$

where $v(t)$ denotes the control input signal. $\bar{u}(t) = [\bar{u}_1, \bar{u}_2, \bar{u}_3, \bar{u}_4]^T$ and$\sigma(t) = \text{diag}\{\sigma_1, \sigma_2, \sigma_3, \sigma_4\}$ are defined fault pattern matrix, i.e, when the j-th actuator fails $\sigma_j(t) = 1$, if there is no failure $\sigma_j(t) = 0$. Substituting Equation

(11) into the second equation of (1), the system model is expressed as

$$J\dot{\omega} = -\omega^{\times} J\omega + D[(I - \sigma(t))v(t) + \sigma(t)\bar{u}(t)] + d(t). \quad (12)$$

For the output $y(t) = q$ to track a given command $y_m(t)$, at least three functional inputs at any time are needed, so that the independent control inputs are enough to make sure that the output can track the arbitrary given output signals. Therefore, no more than one actuator failure can be allowed in the system; the compensable failure cases and the corresponding failure patterns are listed as

(i) no failure case, $\sigma_{(1)} = \text{diag}\{0, 0, 0, 0\}$

(ii) u_1 failure case, $\sigma_{(2)} = \text{diag}\{1, 0, 0, 0\}$

(iii) u_4 failure case, $\sigma_{(3)} = \text{diag}\{0, 0, 0, 1\}$

(iv) u_3 failure case, $\sigma_{(4)} = \text{diag}\{0, 0, 1, 0\}$

(v) u_2 failure case, $\sigma_{(5)} = \text{diag}\{0, 1, 0, 0\}$

2.2.1. Control Objective. For system (1), which has one uncertain failure (9) at most, an adaptive controller $v(t)$ is designed to guarantee system stability and output tracking with guaranteed transient performance. To show the design process of failure compensation control accompanied by satisfactory transient performance, we design the adaptive scheme for the three failure patterns as following:

$$\begin{aligned} \sigma_{(1)} &= \text{diag}\{0, 0, 0, 0\}, \\ \sigma_{(2)} &= \text{diag}\{1, 0, 0, 0\}, \\ \sigma_{(3)} &= \text{diag}\{0, 0, 0, 1\}. \end{aligned} \quad (13)$$

Remark 1. The studied spacecraft of this paper are actuated by four reaction wheels. We can learn from the orientation matrix of reaction wheel given in (3) that three of them are mounted where their spin axes are parallel to the body frame, respectively, and the other one is mounted where its spin axis points to some fixed direction. According to the configuration of reaction wheels, the control design for the u_1 failure case can be expended to address the u_2 and u_3 failure cases, so in this paper, only the three failure cases (13) are taken into account to demonstrate the detail design process.

2.3. Feedback Linearization. For a kind of nonlinear systems with the input and output of $m$dimension

$$\dot{x} = f(x) + g(x)u, y = h(x). \quad (14)$$

Definition 2. The system (14) has a vector relative degree $\{\rho_1, \rho_2, \cdots, \rho_m\}, 1 \le \rho_i \le n$, at a point x_0 for $\forall x$ in a neighborhood of if the below two conditions hold

(i) $L_{g_j} L_f^k h_i(x) = 0$, for all $1 \le j \le m, 1 \le i \le m, 0 < k < \rho_i - 1$ and $L_{g_j} L_f^{\rho_i - 1} h_i(x) \ne 0$, for some $j \in \{1, 2, \cdots, m\}$, and

(ii) the $m \times m$ matrix $G(x)$ is defined as

$$G(x) = \begin{bmatrix} L_{g_1} L_f^{\rho_1 - 1} h_1(x) & \cdots & L_{g_m} L_f^{\rho_1 - 1} h_1(x) \\ L_{g_1} L_f^{\rho_2 - 1} h_2(x) & \cdots & L_{g_m} L_f^{\rho_2 - 1} h_2(x) \\ \cdots & & \cdots \\ L_{g_1} L_f^{\rho_m - 1} h_m(x) & \cdots & L_{g_m} L_f^{\rho_m - 1} h_m(x) \end{bmatrix}. \quad (15)$$

Then the system (14) has the relative degree $\rho = \sum_{i=1}^{m} \rho_i$, with ρ_i being the subrelative degree of the i-th output $y_i = h_i(x)$. If the equilibrium point of system (1) is $x_0 = [0, 0, 0, 0, 0, 0]^T$, we can obtain that $\rho_1 = \rho_2 = \rho_3 = 2$ and the relative degree $\rho = n = 6$. By the twice differentiation to the system output y_i, the control input u_d in the differential equation is expressed in the form of a nonzero factor.

To be specific, the Equation (6) is denoted as

$$\begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \\ \ddot{y}_3 \end{bmatrix} = \begin{bmatrix} F_1(x) \\ F_2(x) \\ F_3(x) \end{bmatrix} + \begin{bmatrix} G_1(x) \\ G_2(x) \\ G_3(x) \end{bmatrix} [u_d + d(t)], \quad (16)$$

where

$$\begin{aligned} F_1(x) &= -\frac{1}{4} q_1 \left(\omega_1^2 + \omega_2^2 + \omega_3^2\right) + \frac{J_1 - J_2}{2J_3} q_2 \omega_1 \omega_2 \\ &\quad + \frac{J_1 - J_3}{2J_2} q_3 \omega_1 \omega_3 + \frac{J_2 - J_3}{2J_1} q_0 \omega_2 \omega_3, \\ F_2(x) &= -\frac{1}{4} q_2 \left(\omega_1^2 + \omega_2^2 + \omega_3^2\right) + \frac{J_2 - J_1}{2J_3} q_1 \omega_1 \omega_2 \\ &\quad + \frac{J_3 - J_1}{2J_2} q_0 \omega_1 \omega_3 + \frac{J_2 - J_3}{2J_1} q_3 \omega_2 \omega_3, \\ F_3(x) &= -\frac{1}{4} q_3 \left(\omega_1^2 + \omega_2^2 + \omega_3^2\right) + \frac{J_1 - J_2}{2J_3} q_0 \omega_1 \omega_2 \\ &\quad + \frac{J_3 - J_1}{2J_2} q_1 \omega_1 \omega_3 + \frac{J_3 - J_2}{2J_1} q_2 \omega_2 \omega_3, \\ G_1(x) &= \left[\frac{q_0}{2J_1}, -\frac{q_3}{2J_2}, \frac{q_2}{2J_3}\right], G_2(x) = \left[\frac{q_3}{2J_1}, \frac{q_0}{2J_2}, -\frac{q_1}{2J_3}\right], G_3(x) \\ &= \left[-\frac{q_2}{2J_1}, \frac{q_1}{2J_2}, \frac{q_0}{2J_3}\right]. \end{aligned} \quad (17)$$

The system (14) can be feedback linearized through differential homeomorphic mapping. Supposing there exists a differential homeomorphic mapping with the form $T(x) = \xi = [h_1(x), L_f h_1(x), h_2(x), L_f h_2(x), h_3(x), L_f h_3(x)]^T$.

Under the afore-mentioned coordinate transform, the system (6) can be transformed into three linear subsystems with the following normal form:

$$\begin{aligned}\dot{\xi}_{11} &= \xi_{12},\\ \dot{\xi}_{12} &= F_1(x) + G_1(x)[u_d + d(t)],\\ \dot{\xi}_{21} &= \xi_{22},\\ \dot{\xi}_{22} &= F_2(x) + G_2(x)[u_d + d(t)],\\ \dot{\xi}_{31} &= \xi_{32},\\ \dot{\xi}_{32} &= F_3(x) + G_3(x)[u_d + d(t)],\\ y &= [\xi_{11}, \xi_{21}, \xi_{31}]^T,\end{aligned} \tag{18}$$

with (11), the system (18) can be described as

$$\begin{aligned}\dot{\xi}_{11} &= \xi_{12},\\ \dot{\xi}_{12} &= F_1(x) + G_{1\sigma}(x)v + \bar{G}_{1\sigma}(x)\bar{u}(t) + G_1(x)d(t),\\ \dot{\xi}_{21} &= \xi_{22},\\ \dot{\xi}_{22} &= F_2(x) + G_{2\sigma}(x)v + \bar{G}_{2\sigma}(x)\bar{u}(t) + G_2(x)d(t),\\ \dot{\xi}_{31} &= \xi_{32},\\ \dot{\xi}_{32} &= F_3(x) + G_{3\sigma}(x)v + \bar{G}_{3\sigma}(x)\bar{u}(t) + G_3(x)d(t),\\ y &= [\xi_{11}, \xi_{21}, \xi_{31}]^T,\end{aligned} \tag{19}$$

where $G_{i\sigma}(x) = G_i(x)D(I - \sigma(t))\bar{G}_{i\sigma}(x) = G_i(x)D\sigma(t)$, and $i = 1, 2, 3$.

2.3.1. Nonlinear Feedback Control Law. The feedback linearization design can be applied to generate an ideal controller, on the condition that the system parameters and fault parameters of a nonlinear system (6) are accessible. From Equation (16), we can get the following equation:

$$\ddot{y}_i = F_i(x) + G_i(x)[u_d + d(t)], \quad i = 1, 2, 3. \tag{20}$$

Considering the uncertainty of external disturbance $d(t)$, we set the control signal as

$$G_i(x)u_d = W_{di} \triangleq -F_i(x) + u_{Li} - G_i(x)\hat{d}(t), \tag{21}$$

where W_{di} denotes the desired control signal generated from a chosen control that is designed for the closed-loop system, u_{Li} is the control law to be proposed, and $\hat{d}(t)$ is the estimator of $d(t)$. Then, the linearized system can be obtained

$$\ddot{y}_i = u_{Li} + G_i(x)\left[d(t) - \hat{d}(t)\right], \quad i = 1, 2, 3. \tag{22}$$

Lemma 3. (State-feedback H_∞ optimal control). *Consider a linear time-invariant system*

$$\begin{aligned}\dot{x} &= Ax + B_1u_w + B_2u,\\ z &= C_1x,\\ y &= C_2x,\end{aligned} \tag{23}$$

where $x \in R^{n_x}$ is the state, $u \in R^{n_u}$ is the control input, $u_w \in R^{n_w}$ is the disturbance, $z \in R^{n_z}$ is the regulated output to be controlled, and $y \in R^{n_y}$ is the measured output. A, B_1, B_2, C_1 , and C_2 are matrices of appropriate dimensions and satisfying the assumptions.

Assumption 4. (A, B_1) is stabilizable; (A, C_2) is detectable; B_1 is column full rank, and C_2 is row full rank; $n_w \le n_y \le n_x$.

If there exists an $\varepsilon < 0$ such that the Riccati equation

$$A^TP + PA - \varepsilon^{-1}PB_2R^{-1}B_2^TP + \gamma^{-1}C_1^TC_1 + \varepsilon S = 0, \tag{24}$$

has a solution $P \ge 0$, where $R \in R^{n_u \times n_u}$ and $S \in R^{n_x \times n_x}$ are given positive-definite matrices.

A state feedback controller $u_s = -((1/2\varepsilon)R^{-1}B_2^TP)x$ can be designed to stabilize the system (23), and the transfer function $G_{wz}(s)$ between disturbance w and output z satisfy the following condition

$$\|G_{wz}(s)\|_\infty < \gamma, \tag{25}$$

for a prespecified constant $\gamma > 0$. It should be noted that γ can be arbitrarily close to the H_∞ optimum by choosing a sufficiently small ε.

Lemma 5. *Let $z = H(s)w$, where $H(s)$ have all their roots in* Re $[s] \ge -\delta/2$, *for any $\delta \ge 0$ and $w \in L^2$, we have*

$$\|z_t\|_2^\delta \le \|H(s)\|_\infty^\delta \|w_t\|_2^\delta, \tag{26}$$

where $\|z_t\|_2^\delta$ is defined as $\|z_t\|_2^\delta =^\Delta \left(\int_0^t e^{-\delta(t-\tau)}[z^T(\tau)z(\tau)]d\tau\right)^{1/2}$ for $\forall z \in [0,\infty) \longrightarrow R^{n}$' and $\delta \ge 0$ and $t \ge 0$. If $H(s)$ is strict, then

$$|z(t)| \le \|H(s)\|_2^\delta \|w_t\|_2^\delta, \tag{27}$$

and if $H(s)$ is a stable n-order transfer function, then

$$\|z_t\|_\infty \le 2n\|H(s)\|_\infty \|w\|_\infty. \tag{28}$$

3. Actuator Failure Compensation Design

In this section, a direct adaptive control scheme is proposed to be combined with a basic control law derived from the modified MRAC design, which is not only adaptive to unknown disturbances but also able to handle uncertain patterns, values, and times of actuator failures. The simplified block diagram is shown in Figure 1.

To achieve the control objective, we will complete four design steps:

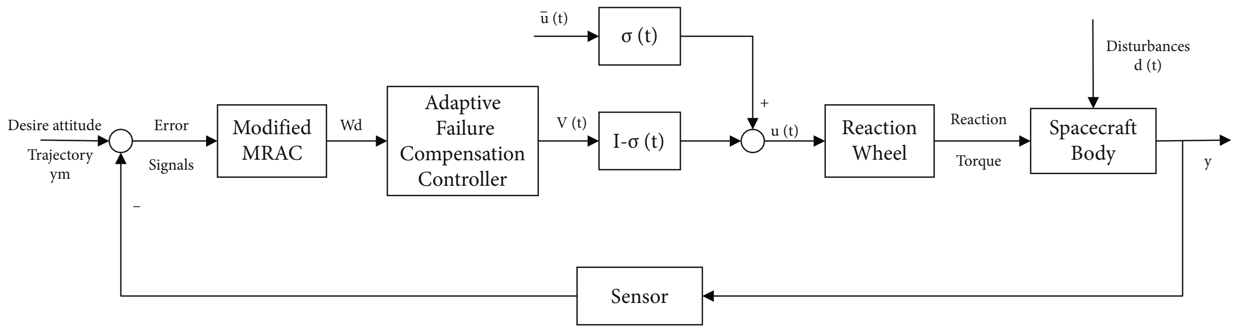


Figure 1: Block diagram of control system.

(1) Derive a desired control signal from modified MRAC technique for the closed-loop system to achieve the desired system performance

(2) Design a composite nominal controller to handle all possible failure patterns simultaneously, with the known parameters of actuator failures

(3) Develop an adaptive control scheme with the estimation of failure parameters updated based on system performance errors, and

(4) Analyze transient and steady-state performance for failure accommodation

In order to obtain an appropriate adaptive law $v(t)$, the error of the control signal equation which is led by the actuator uncertainties is examined. A desired control equation $G_i(x)u_d(t) = W_{di}$ (with $u_d(t) = Du(t)$) is defined to be satisfied by the nominal control signal $v^*(t)$ to be designed in the next section. In the light of (11), we define $u^*(t)$ as $u^*(t) \triangleq (I-\sigma)v^*(t) + \sigma\bar{u}(t)$ and obtain

$$G_i(x)u_d(t) = W_{di} + G_i(x)D(I-\sigma)[v(t) - v^*(t)]. \quad (29)$$

Substituting (21) into (29), we have

$$G_i(x)u_d(t) = -F_i(x) + u_{Li} - G_i(x)\widehat{d}(t) + G_i(x)D(I-\sigma)[v(t) - v^*(t)], \quad (30)$$

and the linearized system $\ddot{y}_i = u_{Li}$ can be rewritten into

$$\ddot{y}_i = u_{Li} + G_i(x)D(I-\sigma)[v(t) - v^*(t)] + G_i(x)\left[d(t) - \widehat{d}(t)\right]. \quad (31)$$

In this paper, we rewrite the linearized subsystem (31) into the transfer function form as

$$y_i = G_{pi}(s)u_{ri} = k_{pi}\frac{Z_{pi}(s)}{R_{pi}(s)}\left[u_{Li} + G_i(x)(I-\sigma)(v-v^*) + G_i(x)\left(d-\widehat{d}\right)\right], \quad (32)$$

with $u_{ri} = u_{Li} + G_i(x)D(I-\sigma)(v-v^*) + G_i(d-\widehat{d})$, and its reference model is

$$y_{mi} = W_{mi}(s)r_i = k_{mi}\frac{Z_{mi}(s)}{R_{mi}(s)}r_i, \quad (33)$$

where $Z_{pi}(s)$, $R_{pi}(s)$, $Z_{mi}(s)$, and $R_{mi}(s)$ are monic Hurwitz polynomials of degree m_{pi}, n_{pi}, , and p_{mi}, respectively, $\rho_i = n_{pi} - m_{pi} = p_{mi} - q_{mi} = 2$. The relative degree of $W_{mi}(s)$ is the same as that of $G_{pi}(s)$.

3.1. Modified MRAC Design. A model reference adaptive controller is constructed as follows

$$u_{Li} = \theta_i^T\omega_i + c_{0i}r_i + u_{ci}, \quad c_{0i} = \frac{k_{mi}}{k_{pi}}, \quad (34)$$

where $\omega_i = [\omega_{i1}^T, y_i]^T$, $\theta_i = [\theta_{i1}^T, \theta_{i2}]^T$, $\omega_{i1} = (\alpha_i(s)/\Lambda_i(s))y$, $\alpha_i(s) = [s^{n_{pi}-2}, s^{n_{pi}-3}, \cdots, s, 1]$, $\Lambda_i(s) = Z_{mi}\lambda_i(s)$, and $\lambda_i(s)$ are monic Hurwitz polynomials of degree $n_{pi} - q_{mi} - 1$; $\theta_i \in R^{2n_i-1}$ is the vector of controller parameters; u_{ci} is a proper H_∞ compensator to be designed later.

Remark 6. To derive the relation between the parameter estimation error of actuator failure and transient performance, we just talk about the uncertainties of actuator failure and disturbance assuming the system parameters are known. If all the system parameters are known, the nominal controller of MRAC can be $u_{Li} = \theta_i^{*T}\omega_i + c_{0i}r_i + u_{ci}$, with known $\theta_i^* = [\theta_{i1}^{*T}, \theta_{i2}^*]^T$.

For a given transform function $G_{pi}(s)$, there is a desired reference value vector $\theta_i^* = [\theta_{i1}^{*T}, \theta_{i2}^*]^T$ to make the following matching conditions valid

$$\frac{c_{0i}k_{pi}Z_{pi}}{R_{pi} - k_{pi}Z_{pi}\left(\left(\alpha_i^T(s)\theta_{i1}^*/\Lambda_i(s)\right) + \theta_{i2}^*\right)} = W_{mi}(s). \quad (35)$$

Multiplying both sides with y_i and using the equation

$R_{pi}(s)y_i = k_{pi}Z_{pi}(s)u_{ri}k_{pi}Z_{pi}$, we have

$$c_{oi}k_{pi}Z_{pi}y_i = W_{mi}(s)\left[k_{pi}Z_{pi}u_{ri} - k_{pi}\left(\frac{\theta_{i1}^{*T}\alpha_i(s)}{\Lambda_i(s)}y_i + \theta_{i2}^*\right)Z_{pi}y_i\right]. \quad (36)$$

Dividing both sides by $k_{pi}Z_{pi}$ we have

$$y_i = \frac{W_{mi}(s)}{c_{0i}}\left[u_{ri} - \frac{\theta_{i1}^{*T}\alpha_i(s)}{\Lambda_i(s)}y_i - \theta_{i2}^*y_i\right]. \quad (37)$$

The spacecraft system parameters are known; that is, in the MRAC law, θ_i is substituted by the desired value θ_i^*. To be more specific, the control u_{Li} is designed as

$$u_{Li} = \frac{\theta_{i1}^{*T}\alpha_i(s)}{\Lambda_i(s)}y_i + \theta_{i2}^*y_i + c_{0i}r_i + u_{ci} = -K_i\xi_i + c_{0i}r_i + u_{ci}, \quad (38)$$

where $K_i = [K_{i1}, K_{i2}]$ are the design controller parameters corresponding with θ_i^*, $\xi_i = [\xi_{i1}, \xi_{i2}]^T$. Substituting (38) into (30), we have

$$G_i(x)u_d(t) = -F_i(x) - G_i(x)\widehat{d}(t) + G_i(x)D(I-\sigma)[v(t) - v^*(t)] - K_i\xi_i + c_{0i}r_i + u_{ci}. \quad (39)$$

Furthermore, the linearized subsystem (18) is described as

$$\begin{aligned}\dot{\xi}_{i1} &= \xi_{i2},\\ \dot{\xi}_{i2} &= -K_i\xi_i + c_{0i}r_i + u_{ci} + G_i(x)D(I-\sigma)[v(t) - v^*(t)] + G_i(x)\left[d(t) - \widehat{d}(t)\right].\end{aligned} \quad (40)$$

We then revise the above equation in the form of state space as follows:

$$\begin{aligned}\dot{\xi}_i &= (A_{ci} - B_{ci}K_i)\xi_i + B_{ci}\left[G_i(x)D(I-\sigma)(v - v^*) + G_i(x)\left(d - \widehat{d}\right) + u_{ci} + c_{0i}r_i\right],\\ y_i &= C_{ci}\xi_i,\end{aligned} \quad (41)$$

where $A_{ci} \in R^{2\times2}$, $B_{ci} \in R^{2\times1}$, and $C_{ci} \in R^{1\times2}$ are the standard form of integrator chains. We also revise the reference model (33) in the state space as follows:

$$\begin{aligned}\dot{\xi}_{mi} &= A_{mi}\xi_{mi} + B_{mi}r_i,\\ y_{mi} &= C_{mi}\xi_{mi},\end{aligned} \quad (42)$$

where A_{mi}, B_{mi}, and C_{mi} are the minimal realization of $W_{mi}(s)$, i.e.,

$$\begin{aligned}A_{mi} &= \begin{bmatrix} 0 & 1 \\ -a_{i1} & -a_{i2}\end{bmatrix} \in R^{2\times2},\\ B_{mi} &= [0, 1]^T \in R^{2\times1},\end{aligned} \quad (43)$$

and $C_{mi} = [1, 0] \in R^{1\times2}$. Denoting $e_i = \xi_i - \xi_{mi}$, we can obtain

$$\dot{e}_i = (A_{ci} - B_{ci}K_i)\xi_i + B_{ci}\left[G_i(x)D(I-\sigma)(v - v^*) + G_i(x)\left(d - \widehat{d}\right) + u_{ci} + c_{0i}r_i\right] - A_{mi}\xi_{mi} - B_{mi}r_i. \quad (44)$$

If the controller parameters K_i is set to satisfy $A_{ci} - B_{ci}K_i = A_{mi}$ and $B_{ci}c_{0i} = B_{mi}$, then

$$\begin{aligned}\dot{e}_i &= A_{mi}e_i + \frac{B_{mi}}{c_{0i}}\left[G_i(x)D(I-\sigma)(v - v^*) + G_i(x)\left(d - \widehat{d}\right) + u_{ci}\right],\\ e_{i1} &= C_{mi}e_i.\end{aligned} \quad (45)$$

With the system model (32), (33) and the controller (34), we obtain

$$y_i = \frac{W_{mi}(s)}{c_{0i}}\left[G_i(x)D(I-\sigma)(v - v^*) + G_i(x)\left(d - \widehat{d}\right) + c_{0i}r_i + u_{ci}\right]. \quad (46)$$

Remark 7. For the tracking error dynamics (45) and output dynamic (46), one may discover that the parameter estimation error items $d - \widehat{d}$ and the controller parameter errors $v - v^*$ can be regarded as the disturbance input and plays an important role in system tracking performance. Hence, the attenuation of the disturbance made by $G_i(x)D(I-\sigma)(v - v^*) + G_i(x)(d - \widehat{d})$ on the closed-loop system is designed by proposing the compensator u_{ci} as an H_∞ optimal controller.

Equation (45) is considered as a special form of (23). With (45) and Lemma 3, a transient performance compensator u_{ci} is designed for the system (45) by the following steps as shown next.

Step 1: initialize $\varepsilon_i > 0$ and $0 < \gamma_i < \|W_{mi}(s)/c_{0i}\|_\infty$, choose two positive-definite matrices $S_{ci} \in \mathfrak{R}^{2\times2}$ and $R_{ci} \in \mathfrak{R}^{1\times1}$, and solve the following Riccati equation

$$y_i = \frac{W_{mi}(s)}{c_{0i}}\left[G_i(x)D(I-\sigma)(v - v^*) + G_i(x)\left(d - \widehat{d}\right) + c_{0i}r_i + u_{ci}\right]. \quad (47)$$

and obtain the state feedback gain

$$y_i = \frac{W_{mi}(s)}{c_{0i}}\left[G_i(x)D(I-\sigma)(v - v^*) + G_i(x)\left(d - \widehat{d}\right) + c_{0i}r_i + u_{ci}\right]. \quad (48)$$

If there is no solution, decrease ε_i and repeat this step again until getting an appropriate positive-definite solution $P_{ci} = P_{ci}^T > 0$ to ensure that the state feedback gain is obtained. The H_∞ compensator which is based on measurement feedback

$$u_{ci} = K_{ci} e_i, \tag{49}$$

is constructed by state feedback gain K_{ci}, which guarantees that the system (45) with $G_i(x)D(I-\sigma)(v-v^*) + G_i(x)(d - \widehat{d})$ disturbance attenuation γ_i is stable.

Step 2: Return to step 1 and reduce the transient performance index until getting a gratifying transient or the optimum γ_i

Remark 8. The standard algorithm can be used to confirm whether the positive-definite solution of a Riccati equation exists. According to the above processes, the optimal compensator of H_∞ can be obtained by selecting sufficiently small ε_i. However, it may cause the gain K_{ci} too large. For the whole control system, the large gain will reduce the stability margin and increase the influence of measurement noise. In practical application, a compromise is usually adopted in the H_∞ compensator design. It turns out that the suboptimum H_∞ compensator for a given $0 < \gamma_i < \| W_{mi}(s)/c_{0i} \|_\infty$ can also achieve a satisfactory transient performance through the following theoretical analysis and simulation.

3.2. Nominal Compensation Design. As mentioned previously, the spacecraft system is turned into three-linear subsystem (18) derived from the feedback linearization technique. To derive the failure compensation control law $u(t)$, we write the three subsystems together as

$$[\ddot{y}_1, \ddot{y}_2, \ddot{y}_3]^T = F(x) + G_D(x)[(I-\sigma)v + \sigma\bar{u}] + G(x)d(t), \tag{50}$$

where $G_D(x) = G(x)D \in R^{3\times4}$ is regarded as control distribution matrix.

3.2.1. Design for no Failure Case. On this condition, $\sigma(t) = \sigma_{(1)} = \text{diag}\{0,0,0,0\}$, $u(t) = v(t)$, and $G_D(x)v(t) = W_d(t)$. The signal $v(t)$ is designed as

$$v(t) = v^*_{(1)}(t) = h_{21}(x) v^*_{0(1)}(t) \tag{51}$$

for a chosen $h_{21}(x) \in R^{4\times4}$, and signal $v^*_{0(1)}(t)$ to be calculated from

$$G_D(x) h_{21}(x) v^*_{0(1)}(t) = W_d(t). \tag{52}$$

The solution $v^*_{0(1)}(t)$ may be derived as

$$v^*_{0(1)}(t) = K_{21}(x) W_d(t), \tag{53}$$

with matrix function $K_{21}(x) \in R^{4\times3}$.

3.2.2. Design for the u_1 Failure Case. In this situation, $\sigma(t) = \sigma_{(2)} = \text{diag}\{1,0,0,0\}$, $u_1 = \bar{u}_1$, and $u_i = v_i$ for $i = 2,3,4$, with $G_D(x) = [G_{D1}, G_{D2}, G_{D3}, G_{D4}] = [G_{D1}, G_{D(2)}] \in R^{3\times4}$ for $G_{D(2)} = [G_{D2}, G_{D3}, G_{D4}] \in R^{3\times3}$, $v = [v_1, v_2, v_3, v_4]^T = [v_1, v^T_{a(2)}]^T \in R^4$ for $v_{a(2)} = [v_2, v_3, v_4]^T \in R^3$, equation $G_D(x) u(t) = W_d(t)$ becomes

$$G_{D1}\bar{u}_1(t) + G_{D(2)} v_{a(2)}(t) = W_d(t). \tag{54}$$

In this situation, the signal v_1 is set to be $v_1 = 0$. A nonsingular matrix function $h_{22}(x) \in R^{3\times3}$ is chosen to set

$$\begin{gathered} v(t) = \left[v_1(t), v^T_{a(2)}(t)\right]^T = v^*_{(2)}(t) = \left[0, v^{*T}_{a(2)}(t)\right]^T, \\ v^*_{a(2)}(t) = h_{22}(x) v^*_{0(2)}(t), \end{gathered} \tag{55}$$

with $v^*_{0(2)}(t) \in R^3$ to be deduced from

$$G_{D1}\bar{u}_1(t) + G_{D(2)} h_{22}(x) v^*_{0(2)}(t) = W_d(t). \tag{56}$$

We can obtain

$$v^*_{0(2)}(t) = K_{22}(x) W_d(t) + K_{221}(x)\bar{u}_1(t), \tag{57}$$

with matrix function $K_{22}(x) \in R^{3\times3}$ and vector $K_{221}(x) \in R^{3\times1}$.

3.2.3. Design for the u_4 Failure Case. Similarly, $\sigma(t) = \sigma_{(3)} = \text{diag}\{0,0,0,1\}$, $u_4 = \bar{u}_4$, and $v_4(t)$ are chosen as $v_4(t) = 0$ and $v_i(t) = u_i(t)$ for $i = 1,2,3$. With $G_D(x) = [G_{D1}, G_{D2}, G_{D3}, G_{D4}] = [G_{D(1)}, G_{D4}] \in R^{3\times4}$ for $G_{D(1)} = [G_{D1}, G_{D2}, G_{D3}] \in R^{3\times3}$ and $v(t) = [v^T_{a(3)}(t), v_4(t)]^T$ for $v_{a(3)}(t) = [v_1(t), v_2(t), v_3(t)]^T \in R^3$, and $G_D(x)u(t) = W_d(t)$ becomes

$$G_{D(1)} v_{a(3)}(t) + G_{D4}\bar{u}_4(t) = W_d(t). \tag{58}$$

The signal $v(t)$ is proposed as

$$\begin{gathered} v(t) = \left[v^T_{a(3)}(t), v_4(t)\right]^T = v^*_{(3)}(t) = \left[v^{*T}_{a(3)}(t), 0\right]^T, \\ v^*_{a(3)}(t) = h_{23} v^*_{0(3)}(t), \end{gathered} \tag{59}$$

with a chosen matrix $h_{23} \in R^{3\times3}$ and a deduced signal $v^*_{0(3)}(t) \in R^3$ from

$$G_{D(1)} h_{23} v^*_{0(3)}(t) + G_{D4}\bar{u}_4(t) = W_d(t). \tag{60}$$

Similarly, we have

$$v^*_{0(3)}(t) = K_{23} W_d(t) + K_{234}\bar{u}_4(t), \tag{61}$$

with matrix function $K_{23} \in R^{3\times3}$ and vector $K_{234} \in R^{3\times1}$.

3.2.4. Composite Control Law. Define three indicator functions χ^*_j, $j = 1,2,3$, which are corresponding to the considered three failure models $\sigma_{(j)}$ and $j = 1,2,3$, respectively.

That is, if $\sigma = \sigma_{(j)}$ and $\chi_j^* = 1$; otherwise, $\chi_j^* = 0$ and $j = 1, 2, 3$. Then, a synthetic control law is obtained by integrating the three individual controllers

$$v^*(t) = \sum_{j=1}^{3} \chi_j^*(t) v_{(j)}^*(t), \tag{62}$$

to handle the three failure cases.

For $v_{a(2)}^*(t) = h_{22} v_{0(2)}^*(t)$, with $v_{0(2)}^*(t)$ in (57), signal $v_{a(2)}^*(t)$ can be further expressed as

$$v_{a(2)}^*(t) = h_{22} K_{22} W_d(t) + h_{22} K_{221} \bar{u}_1(t) \in R^3. \tag{63}$$

For $\bar{u}_1(t)$ in (10), we express

$$\bar{u}_1(t) = \theta_1^{*T} \varpi_1(t) = \varpi_1^T(t) \theta_1^*, \tag{64}$$

where $\theta_1^* = [\bar{u}_{10}, \bar{u}_{11}, \cdots, \bar{u}_{1q_1}]^T \in R^{q_1+1}$, and $\varpi_1(t) = [1, f_{11}(t), \cdots, f_{1q_1}(t)]^T \in R^{q_1+1}$.

Therefore, we have

$$\begin{aligned} \chi_2^*(t) v_{a(2)}^*(t) = {} & \{\chi_{21}^*(t), \chi_{22}^*(t), \chi_{23}^*(t)\} h_{22} K_{22} W_d(t) \\ & + \left[\theta_{1(1)}^{*T}(t)\varpi_1(t)\phi_{2,1}, \theta_{1(2)}^{*T}(t)\varpi_1(t)\phi_{22}, \theta_{1(3)}^{*T}(t)\varpi_1(t)\phi_{23}\right]^T, \end{aligned} \tag{65}$$

where $\chi_{2i}^*(t) = \chi_2^*(t)$, $\theta_{1(i)}^*(t) = \chi_2^*(t)\theta_1^*$, $i = 1, 2, 3$, and $\phi_2 = h_{22} K_{221} = [\phi_{21}, \phi_{22}, \phi_{23}]^T$.

In the same way, to designate $v_{\chi_1(1)}^*(t)$ and $v_{\chi_3(3)}^*(t)$, we express

$$\chi_1^*(t) v_{(1)}^*(t) = \operatorname{diag}\{\chi_{11}^*(t), \chi_{12}^*(t), \chi_{13}^*(t), \chi_{14}^*(t)\} h_{21} K_{21} W_d(t) \tag{66}$$

$$\begin{aligned} \chi_3^*(t) v_{a(3)}^*(t) = {} & \operatorname{diag}\{\chi_{31}^*(t), \chi_{32}^*(t), \chi_{33}^*(t)\} h_{23} K_{23} W_d(t) \\ & + \left[\theta_{4(1)}^{*T}(t)\varpi_4(t)\phi_{31}, \theta_{4(2)}^{*T}(t)\varpi_4(t)\phi_{32}, \theta_{4(3)}^{*T}(t)\varpi_4(t)\phi_{33}\right]^T, \end{aligned} \tag{67}$$

where $\chi_{1i}^*(t) = \chi_1^*(t)$, $i = 1, 2, 3, 4$, $\chi_{3i}^*(t) = \chi_3^*(t)$, and $\theta_{4(i)}^*(t) = \chi_3^*(t)\theta_4^*$, $i = 1, 2, 3$.

3.3. Adaptive Fault Tolerant Control Design. The adaptive version of the nominal control law is as follows (62):

$$\begin{aligned} v(t) = {} & \sum_{j=1}^{3} \chi_j(t) v_{(j)}(t) = \sum_{j=1}^{3} v_{\chi_j(j)}(t) = v_{\chi_1(1)}(t) \\ & + \left[0, v_{\chi_2 a(2)}^T(t)\right]^T + \left[v_{\chi_3 a(3)}^T(t), 0\right]^T. \end{aligned} \tag{68}$$

In view of (65)–(67), we derive

$$v_{\chi_1(1)}(t) \triangleq \operatorname{diag}\{\chi_{11}(t), \chi_{12}(t), \chi_{13}(t), \chi_{14}(t)\} h_{21} K_{21} W_d(t), \tag{69}$$

$$\begin{aligned} v_{\chi_2 a(2)}(t) \triangleq {} & \operatorname{diag}\{\chi_{21}(t), \chi_{22}(t), \chi_{23}(t)\} h_{22} K_{22} W_d(t) \\ & + \left[\theta_{1(1)}^T(t)\varpi_1(t)\phi_{21}, \theta_{1(2)}^T(t)\varpi_1(t)\phi_{22}, \theta_{1(3)}^T(t)\varpi_1(t)\phi_{23}\right]^T, \end{aligned} \tag{70}$$

$$\begin{aligned} v_{\chi_3 a(3)}(t) \triangleq {} & \operatorname{diag}\{\chi_{31}(t), \chi_{32}(t), \chi_{33}(t)\} h_{23} K_{23} W_d(t) \\ & + \left[\theta_{4(1)}^T(t)\varpi_4(t)\phi_{31}, \theta_{4(2)}^T(t)\varpi_4(t)\phi_{32}, \theta_{4(3)}^T(t)\varpi_4(t)\phi_{33}\right]^T, \end{aligned} \tag{71}$$

where $\chi_{ji}(t)$, $\theta_{1(i)}(t)$, and $\theta_{4(i)}(t)$ are the estimates of $\chi_{ji}^*(t)$, $\theta_{1(i)}^*(t)$, and $\theta_{4(i)}^*(t)$, respectively.

From (65)–(71), we obtain

$$v(t) - v^*(t) = \tilde{v}_{\chi_1(1)}(t) + \left[0, \tilde{v}_{\chi_2 a(2)}^T(t)\right]^T + \left[\tilde{v}_{\chi_3 a(3)}^T(t), 0\right]^T, \tag{72}$$

where

$$\begin{aligned} \tilde{v}_{\chi_1(1)}(t) = {} & \{\tilde{\chi}_{11}(t), \tilde{\chi}_{12}(t), \tilde{\chi}_{13}(t), \tilde{\chi}_{14}(t)\} h_{21} K_{21} W_d(t), \\ \tilde{v}_{\chi_2 a(2)}(t) = {} & \operatorname{diag}\{\tilde{\chi}_{21}(t), \tilde{\chi}_{22}(t), \tilde{\chi}_{23}(t)\} h_{22} K_{22} W_d(t) \\ & + \left[\tilde{\theta}_{1(1)}^T(t)\varpi_1(t)\phi_{21}, \tilde{\theta}_{1(2)}^T(t)\varpi_1(t)\phi_{22}, \tilde{\theta}_{1(3)}^T(t)\varpi_1(t)\phi_{23}\right]^T, \tilde{v}_{\chi_3 a(3)}(t) \triangleq \operatorname{diag}\{\tilde{\chi}_{31}(t), \tilde{\chi}_{32}(t), \tilde{\chi}_{33}(t)\} h_{23} K_{23} W_d(t) \\ & + \left[\tilde{\theta}_{4(1)}^T(t)\varpi_4(t)\phi_{31}, \tilde{\theta}_{4(2)}^T(t)\varpi_4(t)\phi_{32}, \tilde{\theta}_{4(3)}^T(t)\varpi_4(t)\phi_{33}\right]^T. \end{aligned} \tag{73}$$

3.3.1. State Error Equations. Introducing $e \in R^6 = [e_1^T, e_2^T, e_3^T]^T = [e_{11}, e_{12}, e_{21}, e_{22}, e_{31}, e_{32}]^T$, $e_{i1} = y_i - y_{mi} = \xi_{i1} - \xi_{mi1}$, $e_{i2} = \dot{\xi}_{i1} - \dot{\xi}_{mi1} = \dot{e}_{i1}$, and $i = 1, 2, 3$. With the three subsystems

$$\begin{aligned}\dot{e}_i = A_{mi}e_i + \frac{B_{mi}}{c_{0i}}[G_i(x)D(I-\sigma)(\nu-\nu^*) \\ - (1/2\varepsilon_i)R_{ci}^{-1}B_{ci}^T P_{ci} e_i + G_i(x)\left(d - \widehat{d}\right)\Big], i = 1, 2, 3,\end{aligned} \tag{74}$$

we have

$$\begin{aligned}\begin{bmatrix}\ddot{e}_{11} + \alpha_{12}\dot{e}_{11} + \alpha_{11}e_{11} \\ \ddot{e}_{21} + \alpha_{22}\dot{e}_{21} + \alpha_{21}e_{21} \\ \ddot{e}_{31} + \alpha_{32}\dot{e}_{31} + \alpha_{31}e_{31}\end{bmatrix} = \frac{1}{c_0}\left[G(x)D(I-\sigma)(\nu-\nu^*) + G(x)\left(d-\widehat{d}\right)\right] \\ - \begin{bmatrix}(1/2\varepsilon_1 c_0)R_{c1}^{-1}B_{c1}^T P_{c1} e_1 \\ (1/2\varepsilon_2 c_0)R_{c2}^{-1}B_{c2}^T P_{c2} e_2 \\ (1/2\varepsilon_3 c_0)R_{c3}^{-1}B_{c3}^T P_{c3} e_3\end{bmatrix}.\end{aligned} \tag{75}$$

If $\sigma = \sigma_{(1)} = \text{diag}\,\{0, 0, 0, 0\}$, with (72) and (73), we rewrite (75) as

$$\begin{aligned}\begin{bmatrix}\ddot{e}_{11} + \alpha_{12}\dot{e}_{11} + \alpha_{11}e_{11} \\ \ddot{e}_{21} + \alpha_{22}\dot{e}_{21} + \alpha_{21}e_{21} \\ \ddot{e}_{31} + \alpha_{32}\dot{e}_{31} + \alpha_{31}e_{31}\end{bmatrix} = \frac{1}{c_0}\left[\sum_{i=1}^{4} G_{Di}\tilde{\chi}_{1i}\nu_{1i} + \sum_{i=1}^{3} G_{D(i+1)}\tilde{\chi}_{2i}\nu_{2i}\right. \\ \left. + \sum_{i=1}^{3} G_{Di}\tilde{\chi}_{3i}\nu_{3i} + \sum_{i=1}^{3} G_{D(i+1)}\widetilde{\theta}_{1(i)}^T\varpi_1\phi_{2i} + \sum_{i=1}^{3} G_{Di}\widetilde{\theta}_{4(i)}^T\varpi_4\phi_{3i}\right] \\ + \frac{1}{c_0}\begin{bmatrix}\sum_{j=1}^{3} G_{1j}\widetilde{\theta}_{dj}^T(t)\varpi_{dj} \\ \sum_{j=1}^{3} G_{2j}\widetilde{\theta}_{dj}^T(t)\varpi_{dj} \\ \sum_{j=1}^{3} G_{3j}\widetilde{\theta}_{dj}^T(t)\varpi_{dj}\end{bmatrix} - \begin{bmatrix}\left(\frac{1}{2\varepsilon_1 c_0}\right)R_{c1}^{-1}B_{c1}^T P_{c1} e_1 \\ \left(\frac{1}{2\varepsilon_2 c_0}\right)R_{c2}^{-1}B_{c2}^T P_{c2} e_2 \\ \left(\frac{1}{2\varepsilon_3 c_0}\right)R_{c3}^{-1}B_{c3}^T P_{c3} e_3\end{bmatrix} \triangleq \tilde{E}_1.\end{aligned} \tag{76}$$

If $\sigma = \sigma_{(2)} = \text{diag}\,\{1, 0, 0, 0\}$, with (72) and (73), we rewrite (75) as

$$\begin{aligned}\begin{bmatrix}\ddot{e}_{11} + \alpha_{12}\dot{e}_{11} + \alpha_{11}e_{11} \\ \ddot{e}_{21} + \alpha_{22}\dot{e}_{21} + \alpha_{21}e_{21} \\ \ddot{e}_{31} + \alpha_{32}\dot{e}_{31} + \alpha_{31}e_{31}\end{bmatrix} = \frac{1}{c_0}\left[\sum_{i=2}^{4} G_{Di}\tilde{\chi}_{1i}\nu_{1i} + \sum_{i=1}^{3} G_{D(i+1)}\tilde{\chi}_{2i}\nu_{2i}\right. \\ \left. + \sum_{i=2}^{3} G_{Di}\tilde{\chi}_{3i}\nu_{3i} + \sum_{i=1}^{3} G_{D(i+1)}\widetilde{\theta}_{1(i)}^T\varpi_1\phi_{2i} + \sum_{i=2}^{3} G_{Di}\widetilde{\theta}_{4(i)}^T\varpi_4\phi_{3i}\right] \\ + \frac{1}{c_0}\begin{bmatrix}\sum_{j=1}^{3} G_{1j}\widetilde{\theta}_{dj}^T(t)\varpi_{dj} \\ \sum_{j=1}^{3} G_{2j}\widetilde{\theta}_{dj}^T(t)\varpi_{dj} \\ \sum_{j=1}^{3} G_{3j}\widetilde{\theta}_{dj}^T(t)\varpi_{dj}\end{bmatrix} - \begin{bmatrix}\left(\frac{1}{2\varepsilon_1 c_0}\right)R_{c1}^{-1}B_{c1}^T P_{c1} e_1 \\ \left(\frac{1}{2\varepsilon_2 c_0}\right)R_{c2}^{-1}B_{c2}^T P_{c2} e_2 \\ \left(\frac{1}{2\varepsilon_3 c_0}\right)R_{c3}^{-1}B_{c3}^T P_{c3} e_3\end{bmatrix} \triangleq \tilde{E}_2\end{aligned} \tag{77}$$

If $\sigma = \sigma_{(3)} = \text{diag}\,\{0, 0, 0, 1\}$, with (72) and (73), we rewrite (75) as

$$\begin{aligned}\begin{bmatrix}\ddot{e}_{11} + \alpha_{12}\dot{e}_{11} + \alpha_{11}e_{11} \\ \ddot{e}_{21} + \alpha_{22}\dot{e}_{21} + \alpha_{21}e_{21} \\ \ddot{e}_{31} + \alpha_{32}\dot{e}_{31} + \alpha_{31}e_{31}\end{bmatrix} = \frac{1}{c_0}\left[\sum_{i=1}^{3} G_{Di}\tilde{\chi}_{1i}\nu_{1i} + \sum_{i=1}^{2} G_{D(i+1)}\tilde{\chi}_{2i}\nu_{2i}\right. \\ \left. + \sum_{i=1}^{3} G_{Di}\tilde{\chi}_{3i}\nu_{3i} + \sum_{i=1}^{2} G_{D(i+1)}\widetilde{\theta}_{1(i)}^T\varpi_1\phi_{2i} + \sum_{i=1}^{3} G_{Di}\widetilde{\theta}_{4(i)}^T\varpi_4\phi_{3i}\right] \\ + \frac{1}{c_0}\begin{bmatrix}\sum_{j=1}^{3} G_{1j}\widetilde{\theta}_{dj}^T(t)\varpi_{dj} \\ \sum_{j=1}^{3} G_{2j}\widetilde{\theta}_{dj}^T(t)\varpi_{dj} \\ \sum_{j=1}^{3} G_{3j}\widetilde{\theta}_{dj}^T(t)\varpi_{dj}\end{bmatrix} - \begin{bmatrix}\left(\frac{1}{2\varepsilon_1 c_0}\right)R_{c1}^{-1}B_{c1}^T P_{c1} e_1 \\ \left(\frac{1}{2\varepsilon_2 c_0}\right)R_{c2}^{-1}B_{c2}^T P_{c2} e_2 \\ \left(\frac{1}{2\varepsilon_3 c_0}\right)R_{c3}^{-1}B_{c3}^T P_{c3} e_3\end{bmatrix} \triangleq \tilde{E}_3\end{aligned} \tag{78}$$

where $G_D = [G_{D1}, G_{D2}, G_{D3}, G_{D4}]$, $\nu_1 = h_{21}K_{21}W_d = [\nu_{11}, \nu_{12}, \nu_{13}, \nu_{14}]^T$, $\nu_2 = h_{22}K_{22}W_d = [\nu_{21}, \nu_{22}, \nu_{23}]^T$, and $\nu_3 = h_{23}K_{23}W_d = [\nu_{31}, \nu_{32}, \nu_{33}]^T$, $\widetilde{\theta}_{dj} = \theta_{dj} - \widehat{\theta}_{dj}$.

From Equations (76)–(78), we can obtain the state error equation

$$\dot{e} = A_m e + B_m \tilde{E}_k = A_m e + B_{m\tilde{E}_k}, \tag{79}$$

where $A_m = \text{diag}\,\{A_{m1}, A_{m2}, A_{m3}\} \in R^{6\times 6}$ and E_{kj} is the j th component of $\tilde{E}_k$, $k = 1, 2, 3$, $B_{m\tilde{E}_k} =$

$[B_{m1}^T\tilde{E}_{k1}, B_{m2}^T\tilde{E}_{k2}, B_{m3}^T\tilde{E}_{k3}]^T \in R^6$, and $B_m = \text{diag}\{B_{m1}, B_{m2}, B_{m3}\} \in R^{6\times3}$.

3.3.2. Adaptive Laws. According to the state error Equation (79), adaptive laws are chosen for the parameter $\hat{\theta}_{di}$ and the parameters $\chi_{1i}(t)$, $\chi_{2i}(t)$, $\chi_{3i}(t)$, $\theta_{1(i)}(t)$, and $\theta_{2(i)}(t)$ of the failure compensator as

$$\dot{\hat{\theta}}_{di} = \frac{1}{c_0}\sum_{j=1}^{3}\Gamma_{di}\varpi_{di}e_{Pj}G_{ji}, \tag{80}$$

$$\dot{\chi}_{1i}(t) = \begin{cases} -\dfrac{\gamma_{1i}}{c_0}\sum_{j=1}^{3}e_{Pj}G_{Di}\nu_{1i}\; i=2,3, \\ -\dfrac{\gamma_{1i}}{c_0}\sum_{j=1}^{3}e_{Pj}G_{Di}\nu_{1i} + f_{\chi_{1i}}\; i=1,4, \end{cases} \tag{81}$$

$$\dot{\chi}_{2i}(t) = \begin{cases} -\dfrac{\gamma_{2i}}{c_0}\sum_{j=1}^{3}e_{Pj}G_{D(i+1)}\nu_{2i}\; i=1,2, \\ \dfrac{\gamma_{2i}}{c_0}\sum_{j=1}^{3}e_{Pj}G_{D(i+1)}\nu_{2i} + f_{\chi_{2i}}\; i=3, \end{cases} \tag{82}$$

$$\dot{\chi}_{3i}(t) = \begin{cases} -\dfrac{\gamma_{3i}}{c_0}\sum_{j=1}^{3}e_{Pj}G_{Di}\nu_{3i}\; i=2,3, \\ -\dfrac{\gamma_{3i}}{c_0}\sum_{j=1}^{3}e_{Pj}G_{Di}\nu_{3i} + f_{\chi_{3i}}\; i=1, \end{cases} \tag{83}$$

$$\dot{\theta}_{1(t)}(t) = \begin{cases} -\dfrac{1}{c_0}\sum_{j=1}^{3}\Gamma_{1i}e_{Pj}G_{D(i+1)}\varpi_1\phi_{2i}\; i=1,2, \\ -\dfrac{1}{c_0}\sum_{j=1}^{3}\Gamma_{1i}e_{Pj}G_{D(i+1)}\varpi_1\phi_{2i} + f_{\theta_{1(i)}}\; i=3, \end{cases} \tag{84}$$

$$\dot{\theta}_{4(t)}(t) = \begin{cases} -\dfrac{1}{c_0}\sum_{j=1}^{3}\Gamma_{4i}e_{Pj}G_{Di}\varpi_1\phi_{2i}\; i=2,3, \\ -\dfrac{1}{c_0}\sum_{j=1}^{3}\Gamma_{4i}e_{Pj}G_{Di}\varpi_1\phi_{2i} + f_{\theta_{4(i)}}\; i=1, \end{cases} \tag{85}$$

where $\Gamma_{di} = \Gamma_{di}^T > 0$, $\Gamma_{1i} = \Gamma_{1i}^T > 0$, $\Gamma_{4i} = \Gamma_{4i}^T > 0$, $\gamma_{1i} > 0$, $\gamma_{2i} > 0$, and $\gamma_{3i} > 0$ are the adaptive gains, and $f_{\chi_{1i}}$ is the projection algorithm. Consequently, based on adaptive laws $\dot{\chi}_{11} = -(\gamma_{11}/c_0)\sum_{j=1}^{3}e_{Pj}G_{D1}\nu_{11} + f_{\chi_{11}}$, we can derive that $0 \le \chi_{11} \le 1$ and $(\chi_{11} - \chi_{11}^*)f_{\chi_{11}} \le 0$. $f_{\chi_{2i}}$, $f_{\chi_{3i}}$, $f_{\theta_{1(i)}}$, and $f_{\theta_{4(i)}}$ have the same characteristics with $f_{\chi_{1i}}$.

3.3.3. Stability Performance Analysis

(i) For period $t \in [T_0, \infty)$, $\sigma = \sigma_{(1)}$. Lyapunov function is defined as

$$V_0 = \frac{1}{2}e^TPe + \frac{1}{2}\sum_{i=1}^{3}\tilde{\theta}_{di}^T\Gamma_{di}^{-1}\tilde{\theta}_{di} + \frac{1}{2}\left[\sum_{i=1}^{4}\tilde{\chi}_{1i}^2\gamma_{1i}^{-1} + \sum_{i=1}^{3}\tilde{\chi}_{2i}^2\gamma_{2i}^{-1} + \sum_{i=1}^{3}\tilde{\chi}_{3i}^2\gamma_{3i}^{-1} + \sum_{i=1}^{3}\tilde{\theta}_{1(i)}^T\Gamma_{1i}^{-1}\tilde{\theta}_{1(i)} + \sum_{i=1}^{3}\tilde{\theta}_{4(i)}^T\Gamma_{4i}^{-1}\tilde{\theta}_{4(i)}\right]. \tag{86}$$

By differentiating V_0 in the interval $[T_0, \infty)$, we can obtain

$$\dot{V}_0 = \frac{1}{2}e^TA_m^TPe + \frac{1}{2}e^TPA_me + \frac{1}{c_0}\left[\sum_{j=1}^{3}\sum_{i=1}^{4}e_{Pj}G_{Di}\tilde{\chi}_{1i}\nu_{1i} + \sum_{j=1}^{3}\sum_{i=1}^{2}e_{Pj}G_{D(i+1)}\tilde{\chi}_{2i}\nu_{2i} + \sum_{i=1}^{4}\gamma_{1i}^{-1}\tilde{\chi}_{1i}\dot{\tilde{\chi}}_{1i} + \sum_{i=1}^{3}\gamma_{2i}^{-1}\tilde{\chi}_{2i}\dot{\tilde{\chi}}_{2i} + \sum_{i=1}^{3}\gamma_{3i}^{-1}\tilde{\chi}_{3i}\dot{\tilde{\chi}}_{3i} + \sum_{i=1}^{3}\tilde{\theta}_{1(i)}^T\Gamma_{1i}^{-1}\dot{\tilde{\theta}}_{1(i)} + \sum_{i=1}^{3}\tilde{\theta}_{4(i)}^T\Gamma_{4i}^{-1}\dot{\tilde{\theta}}_{4(i)}\right., \tag{87}$$

where $e_P = [e_{P1}, e_{P2}, e_{P3}] \in R^{1\times3}$ and e_{Pi} are the $(i+1)$-th column components of $e^TP \in R^{1\times6}$, $i = 1, 2, 3$, and $P = \text{diag}\{P_{c1}, P_{c2}, P_{c3}\}$.

Substituting Equations (80)–(85) into (87), one would have

$$\dot{V}_0 = \frac{1}{2}\sum_{i=1}^{3}e_i^T\left(A_{mi}^TP_{ci} + P_{ci}A_{mi} - \frac{1}{\varepsilon_i}P_{ci}B_{mi}R_{ci}^{-1}B_{mi}^TP_{ci}\right)e_i, \quad t \in [T_0, T_1). \tag{88}$$

Based on of Lemma 3, we can obtain

$$\dot{V}_0 = -\frac{1}{2}\sum_{i=1}^{3}e_i^T\left(\gamma_i^{-1}C_{mi}^TC_{mi} + \varepsilon_iS_i\right)e_i \le 0. \tag{89}$$

(ii) If actuator u_1 fails over the period (T_1, ∞), i.e., $\sigma = \sigma_{(2)}$, we define

$$V_1 = \frac{1}{2}e^TPe + \frac{1}{2}\sum_{i=1}^{3}\tilde{\theta}_{di}^T\Gamma_{di}^{-1}\tilde{\theta}_{di} + \frac{1}{2}\left[\sum_{i=2}^{4}\tilde{\chi}_{1i}^2\gamma_{1i}^{-1} + \sum_{i=1}^{3}\tilde{\chi}_{2i}^2\gamma_{2i}^{-1} + \sum_{i=2}^{3}\tilde{\chi}_{3i}^2\gamma_{3i}^{-1} + \sum_{i=1}^{3}\tilde{\theta}_{1(i)}^T\Gamma_{1i}^{-1}\tilde{\theta}_{1(i)} + \sum_{i=2}^{3}\tilde{\theta}_{4(i)}^T\Gamma_{4i}^{-1}\tilde{\theta}_{4(i)}\right]. \tag{90}$$

By differentiating V_1 in the interval (T_1, ∞) and combining Equations (80)–(84), we can obtain

$$\dot{V}_1 = -\frac{1}{2}\sum_{i=1}^{3} e_i^T \left(\gamma_i^{-1} C_{mi}^T C_{mi} + \varepsilon_i S_i\right) e_i \le 0. \quad (91)$$

(iii) Assume that u_1 is normal, and only u_4 fails at T_1 and remains failed on the interval (T_1, ∞); that is, $\sigma = \sigma_{(3)}$. We define

$$V_2 = \frac{1}{2} e^T P e + \frac{1}{2}\sum_{i=1}^{3} \tilde{\theta}_{di}^T \Gamma_{di}^{-1} \tilde{\theta}_{di} + \frac{1}{2}\left[\sum_{i=1}^{3} \tilde{\chi}_{1i}^2 \gamma_{1i}^{-1} + \sum_{i=1}^{2} \tilde{\chi}_{2i}^2 \gamma_{2i}^{-1} + \sum_{i=1}^{3} \tilde{\chi}_{3i}^2 \gamma_{3i}^{-1} + \sum_{i=1}^{2} \tilde{\theta}_{1(i)}^T \Gamma_{1i}^{-1} \tilde{\theta}_{1(i)} + \sum_{i=1}^{3} \tilde{\theta}_{4(i)}^T \Gamma_{4i}^{-1} \tilde{\theta}_{4(i)}\right]. \quad (92)$$

The time derivative of V_2 is

$$\dot{V}_2 = -\frac{1}{2}\sum_{i=1}^{3} e_i^T \left(\gamma_i^{-1} C_{mi}^T C_{mi} + \varepsilon_{mi} S_i\right) e_i \le 0, \quad t \in [T_1, T_2). \quad (93)$$

With $\dot{V}_k, (k = 0, 1, 2) \le 0$ for three different failure scenarios and the adopted projection scheme of adaptive laws, we can conclude that all the signals in the close-loop system are bounded, and the output error gradually decreases to zero over time.

To sum up, the below theorem is obtained.

Theorem 9. *For the spacecraft system (1) with potential uncertain actuator faults (9) and unknown disturbances (4), controller (68) designed based on an H_∞ transient performance compensator (49), and its parameter adaptive laws (80)–(85) can ensure that the system is stable and perform the given maneuvers asymptotically, if for any failure pattern $\sigma(t)$ belongs to failure pattern set $\Sigma = \{\sigma_{(j)}, j = 1, 2, 3\}$. The following condition holds the following equivalent actuation matrix. $G_\sigma(x) = G(x)D(I - \sigma(t))$ is a full rank in the domain U (definition is $U \subset R^6 \longrightarrow V \subset R^3$).*

3.3.4. Transient Performance Analysis. Then, the transient performance is analyzed by the criteria of the bound of both L_∞ and mean square tracking error at any time.

With (45) and (49), we can get the following output tracking error dynamic equation

$$e_{i1} = \frac{W_{mi}(s)}{c_{0i}}\left[G_i(x)D(I - \sigma)(\nu - \nu^*) + G_i(x)\left(d - \hat{d}\right)\right]. \quad (94)$$

Since the order of the stable reference model $W_{mi}(s)$ is p_{mi}, it can be derived from Lemma 5 that

$$\|e_{i1}(t)\|_\infty \le 2p_{mi}\|W_{mi}(s)\|_\infty \left\|G_i(x)D(I - \sigma)(\nu - \nu^*) + G_i(x)\left(d - \hat{d}\right)\right\|. \quad (95)$$

From Theorem 9, we have $\tilde{\theta}_{di} \in L_\infty$ and $(\nu - \nu^*) \in L_\infty$. With Lemma 3, one can prove that $W_{mi}(s)$ is stable and

$$\|W_{mi}(s)\|_\infty < \gamma_i, \quad (96)$$

and thereby, we have

$$\|e_{i1}(t)\|_\infty \overset{\Delta}{=} \sup_{t \ge 0} |e_{i1}(t)| \le \gamma_i c_i, \quad (97)$$

where $c_i > 0$. Then, according to

$$\int_{t_1}^{t_2} |H(s)x|^2 dt \le \|H(s)\|_\infty^2 \int_{t_1}^{t_2} |x|^2 dt, \quad (98)$$

we have

$$\frac{1}{t}\int_{t_0}^{t_0+t} |e_{i1}|^2 d\tau \le \|W_{mi}(s)\|_\infty^2 \left(\frac{1}{t}\int_{t_0}^{t_0+t} \left|G_i(x)D(I - \sigma)(\nu - \nu^*) + G_i(x)\left(d - \hat{d}\right)\right|^2 d\tau\right) \le \gamma_i^2 c_i. \quad (99)$$

Theorem 10. *For the improved controller (34), the performance index of H_∞ compensator is γ_i, and the output tracking error $e_{i1} = y_i - y_{mi}$ and $i = 1, 2, 3$ of the system (32) satisfies the following inequality condition:*

$$\frac{1}{t}\int_{t_0}^{t_0+t} |e_{i1}|^2 d\tau \le \|W_{mi}(s)\|_\infty^2 \left(\frac{1}{t}\int_{t_0}^{t_0+t} \left|G_i(x)D(I - \sigma)(\nu - \nu^*) + G_i(x)\left(d - \hat{d}\right)\right|^2 d\tau\right) \le \gamma_i^2 c_i. \quad (100)$$

where constant $c_i > 0$.

According to Theorem 10, the transient characteristics rely on the performance level of the H_∞ compensator. Both Theorem 9 and Theorem 10 show that the control objective is reached by our proposed control scheme.

4. Simulations

MATLAB/SIMULINK software has been used to carry out numerical simulations to verify the effectiveness and performance of the proposed control scheme. The nominal moments of inertia parameters $J = \text{diag}\{40.45, 42.09, 42.36\}(kg \cdot m^2)$, orientation matrix of the reaction wheel, and the external disturbances $d(t) = [\sin(0.01t) + 1, 1.5\cos(0.01t) - 1, 2\sin(0.01t) + 1] \times 10^{-3} N \cdot m$ are taken from [36].

4.1. Simulation Conditions. The attitude values at $t = 0$ are given by $q_0(0) = 0.8834$, $q(0) = [0.03, -0.02, -0.03]^T$, and $\omega(0) = [0, 0, 0]^T$. For simulation, the initial values of indicator function and failure parameter estimates are chosen as: $\chi_{1i}(0) = 1$, $i = 1, 2, 3, 4$, $\chi_{2i}(0) = 0$, $\chi_{3i}(0) = 0$, $i = 1, 2, 3$; $\theta_{1(i)}(0) = [0, 0]^T$, and$\theta_{4(i)}(0) = [0, 0]^T$. Basis functions of failure model (10) and disturbance model (4) are $\varpi_1(t) = \varpi_4(t) = [1, \sin(0.25t)]^T \in R^2$ and $\varpi_{d1} = \varpi_{d2} = \varpi_{d3} = [1, \ \sin(0.01t), \ \cos(0.01t)]^T \in R^3$.

The design parameters are chosen as $\gamma_{1i} = 1$, $\gamma_{2i} = 1$, $\gamma_{3i} = 1$, $c_{0i} = c_0 = 1$, $\gamma_i = 1$, $R_i = 2$, $\varepsilon_i = 4$, $i = 1, 2, 3$,

$$
\begin{aligned}
K &= \begin{bmatrix} 12.2 & 3.9 \\ 8.5 & 2.7 \\ 9.1 & 2.3 \end{bmatrix}, \\
\Gamma_{di} &= \begin{bmatrix} 3.3 & 0 & 3.3 \\ 0 & 3.3 & 3.3 \\ 3.3 & 3.3 & 3.3 \end{bmatrix}, \\
\Gamma_{1i} &= \begin{bmatrix} 16.3 & 0 & 0 \\ 0 & 16.3 & 0 \\ 0 & 0 & 16.3 \end{bmatrix}, \\
\Gamma_{4i} &= \begin{bmatrix} 31.4 & 0 & 0 \\ 0 & 31.4 & 0 \\ 0 & 0 & 31.4 \end{bmatrix}, \\
P_{c1} &= \begin{bmatrix} 3.53 & 6.32 \\ 1.05 & 4.39 \end{bmatrix}, \\
P_{c2} &= \begin{bmatrix} 3.53 & 10.51 \\ 1.05 & 6.58 \end{bmatrix}, \\
P_{c3} &= \begin{bmatrix} 3.53 & 4.23 \\ 1.05 & 3.79 \end{bmatrix}.
\end{aligned} \tag{101}
$$

A second-order reference model $W_{mi}(s) = 1/(s^2 + 2s + 1)$ and the reference input signal $r_i(t) = 0$, $i = 1, 2, 3$, are chosen to generate the given command $y_m(t)$ to be tracked by system output $y(t)$.

In the numerical simulation, for comparison, three cases are conducted: (1) attitude tracking control using our proposed modified MRAC based adaptive failure compensation controller (68) (denoted as "MMRAC based FTC"); (2) attitude tracking control using the standard MRAC-based adaptive failure compensation controller (denoted as "SMRAC based FTC") without transient performance compensator; and (3) attitude tracking control using the direct adaptive failure compensation controller (denoted as "DAC-based FTC") in [35]. The attitude tracking responses are analyzed to study the performances of the controllers.

4.2. Simulation Results. To demonstrate the superior performance of the proposed control scheme, two actuator failure conditions are simulated: Case 1—intermittent fault occurring in actuator u_1—and Case 2—Alternate faults occurring in actuators u_1 and u_4.

Case 1. Intermittent fault occurring in actuator u_1. In this case, the following failure conditions are considered as follows:

(i) When $0 \leq t < 100s$, all the reaction wheels function healthily, $u_i(t) = v_i(t)$, $i = 1, 2, 3, 4$

(ii) When $100s \leq t < 200s$, actuator u_1 failed, $u_1(t) = 2\ Nm$ and $u_i(t) = v_i(t)$, $i = 2, 3, 4$

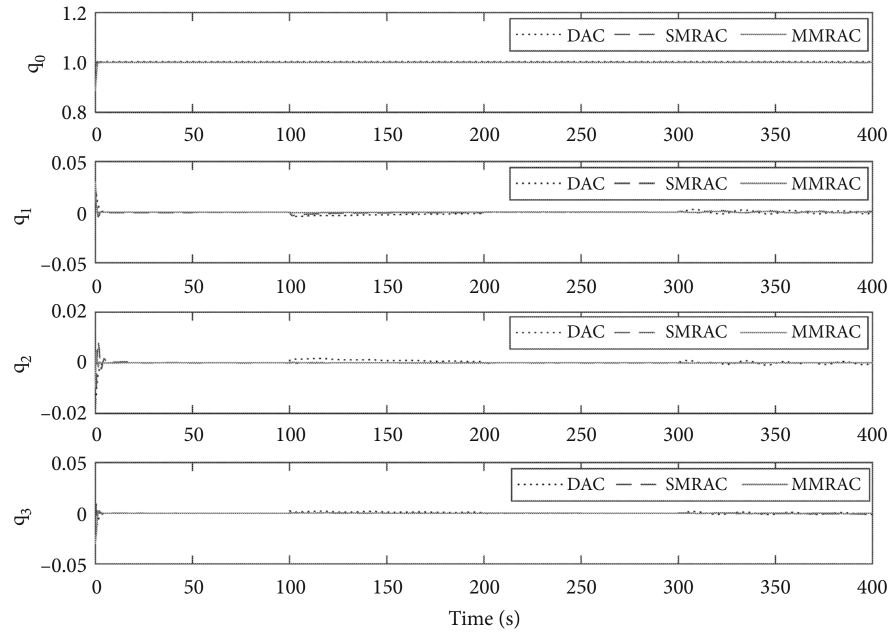
(iii) When $t \geq 200s$, actuator u_1 returns to normal, $u_i(t) = v_i(t)$, $i = 1, 2, 3, 4$

(iv) When $t \geq 300s$, actuator u_1 is out of control, $u_1(t) = 0.75 \sin(0.25t) Nm$, $u_i(t) = v_i(t)$, $i = 2, 3, 4$

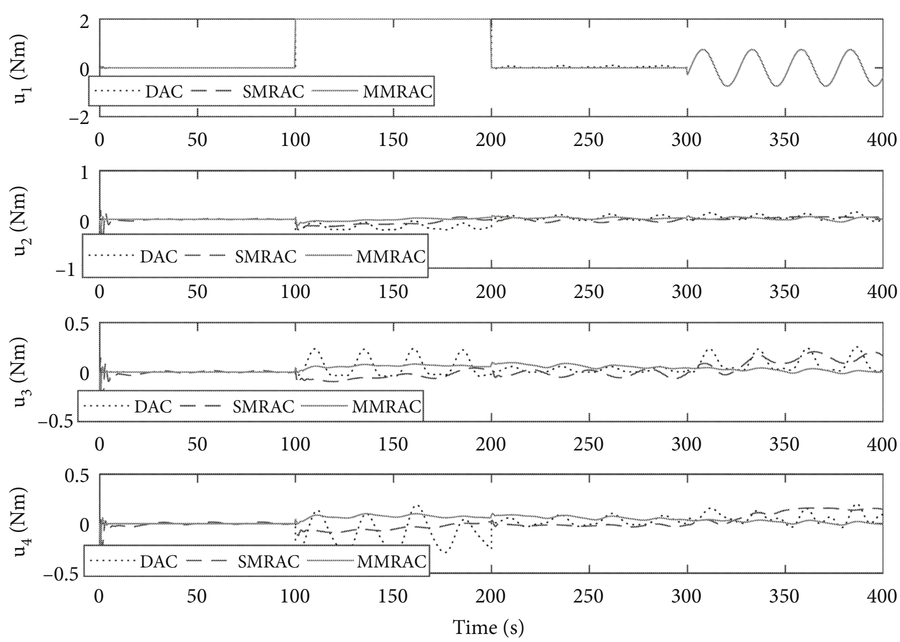
Figure 2(a) shows the simulated results obtained by including the faulty actuators for three controllers, namely the designed MMRAC based fault tolerant controller marked with solid line, the SMRAC-based fault-tolerant controller marked with dashed line, and the DAC-based fault-tolerant controller marked with dotted line. All the three fault-tolerant controllers can compensate for both the constraint and time varying faults, although the system performance degrades to some degree, the overshoot a setting time increase significantly once the failure is introduced. However, the system ultimately regulates the attitude to zero asymptotically; that is, the attitude stabilization maneuver is still performed successfully due to the fault-tolerant performance of the three controllers to uncertain actuator failures. It can be found from Figure 2(c) at the moments when actuator u_1 failed at $t = 100s$ and $t = 300s$ (shown in Figure 2(b), which is corresponding with the simulation conditions), the overshoot of the system implemented by our designed MMRAC-based fault-tolerant controller are smaller than the SMRAC- and DAC-based fault-tolerant controller even before the actuators have failed, and this is because the effect caused by parameter estimation errors of both failure and disturbances on the transient performance has been reduced by the H_∞ compensator. These simulations demonstrate the theoretical result that the desired performance of the system can be achieved by the proposed fault tolerant control even if the faults are unknown in advance.

Case 2. Alternate Faults Occurring in Actuator u_1 and u_4. In this case, the following failure conditions are considered as follows:

(i) When $0 \leq t < 100s$, all the reaction wheels function healthily, $u_i(t) = v_i(t)$, $i = 1, 2, 3, 4$

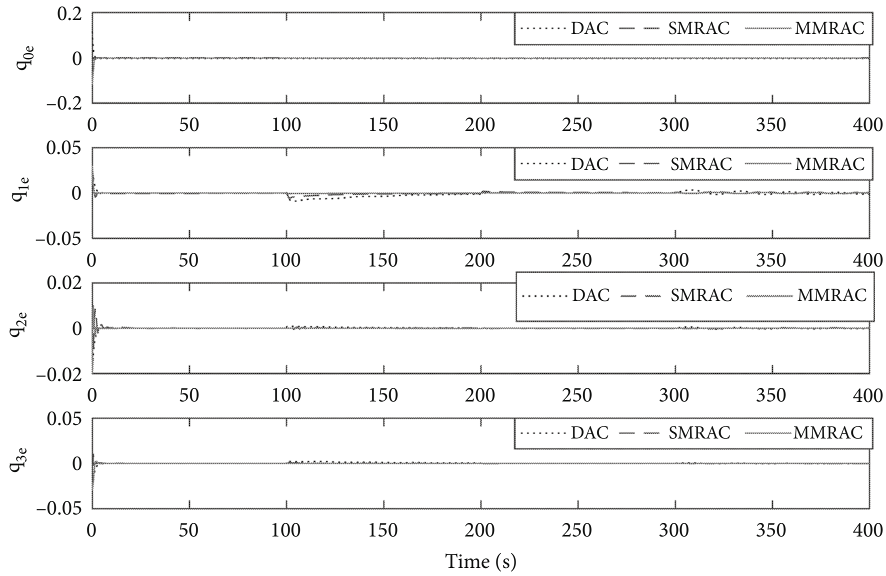


(a) Response of system output

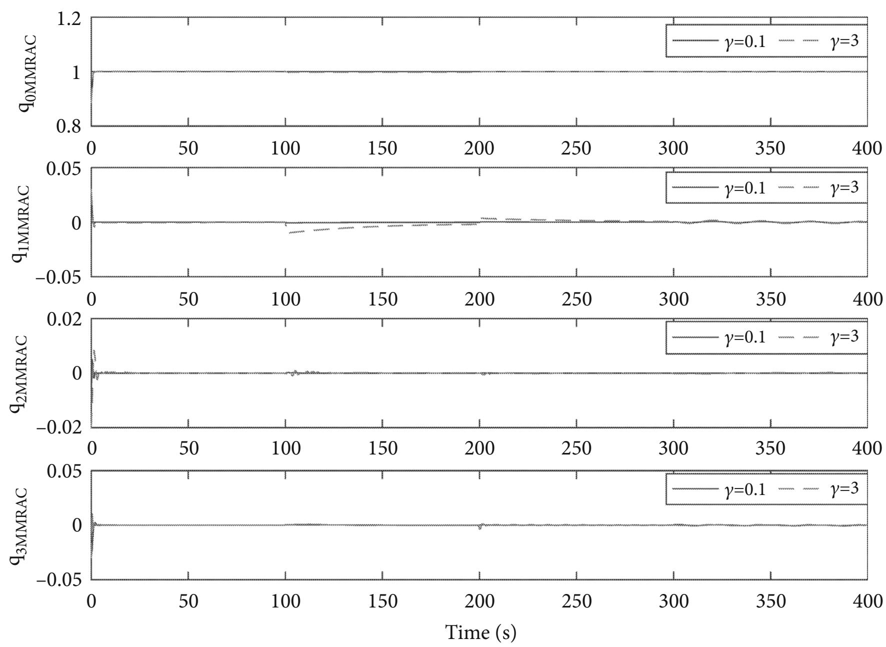


(b) Control input signals for different fault scenarios

Figure 2: Continued.



(c) Responses of tracking errors



(d) System outputs for different γ in the MMAC design

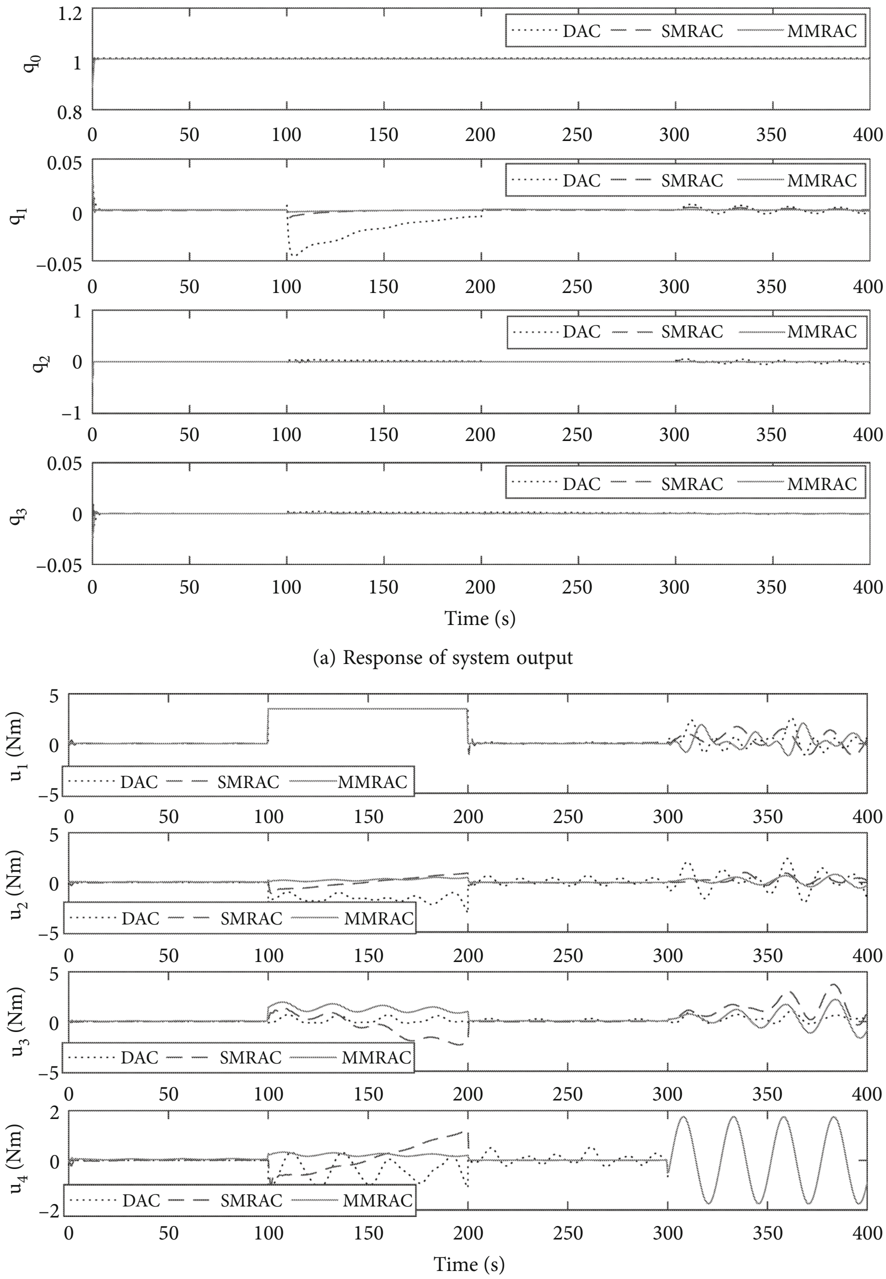
Figure 2: Attitude tracking control using MMRAC-, SMRAC-, and DAC-based fault-tolerant control schemes for Case 1.

(ii) When $100s \le t < 200s$, actuator u_1 failed, $u_1(t) = 3.5Nm$ and $u_i(t) = v_i(t), i = 2, 3, 4$

(iii) When $t \ge 200s$, actuator u_1 returns to normal, $u_i(t) = v_i(t), i = 2, 3, 4$

(iv) When $t \ge 300s$, actuator u_4 is out of control, $u_4(t) = 1.75 \sin(0.25t)\text{Nm}, u_i(t) = v_i(t), i = 1, 2, 3$

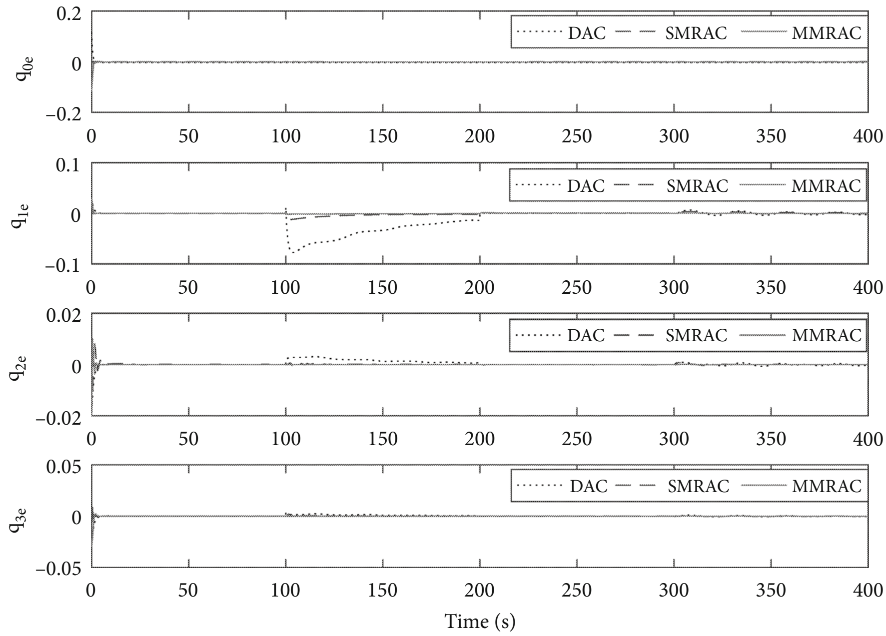
This example represents the severe case in which both two actuators experience failure at the moments of $t = 100s$ and $t = 300s$, respectively. Actually, the failure conditions in Case 2 can be regarded as a mixed pattern of intermittent failure and permanent failure. As shown in Figure 3(b), when u_1 is stuck at the instant $t = 100s$ and becomes normal at $t = 200s$, that is an intermittent failure occurred in actuator u_1, which is activated and inactivated by itself. The system produces an erroneous result when such fault is active during the period of $100s \le t < 200s$ and produces a correct result when it is inactive for $t \ge 200s$. At $t = 300s$, actuator u_4 undergoes time-varying failure and never come back as normal. Figure 3(a) shows the results using the three different control laws based on the same simulation conditions. Clearly, compared with attitude tracking response for Case 1, the attitude control performance deteriorates severely due to the multiple uncertainties of actuator failure, with severe overshoots in the attitude orientation, although the



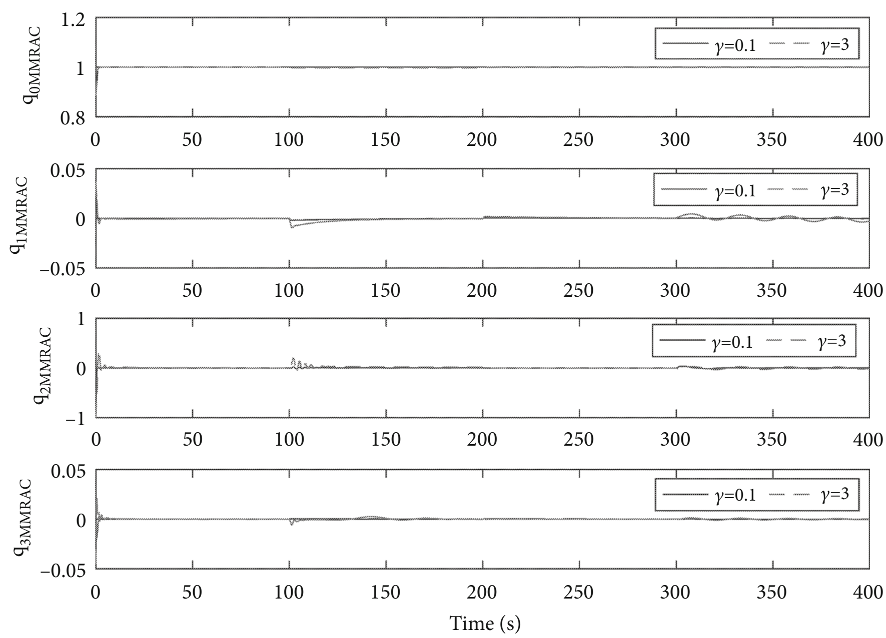
(a) Response of system output

(b) Control input signals for different fault scenarios

FIGURE 3: Continued.



(c) Responses of tracking errors



(d) System outputs for different γ in the MMAC design

Figure 3: Attitude tracking control using MMRAC-, SMRAC-, and DAC-based fault-tolerant control schemes for Case 2.

objective of asymptotic attitude tracking can be achieved finally, as clearly indicated from the output tracking errors present in Figure 3(c).

In summary, for both the normal, intermittent failure and permanent failure cases, the proposed controller significantly improves the normal control performance of the closed-loop attitude system compared to the SMRAC- and DAC-based fault-tolerant control approaches. For the cases with actuator faults, the proposed method gives better transient performance than those controllers without including the transient performance compensator. As the faults become more severe, the proposed controller still guarantees system stability and asymptotic output tracking of a given command.

Furthermore, comparing the modified control scheme with different γ_i $(i = 1, 2)$, it can be found from Figures 2(d) and 3(d) that with smaller $\gamma_1 = 0.1$, the compensator takes more effects on transient oscillations inhibition with a smaller following error of trajectory than that with $\gamma_2 = 3$, which keeps an agreement with the performance analysis by the criterion of L_1 bound or mean squared value.

5. Conclusions

With multiple uncertain actuator faults, the control system of rigid spacecraft attitude is optimized by designing an improved adaptive FTC method in this study. And the major conclusions are as follows. (1) A model reference adaptive control algorithm combing with feedback linearization technology is adopted to propose a basic control law for achieving the desired closed-loop system performance. Then, as an additional item of robust adaptive control, an H_∞ compensator is introduced to optimize the transient performance. (2) Based on the modified basic control design, multiple targeted adaptive failure compensators are designed to handle the corresponding failure patterns. Multiple controllers are fused into a comprehensive controller by using a weighted algorithm, thus multiple uncertain actuator faults are solved. (3) Under different fault conditions with and without additional transient performance improvers, contrastive simulation analysis of output tracking control is adopted to proof the significance of the designed theoretical method. (4) The fault compensation control for spacecraft whose parameters are known, is researched in this paper. The way to overcoming the fault compensation difficulty on the spacecraft with unknown system parameters can be found by the further extended method which is described in this research.

Data Availability

The data (system parameters and simulation parameters) used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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