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Research Article

Computer-Aided Design of Elliptically Focused Bootlace Lens for Multiple Beams

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The design of elliptically focused microwave bootlace lens is investigated and improved design is proposed. The proposed design of the lens is capable of scanning wide area and is compact in size than the other available designs. The phase error is calculated and compared with conventional design of bootlace lens. The results are obtained using Matlab coding and are in accordance to the theoretical values. This design is intended to be used in next generation cellular mobile communication with tilt of required angle at base stations and also it can be used as multiple beam-forming networks for avoiding deafness problem in the ad hoc networks with greater efficiency and saving radiated power.

1. INTRODUCTION

The growing demand of modern communication systems has increased the complexity of the systems for communication and surveillance. The air interfacing is a major part of the communication systems, which needs efficient and compact antenna systems. In radar and communication applications, it is necessary to have wide range coverage system. These systems need generation of directional and multiple beams for scanning wide area with greater efficiency using antenna array. To obtain multiple beams for scanning wide and prescribed area, a multiple beamforming network is required [1]. The multiple beamforming networks are used to control the amplitude and phase at each element of the antenna array. The microwave beamforming networks are preferred for such applications. The Rotman or Bootlace lens is one of the microwave lenses used to scan beams electronically for true time delay scanning antennas [2]. This lens has low circuit complexity and allows real time scanning. In the proposed design of bootlace lens, the contours are elliptical in shape and the focal points do not lie on axis of the lens [3]. The phase shifts that increase from one element to the next and result in an antenna radiation pattern with a single beam pointing into a certain direction in space on the axis of the lens [4]. The proposed design gives better efficiency and scanning capabilities. Results of this design are compared with circular focused bootlace lens and are better than conventional design. The microwave lenses find use in various applications nowadays due to their efficiency and ease of fabrication and installation. In cellular mobile communication, bootlace lens is used at base stations for scanning through the cell with required tilt electronically [5]. The directional antennas are used in ad hoc networks for better efficiency and throughput [6]; but switching between omnidirectional to directional antennas is the cause of concern and makes difficult to adjust power at output ports. The proposed bootlace lens is intended to be used as multibeamforming network in ad hoc networks to be done away with switching of antennas and also eliminates deafness and hidden terminal problems with saving of power [7].

2. LENS GEOMETRY

Design of Elliptically focused bootlace lens is shown in Figure 1. In this lens design, one focal point F_0 is located on the central axis and the two other focal points F_1 and F_2 are symmetrically located on either side on an elliptical focal arc which is called feed contour. The off-axis focal points F_1 and F_2 are located on the focal arc at an angle of $+\beta$ and $-\beta$ from central axis. As shown in Figure 1, I_1 is the inner contour of the lens which is called the array contour. Contour I_2 is a straight line and decides the position of radiating elements of array. Inner contour and outer contour are connected by using TEM mode transmission line $W(N)$.

The lens is designed in such a way that outgoing beams make angles $-\alpha$, 0, and $+\alpha$ with the X-axis from radiating contour when feeds are placed at F_1, F_0, and F_2, respectively. A ray originating from point F_1 on the feed contour reaches the wavefront through a general point $P(X, Y)$ on the

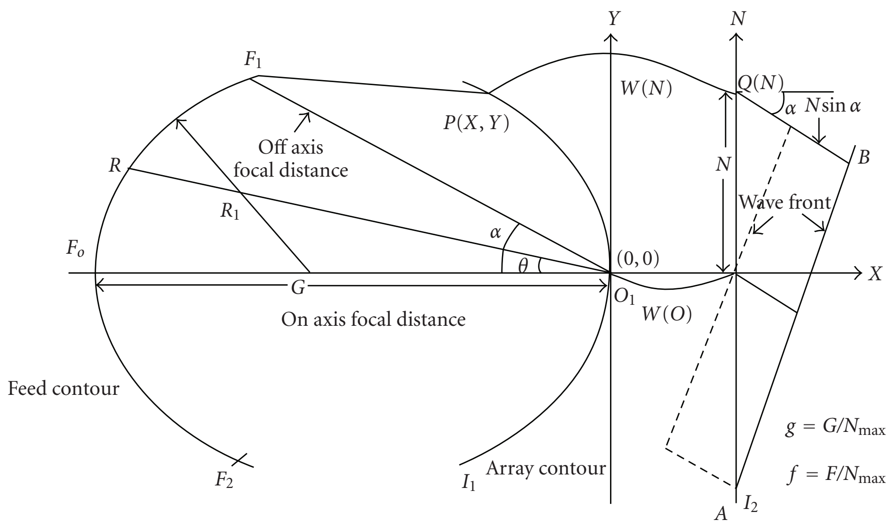

Figure 1: Elliptically focused lens.

inner contour I_1 transmission line $W(N)$, point $Q(N)$ on the outer contour, and then traces a straight line at an angle $-\alpha$ and terminates perpendicular to the wavefront. Also the ray from F_1 may reach the wavefront from F_1 to point O_1, and then trough transmission line $W(0)$ to the wavefront. Same way, the rays from other feed points may reach their respective wavefront to produce required radiating pattern. Inner contour and transmission line are designed from the design equations, which are derived using the fact that at the wavefront, all of these rays must be in phase independent of the path they travel [4]. This requires that the total phase shift in traversing the path to reach the wavefront in each case is equal [4]. This design may further be made compact by reducing the parallel lines. This modification increases scan area but path length will be more than achieved in this case and so is the error. The design equations for this design can be written as follows:

$$\sqrt{\varepsilon_r} F_1 P + \sqrt{\varepsilon_{re}}\, W(n) + N \sin\alpha = \sqrt{\varepsilon_r} F + \sqrt{\varepsilon_{re}}\, W(0), \quad (1)$$

$$\sqrt{\varepsilon_r} F_2 P + \sqrt{\varepsilon_{re}}\, W(n) - N \sin\alpha = \sqrt{\varepsilon_r} F + \sqrt{\varepsilon_{re}}\, W(0), \quad (2)$$

$$\sqrt{\varepsilon_r} F_0 P + \sqrt{\varepsilon_{re}}\, W(n) = \sqrt{\varepsilon_r} G + \sqrt{\varepsilon_{re}}\, W(0), \quad (3)$$

where

$$(F_1P)^2 = (X + F\cos\beta)^2 + (Y - F\sin\beta)^2, \quad (4)$$

$$(F_2P)^2 = (X + F\cos\beta)^2 + (Y + F\sin\beta)^2, \quad (5)$$

$$(F_0P)^2 = (X + G)^2 + (Y)^2, \quad (6)$$

F is distance from F_1 to O_1, called the off-axis focal length. G is distance from F_o to O_1, called the on-axis focal length. N indicates the position of the radiating elements, called the lens aperture. ε_r is substrate dielectric constant. ε_{re} is effective dielectric constant of the transmission lines.

Design parameters are normalized relative to the maximum lens aperture N_{max} and are defined asfollows:

$$f = \frac{F}{N_{max}}, \quad g = \frac{G}{N_{max}}, \quad W = \frac{(W(0) - W(N))\sqrt{\varepsilon_{re}}}{(\sqrt{\varepsilon_r} N_{max})},$$
$$\eta = \frac{N}{N_{max}}, \quad y = \frac{Y}{N_{max}}, \quad x = \frac{X}{N_{max}}. \quad (7)$$

From (4), we get that

$$(F_1P)^2 = X^2 + F^2 + Y^2 + 2XF\cos\beta - 2YF\sin\beta. \quad (8)$$

Normalizing with respect to N_{max},

$$\frac{(F_1P)^2}{(N_{max})^2} = x^2 + f^2 + y^2 + 2xf\cos\beta - 2yf\sin\beta. \quad (9)$$

Similarly,

$$\frac{(F_2P)^2}{(N_{max})^2} = x^2 + f^2 + y^2 + 2xf\cos\beta + 2yf\sin\beta. \quad (10)$$

From (1),

$$\sqrt{\varepsilon_{re}}[W(0) - W(N)] = \sqrt{\varepsilon_r}[(F_1P) - F] + N\sin\alpha. \quad (11)$$

Dividing the above equation by $\sqrt{\varepsilon_r} N_{max}$,

$$w = \frac{[(F_1P) - F]}{N_{max}} + \frac{\eta \sin\alpha}{\sqrt{\varepsilon_r}}, \quad (12)$$

$$\left(\frac{F_1P}{N_{max}}\right) = (w + f) - \frac{\eta \sin\alpha}{\sqrt{\varepsilon_r}}, \quad (13)$$

$$\left(\frac{F_2P}{N_{max}}\right) = (w + f) + \frac{\eta \sin\alpha}{\sqrt{\varepsilon_r}},$$
$$\left(\frac{F_1P}{N_{max}}\right)^2 - \left(\frac{F_2P}{N_{max}}\right)^2 = -4yf\sin\beta. \quad (14)$$

Solving for y, we get

$$y = \frac{\eta \sin \alpha (f + w)}{\sqrt{\varepsilon_r} f \sin \beta}. \tag{15}$$

From (3),

$$\begin{aligned}(w+g) &= \frac{(F_0P)}{N_{\max}},\\ \left(\frac{F_0P}{N_{\max}}\right)^2 &= (x+g)^2 + y^2,\\ (x+g)^2 + y^2 &= (w+g)^2,\\ x^2 + y^2 &= w^2 + 2gw - 2gx.\end{aligned} \tag{16}$$

By equating value of $(F_1P)^2/(N_{\max})^2$ with (13),

$$\begin{aligned}&x^2 + f^2 + y^2 + 2xf\cos\beta - 2yf\sin\beta\\ &\quad = (w+f)^2 + \left(\frac{\eta \sin \alpha}{\sqrt{\varepsilon_{re}}}\right)^2 - \frac{2(w+f)\eta \sin\alpha}{\sqrt{\varepsilon_r}}.\end{aligned} \tag{17}$$

Similarly for $(F_2P)^2/(N_{\max})^2$,

$$\begin{aligned}&x^2 + f^2 + y^2 + 2xf\cos\beta - 2yf\sin\beta\\ &\quad = (w+f)^2 + \left(\frac{\eta \sin \alpha}{\sqrt{\varepsilon_{re}}}\right)^2 + \frac{2(w+f)\eta \sin\alpha}{\sqrt{\varepsilon_r}}.\end{aligned} \tag{18}$$

Adding the above two equations,

$$x^2 + y^2 + 2xf\cos\beta = w^2 + 2wf + \frac{(\eta \sin\alpha)^2}{\varepsilon_r}. \tag{19}$$

Putting value of $x^2 + y^2$ from (16),

$$x = \frac{(\eta \sin\alpha)^2}{(2\varepsilon_r(f\cos\beta - g))} + \frac{w(f-g)}{(f\cos\beta - g)}, \quad x = a + bw, \tag{20}$$

where

$$\begin{aligned}a &= \frac{(\eta \sin\alpha)^2}{(2\varepsilon_r(f\cos\beta - g))},\\ b &= \frac{(f-g)}{(f\cos\beta - g)}.\end{aligned} \tag{21}$$

From (16),

$$x^2 + y^2 - 2gx = w^2 + 2gw. \tag{22}$$

Putting the value of x and y in (22), we get equation in the following form:

$$Aw^2 + Bw + C = 0, \tag{23}$$

where

$$\begin{aligned}A &= b^2 + \frac{\eta^2 \sin^2\alpha}{(\varepsilon_r f^2 \sin^2\beta)} - 1,\\ B &= 2ab + 2\left(\frac{\eta^2\sin^2\alpha}{\varepsilon_r f \sin^2\beta}\right) + 2gb - 2g,\\ C &= a^2 + \eta^2\left(\frac{\sin^2\alpha}{\varepsilon_r \sin^2\beta}\right) + 2ga.\end{aligned} \tag{24}$$

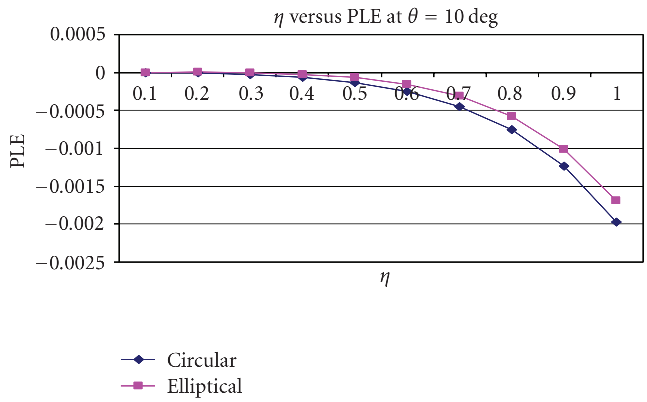

Figure 2: Normalized phase error for scanning angle $\theta = 10$.

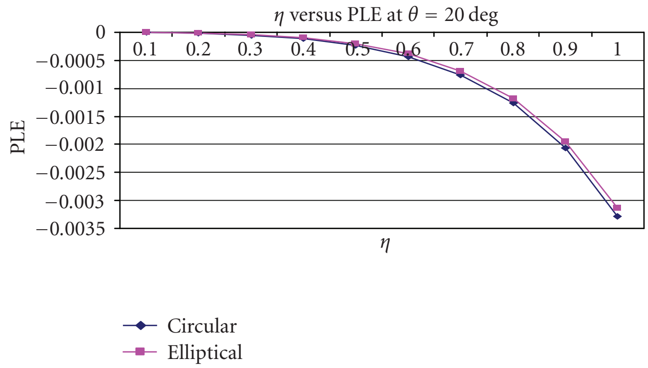

Figure 3: Normalized phase error for scanning angle $\theta = 20$.

For the given value of design parameters F, G, $N_{\max}$, and α, it is required to calculate the value of β, such that the height of the two contours (feed and array contour) be equal. Y, coordinate of array contour, is given by (15). For maximum value of lens aperture,

$$\begin{aligned}N &= N_{\max}, \quad \eta = 1,\\ y_{\max} &= \frac{\sin\alpha(f+w)}{(\sqrt{\varepsilon_r} f \sin\beta)}.\end{aligned} \tag{25}$$

To equalize the height of the two contours Y coordinate of the feed contour, $f \sin\beta$ must be equal to $y_{\max}$, that is,

$$f\sin\beta = \sin\alpha \frac{(f+w)}{(\sqrt{\varepsilon_r} f \sin\beta)}. \tag{26}$$

Using (15), (20), (23), and (26), value of βcan be calculated for given value of α, F, G, $N_{\max}$, and ε_r The lens designed using the calculated value of β will have equal height of the feed and array contour.

$N_{\max}$ is the lens aperture and it depends upon a number of antenna elements and their spacing.

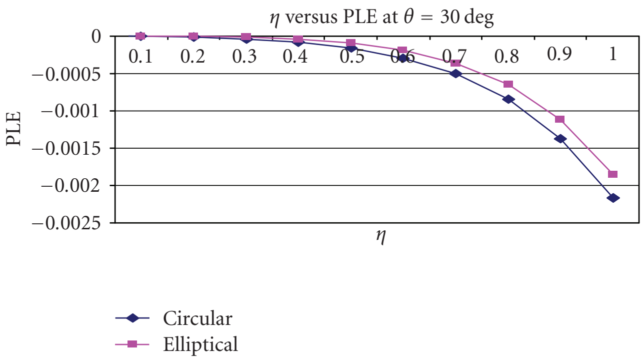

FIGURE 4: Normalized phase error for scanning angle $\theta = 30$.

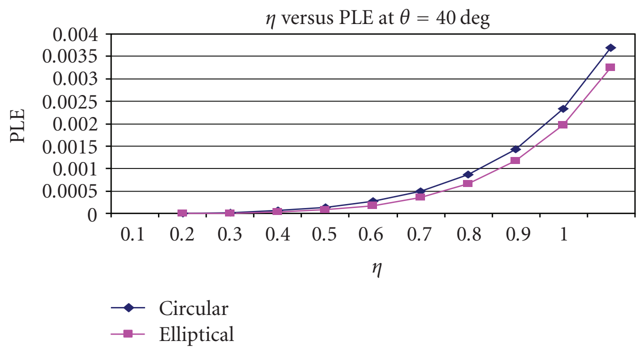

FIGURE 5: Normalized phase error for scanning angle $\theta = 40$.

Co-ordinates of the focal arc are given by the following equation:

$$\frac{(x-m)^2}{a^2} + \frac{(y-n)^2}{b^2} = 1, \tag{27}$$

where $b^2 = a^2(1-e^2)$, (m,n) is the center of the ellipse, and "e" is the eccentricity.

3. PATH LENGTH ERROR

When a feed is placed at one of the focal points on feed contour, the corresponding wave front has no path length error; it is also referred as phase error. But when the feed is displaced from these focal points, the corresponding wave front will have a path length error. However, for wide angle scanning, lens must be focused at all the intermediate points along the focal arc. Let a feed be located at point R in Figure 1 on the focal arc. RO makes an angle θ with the central axis. Let R_a and R_b be the phase shifts from the feed position to the wave front when the ray is passing through $P(X,Y)$ and O, respectively. Then the phase error is given by relation $\delta L = R_a - R_b$, where

$$\begin{aligned} R_a &= \sqrt{\varepsilon_r}(RP) + \sqrt{\varepsilon_{re}}W(N) + N\sin\theta, \\ R_b &= \sqrt{\varepsilon_r}(RO) + \sqrt{\varepsilon_{re}}W(O). \end{aligned} \tag{28}$$

TABLE 1: Comparison of phase errors.

Number	No. of p.l.error (circular)	No. of p.l.error (elliptical)
0.000000	0.000000	0.000000
0.100000	−0.000001	−0.000049
0.200000	−0.000008	−0.000201
0.300000	−0.000027	−0.000462
0.400000	−0.000065	−0.000840
0.500000	−0.000134	−0.001344
0.600000	−0.000253	−0.001983
0.700000	−0.000448	−0.002768
0.800000	−0.000758	−0.003708
0.900000	−0.001240	−0.004818
1.000000	−0.001976	−0.006109

Co-ordinates of point R are given by $(-H\cos\theta_1, H\sin\theta_1)$, where

$$\begin{aligned} \mathrm{Sin}\theta_1 &= \sin\frac{\theta}{\rho}, \\ \rho &= \frac{\mathrm{Sin}\alpha}{\sin\beta}. \end{aligned} \tag{29}$$

4. RESULTS

The results are obtained for elliptical and circular focal arc bootlace lens and plotted in graph comparing both the results. The compared results are shown in Figures 2 to 5 where they show the variation of normalized phase length error (PLE) with the lens aperture η for different scanning angles θ for conventional circular focal arc lens and for the proposed elliptically focused lens. The path length error of both types of lenses is also shown in Table 1 for one angle only. The same way for other angles can be plotted from the results available after simulation. The results shows that the phase error obtained for elliptically focused lens is less than the circular; and hence it will have better performance when used for multiple beamforming networks for antenna array. It is also evident from the results that elliptically focused bootlace lens will have path length error less than circular lens for entire range of variations.

The graphs obtained for various comparative results are given ahead as in Figures 1 to 5.

5. CONCLUSION

It is clear from the simulation results that the proposed design, bootlace lens using elliptical focal arc, is better in terms of phase error which is a basic parameter for true time scanning of the beams. The proposed beamforming network will be compact in size and will have power-saving feature when used with controlled switching network for multiple elements. The path length error for this design is less than conventional Rotman lens. This design will be preferred for multiple beamforming networks in base stations having larger number of sectors in the cell when compared with circular design. It will prove to be a milestone when used in ad hoc wireless networks and vehicular networks.

REFERENCES

[1] W. Rotman and R. Turner, "Wide-angle microwave lens for line source applications," *IEEE Transactions on Antennas and Propagation*, vol. 11, no. 6, pp. 623–632, 1963.

[2] G. Leonakis, "Correction to "wide-angle microwave lens for line source applications"," *IEEE Transactions on Antennas and Propagation*, vol. 34, no. 8, p. 1067, 1986.

[3] D. Archer, "Lens fed multiple-beam arrays," *Microwave Journal*, vol. 18, no. 27, pp. 171–195, 1984.

[4] P. K. Singhal, P. C. Sharma, and R. D. Gupta, "Rotman lens with equal height of array and feed contours," *IEEE Transactions on Antennas and Propagation*, vol. 51, no. 8, pp. 2048–2056, 2003.

[5] S. G. Kim, S. M. Park, C. W. Ro, and J. M. Lee, "Electronic beam tilting of base station antenna: rotman lens fed," in *Proceedings of IEEE Radio and Wireless Conference (RAWCON '98)*, pp. 95–98, Colorado Springs, Colo, USA, August 1998.

[6] R. R. Choudhury, X. Yang, R. Ramanathan, and N. H. Vaidya, "On designing MAC protocols for wireless networks using directional antennas," *IEEE Transactions on Mobile Computing*, vol. 5, no. 5, pp. 477–491, 2006.

[7] G. S. Tomar, S. Verma, and M. Rai, "Modified directional medium access protocol using bootlace lens," in *Proceedings of IEEE Region 10 Annual International Conference (TENCON '06)*, pp. 1–4, Hong Kong, 2006.

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