Hindawi Publishing Corporation
International Journal of Antennas and Propagation
Volume 2011, Article ID 505606, 8 pages
doi:10.1155/2011/505606

*Research Article*

# SVD-Aided Power Allocation and Iterative Detection Scheme for Turbo-BLAST System with Imperfect Channel State Information

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Received 4 November 2010; Revised 20 April 2011; Accepted 9 June 2011

Academic Editor: Ananda Sanagavarapu Mohan



A new technique that combines adaptive power allocation and iterative detection based on singular value decomposition (SVD) is introduced for the modified Turbo-BLAST system with imperfect channel state information (I-CSI). At the transmitter, in order to maximize the capacity performance, the MIMO channel is decomposed into several parallel eigen subchannels by SVD, and then proper power based on the water-filling principle is allocated to every subchannel subject to the total transmit power constraint. At the receiver, the modified MMSE detector taking the CSI imperfection into account is used to remove the coantenna interference, and then the turbo idea is employed for iterative detection to lower the system BER. As a result, the BER performance is effectively enhanced. Numerical results show that the introduced SVD-aided adaptive power allocation method is valid to improve not only the capacity but also the BER performance in the presence of channel state information imperfection, while the iterative detector can further lower the BER results.

## 1. Introduction

Multiple-input multiple-output (MIMO) systems can provide a significant capacity increment over the conventional one through appropriate space-time processing [1, 2]. Bell labs layered space-time (BLAST) structure is one of the promising architectures thanks to the advantages of low detection complexity and high data rates [3]. The new system which combined BLAST structure with turbo idea is called Turbo-BLAST that can offer a reliable and practical solution to high data-rate transmission for wireless communications [4].

One of the attractive features of MIMO system is a multiplexing gain and consequently a higher capacity performance over single-input single-output system. When channel state information (CSI) is perfect, the water-filling power allocation strategy [5–7] is optimized to improve the capacity performance for MIMO systems. Practically, CSI at the receiver is subject to the error due to nonreal time data processing, and so forth [8]. For the modified Turbo-BLAST system, the task of designing the effective link adaptive technique becomes more challenging in the presence of CSI imperfection. Recently, various contributions have also considered the MIMO transceiver design problem for systems with either partial or imperfect CSI at the transmitter [9–12]. In these references, the effect of I-CSI on the capacity performance of MIMO system is investigated in [9, 10]. Based on the perfect and imperfect CSI, the performance of adaptive modulation schemes in MIMO systems is analyzed in [11]. The transmit power allocation for a two-input multiple-output system is presented in [12] by minimizing the BER, but the scheme is only suitable for the systems with two-transmit antennas. The above references, however, consider the uncoded MIMO system only, and thus the superiority of channel coding is not fully utilized. Especially, the Turbo-BLAST structure with superior performance is not considered. As a result, the performance improvement is limited.

Motivated by the reason above, we will study the performance of Turbo-BLAST system in this paper and develop a new SVD-aided adaptive power allocation strategy and

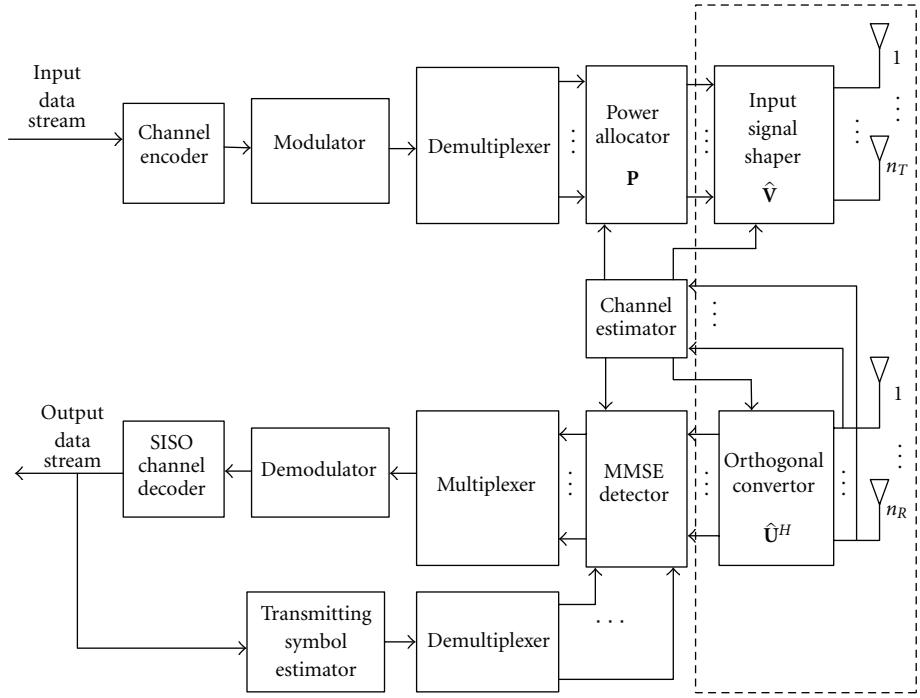


Figure 1: An illustrative diagram of the modified Turbo-BLAST system.

iterative detection scheme for modified Turbo-BLAST system in the presence of CSI imperfection. At the transmitter, aiming at maximizing the capacity performance of the overall system, MIMO channel is decomposed into several parallel eigen subchannels by singular value decomposition (SVD) [13–16], and then appropriate power based on water-filling principle is allocated to each subchannel subject to the total transmit power constraint. At the receiver, the iterative turbo strategy is employed for signal detection to enhance the BER performance. The theoretical analysis and numerical results show that the proposed SVD-assisted transmit power allocation (TPA) approach is an effective method to improve both the capacity and the BER performance. Furthermore, the corresponding BER results of the modified Turbo-BLAST system can be further enhanced through the iterative detection.

This paper is organized as follows: after the introduction in Section 1, the channel model and system description are briefly introduced in Section 2. Section 3 is concerned with SVD-aided power allocation scheme in the presence of imperfect CSI. Section 4 presents an iterative detection algorithm for the modified Turbo-BLAST system with adaptive power allocation. The numerical results are given in Section 5. Finally, Section 6 summarizes the conclusion of the paper.

## 2. Channel Model and System Description

*2.1. Channel Model.* Consider a Turbo-BLAST system with $n_T$ transmit antennas and $n_R$ receive antennas ($n_T \leq n_R$) as depicted in Figure 1. Based on CSI obtained from the feedback channel, the transmitter optimally varies its power allocation parameters as will be detailed in Section 3. We adopt a CSI imperfection model from [8]. The CSI is estimated at the receiver and sent to the transmitter by feedback channel. To assist the receiver in performing channel estimation and symbol detection, known pilot symbols are periodically inserted at the transmitter—a technique that is known as pilot symbol assisted modulation (PSAM) [17]. At the receiver, the samples corresponding to the known pilots are extracted, based on which CSI is interpolated using optimal Wiener filtering [17].

The imperfect CSI is represented by the complex channel matrix $\hat{\mathbf{H}}$, which is related to the true channel matrix $\mathbf{H}$ as

$$\mathbf{H} = \hat{\mathbf{H}} + \Xi, \tag{1}$$

where $\Xi$ is a complex matrix related to the imperfect CSI. Note that the same CSI is known to both the transmitter and receiver. The elements of both $\hat{\mathbf{H}}$ and $\Xi$ can be modeled as independent complex Gaussian variables [8], which subsequently lead to the entries of $\mathbf{H}$ as complex Gaussian variables, and furthermore, $\hat{\mathbf{H}}$ and $\Xi$ are independent to each other [8, 11]. Thus we can express their statistical distributions as $e_{ij} \sim \mathcal{CN}(0, \sigma_e^2)$, $\hat{h}_{ij} \sim \mathcal{CN}(0, 1 - \sigma_e^2)$, and $h_{ij} \sim \mathcal{CN}(0, 1)$, which are all distributed by complex Gaussian law with $\sigma_e^2$ indicating the accuracy of the CSI.

In order to maximize the capacity of the Turbo-BLAST system, the MIMO channel is decomposed into several parallel eigen subchannels by SVD. In practice, true CSI cannot be obtained, so $\hat{\mathbf{H}}$ is treated as the true channel matrix which can be decomposed as

$$\hat{\mathbf{H}} = \hat{\mathbf{U}}\hat{\mathbf{\Lambda}}\hat{\mathbf{V}}^H, \tag{2}$$

where $(\cdot)^H$ denotes a matrix conjugate transpose, $\hat{\mathbf{U}}$ and $\hat{\mathbf{V}}$ are the unitary matrices with left and right singular vector of $\hat{\mathbf{H}}$ as their columns, $\hat{\mathbf{\Lambda}}$ is a nonnegative and diagonal matrix, $\{\sqrt{\hat{\lambda}_i}\}_{i=1}^{n_T}$ are the main diagonal elements of $\hat{\mathbf{\Lambda}}$, and $\{\hat{\lambda}_i\}_{i=1}^{n_T}$ are eigenvalues of $\hat{\mathbf{H}}\hat{\mathbf{H}}^H$, respectively. The left singular vector $\hat{\mathbf{V}}$ is used for the input signal shaping at the transmitter and the conjugate transpose of right singular vector $\hat{\mathbf{U}}$ is used for orthogonal transform at the receiver.

*2.2. System Description.* In this part we propose a system model for a modified Turbo-BLAST system using a novel SVD-assisted power allocation strategy with imperfect CSI.

Figure 1 shows an illustrative diagram of a modified Turbo-BLAST system. At the transmitter, the input data stream is firstly encoded and then mapped into a symbol stream that is subsequently demultiplexed into $n_T$ substreams, and the transmit power at each eigen subchannel is varied according to the instantaneous values and/or statistics of the subchannel gains under an total transmit power constraint.

Let $\mathbf{x} = [x_1, \ldots, x_{n_T}]^T$ and $\mathbf{r} = [r_1, \ldots, r_{n_R}]^T$ denote input and output column vectors at a sampling instant, respectively. Similarly, $\mathbf{n} = [n_1, \ldots, n_{n_R}]^T$ is a column vector, where each component is independent and identically distributed (i.i.d) zero-mean complex Gaussian variables with variance $\sigma_n^2$.

The transmitted signals are received by the $n_R$ antennas at a sampling instant which can be expressed as

$$\mathbf{r} = \hat{\mathbf{U}}^H\left(\mathbf{H}\hat{\mathbf{V}}\mathbf{P}\mathbf{x} + \mathbf{n}\right) = \hat{\mathbf{U}}^H\mathbf{H}\hat{\mathbf{V}}\mathbf{P}\mathbf{x} + \hat{\mathbf{U}}^H\mathbf{n}, \tag{3}$$

where $\mathbf{P} = \mathrm{diag}(\sqrt{P_1}, \ldots, \sqrt{P_{n_T}})$ is the diagonal transmit power matrix with total power constraint $\sum_{k=1}^{n_T} P_k = n_T$.

At the receiver, an iterative detection strategy based on the "turbo" principle is used for the symbol detection. The encoding and demultiplexing for the data stream can be equivalently viewed as a serially concatenated encoding process as depicted in Figure 1. The concatenated code can be decoded using a low-complexity iterative decoder which is similar to the iterative decoder for a serially concatenated turbo code. For the iterative decoding, the optimal decoding process can be separated into two stages, that is, soft-input/soft-output (SISO) detector and SISO channel decoder, respectively, which mutually exchange the extrinsic information sent from one stage to another iteratively until the decoding process converges.

## 3. Power Allocation Scheme with Imperfect CSI

*3.1. Equivalent System Model.* In this subsection, we propose an equivalent system model under the conditions of imperfect CSI for the modified Turbo-BLAST system.

With the channel model and the SVD technique introduced aforementioned, when the CSI is imperfect, the received signal at a sampling instant can be formulated as

$$\begin{aligned}\mathbf{r} &= \hat{\mathbf{U}}^H\left(\hat{\mathbf{H}} + \mathbf{\Xi}\right)\hat{\mathbf{V}}\mathbf{P}\mathbf{x} + \hat{\mathbf{U}}^H\mathbf{n} = \hat{\mathbf{U}}^H\hat{\mathbf{H}}\hat{\mathbf{V}}\mathbf{P}\mathbf{x} + \hat{\mathbf{U}}^H\mathbf{\Xi}\hat{\mathbf{V}}\mathbf{P}\mathbf{x} + \hat{\mathbf{U}}^H\mathbf{n}\\ &= \hat{\mathbf{U}}^H\hat{\mathbf{U}}\hat{\mathbf{\Lambda}}\hat{\mathbf{V}}^H\hat{\mathbf{V}}\mathbf{P}\mathbf{x} + \hat{\mathbf{U}}^H\mathbf{\Xi}\hat{\mathbf{V}}\mathbf{P}\mathbf{x} + \hat{\mathbf{U}}^H\mathbf{n}\\ &= \hat{\mathbf{\Lambda}}\mathbf{P}\mathbf{x} + \hat{\mathbf{U}}^H\mathbf{\Xi}\hat{\mathbf{V}}\mathbf{P}\mathbf{x} + \mathbf{n}'\\ &= \hat{\mathbf{\Lambda}}\mathbf{P}\mathbf{x} + \hat{\mathbf{n}},\end{aligned} \tag{4}$$

where $\mathbf{n}' = \hat{\mathbf{U}}^H\mathbf{n}$, $\hat{\mathbf{U}}$ is the unitary matrix with the right singular vectors of $\hat{\mathbf{H}}$ as its columns, thus $\mathbf{n}' = \hat{\mathbf{U}}^H\mathbf{n}$ is also a column vector of additive Gaussian noise variables, where each component is i.i.d zero-mean complex Gaussian variables with the variance of $\sigma_n^2$, $\hat{\mathbf{n}} = \hat{\mathbf{U}}^H\mathbf{\Xi}\hat{\mathbf{V}}\mathbf{P}\mathbf{x} + \mathbf{n}'$ is an equivalent additive noise consisting of the interference caused by the channel estimation errors and the complex Gaussian noise, respectively. Here we make an approximation that assumes that the entries of x are i.i.d. Gaussian variables, that is, $x_i \sim C\mathcal{N}(0,1)$ [11]. Under this assumption, the variance of the equivalent noise $\hat{\mathbf{n}}$ can be calculated as follows.

Firstly, let $\mathbf{G} = \hat{\mathbf{U}}^H\mathbf{\Xi}\hat{\mathbf{V}}$, the mean and variance of $\mathbf{G}$ can be evaluated as

$$\begin{aligned}\varepsilon\{\mathbf{G}\} &= \varepsilon\left\{\hat{\mathbf{U}}^H\mathbf{\Xi}\hat{\mathbf{V}}\right\} = \hat{\mathbf{U}}^H\varepsilon\{\mathbf{\Xi}\}\hat{\mathbf{V}} = \mathbf{0}_{n_R\times n_T},\\ \varepsilon\left\{\mathbf{G}\mathbf{G}^H\right\} &= \varepsilon\left\{\hat{\mathbf{U}}^H\mathbf{\Xi}\hat{\mathbf{V}}\hat{\mathbf{V}}^H\mathbf{\Xi}^H\hat{\mathbf{U}}\right\} = \varepsilon\left\{\hat{\mathbf{U}}^H\mathbf{\Xi}\mathbf{\Xi}^H\hat{\mathbf{U}}\right\}\\ &= \hat{\mathbf{U}}^H\varepsilon\left\{\mathbf{\Xi}\mathbf{\Xi}^H\right\}\hat{\mathbf{U}} = \sigma_e^2\mathbf{I}_{n_R}.\end{aligned} \tag{5}$$

Clearly, the components of $\mathbf{G}$ are i.i.d zero-mean complex Gaussian variables with the variance of $\sigma_e^2$: thus we can express the statistical distributions as

$$\begin{aligned}g_{ij} &\sim C\mathcal{N}(0,\sigma_e^2),\\ \varepsilon\left\{g_{qk}g_{qi}^*\right\} &= \sigma_e^2\delta(k-i).\end{aligned} \tag{6}$$

Then let $\mathbf{f}$ be denoted as follows:

$$\begin{aligned}\mathbf{f} = \mathbf{G}\mathbf{P}\mathbf{x} &= \begin{bmatrix} g_{11} & g_{12} & \cdots & g_{1n_T}\\ g_{21} & g_{22} & \cdots & g_{2n_T}\\ \vdots & \vdots & \ddots & \vdots\\ g_{n_R1} & g_{n_R2} & \cdots & g_{n_Rn_T}\end{bmatrix}\begin{bmatrix}\sqrt{P_1} & 0 & \cdots & 0\\ 0 & \sqrt{P_2} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \sqrt{P_{n_T}}\end{bmatrix}\\ &\times\begin{bmatrix}x_1\\ x_2\\ \vdots\\ x_{n_T}\end{bmatrix} = \begin{bmatrix}\sum_{k=1}^{n_T} g_{1k}\sqrt{P_k}x_k\\ \sum_{k=1}^{n_T} g_{2k}\sqrt{P_k}x_k\\ \vdots\\ \sum_{k=1}^{n_T} g_{n_Rk}\sqrt{P_k}x_k\end{bmatrix} = \begin{bmatrix}f_1\\ f_2\\ \vdots\\ f_{n_R}\end{bmatrix}.\end{aligned} \tag{7}$$

Clearly, $\mathbf{f}$ is a zero-mean complex Gaussian column vector with its components $f_q = \sum_{k=1}^{n_T} g_{qk}\sqrt{P_k}x_k$, $(q = 1,\ldots,n_R)$, whose variance can be evaluated as

$$\begin{aligned}\sigma_q^2 &= \varepsilon\left(f_q f_q^*\right) = \varepsilon\left\{\left[\sum_{k=1}^{n_T} g_{qk}\sqrt{P_k}x_k\right]\left[\sum_{i=1}^{n_T} g_{qi}\sqrt{P_i}x_i\right]^*\right\}\\ &= \sum_{k=1}^{n_T} P_k\varepsilon\left[\left(g_{qk}g_{qk}^*\right)\left(x_k x_k^*\right)\right] = \sum_{k=1}^{n_T} P_k\varepsilon\left\{\left|g_{qk}\right|^2\right\}\varepsilon\left\{|x_k|^2\right\}\\ &= \sigma_e^2\sum_{k=1}^{n_T} P_k = n_T\sigma_e^2.\end{aligned} \tag{8}$$

Thus the component of $\hat{\mathbf{n}}$ is given as $\hat{n}_q = f_q + n'_q$, $(q = 1,\ldots,n_R)$ by the aforementioned definition with the variance calculated as

$$\begin{aligned}\sigma_{\hat{n}_q}^2 &= \varepsilon\left\{\hat{n}_q\hat{n}_q^*\right\} = \varepsilon\left\{\left(f_q + n'_q\right)\left(f_q + n'_q\right)^*\right\}\\ &= \varepsilon\left\{\left|f_q\right|^2\right\} + \varepsilon\left\{\left|n'_q\right|^2\right\} = n_T\sigma_e^2 + \sigma_n^2.\end{aligned} \tag{9}$$

Given the proposed equivalent system model, a SVD-assisted transmit power allocation scheme aiming at maximizing the capacity of the whole system in the presence of CSI imperfection will be described in the next part.

*3.2. SVD-Aided Power Allocation Strategy.* In the presence of channel state information imperfection, the channel capacity for Turbo-BLAST system with SVD-assisted power allocation scheme can be expressed as [1]

$$C\left(\bar{\hat{\boldsymbol{\lambda}}}\right) = \sum_{k=1}^{n_T}\log_2\left[1 + \frac{P_k\hat{\lambda}_k}{n_T\sigma_e^2 + \sigma_n^2}\right], \tag{10}$$

where $\bar{\hat{\boldsymbol{\lambda}}} = \operatorname{diag}\left[\hat{\lambda}_1,\ldots,\hat{\lambda}_{n_T}\right]$ and $P_k$ is the transmit power allocated for the $k$-th subchannel. The capacity subject to an optimization problem is given as

$$\begin{aligned}&\left[P_1,\ldots,P_{n_T}\right]_{\text{opt}} = \arg\max\left\{\sum_{k=1}^{n_T}\log_2\left[1 + \frac{P_k\hat{\lambda}_k}{n_T\sigma_e^2 + \sigma_n^2}\right]\right\}\\ &\text{subject to}\quad \sum_{k=1}^{n_T} P_k = n_T.\end{aligned} \tag{11}$$

The Lagrange multiplier method is employed to find the optimized power allocation matrix that can maximize the overall capacity with total power constraint. By solving the equation in (11), we can find that the transmit power allocated for the $k$-th subchannel is

$$P_k = \left[\mu - \left(n_T\sigma_e^2 + \sigma_n^2\right)\hat{\lambda}_k^{-1}\right]^+, \tag{12}$$

where $(x)^+ \triangleq \max\{x, 0\}$ and $\mu$ is the Lagrange multiplier restricted by

$$\sum_{k=1}^{n_T}\left[\mu - \left(n_T\sigma_e^2 + \sigma_n^2\right)\hat{\lambda}_k^{-1}\right]^+ = n_T. \tag{13}$$

In a flat fading MIMO channel, the channel capacity is the statistical mean of $\bar{\hat{\boldsymbol{\lambda}}}$ approximately in the presence of CSI imperfection

$$C = \varepsilon_{\bar{\hat{\lambda}}}\left\{C\left(\bar{\hat{\boldsymbol{\lambda}}}\right)\right\} = \varepsilon\left\{\sum_{k=1}^{n_T}\log_2\left(1 + \frac{P_k\hat{\lambda}_k}{\sigma_{\hat{n}}^2}\right)\right\} = \varepsilon\left\{\sum_{k=1}^{n_T}\log_2\left(\mu\hat{\lambda}_k\right)\right\}. \tag{14}$$

Because $\{\hat{\lambda}_i\}_{i=1}^{n_T}$ is i.i.d and has the same probability density function $p(\hat{\lambda}_k)$, formula (14) can be expressed as

$$C = n_T\int_0^{+\infty}\left[\log_2\left(\mu\hat{\lambda}_k\right)\right]^+ p\left(\hat{\lambda}_k\right)d\hat{\lambda}_k. \tag{15}$$

Since singular values are random variables, it is hard to achieve the function of its probability density straightly. We use the algebraic mean to achieve the capacity results in the simulation which can be expressed as

$$\begin{aligned}C &\approx \varepsilon\left\{\sum_{k=1}^{n_T}\log_2\left[1 + \frac{P_k\hat{\lambda}_k}{n_T\sigma_e^2 + \sigma_n^2}\right]\right\}\\ &\approx \frac{1}{N}\sum_{i=1}^{N}\left\{\sum_{k=1}^{n_T}\log_2\left[1 + \frac{P_{k,i}\hat{\lambda}_{k,i}}{n_T\sigma_{e,i}^2 + \sigma_{n,i}^2}\right]\right\},\end{aligned} \tag{16}$$

where $N$ is the number of channel estimation matrix which is used for simulation, $\{P_{k,i}\}_{k=1}^{n_T}$ is the transmit power sequence for $i$-th channel matrix $\hat{\mathbf{H}}_i$, $\{\hat{\lambda}_{k,i}\}_{k=1}^{n_T}$ are eigenvalues of $\hat{\mathbf{H}}_i\hat{\mathbf{H}}_i^{\mathrm{H}}$, $\sigma_{e,i}^2$, and $\sigma_{n,i}^2$ are the variances of channel estimation and AWGN, respectively.

## 4. Iterative Detection Algorithm for the Modified Turbo-BLAST System

In this section, the iterative detection strategy is employed for the modified Turbo-BLAST system with imperfect CSI to enhance the BER performance. The iterative decoding structure of serially concatenated turbo codes provides the principal model for the iterative decoding algorithm [4].

In practice, true CSI cannot be obtained, so the receiver treats $\hat{\mathbf{H}}$ as the true channel matrix and $\hat{\mathbf{n}}$ as the equivalent noise. The iterative detection scheme for the modified Turbo-BLAST system will be described next.

Let $x_k$ be the $k$-th transmitted signal at a sampling epoch, with (4), the received symbol vector $\mathbf{r}$ corrupted by the channel noise and interferences is given as

$$\mathbf{r} = \hat{\boldsymbol{\lambda}}_k\sqrt{P_k}x_k + \hat{\boldsymbol{\Lambda}}_{\bar{k}}\mathbf{P}_{\bar{k}}\mathbf{x}_{\bar{k}} + \hat{\mathbf{n}}, \tag{17}$$

where $\hat{\boldsymbol{\lambda}}_k$ is the $k$-th column of the matrix $\hat{\boldsymbol{\Lambda}}$ and

$$\begin{aligned}\hat{\boldsymbol{\Lambda}}_{\bar{k}} &= \left[\hat{\boldsymbol{\lambda}}_1,\ldots,\hat{\boldsymbol{\lambda}}_{k-1},\hat{\boldsymbol{\lambda}}_{k+1},\ldots,\hat{\boldsymbol{\lambda}}_{n_T}\right],\\ \mathbf{x}_{\bar{k}} &= \left[x_1,\ldots,x_{k-1},x_{k+1},\ldots,x_{n_T}\right]^{\mathrm{T}},\\ \mathbf{P}_{\bar{k}} &= \operatorname{diag}\left(\sqrt{P_1},\ldots,\sqrt{P_{k-1}},\sqrt{P_{k+1}},\ldots,\sqrt{P_{n_T}}\right).\end{aligned} \tag{18}$$

With (18), the decision statistic of $k$-th substream using a linear filter $\hat{\mathbf{w}}_k$ can be expressed as

$$y_k = \underbrace{\hat{\mathbf{w}}_k^{\mathrm{H}}\hat{\boldsymbol{\lambda}}_k\sqrt{P_k}x_k}_{\hat{q}_k} \hat{+} \underbrace{\hat{\mathbf{w}}_k^{\mathrm{H}}\hat{\boldsymbol{\Lambda}}_{\bar{k}}\mathbf{P}_{\bar{k}}\mathbf{x}_{\bar{k}}}_{\hat{d}_k} + \underbrace{\hat{\mathbf{w}}_k^{\mathrm{H}}\hat{\mathbf{n}}}_{\hat{z}_k}, \tag{19}$$

where $\hat{q}_k$, $\hat{d}_k$, and $\hat{z}_k$ are the desired response obtained by the linear filter, the coantenna interference, and the phase-rotated noise, respectively.

The coantenna interference can be removed from $y_k$ by the proposed multisubstream detector and soft interference cancellation based on MMSE principle. The improved estimation of the transmitted symbol $x_k$ can be formulated as

$$\tilde{x}_k = \hat{\mathbf{w}}_k^H\mathbf{r} - \hat{d}_k, \tag{20}$$

where $\tilde{x}_k$ is the estimate of the symbol $x_k$, and $\hat{d}_k$ is the linear combination of the interfering substreams. The estimation error is defined as $\Delta x_k = \tilde{x}_k - x_k$. The weighted vector $\hat{\mathbf{w}}_k$ and the interference estimation $\hat{d}_k$ are optimized by minimizing the mean-square estimation error of $\Delta x_k$ between each substream and the related estimate by the following cost function

$$\left(\hat{\mathbf{w}}_k, \hat{\bar{d}}_k\right) = \arg\min_{\left(\hat{\mathbf{w}}_k, \hat{\bar{d}}_k\right)} \varepsilon\left[\left|\left|\tilde{x}_k - x_k\right|\right|_2^2\right], \tag{21}$$

where the expectation is taken over the equivalent noise $\hat{\mathbf{n}}$ and the statistics of the data sequence $\mathbf{x}$.

We use standard minimization techniques to solve the optimization problem formulated in (21). In order to arrive at the solution, we write the cost function as

$$\begin{aligned}\mathrm{Cost} &= \varepsilon\left[\left|\left|\tilde{x}_k - x_k\right|\right|_2^2\right] = \varepsilon\left[\left|\left|\hat{\mathbf{w}}_k^H\mathbf{r} - \hat{d}_k - x_k\right|\right|_2^2\right] \\ &= \hat{\mathbf{w}}_k^H\varepsilon\left(\mathbf{r}\mathbf{r}^H\right)\hat{\mathbf{w}}_k - \hat{\mathbf{w}}_k^H\varepsilon\left[\mathbf{r}\left(\hat{d}_k + x_k\right)^*\right] \\ &\quad - \varepsilon\left[\mathbf{r}\left(\hat{d}_k + x_k\right)^*\right]^H\hat{\mathbf{w}}_k + \varepsilon\left[\left(\hat{d}_k + x_k\right)\left(\hat{d}_k + x_k\right)^*\right],\end{aligned} \tag{22}$$

where

$$\begin{aligned}\varepsilon\left[\mathbf{r}\mathbf{r}^H\right] &= \varepsilon\left[\hat{\boldsymbol{\lambda}}_k\sqrt{P_k}x_k + \hat{\boldsymbol{\Lambda}}_{\bar{k}}\mathbf{P}_{\bar{k}}\mathbf{x}_{\bar{k}} + \hat{\mathbf{n}}\right. \\ &\qquad \left.\times\left(\hat{\boldsymbol{\lambda}}_k\sqrt{P_k}x_k + \hat{\boldsymbol{\Lambda}}_{\bar{k}}\mathbf{P}_{\bar{k}}\mathbf{x}_{\bar{k}} + \hat{\mathbf{n}}\right)^H\right] \\ &= P_k\hat{\boldsymbol{\lambda}}_k\hat{\boldsymbol{\lambda}}_k^H + \hat{\boldsymbol{\Lambda}}_{\bar{k}}\mathbf{P}_{\bar{k}}\varepsilon\left(x_{\bar{k}}x_{\bar{k}}^H\right)\left(\hat{\boldsymbol{\Lambda}}_{\bar{k}}\mathbf{P}_{\bar{k}}\right)^H \\ &\quad + (n_T\sigma_e^2 + \sigma_n^2)\mathbf{I}_{n_R}, \\ \varepsilon\left[\mathbf{r}\left(\hat{d}_k + x_k\right)^*\right] &= \varepsilon\left[\left(\hat{\boldsymbol{\lambda}}_k\sqrt{P_k}x_k + \hat{\boldsymbol{\Lambda}}_{\bar{k}}\mathbf{P}_{\bar{k}}\mathbf{x}_{\bar{k}} + \hat{\mathbf{n}}\right)\left(\hat{d}_k + x_k\right)^*\right] \\ &= \hat{\boldsymbol{\lambda}}_k\sqrt{P_k} + \hat{\boldsymbol{\Lambda}}_{\bar{k}}\mathbf{P}_{\bar{k}}\varepsilon\left(\mathbf{x}_{\bar{k}}\right)\hat{d}_k^*.\end{aligned} \tag{23}$$

By (23), the cost function in (22) can be further evaluated as

$$\begin{aligned}\mathrm{Cost} = \hat{\mathbf{w}}_k^H&\left\{P_k\hat{\boldsymbol{\lambda}}_k\hat{\boldsymbol{\lambda}}_k^H + \left(\hat{\boldsymbol{\Lambda}}_{\bar{k}}\mathbf{P}_{\bar{k}}\right)\varepsilon\left[x_{\bar{k}}x_{\bar{k}}^H\right]\left(\hat{\boldsymbol{\Lambda}}_{\bar{k}}\mathbf{P}_{\bar{k}}\right)^H\right. \\ &\left. + (n_T\sigma_e^2 + \sigma_n^2)\mathbf{I}_{n_R}\right\}\hat{\mathbf{w}}_k \\ &- \hat{\mathbf{w}}_k^H\left[\hat{\boldsymbol{\lambda}}_k\sqrt{P_k} + \hat{\boldsymbol{\Lambda}}_{\bar{k}}\mathbf{P}_{\bar{k}}\varepsilon\left(x_{\bar{k}}\right)\hat{d}_k^*\right] \\ &- \left[P_k\hat{\boldsymbol{\lambda}}_k^H + \hat{d}_k\varepsilon\left(x_{\bar{k}}\right)^H\left(\hat{\boldsymbol{\Lambda}}_{\bar{k}}\mathbf{P}_{\bar{k}}\right)^H\right]\hat{\mathbf{w}}_k \\ &+ \varepsilon\left[\hat{d}_k\hat{d}_k^* + \hat{d}_k x_k^* + x_k\hat{d}_k^* + x_k x_k^*\right].\end{aligned} \tag{24}$$

The linear combination of interfering substreams $\hat{d}_k$ and the weighted vector $\hat{\mathbf{w}}_k$ is obtained from (24) by letting $\partial\mathrm{Cost}/\partial\hat{d}_k^* = 0$ and $\partial\mathrm{Cost}/\partial\hat{w}_k^* = 0$, respectively,

$$\begin{aligned}\hat{d}_k &= \hat{\mathbf{w}}_k^H\left(\hat{\boldsymbol{\Lambda}}_{\bar{k}}\mathbf{P}_{\bar{k}}\right)\varepsilon\left(\mathbf{x}_{\bar{k}}\right), \\ \hat{\mathbf{w}}_k &= \left[\mathbf{Q} + \mathbf{S} + (n_T\sigma_e^2 + \sigma_n^2)\mathbf{I}_{n_R}\right]^{-1}\left(\hat{\boldsymbol{\lambda}}_k\sqrt{P_k}\right),\end{aligned} \tag{25}$$

where

$$\begin{aligned}\mathbf{Q} &= \left(\hat{\boldsymbol{\lambda}}_k\sqrt{P_k}\right)\left(\hat{\boldsymbol{\lambda}}_k\sqrt{P_k}\right)^H, \\ \mathbf{S} &= \left(\hat{\boldsymbol{\Lambda}}_{\bar{k}}\mathbf{P}_{\bar{k}}\right)\left\{\mathbf{I}_{(n_T-1)} - \mathrm{diag}\left[\varepsilon\left(\mathbf{x}_{\bar{k}}\right)\varepsilon\left(\mathbf{x}_{\bar{k}}\right)^H\right]\right\}\left(\hat{\boldsymbol{\Lambda}}_{\bar{k}}\mathbf{P}_{\bar{k}}\right)^H.\end{aligned} \tag{26}$$

Therefore the weighted vector $\hat{\mathbf{w}}_k$ is used for the modified Turbo-BLAST system in the presence of imperfect CSI. Thus (20) can be rewritten as

$$\begin{aligned}\tilde{x}_k = \hat{\mathbf{w}}_k^H\mathbf{r} - \hat{d}_k &= \left\{\left[\mathbf{Q} + \mathbf{S} + (n_T\sigma_e^2 + \sigma_n^2)\mathbf{I}_{n_R}\right]^{-1}\left(\hat{\lambda}_k\sqrt{P_k}\right)\right\}^H \\ &\quad \times\left[\mathbf{r} - \left(\hat{\boldsymbol{\Lambda}}_{\bar{k}}\mathbf{P}_{\bar{k}}\right)\varepsilon\left(\mathbf{x}_{\bar{k}}\right)\right].\end{aligned} \tag{27}$$

For the first iteration, we assume that $\varepsilon(\mathbf{x}_{\bar{k}}) = \mathbf{0}$, and thus the linear MMSE detection for the $k$-th substream becomes

$$\begin{aligned}\tilde{x}_k &= \left\{\left[\left(\hat{\boldsymbol{\lambda}}_k\sqrt{P_k}\right)\left(\hat{\boldsymbol{\lambda}}_k\sqrt{P_k}\right)^H + \left(\hat{\boldsymbol{\Lambda}}_{\bar{k}}\mathbf{P}_{\bar{k}}\right)\left(\hat{\boldsymbol{\Lambda}}_{\bar{k}}\mathbf{P}_{\bar{k}}\right)^H\right.\right. \\ &\qquad \left.\left. + (n_T\sigma_e^2 + \sigma_n^2)\mathbf{I}_{n_R}\right]^{-1}\left(\hat{\boldsymbol{\lambda}}_k\sqrt{P_k}\right)\right\}^H\mathbf{r} \\ &= \left(\hat{\boldsymbol{\lambda}}_k\sqrt{P_k}\right)^H\left[\left(\hat{\boldsymbol{\Lambda}}\mathbf{P}\right)\left(\hat{\boldsymbol{\Lambda}}\mathbf{P}\right)^H + (n_T\sigma_e^2 + \sigma_n^2)\mathbf{I}_{n_R}\right]^{-1}\mathbf{r}.\end{aligned} \tag{28}$$

Next, we assume that $\varepsilon(\mathbf{x}_{\bar{k}}) \to \mathbf{x}_{\bar{k}}$ with the increasing number of iterations, where the MMSE interference canceller for the modified Turbo-BLAST system using SVD-based power allocation with imperfect CSI is

$$\begin{aligned}\tilde{x}_k &= \left(\hat{\boldsymbol{\lambda}}_k\sqrt{P_k}\right)^H\left[\left(\hat{\boldsymbol{\lambda}}_k\sqrt{P_k}\right)^H\left(\hat{\boldsymbol{\lambda}}_k\sqrt{P_k}\right) + (n_T\sigma_e^2 + \sigma_n^2)\right]^{-1} \\ &\quad \times\left[\mathbf{r} - \left(\hat{\boldsymbol{\Lambda}}_{\bar{k}}\mathbf{P}_{\bar{k}}\right)\mathbf{x}_{\bar{k}}\right].\end{aligned} \tag{29}$$

The MMSE detector gets the estimation of all the transmit signal using (29) and generates soft estimates of the coded bits conditioned on the received signal. The iterative decoder depicts message passing between the MMSE detector and SISO channel decoder. During the first iteration, the initial intrinsic probabilities of all symbol bits are assumed to be 1/2 (i.e., equally likely). The posteriori information is computed and fed back to the SISO channel decoder as the intrinsic information of its all coded bits. Then the SISO modules, in turn, process the soft information and compute refined estimates of soft information on both coded and information bits, based on the trellis structure of the channel codes, which is the channel code constraint. The SISO channel decoders are implemented by using the generalized BCJR algorithm [18].The soft information is sent to MMSE detector for interference cancellation [4]. Steps above are repeated until the decoding algorithm converges.

## 5. Simulation Results

In this section we compare the capacity and the BER performance of the traditional and modified Turbo-BLAST systems, where the proposed adaptive SVD-aided power allocation strategy is adopted in the latter case. At the transmitter, the data stream is first encoded by a rate-1/2 convolutional code with generator (7, 5), then modulated by 4-QAM scheme, whose capacity performance over a large number of channel realizations is exhibited in Figures 2 and 3, extra capacity, which is defined as the capacity gap between the modified Turbo-BLAST system and the conventional one, is given in Figure 4, and finally BER performance is presented in Figure 5, respectively, with the traditional Turbo-BLAST system adopting the equal power allocation strategy, denoted by "EPA", and the modified Turbo-BLAST system employing the proposed transmit power allocation, denoted by "TPA", respectively.

Figure 2 shows that the capacity performance of the modified Turbo-BLAST system outperforms the traditional one in all cases under the same conditions. This indicates that the proposed SVD-aided power allocation strategy is an effective means to improve capacity performance even with the imperfect CSI. For example, at $E_b/N_0 = 10$ dB, there are 0.8 (bit/s/Hz) and 1 (bit/s/Hz) capacity gains for $\sigma_e^2 = 0.04$ and $\sigma_e^2 = 0.1$, respectively.

Figure 3 shows that the capacity degradation is relatively insensitive to the CSI imperfection when CSI is more accurate. Only when $\sigma_e^2$ increases to nearly $10^{-2}$, the capacity starts to fall. The over SNR degrades as the noise contribution due to the use of imperfect CSI increases which determine the detection performance. Figure 3 also shows that the impact of CSI imperfection on the capacity results is higher for larger $E_b/N_0$.

It is clear that from Figure 4, we can see that the extra capacity is higher for low SNR and/or large antenna systems in the presence of CSI imperfection. For instance, at $E_b/N_0 = 4$ dB, there are 3.2 (bit/s/Hz) and 1.3 (bit/s/Hz) capacity gains for the Turbo-BLAST system with eight and seven transmit antennas, respectively.

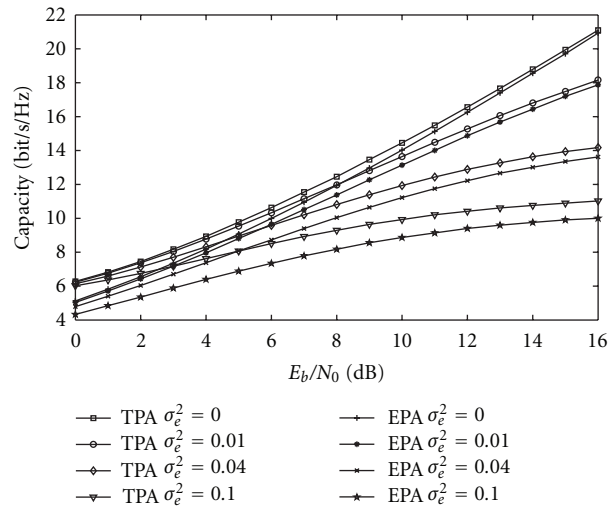


Figure 2: Capacity performance of traditional and modified Turbo-BLAST systems with four transmit and four receive antennas under the conditions of different inaccuracy of channel estimation.

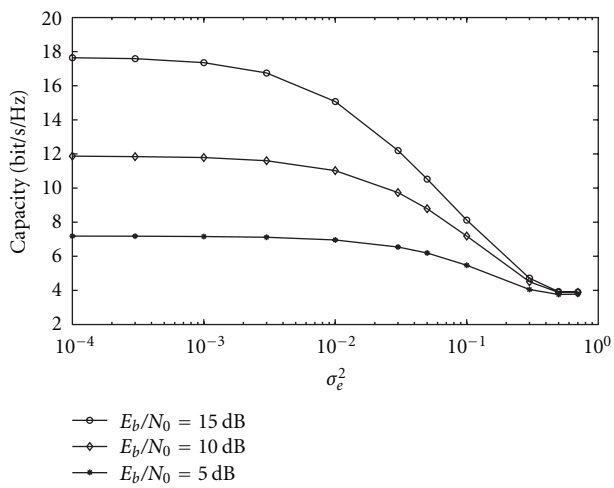


Figure 3: Impacts of CSI on the capacity performance of modified Turbo-BLAST systems with four transmit and four receive antennas in various values of $E_b/N_0$.

It can be observed from Figure 5 that as the BER results concerned, the modified Turbo-BLAST system outperforms the traditional system under the same conditions, regardless of the status of CSI. This implies that the proposed power allocation strategy is also valid to improve BER performance even with the imperfect CSI. For example, at a BER of $10^{-4}$, we can see that there are 5 dB gains for the modified system over the traditional one in the 1st detection iteration under the condition of perfect CSI, that is, $\sigma_e^2 = 0$. Moreover, the BER results are quickly lowered with the increasing detection iterations. For instant, at a BER of $10^{-4}$, there are 3 dB extra gains attained for the modified Turbo-BLAST system using TPA scheme through 3rd iterative detection under the condition of perfect CSI.

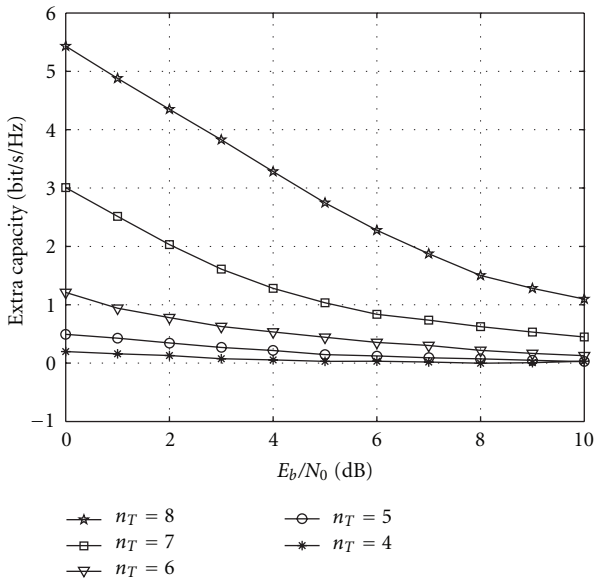


Figure 4: Extra capacity results between modified and traditional Turbo-BLAST systems with eight receive antennas and various transmit antennas configuration under the conditions of imperfect CSI ($\sigma_e^2 = 0.01$).

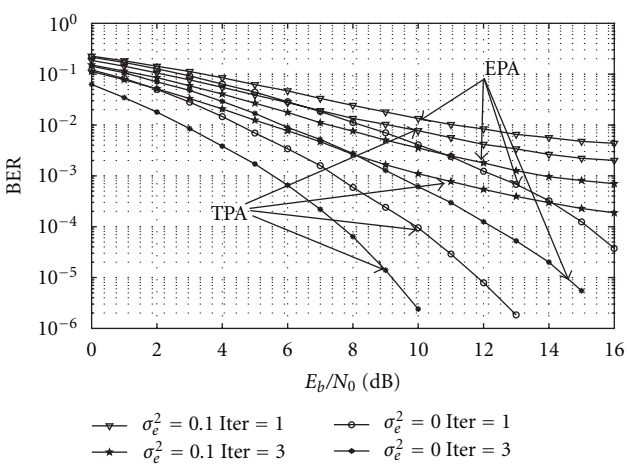


Figure 5: BER performance of traditional and modified Turbo-BLAST systems with four transmit and four receive antennas under the conditions of different inaccuracy of channel estimation.

## 6. Conclusion

In this paper, we propose a new strategy that combines SVD technique, transmit power allocation, and iterative detection techniques to improve both the capacity results and the BER performance for the modified Turbo-BLAST system.

Based on the equivalent system model used in this paper and the derived variance of the equivalent noise, the SVD-assisted transmit power allocation strategy is theoretically obtained. The Lagrange multiplier method is then employed to find the optimized power allocation matrix which can maximize the capacity for the whole system subject to the total transmit power constraint. Finally, the iterative detection technique is adopted for the modified Turbo-BLAST system to enhance the BER performance. Simulation results show that the newly introduced method is effective to enhance not only the capacity results but also the BER performance of Turbo-BLAST system in both cases of perfect and imperfect CSI.

## Acknowledgments

This paper is supported by Doctoral Fund of Ministry of Education of China (no. 20093218120021), Open Research Fund of National Mobile Communications Research Laboratory, Southeast University (no. N200904), and NUAA Research Funding (NS 2010113). The authors would like to thank the anonymous reviewers for their valuable comments which improve the quality of this paper greatly.

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