Review Article

Trends in Adaptive Array Processing

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Enormous progress has been made during the past five decades in the area of adaptive array processing. Increased computational power has resulted in many practical applications of optimum algorithms. The present paper deals with many facets of array signal processing and adaptive beam forming. It provides a comprehensive description of various beam-forming schemes, adaptive algorithms to adjust the required weighting on antenna elements, direction-of-arrival estimation methods, including their performance comparison. The effects of various types of errors on the performance of an array system are illustrated along with their remedial measures. Since array signal processing has widespread applications, the study is carried out across various disciplines.

1. Introduction

Array processing plays an important role in many diverse application areas. Most modern radar and sonar systems rely on antenna arrays as an essential component of the system. Adaptive arrays are capable of spatial filtering, which makes it possible to receive energy from a particular direction while simultaneously blocking it from another direction. This property makes the adaptive arrays a very effective means for detecting and locating an underwater source of sound such as submarine without using active sonar. Many communication systems utilize the phased arrays or multiple beam antennas for their performance objectives. Seismic arrays are widely used for oil exploration and detection of underground nuclear tests. Various medical diagnosis and treatment techniques exploit arrays because of their capability to transmit energy directly towards a desired direction. Radio astronomy utilizes very large antenna arrays to achieve resolution goals.

Adaptive beamforming is a versatile approach for enhance-ment of a signal of interest (SOI) while suppressing interference signals and noise at the output of the array of sensors. The fundamental of such systems is not new; they essentially depend on Weiner's filter theory. Earlier their application in practice had been limited both by technology and by the absence of robust algorithms rapid enough for real-time operation. Rapid developments in the past decades in the fields of electronic components and computer technology have changed the situation by offering the possibility of complicated signal processing in real time at economical costs. Adaptive beamforming technology has been successfully utilized in a wide range of applications in both defense and civilian areas. An array comprises of a set of sensors, the outputs of which are combined in to produce a desired effect. By tuning the amplitude and phase of the wavefronts at each sensor, it is possible to electronically steer the beam in the desired direction and to place nulls in other directions [1]. It is well known that the receiving array antenna can steer its main beam toward any direction by adjusting the complex weight of each element. However it is difficult to steer the beam and null simultaneously towards two prespecified directions, with a single set of weights. This ability of array depends on several factors, namely, element positions, orientation of elements, antenna patterns of elements, polarization of signals, and the direction of the beams and/or nulls [2]. The coefficient called spatial correlation includes these factors and completely characterizes array beam pointing and nulling [3]. In adaptive arrays, the beam pattern is optimized so that maximum gains are offered in specific directions corresponding to the desired signal, while maximum attenuation is placed in specific directions that

correspond to the undesired interference signal or jammer. This simple principle is effective only when one is dealing with narrowband signals such that they can be adequately characterized by single phase and single amplitude terms.

For broadband signals, care must be taken in order to equalize the matched amplitude and frequency responses of each channel, including the group delay in the paths to a summing point. The lack of path matching leads to amplitude and phase distortions across the cancellation band of the wanted signal. This requires some form of frequency-dependent weighting [4]. A broadband adaptive beamformer consists of a multi-input single output linear combiner and an adaptive algorithm that adjusts the weights optimally [5]. Broadband beamforming is employed when the nature of the signals of interest is wideband. One advantage of the narrowband system is that the array can be steered simply by phase changes, whereas the broadband system involves physical delay. Thus the broadband system requires simultaneous processing of a large number of samples from each sensor and is more complex. An alternative approach to obtain optimum solution in adaptive processing is the use of constraints. It is assumed in a conventional design of adaptive array algorithms that there are no desired signal components in the beamformer training data. However this assumption is valid only for specific applications.

If signal-free beamformer training snapshots are available, the adaptive algorithms are known to be quite robust vis a vis the errors in the steering vector of the desired signal [6]. However the situation is entirely different if the desired signal is present in the error signal. In this case, the beamformer suffers from the *signal cancellation* phenomenon. This results in severe degradation in performance of the array and the rate of convergence. The currently available techniques for adaptive array are mostly based on the assumption of an accurate knowledge of the array response to the desired signal. This may result in drastic degradation of the performance, if the assumptions made about the environment and antenna array are wrong or inaccurate [7].

In the present paper, a comprehensive description of various available adaptive algorithms for weight estimation, beam-forming techniques, including the comparison of their performance is provided in Sections 2 and 3, respectively. Section 4 describes the effects of various types of errors on the performance of an array system along with their remedies. Various techniques for enhancement of robustness of adaptive arrays are also discussed. Finally in Section 5, the role of various parameters in deciding the interference suppression capabilities of array is described. A schematic diagram (Figure 1) shows the flow of various concepts and approaches outlined here. The paper contains extensive references related to the application of adaptive arrays in various disciplines.

2. Algorithms for Adaptive Arrays

The optimal weights of an array, which maximizes the output SNR in the absence of errors, are in general computed using the array correlation matrix of noise and the steering vector in the look direction. Several algorithms have been proposed and studied for adaptive beamforming. There are two main approaches leading to three widely used classes of algorithms, for the adaptive estimation of optimum mean square error (MSE) parameters based on the available data set. The first one is based on the stochastic approximation of the method of steepest descent and is known as the Least Mean Squared (LMS) family. These are gradient-based optimization algorithms employing a noisy estimate of the required gradient vector. The second class consists of matrix inversion algorithms, which are based on the inversion of the sample correlation matrix of the array processor. Yet another class is based on the stochastic approximation of the Gauss-Newton method and is known as the Recursive Least Squares (RLSs) family. They utilize the matrix inversion lemma to develop sample-by-sample updating of the inverse of sample correlation matrix.

The original work in the area of adaptive array algorithms was done by [8–10]. The Shor algorithm [8] maximizes the signal-to-noise ratio (SNR) at the array output, where noise includes the interference. Since both the desired signal and interference ought to be known, the Shor's algorithm was difficult to implement. The Applebaum algorithm [9] too maximizes the SNR, but under the assumption that the desired signal is absent most of the time, as in a pulsed radar. The *Applebaum array* is more practical to implement and has been used in various radar system clutter and interference rejection problems. The adaptive array concept due to Widrow et al. [10], which is known as the LMS algorithm, minimizes the mean square error between the actual array output and a desired array output, called the reference signal. The LMS algorithm minimizes the waveform distortion (where the reference signal is the replica of the desired signal), whereas the Applebaum algorithm maximizes SNR. The difference between the weights estimated by the adaptive algorithm and the optimal weights is further characterized by the ratio of the average excess steady-state MSE and the minimum mean square error (MMSE). This is referred to as the *misadjustment*, which is a dimensionless parameter and a measure of the algorithm performance. It essentially results from the use of noisy estimate of the gradient. The resulting noise is referred to as the *misadjustment noise* [11]. As the step size is increased, the misadjustment noise also gets increased. On the other hand, it leads to faster convergence. Moreover, instead of having a single step size for an entire weight vector, one may select a variable step size for each weight separately, leading to an increased convergence of the algorithm.

2.1. Least Mean Square Algorithm. The original LMS algorithm [12, 13] does not require any constraint on the weights and is thus referred to as the unconstrained LMS algorithm. The constrained LMS algorithm has been applied to adaptive beamforming by many researchers [5, 14, 15]. The analyses of these algorithms include the transient behavior of the weights, the convergence, and the misadjustment analysis. Adaptive array processing employing a constrained LMS algorithm requires an unbiased estimate of the gradient of
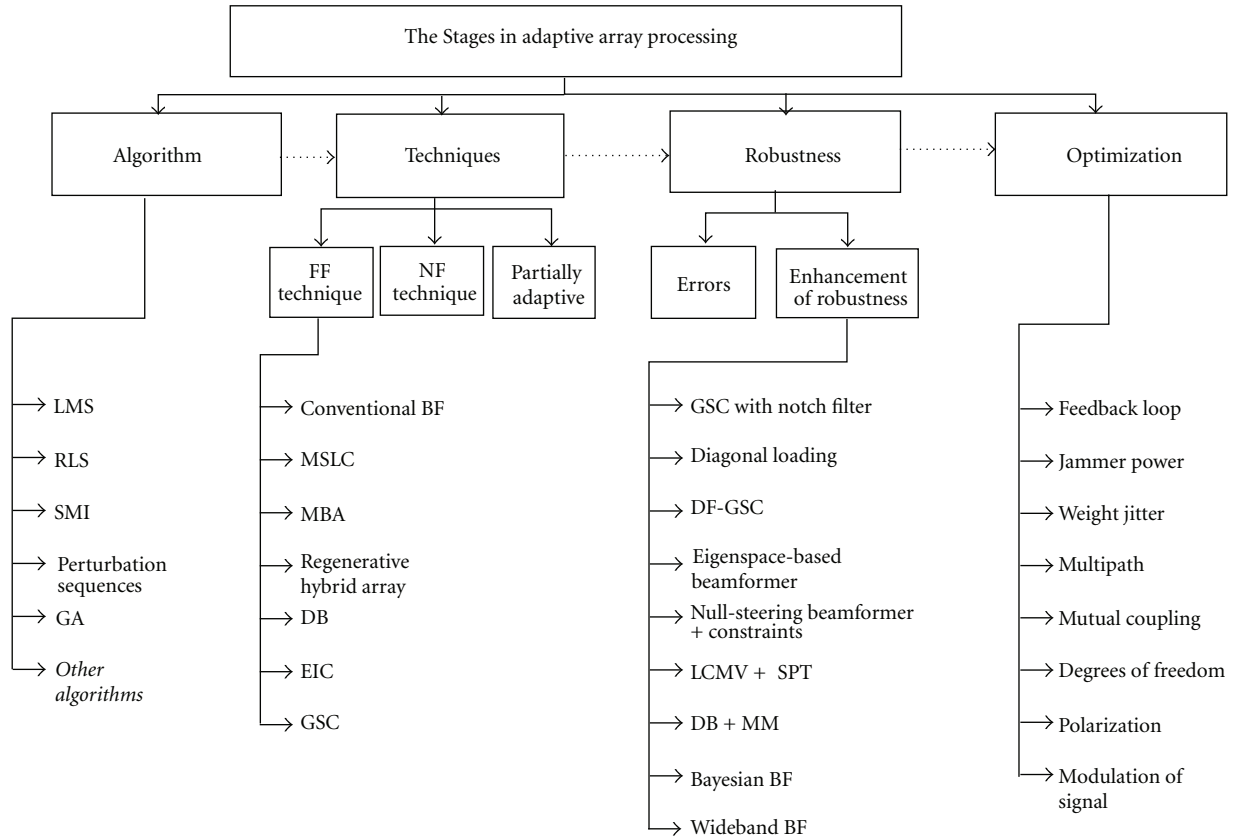
FIGURE 1: Schematic chart for adaptive array processing.

the output power with respect to the array weights. There are a number of schemes for determining the unbiased estimate of this gradient. Though in every scheme the gradient is unbiased, the covariance of the estimated gradient with each method is however different and thus the transient and the steady state behavior of the constrained algorithm are different in each case.

2.1.1. Standard LMS. The LMS algorithm updates the weights at each iteration by estimating the gradient of the quadratic surface and then moves the weights in the negative direction of the gradient by a small amount. The constant determining this amount is referred to as the step size. When this step size is small enough, the process leads the estimated weights to approach the optimal value. This is referred to as standard LMS algorithm [16].

2.1.2. Structured Gradient LMS. An alternative method, referred to as the structured gradient LMS, is by Godara and Gray [17]. This algorithm exploits the Toeplitz structure of the array correlation matrix (ACM) for estimating the gradient. The noisy estimate of the ACM, used in the standard algorithm, is not constrained to have this structure. The convergence rate and the required step size are found to be sensitive to the signal power in the look direction. This signal sensitivity is greatly reduced by structured gradient method. Further, in the presence of strong signal in the look

direction, the standard algorithm requires a smaller step size, and hence a slower rate of convergence than the structured gradient LMS algorithm.

2.1.3. Improved LMS. In this algorithm, the gradient is not only estimated from ACM, but also uses all the previously available samples for updating the weights of the array. Figure 2 demonstrates the learning curves of decision feedback generalized sidelobe canceller (DFGSC) scheme of adaptive arrays for standard LMS, structured LMS, and improved LMS algorithms. It is apparent from the graph that improved LMS and structured LMS algorithms have far better performance than the standard LMS algorithm.

2.1.4. Normalized LMS. For certain conditions, the convergence time of the adaptive array can be longer than the radar dwell time [16]. It is found that because of long convergence time a single low power jammer can be more effective than higher power jammers. Although the techniques based on various types of covariance matrix are available, their implementation cost is quite high. The normalized LMS (NLMS) algorithm [18] can substantially decrease the convergence time with same implementation cost as of conventional LMS. It is shown that, by neglecting the thermal noise, the convergence behavior of normalized LMS algorithm becomes independent of the jammer power for the single jammer case. For multiple jammers, the convergence time is
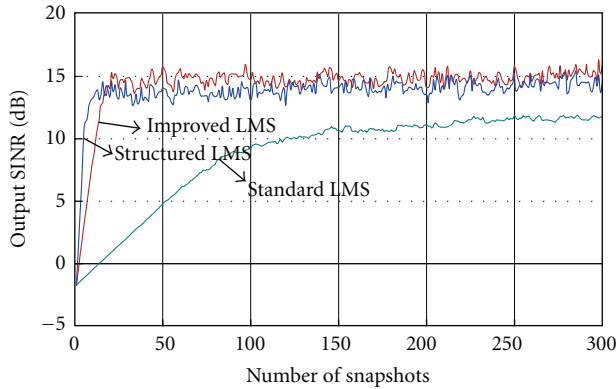
FIGURE 2: Learning curve of DF-GSC scheme of adaptive array using different LMS algorithms.

FIGURE 3: Mean output noise power versus the iteration number. 10 elements, $\lambda/2$ spacing, phi = 90°; 2 jammers: 72°, power ratio (p.r.): 100; 98°, p.r.: 1.

unchanged when each jammer power changes by the same factor [19]. Another form of normalized LMS algorithm, proposed by Bershad [20], is a variation of the constant-step-size LMS algorithm. It uses the data-dependent step size at each iteration and avoids the need for estimating the eigenvalues of the correlation matrix or its trace. This algorithm has a better convergence speed and is less sensitive to signal compared to the standard LMS.

2.1.5. Other Forms of LMS. A recursive LMS algorithm uses all the previous samples and updates the correlation matrix upon the arrival of a new sample. This matrix is then used for estimating the required gradient. Ogawa et al. [21] analyzed the steady-state performance of the LMS adaptive array in which a pilot signal is used as a reference signal. The LMS array accepts all the signals except the reference pilot signal, which is taken to be an interference. In order to separate the pilot signal from other components easily, its frequency is taken to be different from that of the information signal. It has been shown that such array seldom suppresses the desired signal. Other algorithms that have been proposed for adaptive arrays include recursive algorithms, namely, least squares estimation techniques, additional modified versions of LMS algorithms, and covariance matrix inversion methods.

2.2. Recursive Least Square (RLS) Algorithm. This algorithm replaces the gradient step size in conventional LMS algorithm by the gain matrix. The RLS algorithm offers better convergence rate, steady-state MSE, and the parameter tracking capability over the LMS-based algorithms. However its complexity is proportional to the squared number of the unknown system parameters. To overcome this drawback, a numerically stable RLS algorithm, called the QR-decomposition RLS (QRD-RLS), was proposed [22]. In this algorithm, the QR decomposition of the input data matrix is computed using *Givens rotation* and the least squares (LSs) weight vector is solved by back substitution. The back substitution however is a costly operation to be performed in the array structure. So an inverse QRD-RLS algorithm was proposed where the LS weight vector is computed without
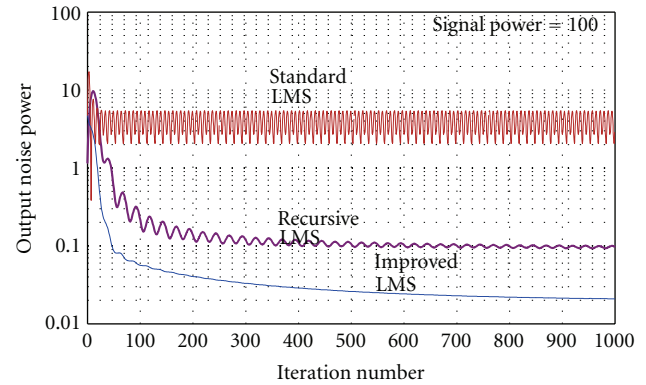
the back substitution [23, 24]. The algorithm, which has better convergence performance than RLS algorithm in the presence of the strong look-direction signal, is the improved LMS [25]. It uses the structured method to estimate the correlation matrix using all the samples (Figure 3). Many other forms of RLS algorithms have been proposed that improve the computational efficiency [26].

2.3. Sample Matrix Inversion Algorithm. The sample matrix inversion (SMI) algorithm is probably used most often when rapid convergence is required. SMI algorithm overcomes many convergence problems of the feedback-loop adaptive arrays [27] and is an attractive alternative to the LMS algorithm. It uses an estimate of the Weiner weight solution to determine the optimum weights. In this method, the noise covariance matrix must be nonsingular so as to be invertible. The time required to adapt is fixed in this method. It is not the eigenvalue but the eigenvalue spread of the covariance matrix that controls the weights. There are modified SMI algorithms, which construct adapted weight vectors in the case of singular noise covariance matrices. For example, the technique using Buehring's orthogonal projection method [28] involves the iterative estimation of the weight vectors and has an accelerated rate of convergence.

2.4. Algorithms Using Perturbation Sequences. Algorithms for adjusting weights of an adaptive array are usually based on gradient estimates. Many of these techniques require measurement or estimation of all the array element signals. In situations where this access is not available or not economical, one could estimate the required gradient using perturbation schemes. The development of single-port algorithms, which sample the beamformer output only, has made the implementation of adaptive array simpler and economical. Gradient estimation in single output beamformers can be achieved by perturbing the adaptive weights and correlating the resultant power sequence with the perturbation sequence [12].

2.5. Genetic Algorithms. Genetic algorithms (GAs) have also been employed to optimize adaptive arrays [29]. They have

been found to be more efficient than the method of steepest descent [30]. The literature survey has shown that the GA-controlled arrays are not yet used in real systems. A GA can be used to place a null in an array pattern in a specified location [31]. However the GAs cannot be used to control array patterns in the presence of unknown interference. Another application of GA lies in finding fault detection in the radiating elements of the array [32]. An advanced operator GA, called the dominance and diploidy GA (D&DGA), was proposed by Weile and Michielssen [33] for controlling an antenna array capable of both sensing and adapting to constantly changing conditions. The incorporation of diploidy and dominance in the algorithm was shown to improve the real-time performance of the algorithm in presence of changing interference. It was shown that D&DGA has elementary learning and memory capabilities, which can be explored for diverse applications.

2.6. Other Algorithms. One can infer that most of the adaptive algorithms are based on the covariance matrix of interference [34]. These algorithms suffer from two major drawbacks. Firstly, they require independent, identically distributed data to estimate the covariance matrix of the interference. But in early warning radar applications, the statistics of the interference may fluctuate rapidly over a short distance. This limits the availability of the homogeneous data resulting in the errors in the covariance matrix, which in turn reduces the ability to suppress the interferences. The second drawback is that the estimation of the covariance matrix requires the storage and processing of the data, which is computationally intensive. This covariance matrix computation can be avoided by the use of data-domain algorithms [35, 36].

Farina and Flam [37] proposed a method to normalize the step parameter and perturbation amount used for gradient search adaptive algorithms. The gradient search algorithm differs from other adaptive algorithms in that it does not require the knowledge of received signal. The conventional gradient search algorithm performs well only for a very limited range of jammer-to-noise ratio (JNR), while the normalized gradient search algorithm is reported to perform well against a wide range of JNRs, and has thus proven to be robust. An algorithm known as a *sign algorithm* [38], where the error between the array output and the reference signal is replaced by the sign, is computationally less complex than the LMS algorithm.

Since adaptive beamforming has to be performed in real time, low-cost algorithms should be preferred [39]. The LMS-based schemes offer adaptive processing at a lower cost, and thus they are preferred for real-time implementation of the adaptive beam-forming processing task. Computational savings, without the sacrifice of the performance, is a primary interest for the design of high-speed adaptive array systems, where the processing power of several GOPS (Giga operations per second) is required. On the other hand, in slow sampling rate adaptive beamforming systems, for example, in speech processing and hearing aids applications, where the special VLSI ASICs are required for small size and low power implementation, the reduction in the computational complexity is of main interest.

3. Techniques for Adaptive Array Processing

Array signal processing involves manipulations of signals induced on the elements of an array. There are various schemes to select the weights of the beamformer, each with its own characteristics and limitations. A conventional beamformer is a simple beamformer, sometimes referred as the delay-and-sum beamformer, with all its weights being of equal magnitudes. The phases are selected to steer the array in a particular direction, known as the look direction. The mean output power of the conventional beamformer steered in the look direction is equal to the power of the source in the look direction. It is similar to the mechanical steering except that it is done electronically by adjusting the phase.

Null-Steering Beamformer. This technique is used to cancel a plane wave arriving from a known direction and thus produce a null in the response pattern, in the direction of arrival (DOA) of the plane wave [40]. It involves signal estimation through the steering of conventional beam in the known direction of the source and the subtraction of its output from each element. The use of delay-and-sum beamforming network along with shift registers provides the required delay at each element such that the signal arriving from the beam-steering direction appears in phase after delay. The process is very effective in canceling strong interference and may be repeated for multiple interference cancellation.

Optimal Beamforming. Conventional and null-steering beamformer requires the knowledge of the directions of interference sources, and the output SINR is not maximized by the estimated weights [40]. Optimal beamformer overcomes these limitations. The weights are selected by minimizing the mean output power of the processor while maintaining the unity response in the look direction. The constraint included takes care that the signal remains undistorted. Further, the performance of the processor in terms of its output SNR and the array gain is not affected by the look-direction constraint, as it only scales the weights. Alternatively, adaptive array processing can be divided in two categories: element-space processing and beam-space processing [40]. In element-space processing, the signals derived from the elements are weighted and summed to produce the array output. The beam-space processing is a two-stage scheme where the first stage takes the array signals as input and produces a set of multiple outputs. These outputs are then weighted and combined to produce the array output.

In general, beam-space processing arrays are used in situations where the number of interferences is much less than the number of elements. They offer computational advantage over the element-space processing array. For element-space processing case, the constraints on the weights are imposed to prevent the signal arriving from the look direction from being distorted and thus to make the

array more robust against errors. A performance comparison of an element-space processor and beam-space processor for the case of a single interference case is presented by Godara [41]. The beam-space processor is reported to produce superior performance in the presence of look direction errors. Beam-space processors have been studied under many different names, including *Howell-Applebaum array* [9, 42], *Generalized sidelobe canceller* [43], partitioned processor [44, 45], partially adaptive arrays [46], and multibeam antennas [47]. One of the advantages of beam-space processing is that the number of degrees of freedom of an array that are used to achieve adaptivity is proportional to the number of the unwanted signals rather than the number of the antenna elements. In the absence of errors, both processing schemes yield identical results.

3.1. Far-Field Adaptive Processing. The adaptive receiving array for radar application was developed by Applebaum [9]. Similar techniques have been used in communication systems, in which the mean square error between the array output and a transmitted pilot signal that is known *a priori* is minimized.

3.1.1. Conventional Beamformers. Both Applebaum and LMS adaptive array techniques based on statistical methodologies were applied to analog signals for continuous operation and iteratively nulling the jammers. The LMS array [10] relies on a locally generated reference signal to guide feedback loops that generate the array weights. The Applebaum array uses a steering vector to control the feedback loops. This approach is vastly simpler as it does not require reference signal. The difficulty with this method is that the designer must know where to point the beam. Thus *a priori* knowledge of the arrival angle of the desired signal is required. When this angle is not known, one can choose the Applebaum array in the form of *power inversion array* [48], where steering vector turns one element on and the rest off. The turned-on element is so chosen that its element pattern covers some large sector of space from which desired signals may arrive. There is no steering of beam towards the desired signal involved. However the performance of power inversion array is poorer than that of the Applebaum array with a properly steered beam. It can accommodate only limited dynamic range of the desired signal. Compton [49, 50] has analyzed the performance of steered beam adaptive array as a function of beam-pointing error. It has been investigated as to how close the steered beam ought to be to the actual desired signal arrival angle for good performance. The algorithm employing the MSE criterion as a performance measure was further developed by Griffiths [13] and Frost [5]. The processors utilize tapped delay line networks and are applicable to broadband signals.

Bar-Ness [51] compared the performance of a constrained steered beam interference canceller and the LMS interference canceller to conclude that these two methods are complimentary to each other. Haber et al. [52] made use of this suggestion to present an adaptive array utilizing prior knowledge of both the approximate signal arrival
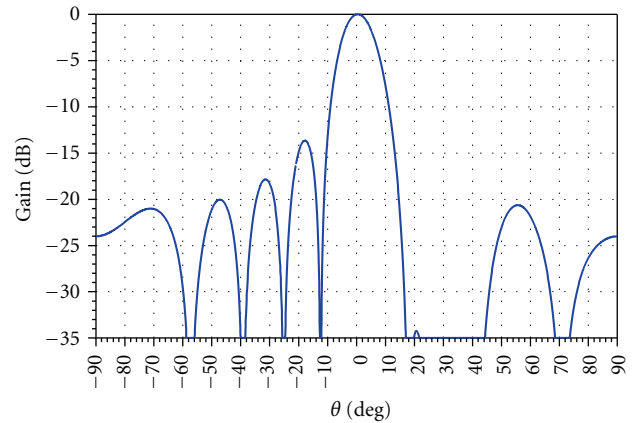

FIGURE 4: Adapted pattern of 10-element linear array with 4-narrowband jammers located in sidelobe region ($18°$, $25°$, $33°$, $42°$).

direction and signal characteristics. The method proposed combines the feature of a directionally constrained array and an array with a self-generated reference signal. The inclusion of a self-generated reference circuit is shown to reduce the sensitivity to pointing error, especially for the array having zero-order directional constraint. This improvement is due to the reduction of the desired signal component fed back to the sidelobe canceling circuit.

It is known that the LMS adaptive arrays, in principle, are capable of receiving signals and nulling interference from any angles. Whatever the signal arrival angles, the LMS array yields the maximum SINR attainable for a given set of elements. However, the SINR for specific arrival angles depends on the element patterns and spacing [53]. The eigenvalue behavior is important even if the adaptive array uses a weight control technique other than the LMS algorithm. Zohar [54] performed the steady-state analysis of adaptive arrays by representing the overall adapted pattern (Figure 4) as a linear combination of simpler basis patterns, each of which is a function of single source only. It is shown that unlike Gabriel's retrodirective eigenvector approach [55], the approach is applicable to an arbitrary, three-dimensional array configuration with arbitrary, different, antenna patterns for its element. Gupta [56] considered the case of multiple desired signals (Figure 5). It is assumed here that the quiescent pattern of the array is a multiple beam pattern with independent beams in each signal direction, taking care that each weight is according to the signal strength. It is shown that the performance of a steered beam adaptive array depends on the range of input signal strengths and the choice of the steering vector.

3.1.2. Multiple Sidelobe Canceller. The multiple sidelobe canceller (MSLC) is one of the earliest adaptive processing techniques used in canceling the interference from multiple sources located in the sidelobes of radar beam. Its action can be viewed as the adaptive combination of the unadapted main beam and a retrodirective or perturbation beam. The classical analysis of the retrodirective beam [56] is done in terms of the optimum weights, determined from known
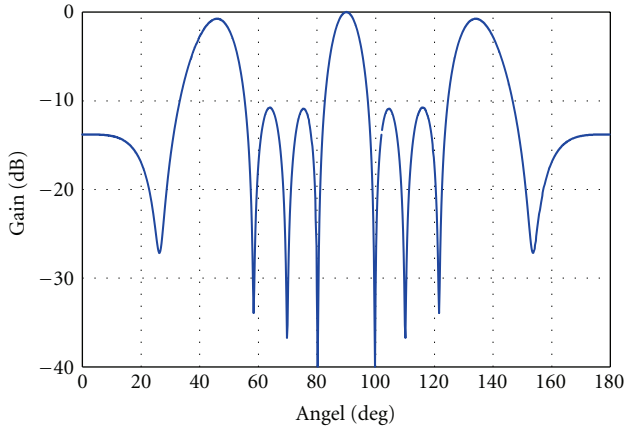
FIGURE 5: Adapted pattern of 10-element linear array with 3 desired signals (45°, 90°, 135°).
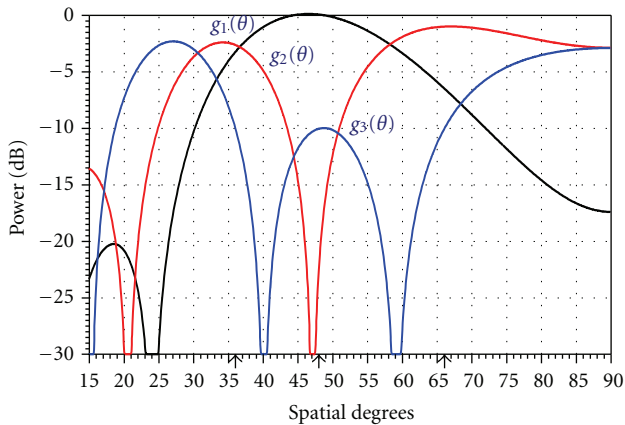

FIGURE 6: Retrodirective eigenvector beams $g_1(\theta)$, $g_2(\theta)$, and $g_3(\theta)$ for continuously distributed jammers in the sidelobe region from 30° to 90° of 8-element linear array; compare [55].

FIGURE 7: Adapted beam pattern of an antenna array with two desired signals (−20°, 20°) and four interfering sources (−60°, −40°, 5°, 40°). Both the interfering signals at −40° and 5° are correlated to signal at −20° with $\alpha = (0.1, -0.23)$ and $(-0.15, 0.08)$, respectively. $c = [1 \ 1 \ 0 \ 0]$; [62].

[60, 61] investigated the behavior of such arrays and found that the performance of the spatially smoothened adaptive arrays depends on the number of the subarrays, the angle separation, relative power, and the initial phase difference between the desired signal and the coherent jammer. Purswani et al. [62] used matrix reconstruction algorithm to decorrelate the signals and get the desired adapted pattern (Figure 7).

The principle of adaptive arrays is also applied in monopulse processing for the DOA measurements [63]. Their analysis makes use of maximum likelihood theory of angle estimation that leads to adaptive sum and difference beams. These beams formed by adaptive receiving array techniques automatically null, both the MLJ and SLJ interference sources. The disadvantage of this technique is that, along with the adaptive nulling, the antenna patterns tend to get distorted. Fante [64] attempted to improve this technique including the constraint of preserving the monopulse slope in the optimization of difference beam.

3.1.3. Multiple Beam Antenna System. One of the basic limitations of Applebaum-Howells processor is the adaptation speed. It has slower response towards the weak interfering sources. In addition, as the desired nulling bandwidth increases, the interference rejection capability inherent in the particular antenna configuration decreases. The spread in adaptation times against range of interference power levels can be viewed as the consequence of the spread in the eigenvalues of the interfering source covariance matrix defined at the antenna output ports. One way of accelerating the convergence is to decouple the inputs to the adaptive feedback system, which corresponds to diagonalizing the covariance matrix. An alternate way of decoupling the output ports is to incorporate the output ports with multiple beam antenna (MBA) system [65]. This leads to partially

covariance matrix *a priori*. The retrodirective beam can be decomposed into eigenvector beams (Figure 6) depending upon the eigenvectors of covariance matrix of the auxiliary array. The MSLC consists of a single high-gain main antenna to which a number of small auxiliary elements are summed. By controlling the adaptive weights in the auxiliary elements, the jamming signals may be nulled out. If the signal information is inaccurate, or the desired signal and the interference signal are correlated, the desired signal cancellation may take place. Remedy for this problem can be a constrained adaptation of the auxiliary array. Robust methods have been developed to reduce the effect of inaccurate signal knowledge in adaptation process [57]. The methods for reducing the signal cancellation due to correlated and coherent jamming have also been suggested [58].

Shan and Kailath [59] used the approach of spatial smoothing for correlated interferers. In their approach, the antenna array is grouped into subarrays. It is shown that the rank of the noise-free spatially-smoothed covariance matrix is the same as the number of the sources. Yeh et al.
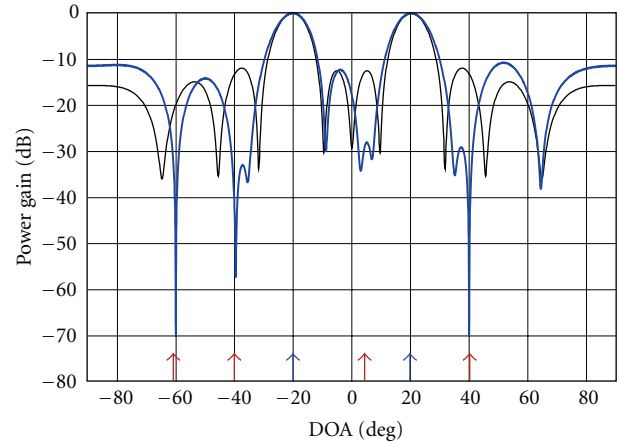
diagonalized covariance matrix that is a diagonally dominant matrix and has an improved dynamic performance [66].

3.1.4. Regenerative Hybrid Array.

For data communications, a regenerative hybrid array has been proposed without the use of the spread spectrum technique [67]. It consists of Applebaum-type array and a regenerative reference loop in which the reference signal is generated through a detection-generation process. In case the desired signal is not present the weight of the regenerative hybrid array converges to that of Applebaum array. Since it is based on the gradient-search algorithm, it converges slowly. Yeh et al. [68] applied the least squares method for regenerative hybrid array for more rapid convergence. Recursive algorithms such as QR decomposition can be used to update the weights. It is shown that the regenerative hybrid array is much less sensitive to steering vector errors than the Applebaum array.

3.1.5. Digital Beamformer.

With the advent of digital technology, these techniques were reemployed using digitally sampled data. For digital signals, approaches like the method of conjugate gradient [35, 69] and the method of least squares [70] have replaced the LMS algorithm for the enhancement of the convergence speed of the algorithm. It is shown that conjugate gradient method utilizes simpler data matrix and is computationally more stable to the solution of ill-conditioned covariance matrix formed by the signal and the jammers.

Sarkar et al. [71] proposed a direct data-domain approach, based on the spatial samples of the data. The adaptive analysis is done on a snapshot-by-snapshot basis, and thus nonstationary environments, including the multipath environments can be handled efficiently. The assumption of *a priori* knowledge of the signal, that is, its direction of arrival, is considered. The problem of signal cancellation is dealt using two methods, one through the main-beam constraints and the other through the norm of the weight vectors.

3.1.6. Generalized Sidelobe Canceller.

One of the most common criteria used in beamformer for suppression of interfering signals and noise is the linearly constrained minimum variance (LCMV) beamformer. This was first proposed by Frost [5]. An efficient implementation of LCMV was done by Griffiths and Jim [43] in proposing generalized sidelobe canceller (GSC). The basic model was of Applebaum and Chapman [42]. The optimum array parameters are adaptively estimated based on the available data set and using an LMS adaptive filter results in an LMS GSC adaptive scheme. Since the GSC uses an unconstrained rather than a constrained algorithm, it may be possible to adapt the weights much faster [72]. In order to improve the convergence rate of the original LMS GSC scheme, discrete unitary transforms, such as the discrete Fourier transforms (DFTs), have been utilized for the decorrelation of the input data [73, 74]. Although the transformed domain (TD) LMS GSC algorithms may have increased convergence rates for some classes of input signals, the computational complexity tends to remain similar to that of the original LMS schemes. Glentis
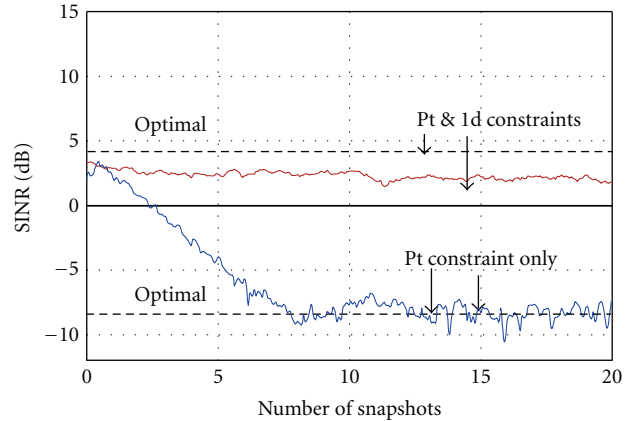

FIGURE 8: Learning curves for GSC with DOA mismatch utilizing point constraint only and first-order derivative constraint [7].

[75] attempted to reduce the computational complexity of both GSC-LMS and TD-LMS GSC by treating the complex signals as a pair of real signals and using the algorithmic strength reduction technique [76]. Although GSC has less computational complexity, it is quite sensitive to even a small mismatch of DOA of the desired signal (Figure 8). Due to this mismatch the GSC tends to misinterpret the desired signal component by nulling instead of maintaining distortionless response towards it. In addition, in most of the applications, the desired signal is absent in training period of the beamformer. This requires the beamformer to be more robust against mismatch errors in the array response.

3.2. Partially Adaptive Array.

In partial adaptive array [77] only a fraction of the array elements are weighted and the output is obtained by summing the adaptively weighted element voltages and fixed weighted element voltages. Partial adaptivity is of considerable interest since it is less costly to implement but has performance similar to the full adaptive case. Reducing adaptive dimension also leads to a faster response [78]. But this may degrade the cancellation performance of the array. Chapman [77] has investigated several partially adaptive approaches for canceling distributed interferences. It was found that the effect of the random sidelobe level caused by element weighting errors adversely affects the performance, which is in inverse proportion to the number of the adaptive elements. This is along the expected lines since the distributed interference model requires more degrees of freedom similar to those of fully adaptive array. The partial adaptive arrays are an outgrowth of the well known multiple sidelobe cancellation system (MSLC) technique. A variety of approaches have been suggested for the design of partially adaptive arrays [46]. It has been shown that for a narrowband signal environment and a unity gain constraint in the desired signal direction, one can get a fully adaptive performance using a number of adaptive weights equal to the rank of the spatially correlated portion of the interference correlation matrix. This requires knowledge of eigenstructure of interference correlation matrix.

Veen [79] presented a procedure for designing partially adaptive arrays having nearly fully adaptive performance under the steady-state conditions. Instead of adaptive estimation of the interference eigenstructure, the eigenstructure of an averaged correlation matrix, which spans the interference scenarios of interest, is utilized. In order to place more nulls in the far-field pattern, Haupt [80] presented simultaneous nulling in sum and difference patterns of a monopulse antenna, using full phase-only nulling (FPON) algorithm. Since the algorithm assumes that the adaptive phases are small, its performance is subject to phase shifter quantization errors. Its application is limited to an array equipped with high-priced, high-resolution phase shifters. For further improvement in adaptive nulling, Chang et al. [81] presented a partial phase-only nulling (PPON) algorithm in which the phase change of the phase shifter is enlarged with focus on a subset of elements, so that the nulls can be steered to the direction of jammers. Although the PPON algorithm requires low-resolution phase shifters, its nulling performance is as good as the FPON algorithm using high-resolution phase shifters.

4. Robustness in Adaptive Arrays

Adaptive arrays are basically data-dependent beamformers that select the weight vectors as a function of the data to optimize the performance subject to various constraints. However, they are much sensitive to errors. Much effort has been put over past three decades to design the robust beamformers. The application areas of the robust beamformer are also expanding. Examples of new areas include smart antennas in wireless communications, handheld ultrasound imaging systems, and directional hearing aids.

4.1. Errors in Adaptive Array Processing. In real applications, one can neither easily generate the reference signal for the LMS array, nor have complete knowledge about the steering vector for the Applebaum array. It has been found that Applebaum array can be used only for limited types of signals, for example, the spread spectrum signal [82]. Furthermore, the steering vector may be obtained by using the reference signal. One of the ways for this is to let an LMS array use a reference signal to adapt to a desired signal during an interval without interference and then estimate the steering vector [83] for the later period using adapted weights, when the interference is present. Such a steering vector will contain random errors due to the noise present in the array feedback loops when the weights are measured. These random errors occur in the weight vector of both Applebaum array and LMS array. The effects of random errors on the performance of the beamformers have been studied by many authors. These errors include the error due to signal fluctuation [84], weight quantization [85], element failure [86], imperfect knowledge of the signal direction [87], the weight errors [88], and the steering vector errors. The analyses so far carried out are concerned with the effect of errors on the beam pattern, sidelobe level of the beamformer, and the array gain.

If the desired signal is weak, the array pattern is not affected much by the signal. The reason for this is that the desired signal is not present long enough to have much impact on the weights and thus the minor errors in the steering vector are of little consequence. In many situations, the desired signal may be present continuously and may be also strong. Such an array becomes more sensitive to errors in the steering vector. For a continuous desired signal, array feedback minimizes the output desired signal power. This is done by simply lowering the magnitude of the array pattern and choosing proper steering vector components thereby reducing the array output desired power, interference power, thermal noise power proportionally, and leaving the output SINR unaffected. In case of not choosing the steering vector correctly, the array feedback may null the desired signal without reducing the output thermal noise or interference power. This results in rapid drop of output SINR, making array sensitive towards the steering vector errors. Another type of steering vector error is the beam pointing error. It occurs when the actual desired signal arrival angle is different from its assumed or estimated value [50]. Compton [83] presented a study of the effect of the random errors in the steering vector of the Applebaum array. The steering vector was chosen to produce a beam in the proper direction, but each component was assumed to have a random error, uncorrelated from one element to another. It was shown that the array output SINR becomes more sensitive to steering vector errors as the number of elements in the array is increased and as the received SNR becomes larger. The variance of the steering vector that may be tolerated was found to depend on the required desired signal dynamic range.

Godara [15] considered the random errors, namely, the weight vector errors (WVEs) and the steering vector errors (SVEs), which can be modeled as additive stochastic processes of zero mean and are uncorrelated from element to element. The effect of these errors was studied on two processors, namely, the noise-alone matrix inverse (NAMI) processor and the signal-plus-noise matrix inverse (SPNMI) processor, for a general array configuration. The two processors are observed to be very sensitive to the interference power in the presence of the WVE whereas this is not so in the case of SVE. For SVE, the SPNMI processor is found to be very sensitive to the input signal power. Lin and Barkat [89] suggested a hybrid adaptive array (a combination of LMS array and Applebaum array), in order to minimize the effect of the random errors in the weight vectors. It was shown that a hybrid array maximizes the output SINR and gives a better result than the LMS array and Applebaum array, if each was used separately. Further, the weighting vector algorithm does not require any prior knowledge or direct measurements of the random errors. The weight vectors containing random errors are scaled and combined to yield new weight vector.

4.2. Enhancement of Robustness in Adaptive Arrays. If array parameters are perturbed from ideal conditions under which the theoretical performance of the system is predicted, the performance of the system may get degraded through the reduction of the array gain and alteration in the beam

pattern. Various schemes have been proposed to overcome these problems and to enhance the array system performance under nonideal conditions. A survey of robust signal processing techniques in general was conducted by Kassam and Poor [90].

4.2.1. Generalized Sidelobe Canceller with Notch Filter. For enhancement of the robustness of the beamformer, Takao and Boon [91] incorporated a notch filter into the conventional GSC that leads to the exclusion of the desired signal component in the beamformer input. The proposed structure of GSC lessens the deviation of adaptive weights from the optimum value and thus improves the convergence rate. The disadvantage of the structure is its complexity. It requires a sharp notch filter and a slave array for recovery of the desired signal. However, there are many other approaches to design a robust adaptive array, namely, structure with mainbeam constraints or eigenspace-based structures.

4.2.2. Diagonal Loading. Regularization methods such as diagonal loading [92] have also been used for beamforming. In this method, the sum of the weighted array output plus a penalty term, proportional to the square of the norm of the weight vector, is minimized. This makes the gain in the angle of arrival equal to unity. This technique protects the mainbeam at the expense of the sidelobe nulling. The main shortcoming of this approach is that there is no appropriate way of choosing the loading parameter. However it can be used along with subspace constraints for better performance.

4.2.3. Decision Feedback Generalized Sidelobe Canceller. A new robust beamformer called DF-GSC was proposed by Lee and Wu [7]. The purpose was to overcome the mismatch errors, signal cancellation, and slow convergence problem. The proposed structure consists of a blind equalizer and a feedback filter along with GSC structure. The structure eliminates the desired signal from the error signal. The weights of GSC are calculated using LMS algorithm. Since the error signal does not contain the desired signal, larger step size may be used for faster convergence (Figure 9).

4.2.4. Null-Steering Beamformers with Constraints. In narrowband adaptive array processing, the null-steering techniques are popular due to their rapid convergence behavior [93]. Khanna and Madan [94] have proposed a narrowband linearly constrained adaptive array based on such a beamformer in which the residue power from each stage is locally minimized by controlling the weights of that stage. The perturbation and estimation-based algorithms have also been applied on a beamformer with only phase shifters and investigated for a power inversion array [95].

4.2.5. Linearly Constrained Minimum Beamformer (LCMV) with Split Polarity Transformation. This structure is introduced by Lu and He [96] for coherent interference environment. It consists of a split-polarity transformation (SPT) processor and a conventional LCMV beamformer. Thus with the aid of SPT processor, the signal cancellation due to
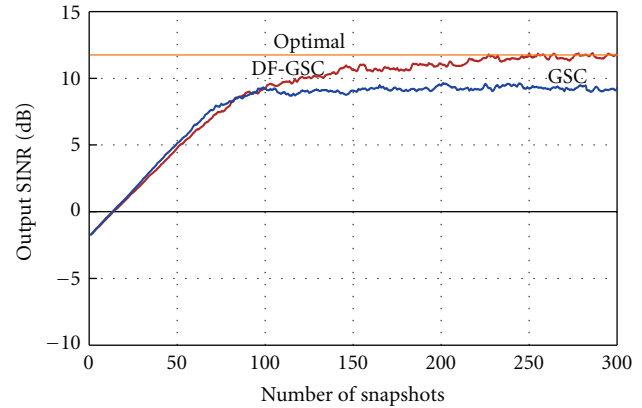

FIGURE 9: Learning curves for GSC and DF-GSC showing the comparison of convergence rate. Compare Lee and Wu [7].

correlation between the desired signal and interference is totally eliminated. The SPT-LCMV beamformer is shown to be robust against the direction uncertainty in the assumed look direction.

4.2.6. Digital Beamformer with Mainlobe Maintenance. Yu and Murrow [97] proposed a radar digital beamforming (DBF) architecture and processing algorithms for nulling the signal from a mainlobe jammer and multiple sidelobe jammers while maintaining the angle estimation accuracy on the target. The proposed architecture consists of an SLJ adaptive array followed by an MLJ canceller. The approach involves the cascading of an adaptive DBF subarray architecture and a multiple lobe canceller (MLC).

The mainlobe maintenance (MLM) is essential part of such two-stage adaptive processing architecture, since the SLJs are canceled in the first stage while the MLJ in the second stage. Several MLM techniques are available such as structured covariance matrix using subspace constraint [98, 99], diagonal loading [92], MLJ filtering, blocking matrix [100], and covariance matrix tapers [101]. These techniques allow the desired signal to arrive from a region in the DOA space rather than just from a single direction and are based on the assumptions that strength of the desired signal and the interval over which the DOA can vary. The robustness to DOA uncertainty is enhanced at the expense of reduction in noise and the extent of interference suppression.

4.2.7. Bayesian Beamformer. An adaptive beamformer based on a Bayesian approach was developed by Bell et al. [102], for improved robustness to pointing errors. It consists of set of minimum variance distortionless response (MVDR) beamformers and minimum variance (MV) spatial spectral estimators. SMI algorithm is used here for the implementation. It is shown that assuming the DOA to be a discrete random variable with a probability density function (pdf) known *a priori*, the beamformer is capable of balancing the use of the observed data and *a priori* knowledge about the source DOA. It is shown that, for high SNR, the Bayesian beamformer places more emphasis on the observations, and

its performance is same as that of more computationally complex DF-based and subspace beamformers. At low SNR, the proposed beamformer uses *a priori* knowledge about the DOA interval. Since each individual MVDR beamformer in the Bayesian beamformer employs only one directional constraint, the degrees of freedom are not sacrificed as in the case of LCMV beamformer.

4.2.8. Wideband Beamformers. For broadband applications, tapped delay line transversal filters [5] instead of quadrature weights are employed for processing of the array elements. This is because the number of variables to be adaptively controlled is less than required in equivalent narrowband array system. However, the use of tapped delay line processing leads to wider spread of eigenvalues of the covariance matrix [103]. On employing the steepest descent algorithm, the convergence behavior can be improved if the null-steering techniques are employed along with the tapped delay line arrays [104].

4.2.9. Other Techniques. J. H. Lee and Y. T. Lee [105] used the signal cyclostationarity for achieving the robustness of the beamformer but it suffers from high computational complexity and requires *a priori* information about some auxiliary parameters. Another approach called the worst-case performance optimization (WCPO) was proposed [106–108] making use of the second-order cone (SOC) programming. But WCPO approach cannot be directly applied to the GSC scheme. Another method is through subband decomposition [109].

For multiple jammers environment, the conventional Frost beamformer may have some problems. In order to improve the nulling capability and the convergence rate, the adaptive transformed domain normalized LMS algorithm with discrete cosine transform (DCT) and discrete Hartley transform (DHT) for broadband adaptive array structure was proposed [110]. Resende et al. [111] further improved the performance of broadband beamformer by using the linearly constrained robust fast least square (FLS) beamforming algorithm. Chern and Chang [112] developed a linearly constrained (LC) RLS array beam-forming algorithm, based on the inverse QR decomposition, for moving jammers. The difference between his approach and the FLS is that the adaptation gain is evaluated using Givens rotation. Techniques that improve the robustness of data-driven beamformers in the presence of moving and spatially spread sources by incorporating additional linear constraints have been also proposed [113, 114].

5. Optimization of the Performance of Adaptive Arrays

For the design and analysis of arrays, it is invariably necessary to make assumptions regarding the array and sources incident on the array. In practical situations, the assumptions of plane wave, perfectly matched sensor channels, knowledge of the direction of arrival of the desired signal, and the position and polar response of the array elements, do not hold. Thus the performance of optimum antenna array processors based on power minimization subject to linear look direction constraints degrades significantly. In order to optimize the performance, various linear and nonlinear constraints have been included by researchers [115–119]. The implication of these constraints is that the array pattern has a unity response in the look direction. On specifying the additional constraints, such as derivative constraints, the pattern gets broadened [120–122]. Griffiths and Buckley [123] presented a method for quiescent pattern control in linearly constrained adaptive arrays. The proposed method basically mixes the adaptive and deterministic design techniques. The method is based on the use of a generalized sidelobe canceller (GSC) and broadband array.

Godara and Cantoni [14] analyzed the convergence and transient behavior of the weight covariance matrix of the constrained LMS algorithm and derived exact expressions for the misadjustments for the real, the complex, and the perturbation case. The scheme employed includes the gradient estimation when all the array signals are accessible as well as the gradient estimation using perturbation sequences for the cases where the array signals are inaccessible. Park and Sarkar [124] included the multiple look direction constraints to a deterministic eigenvalue approach in order to prevent signal cancellation in one-dimensional adaptive nulling problem. Due to addition of multiple look directions, the targets can be detected even in the presence of noise and clutter.

5.1. Feedback Loop. Both the LMS and Applebaum arrays use feedback loops to generate the array weights. These feedback loops may be of analog type, or in sampled data systems, they may be digital. Moreover, the optimum weight vector of the Applebaum array is proportional to the optimum weight vector of the LMS array. The feedback-loop algorithms generally converge slowly when there is large spread of eigenvalues of the input signal covariance matrix. The LMS algorithms can be used in systems where the desired signal is always present and a reference signal that is correlated with the desired signal can be generated at the receiving end. The Applebaum and SMI arrays are used when the desired signal direction is known. Therefore, the LMS array is often used in communications system, while the Applebaum and SMI arrays are typically used in the radar systems.

For an adaptive array implemented with continuous control loops, the speed of response of array depends on the received signal power. The array responds slowly to a weak signal and rapidly to a strong signal. When both the weak and strong signals are present, the weight transients include both the fast and slow terms, with their speed being proportional to the eigenvalue spread of the covariance matrix. This makes it difficult for the array to accommodate a wide range of signal powers. Compton [49] presented an improved adaptive array feedback loop that produces the same steady state weights as the LMS algorithm, but has the property that its time constants are nearly independent of signal power or eigenvalue spread. In order to increase the convergence speed of feedback-loop adaptive arrays, Ganz [125] presented novel adaptive array architecture.

The proposed architecture consists of cascaded structure having two layers of modified discrete-time Applebaum arrays [13]. The hardware implementation of such cascaded array is possible using parallel processors.

5.2. Jammer Power. The output SINR of an adaptive array is also a function of the jammer power. Gupta [126] analyzed the effect of the jammer power on the performance of adaptive arrays and found that the output SINR decreases with increasing jammer power. Figure 10 shows the effect of jammer power and jammer direction on the output noise power of the array. For wideband jammer, it eventually goes to zero. Unlike the continuous wave (CW) jammers, the wideband jammers do not go through power inversion. Instead as the power of jammer is increased, the interference-to-noise (INR) power at the array output undergoes oscillations. Beyond a certain power level, the output INR monotonically increases with increasing power level of the jammer. The array is fully constrained and the output SINR drops sharply. The performance of adaptive array (Figure 11) depends on the capability of the adaptive algorithm employed for array processing and weight estimation [127].

5.3. Weight Jitter. Weight jitter is one of the important effects that can cause an increase in the sidelobe level of the adapted pattern when a low-sidelobe pattern is specified by the steering vector [128]. In order to improve the convergence speed, LMS algorithm is often used in conjunction with pre-processing structures such as Davies beamformer [95] and Nolen beamformer [129]. Ko [130] analyzed the statistics of weight jittering noises generated when the LMS is used to update each stage of an adaptive Gram-Schmidt processor in interference canceling adaptive arrays. In the proposed structure, the weight jittering noises do not accumulate as expected, but actually cancel one another and decrease in magnitude as the optimal powers become smaller from one stage to another. The synchronization of processing of weights is essential; otherwise the jittering noise generated may become independent, accumulate, and deteriorate the system performance significantly.

5.4. Multipath Effect. Most of the studies on adaptive cancellation of strong jammers consider only the direct signal from the jammer and ignore any multipath components. Airborne surveillance radars operate in an environment that may include the clutter, standoff jammers, and diffuse jammer multipath. In the multipath propagation environment, the desired signal is highly correlated with the interferences [122]. In past decade, considerable efforts have been made to overcome the coherency difficulties. Widrow et al. [58] used a subtractive preprocessor to remove the desired signal during the adaptation process. This method however may fail to null for more than one coherent jammer. Researchers tried to destroy the coherency by using a class of subarray averaging techniques [59, 131]. An algorithm, which determines source directions in coherence multipath conditions by a maximum-likelihood method and computes optimum
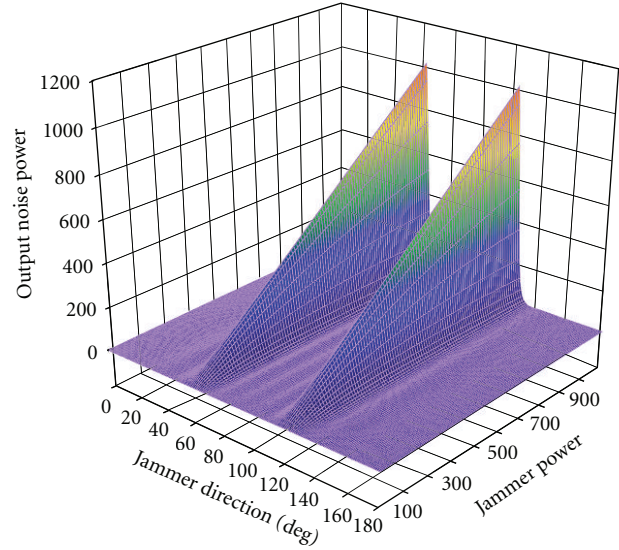

FIGURE 10: Output noise power of the antenna array versus jammer direction and jammer power. Two desired signals (60°, 120°, power: 0 dB each) and one probing source with variable angle of arrival and variable power are considered [127].
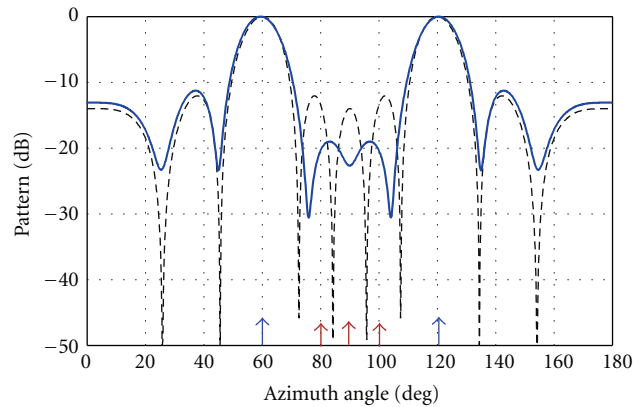

FIGURE 11: Adapted pattern of the array using improved LMS algorithm. Two desired signals (60°, 120°, power ratio of 1 each) and 3 continuously wideband probing sources (80°, p.r.: 800; 90°, p.r.: 600; 100°, p.r.: 800) with 2% bandwidth and 3 spectral lines are considered.

beam-steering coefficients for SNR enhancement by interference nulling, was given by Hudson [132]. An estimator and beamformer of such type can be used, not only in radio communications but also in underwater acoustics, where the least squares adaptive beamformers are not used. The main drawback of this method is the excessive computing for maximizing the likelihood function. An alternative to this method may be suboptimum methods based on eigenstructure analysis or Prony's method [133].

Certain techniques of redundancy averaging and enhanced redundancy are proposed to put nulls in the direction of arrivals (DOA) of coherent interferences [25, 134]. However, steering the nulls in the directions of the coherent signals is not desirable. A beamformer should be able to
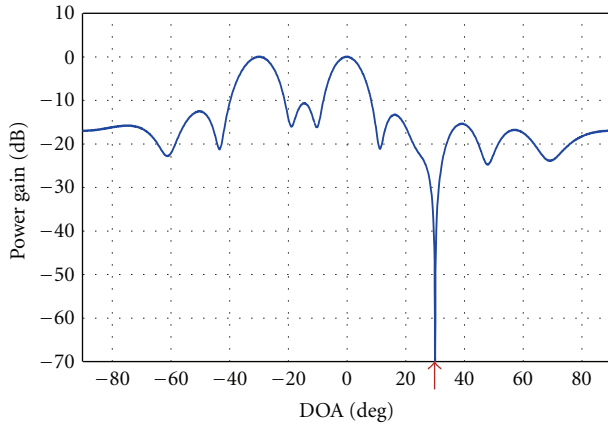
FIGURE 12: Adapted beam pattern of an antenna array with two desired signals $(-30°, 0°)$ and one interfering source $(30°)$. Interfering signal is correlated to signal at $-30°$. $\alpha = (-0.15, 0.05)$, $c = [1\ 1\ 0]$; compare [136].

FIGURE 13: Pattern synthesis of 2-element array, $\lambda/2$ spacing, $\Phi = 0°$, power: 40 dB; 1 jammer: $50°$, power: 0 dB; USV: universal steering vector, CSV: conventional steering vector [141].

constructively combine these signals instead of canceling all but one of them so as to avoid any loss of information. Gonen and Mendel [135] developed a cumulant-based blind beamformer to overcome this problem in the presence of coherent multipath propagation. Another approach consisting of matrix reconstruction scheme along with an iterative algorithm is proposed by Lee and Hsu [136] to maintain the array efficiency even in the presence of coherent signals (Figure 12). Yeh and Wang [61] used an arbitrarily configured array. In their approach, DOAs of coherent interferences either are known *a priori* or are estimated by applying high-resolution methods to the observed data prior to the beamforming. The weights are then adapted subject to null constraint in those interference directions. The beamformer is adaptive to the change in incoherent interferences but not to that of coherent interferences. Morgan and Aridgides [137] addressed the multipath problem by using the fact that the multiple received signals are correlated in time and thus can be removed by incorporating a tapped delay line in each auxiliary channel, providing an adaptive cancellation loop for each tap.

In order to suppress, direct jammer signal and multiple reflections, one can add either more spatial degrees of freedom or more temporal degrees of freedom, or a combination of both. Fante [138] proposed a hybrid system that uses both bandwidth partitioning and adaptive FIR filters for cancellation of specular and diffuse multipath. The time-domain approach can also be used to reduce the multipath problem [139]. This is done through generation of the replica of the interference and then passing it through a delay line. This approach is reasonably effective for main beam diffuse jammer multipath, but not for those multipaths entering through the sidelobes. It is shown that, using all the auxiliary beams, an effective cancellation can be performed both in beam space and element space, with considerable reduction in processing requirements. Lo and Litva [140] presented a beam-space nulling technique for enhancement of reception of the desired signal in a multipath environment. The technique is based on a well known vector relationship
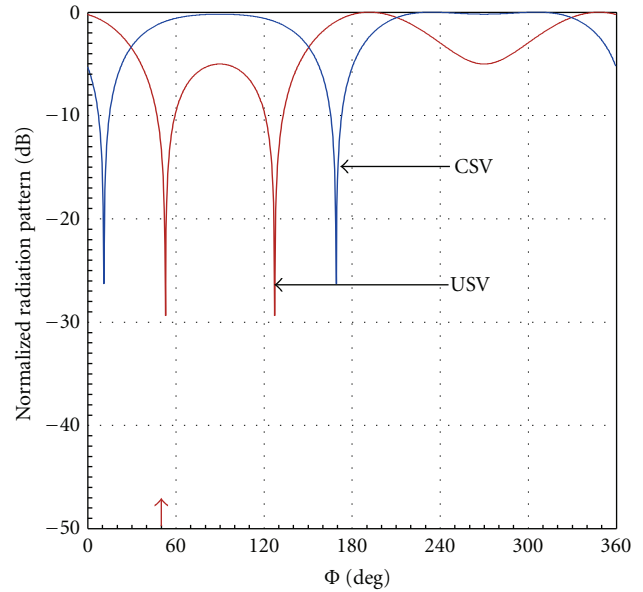
in beam space. The auxiliary beams are derived from a set of orthogonal vectors, which are obtained by a vector transformation.

5.5. Mutual Coupling Effect. Most of the adaptive algorithms use the assumption that the elements of receiving array are independent isotropic point sensors that sample but do not reradiate. Further it is assumed that the array is isolated from its surroundings. However, in a real system, each array element is of finite size and also reradiates the incident fields. The reradiated fields interact with other elements causing the elements to be mutually coupled. The mutual coupling between antenna elements affects the performance of an adaptive array, especially when the interelement spacing is small (Figure 13). Many authors have analyzed this mutual coupling effect in different types of adaptive arrays, namely, LMS and Applebaum arrays [142], for a power inversion array [143] and adaptive Yagi array [144]. The aim of these studies varied from accurate signal recovery [145] to elimination of mutual coupling on DOA estimation [146]. It is shown that the mutual coupling does affect the performance of adaptive arrays, even for large interelement spacing, that is, for spacing more than half a wavelength. It is found that the failure to include the mutual coupling degrades the performance of Applebaum-type adaptive arrays much more than in case of LMS arrays [141].

5.6. Consumption of Degrees of Freedom. The main aim of various investigations on adaptive nulling is to determine an estimate of the maximum power of jammers that can be tolerated. These studies will be scenario dependent because of the consumption of only few of degrees of freedoms. It is well known that no adaptive pattern shaping can result
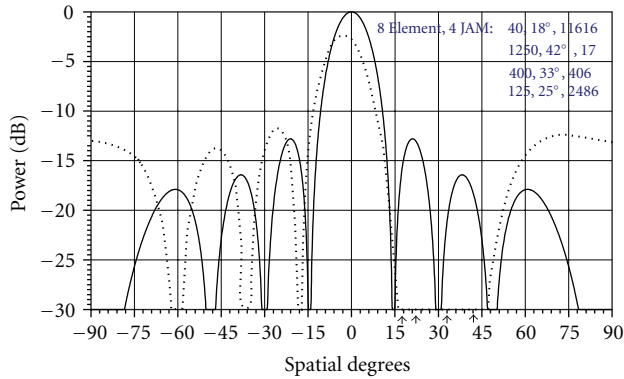
FIGURE 14: Validation for the interference suppression for four narrowband sources located in sidelobe region. Compare Gabriel [55].

if all the degrees of freedom are consumed. Sources that are grouped closely in terms of nulling the antenna half-power beamwidth will tend to use less degree of freedom than the widely separated sources. Gabriel [55] studied this effect by allowing the incorporation of the beam-steering weights into the adaptive control loops, which specify the desired radiation pattern in the absence of interference. The consumption of degrees of freedom was related to the number of interference covariance matrix eigenvalues greater than unity (Figure 14). For each eigenvalue greater than unity, it is shown that the associated eigenvector contributes to the formation of the pattern nulls.

5.7. Polarization of the Incoming Signals. It is not only the angle of arrival of the signal but also the polarization, which an array must take care of. If the antenna elements of a radar system are capable of receiving two orthogonal polarizations simultaneously, it is possible to synthesize a desired polarization by the linear combination of the signals from the two channels. This can then be used to track the polarized signal in unpolarized noise. In case of signal with fluctuating polarization, past samples of the received waveform can be used to estimate the polarization. Similarly, it is possible to suppress the clutter or polarized interference, by synthesizing a receiver polarization that is orthogonal to that of undesired signal.

Compton [147] studied the performance of adaptive array consisting of a tripole, when the interference source is a cross-polarized jammer. It has been shown that as long as the desired signal is not linearly polarized, the tripole effectively eliminates a single interference signal, regardless of angle of arrival or polarization except for certain conditions. Further it was found that the adaptive array was least susceptible to such jamming if the desired signal was circularly polarized. Raghavan et al. [148] proposed the use of the signal-free secondary vectors for estimating the polarization of clutter/interference adaptively. These polarimetric clutter suppression strategies may be useful for detection of targets where the clutter has a high degree of polarization and where the desired signal and clutter cannot be separated in other domains. The Stokes vector has been extensively used for

electromagnetic polarimetry. The direction of the estimated Stokes vector provides the information for determining the polarization of the receiving antenna for minimizing (or maximizing) the received power. In order to analyze the performance of adaptive algorithms, it is necessary to know the statistical distribution (e.g., pdf) of the estimated Stokes vector [149]. The density function is used to obtain the accuracy of the estimated direction of the Stokes vector and average level of clutter/interference suppression.

5.8. Modulation in Desired Signal and Jammers. The performance of LMS adaptive array is dependent on the modulated envelope of the interference as well. If the interference is modulated at a rate slow enough to be tracked by the array feedback, it can cause the weights to vary continuously and thus will not be able to reach the steady state. This will in turn make output SINR vary continuously and result in modulated desired signal. Methods have been developed for using adaptive arrays with ordinary amplitude modulated (AM) signals [150], frequency shift keyed (FSK) signals, phase shift keyed (PSK) spread spectrum signals [82], and quadriphase PSK spread spectrum signals [151]. All these techniques have been experimentally demonstrated.

Compton [53] has studied the effect of a pulsed interference signal on an adaptive array. It was shown that if the differential phase shifted keyed (DPSK) signal is incident on an adaptive array, the bit error probability increases as compared to the CW interference. For double-sideband, suppressed carrier amplitude modulated (DSBSC-AM) interference signal [152], however, the array modulates the desired signal envelope but the effect on bit error probability is not of concern. These effects were further studied for general type of envelope modulation [153]. It was assumed that the interference has only envelope modulation, that is, there is no phase modulation and that the interference modulation is periodic. Acar and Compton [154] attempted to analyze the performance of adaptive array with frequency-hopped signals. It has been shown that the frequency hopping has several adverse effects on LMS array. It causes modulation of both the amplitude and phase in the received signal. It also results in variation of output SINR with time and hence the bit error probability for the demodulated signal is enhanced.

6. The Emerging Trends

The theoretical origin of adaptive array essentially connects to Weiner's filter theory. During initial period, adaptive array applications were limited due to technology and robust adaptive algorithms for real-time operation. However, the contemporary scenario is completely altered. Recent advances in electronics and computers enable handling of complicated signal processing in real time with ease. Adaptive beam-forming technology has been successfully implemented both in defense and civilian sector. The following subsections summarize the trends in adaptive algorithms, beam-forming techniques, and the methods for error optimization and robustness enhancement in adaptive arrays.

6.1. Emerging Trends in Adaptive Algorithms. Several adaptive algorithms have been proposed in the literature. These algorithms can be broadly divided into three classes: LMS algorithms, matrix inversion algorithms, and RLS algorithms. Both the unconstrained and constrained algorithms exist within the set of LMS algorithms. The adaptive array processing in constrained algorithms requires unbiased estimate of gradient of output power with respect to the weights. There are several schemes available in open domain to determine the unbiased gradient. Further, the performance of algorithm depends on the *misadjustment*, difference between the adaptive weights estimated and the optimal weights. This dimensionless parameter depends on the step size. The lower is the step size, lesser is the misadjustment noise. However this leads to slower convergence rate. This trade-off can be avoided by having variable step size instead of a single step size for an entire weight vector. The RLS algorithm uses the gain matrix instead of gradient step size. This offers faster convergence rate, parameter tracking capability, and steady-state MSE. However it has the drawback of complexity. Other forms of RLS algorithm like the numerically stable QRD-RLS algorithm or the inverse QRD-RLS algorithm are emerging with potential for improved computation efficiency. The algorithms using perturbation sequences eliminate the estimation of the projection operator in weight calculation and hence reduce the computational complexity. Apart from these analytical methods, genetic algorithms (GAs) have become popular choice to optimize the adaptive arrays. These algorithms are demonstrated to be efficient in constantly changing signal environment and detecting faults in the array. Several algorithms, such as data-domain algorithms and perturbation search algorithms, do not require covariance matrix processing and perform well even in rapidly fluctuating scenario. The hardware implementation of these adaptive algorithms is a challenging task. Since adaptive beamforming is done in real time, computationally low-cost but high-speed algorithms are desired leading to the preference for LMS-based algorithms. Computational complexity and memory requirements are also of interest.

6.2. Trends in Beam-Forming Techniques in Adaptive Array Processing. The most basic technique for adaptive array processing is a conventional beamformer, in which phase is used to electronically steer the beam. Null-steering beamformer involves the signal estimation through the steering of the conventional beam in the known source direction and the subtraction of its output from each element. However, these schemes require *a priori* information regarding DoA and power level of the impinging signals. In an optimal beamformer, the weights are selected by minimizing the mean output power while at the same time maintaining sufficient gain in the look direction. Further, the performance of the beamformer is not affected by the look-direction constraint, as it only scales the weights. The beam-space processing is used for signal scenario consisting of fewer interfering sources. In element-space processing, the self-nulling is prevented by imposing constraints, proving the scheme to be more robust against the mismatch errors. If a self-generated

reference circuit is included in a constrained steered-beam interference canceller, the sensitivity to the pointing error can be reduced. This in turn improves the performance of the array processor owing to the reduction of the desired signal component fed back to the sidelobe canceling block. Further, by choosing the steering vector appropriately, the scheme can be used to generate multiple beams towards each of the desired directions.

The signal cancellation due to correlated impinging signals on MSLCs can be mitigated by spatial smoothing or covariance matrix reconstruction algorithms. If the interfering signals fall within the mainlobe, MSLC scheme proves to be ineffective. Alternate solution is to use the high gain difference beam for nulling mainlobe jammers. The regenerative hybrid array along with efficient recursive algorithms is less sensitive to steering vector errors than the conventional Applebaum array. This makes the hybrid array with recursive weight adaptation preferred choice. The speed of response in adaptive arrays depends on the dynamic range of power levels and bandwidth of impinging signals. The processor shows slower response towards the weak and wideband interfering sources. The delay in adaptation is due to the spread in the eigenvalues of the interfering source covariance matrix defined at the antenna output ports. If the input ports are decoupled with the feedback system of the processor, the speed of the response of array is improved. The LCMV beamformer is yet another commonly employed scheme for suppression of interfering signals and noise. The optimum array parameters are adaptively estimated based on the available data set, with the use of an LMS adaptive filter resulting in an LMS GSC adaptive scheme. The convergence rate of an LMS based GSC scheme depends on the eigenvalue spread of the correlation matrix of the input data. The adaptive array processing with digitally sampled data has shown incredible improvement in convergence speed. Approaches like the method of conjugate gradient and the method of least squares have replaced the LMS algorithm. These methods are shown to be computationally more stable to the solution of the ill-conditioned covariance matrix. In order to improve the convergence rate of the LMS GSC scheme, Discrete Unitary Transforms, such as DFT, have been utilized for the decorrelation of the input data. The computational cost of the method can be further reduced by treating the complex signals as a pair of real signals and using the algorithmic strength reduction technique. It is well known that, in adaptive arrays, the number of available degrees of freedom controls the computational complexity. In order to reduce the computational cost (and hence faster response), partial adaptive arrays are emerging as a preferred choice. The partial adaptive arrays are outgrowth of the well known MSLC technique. One can achieve the adaptive performance fully by using the number of adaptive weights equal to the rank of the spatially correlated portion of the interference correlation matrix.

6.3. Methods of Error Optimization and Robustness Enhancement in Adaptive Arrays. The adaptive array performance gets degraded due to several types of random errors, namely,

signal fluctuation, weight quantization, element failure, the errors in weights and steering vector errors, and so forth. If the desired signal is weak, it does not affect the weights generating the adapted pattern. However, if the desired signal is strong, the array becomes more sensitive to errors. This can be avoided with the feedback block of the processor that minimizes the output desired signal power and hence results in an unaffected output SINR. In beam pointing error, the steering vector components deviate slightly from the desired signal, resulting in the degradation of output SINR. This effect becomes worse for large arrays. The power level of the desired signal and interfering signal plays an important role in the performance of adaptive array processor in the presence of weight vector and steering vector errors. Hybrid adaptive array, that is, combination of schemes, such as combination of LMS and Applebaum array, minimizes the effect of random error on the performance of array. The mutual coupling between antenna elements usually affects the performance of an adaptive array, especially when the interelement spacing is small. Other parameters, namely, multipath, polarization, jammer power, too affects the suppression capabilities of adaptive arrays. Moreover the performance of adaptive array is dependent on the modulated envelope of the interference. If the interference is modulated at a rate slow enough to be tracked by the array feedback, it can cause the weights to vary continuously and thus will not reach the steady state. This in turn makes output SINR vary continuously and results in modulated desired signal. The polarization of the incoming signal is also an important factor in controlling the interference suppression in adaptive arrays. It is possible to suppress the clutter (or polarized interference), if the receiver polarization is orthogonal to that of the undesired signal. Moreover the adaptive array is least susceptible to such jamming if the desired signal is circularly polarized. A number of mainlobe maintenance (MLM) techniques are available in open domain such as structured covariance matrix using subspace constraint, modified dominant mode rejection, MLJ filtering, covariance matrix tapers, and so forth. In these techniques, the desired signal is allowed to arrive from an arbitrary direction. However the robustness to DOA uncertainty in such a technique is enhanced at the expense of a reduction in noise and interference suppression. Adaptive beamforming, which assumes uncorrelated signal sources, suffers from signal cancellation in presence of coherent signals. The techniques like cumulant-based blind beamformer and matrix reconstruction algorithm overcome this problem. The time-domain approach and beam-space nulling technique are also known to reduce the multipath problem. There are several approaches being proposed for enhancing the robustness in the adaptive array, namely, GSC with notch filter, SCs with mainbeam constraints, blind equalization, WCPO, and diagonal loading.

To summarize, enormous effort has been focused particularly since last decade on the optimization of the performance of the beamformer so as to cater to the complex signal environment. This includes modification of the algorithms, digitization of signal processing, and blind adaptation in beamforming schemes as the emerging techniques.

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