Hindawi Publishing Corporation
International Journal of Antennas and Propagation
Volume 2014, Article ID 162421, 7 pages
http://dx.doi.org/10.1155/2014/162421

Research Article

Analyzing Electromagnetic Systems on Electrically Large Platform Using a GTM-PO Hybrid Method with Synthetic Basis Functions

Shang Xiang, Gaobiao Xiao, and Junfa Mao

Key Laboratory of Ministry of Education of Design and Electromagnetic Compatibility of High-Speed Electronic Systems, Shanghai Jiao Tong University, Shanghai 200240, China

Correspondence should be addressed to Shang Xiang; bingoxiang917@sjtu.edu.cn

Received 27 November 2013; Revised 24 January 2014; Accepted 30 January 2014; Published 3 March 2014

Academic Editor: Ladislau Matekovits

A hybrid method of generalized transition matrix (GTM) and physical optics (PO) with synthetic basis functions (SBF) is proposed to analyze electromagnetic systems on electrically large platforms. Based on domain decomposition method (DDM), the proposed approach is to divide the whole problem into a GTM region and a PO region. The GTM algorithm can simulate antennas and scatterers accurately, and the PO algorithm is applied to obtain current distribution on the electrically large platform. With the characteristics extraction technique using SBFs on the GTM models, the number of unknowns can be greatly reduced and the computational efficiency can be further improved. PO region is regarded as an environment background and the unknowns in the PO region need not to be directly solved. Numerical examples will be shown to demonstrate the feasibility of the hybrid method.

1. Introduction

Analyzing the electromagnetic problem of an electromagnetic system working under various complex environments is still a challenging work worthy of research. The electromagnetic systems on electrically large platforms are such kinds of problems, where the interactions of systems and platforms should be evaluated efficiently. Method of moments (MoM) [1] based on the integral equations is often employed in these situations. However, it is usually time consuming and memory consuming due to the large number of unknowns generated in the discretization process with a typical spatial sampling rate of $\lambda/10$.

Recent development of domain decomposition methods [2–4] combined with the accelerating methods or model order reduction methods provides us with a more efficient way to obtain field solution. DDM is a kind of flexible strategy for solving multiscale problems and requires lower computational cost. When the DDM is applied, the part with fine structures in the system is divided into many subdomains, and the electromagnetic fields of each subdomain are solved independently using suitable method. Then the coupling effects among all subdomains are calculated to get the electromagnetic characteristics of the entire system.

In this paper, we adopt the generalized transition matrix (GTM) algorithm [5] to describe the electromagnetic characteristics of the subdomains in the systems, except the electrically large platform. By choosing a reference surface containing the fine target object, an associated generalized transition matrix is defined uniquely to describe the relationships between the rotated tangential components outgoing field and the rotated tangential incident field components on the reference surface. The generalized transition matrix defined on the reference surface with a regular shape can be obtained easily. Once the associated generalized transition matrix is obtained, the interaction between objects is replaced by the mutual coupling of the rotated tangential fields on different reference surfaces [6] so that it accomplishes the purpose of reducing unknowns, especially when the target object has very fine details. Actually more substantive efficiencies can be achieved by integrating iterative solvers, such as fast multipole method (FMM) [7] and the adaptive integral method (AIM) [8], into the GTM method when dealing with

the coupling effects among all blocks. This issue, however, will not be addressed in the present paper.

Using the reference surface enables us to separate an object with other ones and the environment by a definitely specified boundary with known boundary conditions. The electromagnetic characteristics can be efficiently expressed by a much coarser mesh reference surface. The GTM model can be obtained through many methods, such as the surface integral equations (SIE), the volume integral equations (VIE), and the finite volume method (FVM). Different from the methods based on iterative schemes with fast factorization, the kernel-free GTM method can be applied to various subblocks, even if there exists no explicit Green's function in the media of those subblocks.

Meanwhile, physical optics formulation is used to evaluate the currents on the electrically large platform. The PO region is regarded as an environment background. It is not required to solve the currents in the PO region simultaneously with the unknowns on the reference surfaces of subdomains.

After the coupling matrix is constructed, high-order basis functions such as the synthetic basis functions (SBF) [9] on the GTM model can be applied to further extract the effective electromagnetic characteristics of the subdomain. The synthetic function expansion technique, aiming to reduce the number of unknowns by aggregating standard basis functions, is applied in analyzing problems with complex structures such as the slot configurations [10] and it is combined with other acceleration techniques as described in [11]. The proposed method with SBFs is much more efficient than the existing hybrid MoM-PO techniques [12–15], which are based on low-order, small-domain discretization of both MoM and PO regions. It is noteworthy that the construction of SBFs is based on the solution space of the GTM model of a subdomain instead of its original structure. The solution space is obtained by the responses from the excitations of a series of independent auxiliary sources outside the corresponding reference surface. SBFs for the subdomain are generated by choosing a threshold of the eigenvalues. The number of SBFs is usually much smaller than that of the low-order basis functions for the subdomain. With the SBF method, the computational efficiency may be largely improved due to the reduction of the number of unknowns, while the computational accuracy is not deteriorated significantly.

In Section 2, the basic concept and construction of GTM model are described at first. And then the hybrid formulation derivation with the PO method is discussed. At last, the SBF implementation is introduced. Numerical examples are provided in Section 3.

2. Theory of the Hybrid Method

2.1. Generalized Transition Matrix. Consider an object with arbitrarily shaped structure analyzed in the free space as shown in Figure 1. We define a regular reference surface around it. For some objects with regular shapes, such as spheres or cubes, their natural boundaries can be used as

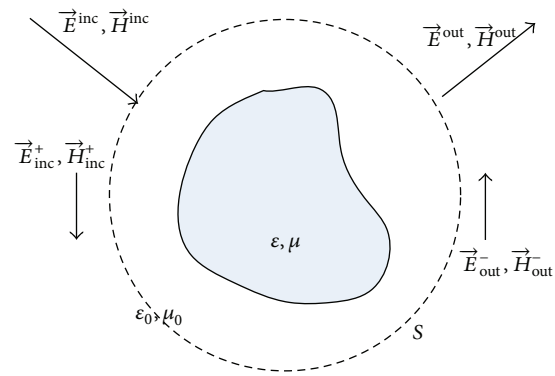

Figure 1: A target object in free space bounded by a reference surface.

the reference surface for simplicity. The permittivity and permeability of the free space and the object are ε_0, μ_0 and ε, μ, respectively. Based on the equivalence theorem, the target object is equivalent to free space region with surface equivalence currents.

The normal unit vector $\hat{a}_n$ of the reference surface S points outward. $\vec{E}^{\text{inc}}$, $\vec{H}^{\text{inc}}$ are incident fields illuminating on S and $\vec{E}^{\text{out}}$, $\vec{H}^{\text{out}}$ are resultant outgoing fields aroused by the equivalent sources on the reference surface. Denote the rotated tangential components of the incident field and the outgoing field on the reference surface S by $\mathbf{X}^+$ and $\mathbf{X}^-$ as

$$\begin{aligned} \mathbf{X}^+ &= \begin{bmatrix} \mathbf{E}^+ \\ \mathbf{H}^+ \end{bmatrix} = \begin{bmatrix} \mathbf{E}^{\text{inc}} \times \hat{a}_n \\ \hat{a}_n \times \mathbf{H}^{\text{inc}} \end{bmatrix}\Bigg|_S, \\ \mathbf{X}^- &= \begin{bmatrix} \mathbf{E}^- \\ \mathbf{H}^- \end{bmatrix} = \begin{bmatrix} \mathbf{E}^{\text{out}} \times \hat{a}_n \\ \hat{a}_n \times \mathbf{H}^{\text{out}} \end{bmatrix}\Bigg|_S. \end{aligned} \tag{1}$$

The outgoing fields $\mathbf{X}^-$ can be determined from $\mathbf{X}^+$ if the internal structure of the block is a scatterer, which is modeled as a generalized one-port device, with its reference surface being selected to coincide with S. However, if there are sources inside the reference surface, $\mathbf{X}^-$ is not only aroused by the external incident field, but also attributed by the internal excitations. Therefore, the rotated tangential components of the outgoing field $\mathbf{X}^-$ on the reference surface S can be expressed by the linear superposition of the two kinds of responses from the external and internal signals.

Approximating surface S with a set of mesh structure and expanding all fields with vector basis functions $\vec{\mathbf{f}}_n(\mathbf{r})$ yield

$$\vec{\mathbf{E}}^{\pm}(\mathbf{r}) = \sum_{n=1}^{N} e_n^{\pm} \vec{\mathbf{f}}_n(\mathbf{r}), \qquad \vec{\mathbf{H}}^{\pm}(\mathbf{r}) = \sum_{n=1}^{N} h_n^{\pm} \vec{\mathbf{f}}_n(\mathbf{r}). \tag{2}$$

In the case that there is only one source I in the target block, the rotated tangential components $\mathbf{X}^-$ of the outgoing field and the rotated tangential components $\mathbf{X}^+$ of the incident field can be related by

$$\mathbf{X}^- = \mathbf{T} \cdot \mathbf{X}^+ + \mathbf{M}I. \tag{3}$$

$\mathbf{T}$ is the generalized transition matrix which connects the rotational components of the scattered field and the incident

field on the reference surface directly. The column vector $\mathbf{M}$ is defined as the mapping vector which is constructed to map the internal source I onto the reference surface as the rotated tangential components of the radiation field. For elements with multiple feeding sources, I becomes a vector, and $\mathbf{M}$ becomes a matrix. Since the details of formulas can be found in [7, 16], they are not listed in this paper for the sake of simplicity.

According to Huygens' principle, the outgoing fields from the block at observation point outside that block are calculated with the following surface integral equation (SIE) on its reference surface

$$\vec{\mathbf{E}}^{\text{out}}(\vec{\mathbf{r}}) = \int_S \left[-j\omega\mu_0 \overline{\overline{\mathbf{G}}}_0 \cdot \vec{\mathbf{H}}^- - \nabla \times \overline{\overline{\mathbf{G}}}_0 \cdot \vec{\mathbf{E}}^-\right] dS', \quad (4)$$

$$\vec{\mathbf{H}}^{\text{out}}(\vec{\mathbf{r}}) = \int_S \left[-j\omega\varepsilon_0 \overline{\overline{\mathbf{G}}}_0 \cdot \vec{\mathbf{E}}^- + \nabla \times \overline{\overline{\mathbf{G}}}_0 \cdot \vec{\mathbf{H}}^-\right] dS', \quad (5)$$

where $\overline{\overline{\mathbf{G}}}_0 = (\overline{\overline{\mathbf{I}}} + (\nabla\nabla)/k^2)g_0$ is the dyadic Green's function and g_0 is the scalar Green's function of background medium.

At a given frequency point, $\mathbf{T}$ and $\mathbf{M}$ are independent of the incident fields and the internal sources. It can fully characterize the scattering and radiation characteristics of a target bounded by the specified reference surface and can be used to replace the object in evaluating fields outside the reference surface. The reference surface does not only provide a clear interface to divide the interior and exterior area of a block but also provides a "port" on which various forms of port variables can be defined.

It indeed costs more time to construct the GTM model of an object than to solve it directly using conventional methods; however, the advantage shows up when solving problems of multimodule system especially when the parallel technique is applied. The GTM method projects the characteristics of the target onto the regular reference surface to reduce the number of unknowns and to improve the computational efficiency.

2.2. Hybrid Formulation. Consider a complex system on an electrically large platform. The problem is divided into a GTM region and a PO region. The GTM region contains all objects with fine mesh structures, with its subblocks described with GTM models. The PO region contains the platform, with the PO current on its illuminated surface given by [12]

$$\mathbf{J}^{\text{PO}} = 2\hat{n} \times \mathbf{H}^{\text{in}}, \quad (6)$$

where $\hat{n}$ denotes the normal vector on the surface.

Considering the system in Figure 2, we divide the system into M GTM blocks and a PO region according to its natural boundary. Each GTM block is surrounded by its reference surface where the GTM model is built independently. The information of the mth block participating interaction with the others is only conveyed by the calculated GTM model with the GTM $\mathbf{T}_m$ and the mapping vector $\mathbf{M}_m$. The rotated tangential components of the outgoing field of block-n on the reference surface are denoted as $\mathbf{X}_n^-$, by which the outgoing fields in the space can be obtained. And the electrically large plate is named as the 0th block, with its outgoing field calculated by the PO currents. Through the field transmission

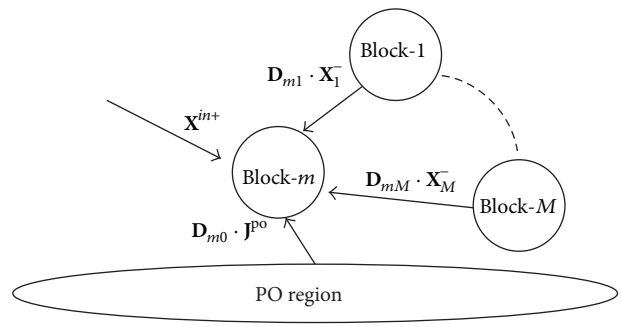

Figure 2: Incident fields on block-m from the external excitation, the PO region, and all the other GTM blocks.

between blocks, the outgoing fields of block-n become part of the excitation fields on the other blocks and the PO region. The total incident fields are the superposition of the natural excitation from external environment, the outgoing fields from all the other blocks, and the PO region, which we can apply to establish the iterative equations. In this paper, we use N_p RWG basis functions to discretize the PO currents on the electrically large platform.

Taking block-m as the target block as shown in Figure 2, the total incident field on its reference surface S_m is defined as

$$\mathbf{X}_m^+ = \mathbf{X}_m^{\text{in}+} + \mathbf{D}_{m0} \cdot \mathbf{J}^{\text{PO}} + \sum_{n=1, n\neq m}^{M} \mathbf{D}_{mn} \cdot \mathbf{X}_n^-, \quad (7)$$

where $\mathbf{D}_{mn}$ is defined as the field transmission matrix from block-n to block-m. All the entries in matrix $\mathbf{D}_{mn}$ are evaluated with double surface integrals, which can be found in [6]. $\mathbf{D}_{m0}$ represents the field transmission from the PO region to the mth GTM block. $\mathbf{X}_m^{\text{in}+}$ describes the rotated tangential components of the external interference excitation field which needs to be considered only when the electromagnetic system exposed in an external interference fields. The PO current $\mathbf{J}^{\text{PO}}$ includes two parts,

$$\mathbf{J}^{\text{PO}} = \mathbf{J}_0^{\text{in}} + \sum_{n=1}^{M} \mathbf{D}_{0n} \cdot \mathbf{X}_n^-, \quad (8)$$

where $\mathbf{J}_0^{\text{in}}$ is the PO current on the plate aroused by external interference excitation magnetic field, which is expressed by (6) and is expanded with vector basis functions on the plate. $\mathbf{D}_{0n}$ is the transmission matrix which accounts for the PO current caused by the scattered magnetic fields associated with the rotated tangential fields on the nth GTM reference surface. By virtue of (2), (5), and (6), we can derive the discretized form of the generalized surface integral equation defined on reference surface S_m as follows:

$$\begin{aligned}(\mathbf{U} - \mathbf{T}_m\mathbf{D}_{m0}\mathbf{D}_{0m}) \cdot \mathbf{X}_m^- &= \mathbf{T}_m \sum_{n=1, n\neq m}^{M} (\mathbf{D}_{mn} + \mathbf{D}_{m0}\mathbf{D}_{0n}) \cdot \mathbf{X}_n^- \\ &\quad + \mathbf{T}_m\mathbf{D}_{m0} \cdot \mathbf{J}_0^{\text{in}} + \mathbf{M}_m I_m + \mathbf{T}_m \cdot \mathbf{X}_m^{\text{in}+},\end{aligned} \quad (9)$$

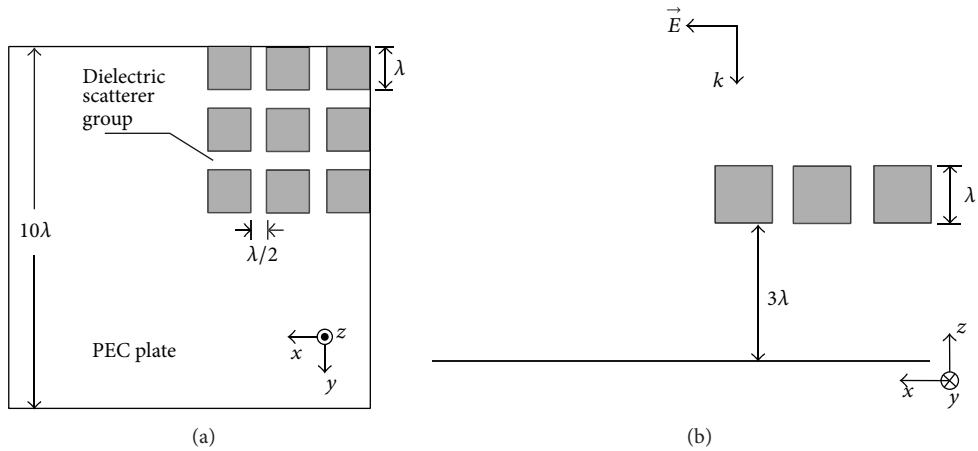

Figure 3: (a) Top view and (b) side view of the scatterer group of 3 × 3 dielectric bodies set above the electrically large PEC platform.

where **U** is the identity matrix. The dimension of the coefficient matrix is determined only by the mesh of the GTM reference surfaces, excluding the mesh of PO region. The PO region is considered as a background with its currents handled as intermediate variables in the solving process. By solving (9), we can get the total tangential components of the outgoing field on each reference surface of the GTM blocks, and the PO current coefficient vector $\mathbf{J}^{\mathrm{po}}$ can be obtained from (8). Hence, both current distributions in the PO region and GTM region are obtainable. The outgoing fields of the whole system can be calculated by (**??**). Compared to the conventional MoM method where the whole structure is discretized with fine meshes, in GTM-PO method, the number of unknowns involved in the final linear system is significantly reduced.

2.3. SBF Implementation. The SBFs [9] can be used to extract the most important electromagnetic characteristics of the GTM models in order to further reduce the system scale. Two important processes are introduced in generating SBFs for a block: (1) adopting equivalent auxiliary sources to obtain the solution spaces and (2) applying singular value decomposition (SVD) to generate characteristic vectors and create SBFs.

We denote the kth SBF of the mth GTM model as

$$\vec{F}_k^{(m)} = \sum_{i=1}^{N_g} a_{i,k}^{(m)} \vec{f}_i^{(m)}, \quad k = 1, 2, \ldots, K, \tag{10}$$

where the SBF $\vec{F}_k^{(m)}$ is an aggregation of the lower order basis functions $\vec{f}_i^{(m)}$ on the reference surface of the GTM model and $a_{i,k}^{(m)}$ is the kth column vector of the singular matrix obtained by SVD. The coefficients to construct SBFs can be chosen by setting a reasonable threshold. Usually the number of SBFs is far less than that of basis function on the reference surface.

The undetermined field tangential components on the reference surface can be replaced by expansion of the synthetic basis functions

$$\mathbf{X}_m^- = \sum_{k=1}^{K} \varphi_k^{(m)} \vec{F}_k^{(m)}. \tag{11}$$

A small amount of SBFs can effectively indicate the characteristics of a GTM model. Substituting (11) into (9), the dimension of the coefficient matrix determined by the numberof SBFs can be further reduced.

3. Numerical Results

The essential concept of hybrid method can be described in the following way: firstly, divide the whole region into a GTM region and a PO region. Excluding the unknowns in the PO region, the dimension of the coefficient matrix is determined by the mesh on the GTM reference surfaces. Afterwards, the application of SBFs can extract the characteristics of GTM blocks in order to further reduce the dimension of the system. The current on the PO region is directly determined by the total tangential magnetic fields on the surface, only those currents on the reference surfaces of the GTM blocks have to be solved from the linear equation. The merits of the hybrid GTM-PO method show up especially when the objects have complex fine structures or the platform is very large.

Here we take a dielectric array on an electrically large platform as an example. It is difficult to use the conventional RWG-MoM, because the dimension of the linear system is too large to solve. In the following, the proposed hybrid method is applied and numerical results are presented to illustrate the accuracy of the GTM-PO method with SBFs.

For the model shown in Figure 3, there are 9 periodic dielectric cubes placed on a square PEC plate. The system is illuminated by a θ-polarized plane wave of 300 MHz that propagates in the negative z direction plotted in Figure 3(b). The relative dielectric constant ε_r of the dielectric cubes is 3. The edge length of each homogeneous dielectric body and the PEC plate is 1λ and 10λ, respectively. The dielectric array is placed 3λ away from the large plate in positive z

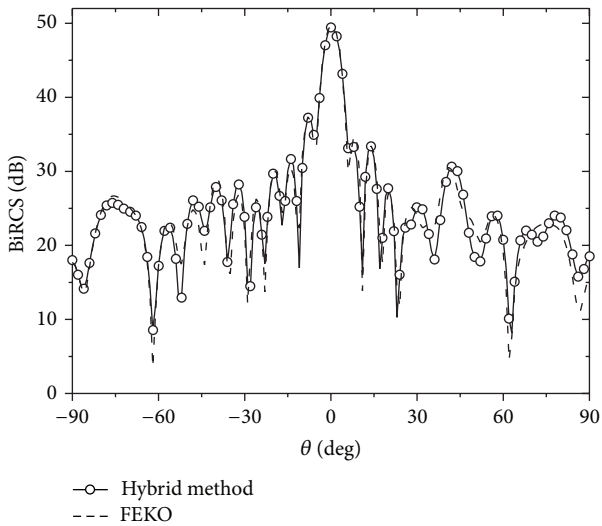

Figure 4: BiRCSs of the dielectric group on a large PEC platform.

direction. The distance between adjacent dielectric elements is 0.5λ. In the implementation of the conventional MoM for discretization, each dielectric element is meshed into 782 RWGs and the PEC plate is meshed into 3544 RWGs. Generally, we construct the PMCHW equations to solve a homogeneous dielectric scatterer. Note that both the electric field and the magnetic field are needed to be calculated, and the number of unknowns in the linear system should be twice the number of basis functions. Therefore, the order of the final matrix constructed by conventional MoM method is 17620. Each dielectric element is defined as an individual GTM model with the corresponding natural boundary as its GTM reference surface. Each reference surface is coarsely meshed into 394 RWGs. 312 auxiliary sources are applied for the construction of solution space. In SVD process, by setting a threshold of singular value ratio as 0.02, we take the first 50 eigenfunctions, corresponding to the first 50 biggest eigenvalues, as the coefficients to construct SBFs. The determination of coefficients needs to balance the demand of precision and order reduction. Then the final coefficient matrix is constructed applying SBFs to (9), resulting in a 450×450 linear matrix, much smaller than the 17620×17620 linear matrix in the RWG-MoM solution.

Figure 4 shows that the result of RCS obtained by the proposed hybrid method has a good accuracy and gains coincident result with the commercial software. The results of RCS include the influences from both the large PEC platform and the dielectric array. The error is mainly raised by the PO method since it is an approximate method. The error may be reduced by taking into account the Fock currents in the shadow region and the fringe wave currents at the edge of the large PEC plate. The number of unknowns using the proposed method with SBFs is only 2.55% of those using the conventional RWG-MoM method in this example.

For antennas with excitation sources working on the electrically large platform, the proposed method also has its advantages.

In the following example, a practical Vivaldi antenna working at 2.4 GHz is placed 2λ away from the middle of a PEC plate in z direction as shown in Figure 5(a). The currents in the PO region are excited by the radiation of the currents in the GTM region.

The length and width of the antenna are, respectively, 18 cm and 7 cm. The geometry size of the Vivaldi antenna is listed as follows: $W_t = 7$ cm, $W_s = 0.3$ cm, $W_k = 0.2$ cm, $L_t = 8.5$ cm, $L_s = 0.3$ cm, $d_1 = 0.7$ cm, $d_2 = 2$ cm, $R_1 = 1$ cm, $R_2 = 1.5$ cm, and $\theta_1 = 77^\circ$, $\theta_2 = 31^\circ$ referring to Figure 5(c). The relative dielectric constant ε_r of the substrate is 2.2 and the thickness t is 0.4 cm. There are 1593, 449, and 50 RWG basis functions meshed on the dielectric, the ground, and the feed line, respectively. The GTM reference surface is set 0.25λ close to the natural boundary of the antenna element as shown in Figure 5(b). It is triangularly meshed, resulting in 512 basis functions. Then, by setting a threshold of singular value ratio as 0.01, 32 SBFs is chosen after SVD process. The square PEC plate with its edge length of 5λ is meshed by 886 RWGs. The radiation patterns in xoz plane calculated by the proposed hybrid method and the commercial software are compared in Figure 6.

The proposed method has an acceptable agreement with FEKO at the main lobe. Some errors, within 3 dB, appear in the back lobe of the antenna. According to Figure 6, when the PEC plate is analyzed by the conventional MoM and the antenna is analyzed by the GTM method with SBFs, the results are in good agreement with the results of MoM. Therefore, we can conclude that the main factor of the error generation comes from the PO method in our proposed method. The number of unknowns by the proposed hybrid method in this example is only 0.71% of the conventional RWG-MoM method.

The accuracy can be judged by the relative errors of radiation field in the xoz plane as

$$\mathrm{Err} = \left(\frac{\sum_{i=1}^{360} \left| rE'(\varphi_i) - rE_{\mathrm{MoM}}(\varphi_i) \right|^2}{\sum_{i=1}^{360} \left| rE_{\mathrm{MoM}}(\varphi_i) \right|^2} \right)^{0.5} \times 100\%, \quad (12)$$

where $\varphi_i = i\pi/360$, $i = 1, 2, \ldots, 360$, and rE' means radiation field calculated by different hybrid methods. The quantitative analyses of different methods are made by the relative errors and the number of unknowns, based on the data listed in Table 1. The proposed method makes a great compression of the final matrix, and the relative error, 1.52%, is the largest among all the methods but is still within the acceptable limits. The SBF approach applied to the GTM model does not bring obvious deterioration of the accuracy. When the PEC plate is analyzed by the conventional MoM, the accuracy can be effectively guaranteed according to the data in the table; however, the dimension of the matrix is larger than that generated by the GTM + SBFs method with PO method. When the platform is very large with dense meshes, the merits of the proposed hybrid method show up significantly.

4. Conclusion

In this paper, a novel hybrid method of GTM-PO with SBFs based on the DDM is introduced. The GTM models are used to accurately describe the electromagnetic characteristics of

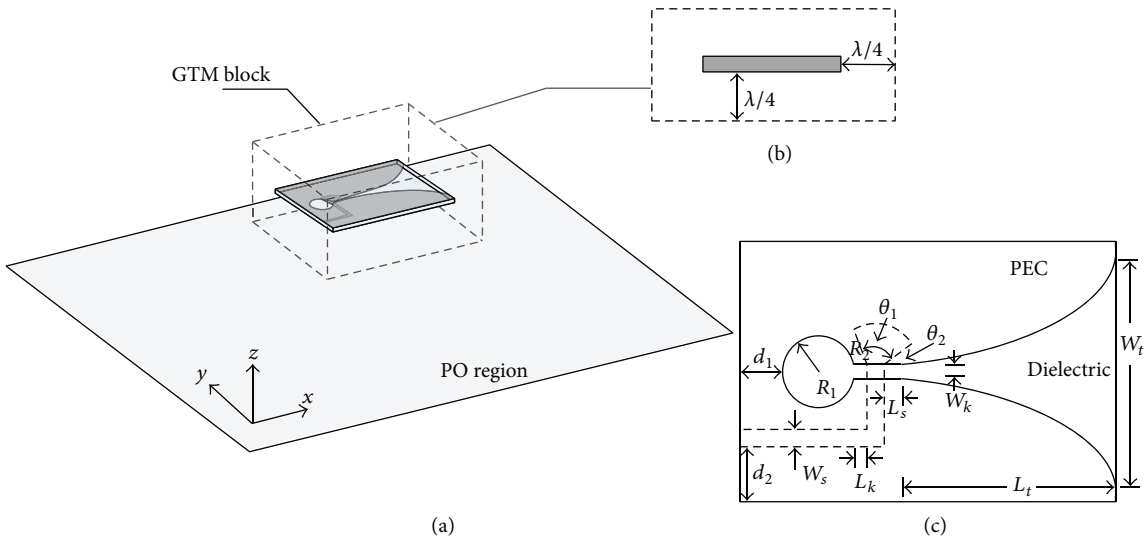

Figure 5: (a) The sketch of the antenna on the PEC plate and (b) side view of the GTM block. (c) Geometry parameters of the Vivaldi antenna.

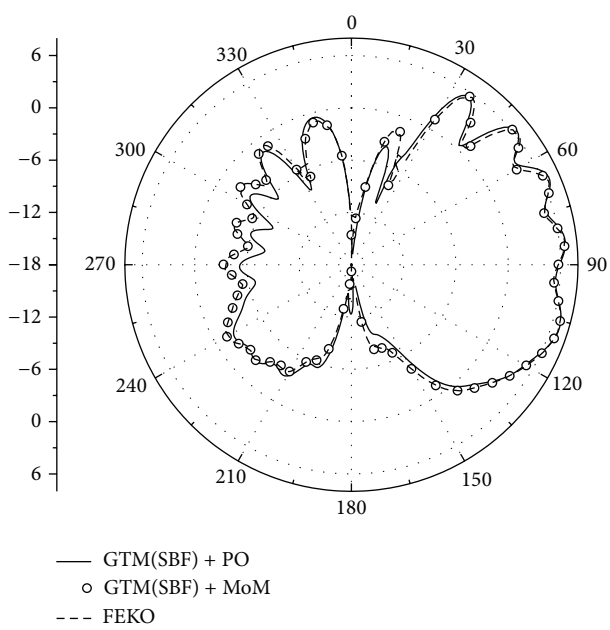

Figure 6: Radiation patterns in the *xoz* plane of the Vivaldi antenna element on a large PEC plate.

target objects with fine structures, while the PO method is used to analyze the electrically large platform. SBFs are applied to extract features of GTM models by the SVD process on the solution spaces. The order of the final coefficient matrix is determined by the total number of SBFs and is greatly reduced. In the proposed method, the PO region is considered as the background and the PO currents are handled as intermediate variables in the solving process. The proposed hybrid method greatly improves the computational efficiency with an acceptable accuracy.

Table 1: Comparison of the computation results of the example in Figure 5.

Method	Number of unknowns	Relative error
MoM	4571	—
GTM + MoM#	1910	0.39%
GTM(SBF) + MoM#	918	0.41%
GTM + PO*	1024	1.46%
GTM(SBF) + PO*	32	1.52%

#The PEC plate is calculated by conventional MoM.
*The PEC plate is calculated by PO method.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgment

This work is supported by SAST foundation 2013 and the National Science Foundation of China (61234001).

References

[1] S. M. Rao, D. R. Wilton, and A. W. Glisson, "Electromagnetic scattering by surface of arbitrary shape," *IEEE Transactions on Antennas and Propagation*, vol. 30, no. 3, pp. 409–418, 1982.

[2] S.-C. Lee, M. N. Vouvakis, and J.-F. Lee, "A non-overlapping domain decomposition method with non-matching grids for modeling large finite antenna arrays," *Journal of Computational Physics*, vol. 203, no. 1, pp. 1–21, 2005.

[3] A. M. Van De Water, B. P. De Hon, M. C. Van Beurden, A. G. Tijhuis, and P. De Maagt, "Linear embedding via Greenâs operators: a modeling technique for finite electromagnetic band-gap structures," *Physical Review E*, vol. 72, no. 5, Article ID 056704, 2005.

[4] M.-K. Li and W. C. Chew, "Wave-field interaction with complex structures using equivalence principle algorithm," *IEEE Transactions on Antennas and Propagation*, vol. 55, no. 1, pp. 130–138, 2007.

[5] G. Xiao, J. Mao, and B. Yuan, "Generalized transition matrix for arbitrarily shaped scatterers or scatterer groups," *IEEE Transactions on Antennas and Propagation*, vol. 56, no. 12, pp. 3723–3732, 2008.

[6] G. Xiao, J.-F. Mao, and B. Yuan, "A generalized surface integral equation formulation for analysis of complex electromagnetic systems," *IEEE Transactions on Antennas and Propagation*, vol. 57, no. 3, pp. 701–710, 2009.

[7] J. M. Song and W. C. Chew, "Multilevel fast-multipole algorithm for solving combined field integral equations of electromagnetic scattering," *Microwave and Optical Technology Letters*, vol. 10, no. 1, pp. 14–19, 1995.

[8] E. Bleszynski, M. Bleszynski, and T. Jaroszewicz, "AIM: adaptive integral method for solving large-scale electromagnetic scattering and radiation problems," *Radio Science*, vol. 31, no. 5, pp. 1225–1251, 1996.

[9] L. Matekovits, V. A. Laza, and G. Vecchi, "Analysis of large complex structures with the synthetic-functions approach," *IEEE Transactions on Antennas and Propagation*, vol. 55, no. 9, pp. 2509–2521, 2007.

[10] L. Matekovits, G. Vecchi, M. Bercigli, and M. Bandinelli, "Synthetic-functions analysis of large aperture-coupled antennas," *IEEE Transactions on Antennas and Propagation*, vol. 57, no. 7, pp. 1936–1943, 2009.

[11] A. Freni, P. De Vita, P. Pirinoli, L. Matekovits, and G. Vecchi, "Fast-factorization acceleration of MoM compressive domain-decomposition," *IEEE Transactions on Antennas and Propagation*, vol. 59, no. 12, pp. 4588–4599, 2011.

[12] U. Jakobus and F. M. Landstorfer, "Improved PO-MM hybrid formulation for scattering from three-dimensional perfectly conducting bodies of arbitrary shape," *IEEE Transactions on Antennas and Propagation*, vol. 43, no. 2, pp. 162–169, 1995.

[13] C. S. Kim and Y. Rahmat-Samii, "Low profile antenna study using the physical optics hybrid method (POHM)," in *Proceedings of the Antennas and Propagation Society Symposium*, vol. 3, pp. 1350–1353, London, ON, Canada, June 1991.

[14] R. E. Hodges and Y. Rahmat-Samii, "An iterative current-based hybrid method for complex structures," *IEEE Transactions on Antennas and Propagation*, vol. 45, no. 2, pp. 265–276, 1997.

[15] F. Obelleiro, J. M. Taboada, J. L. Rodrıguez, J. O. Rubinos, and A. M. Arias, "Hybrid moment-method physical-optics formulation for modeling the electromagnetic behavior of on-board antennas," *Microwave and Optical Technology Letters*, vol. 27, no. 2, pp. 88–93, 2000.

[16] S. Xiang, G. B. Xiao, X. Z. Tian, and J. F. Mao, "Analysis of large-scale phased antenna array with generalized transition matrix," *IEEE Transactions on Antennas and Propagation*, vol. 61, no. 11, pp. 5453–5464, 2013.