

## Research Article

# **Energy-Efficiency Analysis of Per-Subcarrier Antenna Selection** with Peak-Power Reduction in MIMO-OFDM Wireless Systems

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Received 8 January 2014; Accepted 14 May 2014; Published 11 June 2014

Academic Editor: Ahmad Safaai-Jazi

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The use of per-subcarrier antenna subset selection in OFDM wireless systems offers higher system capacity and/or improved link reliability. However, the implementation of the conventional per-subcarrier selection scheme may result in significant fluctuations of the average power and peak power across antennas, which affects the potential benefits of the system. In this paper, power efficiency of high-power amplifiers and energy efficiency in per-subcarrier antenna selection MIMO-OFDM systems are investigated. To deliver the maximum overall power efficiency, we propose a two-step strategy for data-subcarrier allocation. This strategy consists of an equal allocation of data subcarriers based on linear optimization and peak-power reduction via cross-antenna permutations. For analysis, we derive the CCDF (complementary cumulative distribution function) of the power efficiency as well as the analytical expressions of the average power efficiency. It is proved from the power-efficiency perspective that the proposed allocation scheme outperforms the conventional scheme. We also show that the improvement in the power efficiency translates into an improved capacity and, in turn, increases energy efficiency of the proposed system. Simulation results are provided to validate our analyses.

## **1. Introduction**

The last few years have seen an increasing demand for very fast data speeds in wireless multimedia applications. Meanwhile, reducing energy consumption in wireless networks has become a problem of concern among academic and industrial communities. As a result, high-speed systems with high-energy efficiency has emerged as a main stream for designing future wireless networks [1, 2]. The potential for improving energy efficiency in wireless systems could be in component level, link level, or network level [1]. From a physical layer viewpoint, it is well known that high-power amplifier (HPAs) is a major source of RF (radio frequency) power consumption. For example, in mobile networks, HPAs consume up to 50%–80% of overall power at a base station [1, 3]. Thus, increasing power efficiency of HPAs is of importance to achieve high energy-efficient wireless networks.

To date, MIMO-OFDM (multi-input multi-output orthogonal frequency division multiplexing) has been considered as a key technique for high-speed wireless transmission [4]. This is mainly because OFDM could offer high spectral efficiency and robustness against intersymbol interference in multipath fading channels. Also, an increase in capacity and/or diversity gains could be achieved with MIMO [5, 6]. In fact, MIMO-OFDM has been adopted in current and future standards, such as WiMAX, WLAN 802.11n, or LTE-Advanced. Among various MIMO schemes, antenna selection appears to be promising for OFDM systems. This MIMO scheme requires a low-cost implementation and small amount of feedback information, compared to other precoding or beamforming methods [7].

Many research works have considered the application of antenna selection in OFDM systems; for example, see [8–12]. To achieve a large coding gain resulting from the frequency-selective nature of the channels, a per-subcarrier antenna selection (i.e., selecting antennas on each subcarrier basis) is applied [9]. However, as this method selects antennas independently on each subcarrier, a large number of data symbols may be allocated to some antennas, depending on the channel condition. As a result, the peak power and average power of the signals on these antennas might be very large, whereas those on the other antennas might be small. The fluctuation of the powers clearly affects the power efficiency of HPAs or distorts signals, which in turn reduces the potential benefits of the antenna selection OFDM systems [13].

One possible approach to deal with the problem of imbalance allocation of data subcarriers is selecting antennas under a constraint that the number of data subcarriers allocated to each antenna is equal. Some research works have studied such a constrained selection approach in the literature; for example, see [10-12]. In [10], an ad hoc algorithm was developed to realize the constrained selection scheme. Meanwhile, the authors in [11] considered linear optimization to devise the constrained selection scheme. It was shown that the selection scheme based on optimization could offer a better performance than the suboptimal solution in [10]. In [12], we generalized the approach in [11] to the system with an arbitrary number of multiplexed data streams and analyzed the performance directly in nonlinear fading channels. However, all of these works only study the efficacy of the constrained selection scheme from error-performance perspective. Moreover, even though the same number of data subcarriers is allocated to each transmit antenna (i.e., all antennas have an equal average power), an occurrence of high peak-to-average power ratio (PAPR) still affects the system. Despite an increasing concern about energy consumption in wireless networks, to the best of our knowledge, a research on energy efficiency in the context of antenna selection MIMO-OFDM systems is still missing.

It is also essential to emphasize the need of research on improving energy efficiency in antenna selection MIMO-OFDM systems. First, per-subcarrier antenna selection MIMO configurations need multiple active RF (radio frequency) chains, which immediately raises a concern about energy consumption compared to single RF-chain OFDM systems. Second, OFDM inherently suffers from a high PAPR problem, which leads to a poor power efficiency of HPAs [13, 14]. In per-subcarrier antenna selection OFDM systems, this effect is intuitively more serious due to a combination of the high PAPR and power fluctuation resulting from an imbalance allocation of data subcarriers. This problem also becomes crucial in the OFDM systems where linear scaling (i.e., scale the peak power of the time-domain OFDM signals to the saturation level of the HPAs [15–17]) is implemented to realize OFDM transmissions with no nonlinear distortions.

In this paper, per-subcarrier antenna selection MIMO-OFDM systems with linear scaling are investigated from an energy perspective for the first time. The important contributions of this work include (a) the analysis of power efficiency of HPAs and energy efficiency and (b) the proposed strategy to improve these useful performance metrics. The main results are summarized as follows.

 A two-step data allocation strategy is proposed to deliver a maximum overall power efficiency of HPAs. This strategy consists of an equal allocation of data subcarriers among transmit antennas based on linear optimization and a peak-power reduction algorithm via cross-antenna permutations.

- (2) Analytical expressions characterizing the achieved power efficiency of HPAs, including the CCDFs (complementary cumulative distribution function) and the average power efficiency, are derived. It is proved that, from the power-efficiency viewpoint, the proposed allocation scheme outperforms the conventional scheme.
- (3) The improvements in capacity and energy efficiency resulting from the improved power efficiency of HPAs are analyzed.

Numerical results are provided to verify the analyses as well as demonstrate the benefits in terms of the power efficiency of HPAs and capacity as well as energy efficiency.

The remainder of the paper is organized as follows. In Section 2, a per-subcarrier antenna subset selection MIMO-OFDM system with linear scaling is described. In Section 3, a data allocation strategy that could allocate evenly data subcarriers across antennas with a low peak power is proposed. Analysis of power efficiency is carried out in Section 4. The achievable capacity and energy efficiency are considered in Section 5. Numerical results are provided in Section 6. Finally, Section 7 concludes the paper.

*Notation*. A bold letter denotes a vector or a matrix, whereas an italic letter denotes a variable.  $(\cdot)^T$ ,  $(\cdot)^H$ ,  $E\{\cdot\}$ , and det $(\cdot)$  denote transpose, Hermitian transpose, expectation, and determinant of a matrix, respectively.  $\otimes$  denotes the Kronecker product.  $\mathbf{I}_n$  indicates the  $n \times n$  identity matrix, and  $\mathbf{1}_K$  is a  $K \times 1$  vector of ones.  $\mathfrak{R}$  indicates the set of real numbers.

## 2. Antenna Selection for MIMO-OFDM Systems with Linear Scaling

2.1. System Model. We consider a MIMO-OFDM system with K subcarriers,  $n_T$  transmit antennas, and  $n_R$  receive antennas as shown in Figure 1. At the transmitter, the input data are demultiplexed into  $n_D$  independent data streams. Each data stream is then mapped onto M-QAM (M-ary quadrature amplitude modulation) constellations. For the kth subcarrier, we denote  $u_l^k$  and  $x_i^k$ ,  $1 \leq l \leq n_D$ ,  $1 \leq i \leq n_T$ ,  $0 \le k \le K - 1$ , to be the symbols that the subcarrier block takes at its *l*th input and outputs at its *i*th output, respectively. The allocation block assigns the elements of  $\mathbf{u}_k = [u_1^k, u_2^k, \dots, u_{n_D}^k]^T$  to  $n_D$  selected antennas at the *k*th subcarrier based on feedback information. As a result, only  $n_D$  elements in a vector  $\mathbf{x}_k = [x_1^k, x_2^k, \dots, x_{n_T}^k]^T$  are assigned values from  $\mathbf{u}_k$ , whereas the others are zeros. It is assumed that  $E\{\mathbf{u}_k\mathbf{u}_k^H\} = \sigma^2 \mathbf{I}_{n_D}$ . The output sequences from the subcarrier allocation block are then fed into K-point IFFT (inverse fast Fourier transform) blocks. In this paper, the Nyquist sampling signal is considered. Thus, the discretetime baseband OFDM signals can be expressed as

$$s_i(n) = \frac{1}{\sqrt{K}} \sum_{k=0}^{K-1} x_i^k e^{j2\pi nk/K}, \quad 0 \le n \le K-1, \ 1 \le i \le n_T.$$
(1)



FIGURE 1: A simplified block diagram of a MIMO-OFDM system with per-subcarrier transmit antenna selection.

For simplicity, we consider ideal predistortion HPAs (i.e., soft envelope limiters) with a unity gain and class-A operation. To deliver the maximum power efficiency with no nonlinear distortions in the system with nonlinear HPAs, the peak power across transmit antennas is linearly scaled to the saturation level  $P_{sat}$  of the HPAs. In addition, as feedback information (i.e., the selected antenna indices which are calculated based on the channel state information) is deployed by the transmitter, all transmit branches are scaled with the same scaling factor [16]. Thus, the signal after linear scaling can be expressed as

$$\widetilde{s}_i(n) = \sqrt{\alpha} s_i(n), \qquad (2)$$

where the scaling factor  $\alpha = P_{sat}/P$ , and the peak power across antennas  $P = \max\{s_i(n)|n = 0, \dots, K - 1; i = 1, \dots, n_T\}$ . Each time-domain OFDM signal is then amplified by the HPA before being transmitted via its corresponding transmit antenna.

At the receiver, the received signal at each antenna is fed into the FFT block after the GI (guard interval) is removed. The system model in the frequency domain corresponding to the *k*th subcarrier can be expressed as

$$y_k = \sqrt{\alpha} \mathbf{H}_k \mathbf{x}_k + \mathbf{n}_k$$
  
=  $\sqrt{\alpha} \mathbf{H}_k \mathbf{u}_k + \mathbf{n}_k$ , (3)

where

$$\mathbf{x}_{k} = \begin{bmatrix} x_{1}^{k} & x_{2}^{k} & \cdots & x_{n_{T}}^{k} \end{bmatrix}^{T},$$

$$\mathbf{H}_{k} = \begin{bmatrix} h_{1,1}^{k} & h_{1,2}^{k} & \cdots & h_{1,n_{T}}^{k} \\ h_{2,1}^{k} & h_{2,2}^{k} & \cdots & h_{2,n_{T}}^{k} \\ \cdots & \cdots & \cdots & \cdots \\ h_{n_{R},1}^{k} & h_{n_{R},2}^{k} & \cdots & h_{n_{R},n_{T}}^{k} \end{bmatrix},$$

$$\mathbf{y}_{k} = \begin{bmatrix} y_{1}^{k} & y_{2}^{k} & \cdots & y_{n_{R}}^{k} \end{bmatrix}^{T},$$

$$\mathbf{n}_{k} = \begin{bmatrix} n_{1}^{k} & n_{2}^{k} & \cdots & n_{n_{R}}^{k} \end{bmatrix}^{T}.$$
(4)

In the above equations,  $h_{j,i}^k$  indicates the channel coefficient between the *i*th transmit antenna and the *j*th receive antenna. The effective channel matrix  $\underline{\mathbf{H}}_k$  is obtained by eliminating the columns of  $\mathbf{H}_k$  corresponding to the unselected transmit TABLE 1: Antenna subsets ( $n_T = 4$ ,  $n_D = 2$ , and  $\Gamma = 6$ ).

γ	$\Gamma_{\gamma}$
1	{1,2}
2	{1,3}
3	$\{1, 4\}$
4	{2,3}
5	$\{2, 4\}$
6	{3,4}

antennas.  $y_j^k$  and  $n_j^k$  denote the received signal and the noise at the *j*th receive antenna, respectively. Here, the noise is modeled as a Gaussian random variable with zero mean and  $E\{\mathbf{n}_k \mathbf{n}_k^H\} = \sigma_n^2 \mathbf{I}_{n_k}$ . We assume that the receiver can perfectly estimate the channel coefficients, for example, using a blocktype pilot arrangement for channel estimation [18].

2.2. Per-Subcarrier Antenna Subset Selection. In a MIMO-OFDM system with the conventional per-subcarrier antenna subset selection, antenna subsets are selected independently for each subcarrier [9]. On each subcarrier, only  $n_D$  antennas out of  $n_T$  available transmit antennas are active. Denote  $\Gamma_{\gamma}, \gamma = 1, 2, ..., \Gamma$ , to be the  $\gamma$ th subset consisting of  $n_D$ selected transmit antennas, where  $\Gamma = \binom{n_D}{n_T} = n_T!/n_D!(n_T - n_D)!$  is the number of all possible  $n_D$ -element subsets. Each subset consists of  $n_D$  transmit antenna indices that are chosen based on the feedback information from the receiver. For example, when  $n_T = 4$  and  $n_D = 2$ , then  $\Gamma = 6$ , and all possible subsets  $\Gamma_{\gamma}, \gamma = 1, 2, ..., 6$ , are defined in Table 1. The choice of the best antenna subset depends on a particular selection criterion.

Several antenna selection criteria, such as maximum capacity or maximum SNR (signal-to-noise ratio) [19] can be extended to this system. In this paper, we consider the capacity criterion. Accordingly, the optimal subset at the kth subcarrier is determined by maximizing the mutual information of the kth subchannel; that is,

$$\Gamma_{\gamma^*}(k) = \arg \max_{\Gamma_{\gamma}, \gamma = 1, \dots, \Gamma} I_{\gamma}^k,$$
(5)

where

$$I_{\gamma}^{k} = \log_{2} \left( \det \left( \mathbf{I}_{n_{R}} + \frac{\rho}{n_{D}} \underline{\mathbf{H}}_{k} \underline{\mathbf{H}}_{k}^{H} \right) \right)$$
(6)

is the instantaneous mutual information associated with the *k*th subchannel [5]. Here,  $\rho = P_t/\sigma_n^2 = \alpha n_D \sigma^2/\sigma_n^2$ , and  $P_t = \alpha n_D \sigma^2$  is the total transmit power per subchannel. Also, we have assumed in (6) that the transmit power is allocated uniformly across antennas. This is due to the fact that the feedback information in our system is only the selected antenna indices (i.e., not sufficient enough to perform power allocation algorithms across subcarriers as well as antennas). The average mutual information across subcarriers can now be expressed as

$$I\left(\rho,\underline{\mathbf{H}}\right) = \frac{1}{K} \sum_{k=0}^{K-1} I_{\gamma^*}^k.$$
(7)

## 3. A Proposed Strategy for Peak-Power Reduction

In Section 2, we have described the MIMO-OFDM system with the conventional selection scheme. It can be noted that the number of data subcarriers assigned to each transmit antenna might be significantly different depending on the channel condition. In the system with identical linear scaling, to achieve the maximal overall power efficiency of HPAs, the peak power across antennas should be as small as possible. We note that the peak power of the signal on each branch depends on both the transmitted constellation symbols and the number of data subcarriers allocated in each OFDM symbol (cf. (1)). Thus, it is not sufficient to reduce the peak power by solely implementing PAPR reduction techniques. More specifically, PAPR reduction techniques themselves cannot solve the problem of imbalance allocation of data subcarriers across antennas. To reduce the peak power across antennas, we propose a two-step strategy consisting of the following.

*Step 1.* Allocate the same number of data subcarriers to all transmit antennas (i.e., selecting antennas under a constraint that all antennas have the same number of data subcarriers as illustrated in Figure 2(b)). Once this is achieved, the time-domain signals on all transmit branches have the same average power. Moreover, as we will mathematically prove in Section 4.1, the peak power across antennas is reduced.

*Step 2.* Reallocate data symbols across antennas. This process will alter the statistical distribution of signals and thus further reduce the peak power.

We note that this paper does not focus on developing original techniques for either equal allocation of data subcarriers or peak-power reduction. Instead, we are interested in analyzing power efficiency of HPAs and energy efficiency in per-subcarrier antenna selection OFDM systems, which has not been considered so far. The two steps in our proposed strategy, described in Sections 3.1 and 3.2, are accomplished by extending the suitable approaches available in the literature to the context of the considered system. 3.1. Optimal Equal Allocation of Data Subcarriers. The optimal constrained antenna selection scheme based on linear optimization was considered in [11, 12] to improve error performance of OFDM systems suffering nonlinear distortions due to HPAs. We now consider this method for the first step of our strategy to achieve a better power delivery in a linearly scaled MIMO-OFDM system. Specifically, we define a variable  $z_{\gamma}^{k}$ , where  $z_{\gamma}^{k} = 1$ , if  $\Gamma_{\gamma}$  is chosen for the *k*th subcarrier, and  $z_{\gamma}^{k} = 0$  otherwise. Also, denote  $c_{\gamma}^{k}$  to be the cost associated with the chosen subset  $\Gamma_{\gamma}$ . Here,  $c_{\gamma}^{k} = I_{\gamma}^{k}$  as the maximum capacity criterion is considered. By denoting vectors  $\mathbf{z} = (z_{1}^{0}, \ldots, z_{\Gamma}^{0}, z_{1}^{1}, \ldots, z_{\Gamma}^{1}, \ldots, z_{\Gamma}^{K-1}, \ldots, c_{\Gamma}^{K-1})^{T} \in \{0, 1\}^{K\Gamma \times 1}$ , and  $\mathbf{c} = (c_{1}^{0}, \ldots, c_{\Gamma}^{0}, c_{1}^{1}, \ldots, c_{\Gamma}^{1}, \ldots, c_{\Gamma}^{K-1}, \ldots, c_{\Gamma}^{K-1})^{T} \in \Re^{K\Gamma \times 1}$ , an optimal solution for an equal allocation of data subcarriers is obtained by solving the following linear optimization problem [11, 12]:

$$\max_{\mathbf{z}\in\{0,1\}^{KT\times 1}} \mathbf{c}^{T} \mathbf{z},$$
(8)

subject to 
$$Az = a$$
,

where  $\mathbf{A}_1 = \mathbf{I}_K \otimes \mathbf{1}_{\Gamma}^T \in \{0, 1\}^{K \times K\Gamma}, \mathbf{A}_2 = \mathbf{1}_K^T \otimes \mathbf{I}_{\Gamma} \in \{0, 1\}^{\Gamma \times K\Gamma}, \mathbf{A} = (\mathbf{A}_1^T, \mathbf{A}_2^T)^T \in \{0, 1\}^{(K+\Gamma) \times K\Gamma}, \mathbf{a} = (\mathbf{1}_K^T, \boldsymbol{\lambda}^T)^T$ , and  $\boldsymbol{\lambda} = (\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2, \dots, \boldsymbol{\lambda}_{\Gamma})^T$ , where  $\boldsymbol{\lambda}_{\gamma}$  is the number of times that the subset  $\boldsymbol{\Gamma}_{\gamma}$  is selected. The constraint in (8) means that only  $n_D$  antennas are allowed to transmit data symbols on each subcarrier and all transmit antennas have the same number of allocated data subcarriers. It is important to note that this binary linear optimization problem can be relaxed to linear programming with integral solutions [12]. Hence, (8) can be solved efficiently by the known linear programming methods, such as the simplex methods or interior point methods [20].

3.2. Data Allocation with Peak-Power Reduction. To further reduce the peak power of the whole system, various available PAPR reduction techniques (e.g., see [14] and the references therein) can be now adopted. In this paper, we are interested in a selected mapping (SLM) technique [21] as SLM is a distortionless PAPR technique that could achieve a good PAPR reduction [14, 21]. One of the most important steps in SLM is creating a set of candidates that represents the same data information. To exploit the available degrees of freedom in multiple-antenna systems for peak-power reduction, we propose to create a set of candidates using cross-antenna permutations. In the literature, a SLM-based scheme that could exploit the available degrees of freedom was first developed in [22]. The scheme in [22] creates candidates by performing cross-antenna rotation and inversion (CARI) based on a defined random matrix. However, that scheme is proposed for an Alamouti code based MIMO-OFDM system only. In a per-subcarrier antenna selection OFDM system, CARI cannot be implemented directly as only  $n_D$  out of  $n_T$ antennas are active on each subcarrier. To create candidates in our scheme, we perform cross-antenna permutations instead of CARI. In addition, we utilize an antenna allocation pattern that is already known by the transmitter and receiver, instead of storing a defined random matrix at both transmitter and



FIGURE 2: Illustration of per-subcarrier antenna subset selection ( $n_T = 4$ ,  $n_D = 2$ , and K = 12).



FIGURE 3: Illustration of cross-antenna permutations ( $n_T = 4, n_D = 2$ , and K = 4).

receiver as in [22]. The proposed algorithm is described as follows.

- (1) Create W candidates by performing cross-antennas permutations. An illustration of this process in the system with  $n_D = 2$ ,  $n_T = 4$ , and K = 4 is shown in Figure 3. Accordingly, the first candidate is the original data allocation. The second candidate is obtained by permuting all symbols on the first antennas with their associated symbols on the other antennas. The third and fourth candidates are created in a similar manner. To obtain a larger number of candidates, all symbols on the antenna that are going be permutated need first have their phase rotated (i.e., being multiplied with an element of a phase set, e.g., a 4-phase set is  $\{0, \pi/2, \pi, 3\pi/2\}$ ).
- (2) Calculate the peak powers of all available candidates.
- (3) Select the candidate with the minimum peak power for transmission.

To recover the transmitted data, the transmitter needs to inform the receiver which candidates have been selected. Thus, the number of side information bits in this scheme is  $\log_2 W$ , which is similar to that in [22].

*3.3. Complexity Considerations.* In this subsection, the complexity of the proposed allocation scheme is compared to that of the conventional (imbalance) allocation scheme. In the first step of the proposed scheme, to realize an equal allocation of data subcarriers, the optimization problem in (8)

needs to be solved at the receiver. We note that this linear optimization problem can be solved in polynomial time. More specifically, the complexity to solve this problem using interior point methods can be reduced to  $O([(K\Gamma)^3/\ln(K\Gamma)]\xi)$ , where  $O(\cdot)$  denotes an order of complexity and  $\xi$  is the bit size of the optimization problem [23]. In addition, it is noted that this step is transparent to the transmitter (i.e., no additional complexity is required at the transmitter). In the second step, a major additional complexity lies in the required IFFT operations due to additional W candidates. As an K point-IFFT requires  $K \log_2 K$  complex additions and  $(K/2) \log_2 K$ complex multiplications, the numbers of complex additions and complex multiplications in the conventional scheme are  $n_T K \log_2 K$  and  $n_T (K/2) \log_2 K$ , respectively. Meanwhile, in the proposed scheme with W candidates,  $Wn_T K \log_2 K$  complex additions and  $Wn_T(K/2)\log_2 K$  complex multiplications are required. However, as we will show analytically in Section 4 and numerically in Section 6, an improvement in the peakpower reduction reduces when W becomes large. Thus, a small value of W is generally chosen, which does not incur much additional complexity. Finally, the amount of feedback information in the proposed scheme is similar to that in the conventional scheme.

#### 4. Analysis of Power Efficiency

4.1. Statistical Distribution of Peak Power. Before proceeding to analyze the power efficiency of HPAs, we need to investigate the distribution of the peak power of the MIMO-OFDM signals (i.e., the peak power across transmit antennas). We consider the complementary cumulative distribution function (CCDF) of the peak power, defined as the probability that the peak power P exceeds a given threshold  $P_0$ ; that is,

$$\operatorname{CCDF}^{P}(P_{0}) = \Pr\left(P > P_{0}\right). \tag{9}$$

Note that although a procedure for calculating CCDF of PAPR in OFDM systems is known, to the best of our knowledge, all the CCDF expressions with respect to MIMO-OFDM signals available in the literature assume that all data subcarriers are active, which can be considered as a special case in the considered system when all the transmit antennas have the same number of allocated data symbols. In the following, we calculate the CCDF of the peak power in our system.

Let us begin with the discrete-time OFDM signal  $s_i(n)$ , n = 0, 1, ..., K-1, corresponding to the *i*th transmit antenna. The peak power of this signal is defined as

$$P_i = \max_{0 \le n \le K-1} |s_i(n)|^2.$$
(10)

For analytical tractability, we assume that both the real part and imaginary part of the signal  $s_i(n)$  are asymptotically independent and identically distributed Gaussian random variables. Note that this assumption, which is based on the central limit theorem [24], only holds when the number of assigned data subcarriers on the *i*th antenna, denoted as  $K_i$ , is large enough. As a result,  $|s_i(n)|$  follows the Rayleigh distribution, and  $|s_i(n)|^2$  has a chi-square distribution with two degrees of freedom. The probability density function of the signal  $|s_i(n)|^2$  can be expressed as [24]

$$p_{|s|^{2}}\left(\left|s_{i}\right|^{2}\right) = \frac{1}{\sigma_{K_{i}}^{2}}e^{-|s_{i}|^{2}/\sigma_{K_{i}}^{2}},\tag{11}$$

where  $\sigma_{K_i}^2 = \sigma^2 K_i / K$  is the variance of the signal  $|s_i(n)|$ . Note that  $\sum_{i=1}^{n_T} K_i = n_D K$ ; thus, we have  $\sum_{i=1}^{n_T} \sigma_{K_i}^2 = n_D \sigma^2$ . The CDF (cumulative distribution function) of the signal  $|s_i(n)|^2$  is given as

$$\Pr\left(\left|s_{i}\right|^{2} \leq \theta\right) = 1 - e^{-\theta/\sigma_{K_{i}}^{2}}, \quad \theta \geq 0.$$
(12)

Suppose that *K* samples of  $|s_i(n)|$ , n = 0, 1, ..., K - 1, are independent; the CDF of the peak power  $P_i$  can be expressed as

$$CDF^{P_{i}} = Pr(P_{i} \leq P_{0})$$

$$= Pr(|s_{i}(0)|^{2} \leq P_{0})Pr(|s_{i}(1)|^{2} \leq P_{0})\cdots$$

$$Pr(|s_{i}(K-1)|^{2} \leq P_{0})$$

$$= (1 - e^{-P_{0}/\sigma_{K_{i}}^{2}})^{K}.$$
(13)

In MIMO-OFDM systems with linear scaling, the peak power across transmit antennas *P* can be defined as

$$P = \max_{1 \le i \le n_T} P_i. \tag{14}$$

Given the statistical independence of data among transmit antennas, which is the case in the considered spatial multiplexed OFDM system, the CDF of the peak power P is calculated as

$$CDF^{P} = Pr(P \le P_{0})$$

$$= Pr(P_{1} \le P_{0}) Pr(P_{2} \le P_{0}) \cdots Pr(P_{n_{T}} \le P_{0})$$

$$= \prod_{i=1}^{n_{T}} (1 - e^{-P_{0}/\sigma_{K_{i}}^{2}})^{K}.$$
(15)

Therefore, the CCDF of the peak power of the antenna selection MIMO-OFDM signals can be expressed as

$$CCDF_{\text{imbalance}}^{P}(P_{0}) = 1 - CDF^{P}$$
$$= 1 - \prod_{i=1}^{n_{T}} \left(1 - e^{-P_{0}/\sigma_{K_{i}}^{2}}\right)^{K}.$$
 (16)

In the MIMO-OFDM system with a power balancing constraint, the number of allocated data subcarriers per transmit antenna is equal to one another (i.e.,  $K_i = n_D K/n_T := \overline{K}, \forall i = 1, 2, ..., n_T$ ). Thus, the variances of the signals are  $\sigma_{K_i}^2 = n_D \sigma^2/n_T := \sigma_{\overline{K}}^2, \forall i = 1, 2, ..., n_T$ . As a result, the CCDF expression could be simplified to as

$$\text{CCDF}_{\text{balance}}^{P}(P_{0}) = 1 - \left(1 - e^{-P_{0}/\sigma_{K}^{2}}\right)^{n_{T}K}$$
. (17)

A comparison of the CCDF of the peak powers in the two systems is presented in the following theorem.

**Theorem 1.** In MIMO-OFDM transmission schemes that consist of inactive data subcarriers (e.g., per-subcarrier antenna selection), the probability of occurrences of high peak power is the smallest when the same number of data symbols is allocated to all transmit antennas; that is,

$$CCDF_{balance}^{P}(P_{0}) \leq CCDF_{imbalance}^{P}(P_{0}).$$
 (18)

*Proof.* The proof is given in Appendix A.

When the peak-power reduction algorithm proposed in Section 3.2 is implemented in the MIMO-OFDM system with a power-balancing constraint, the CCDF of the peak power can be expressed as

$$CCDF_{balance+reduced}^{P}(P_{0}) = \left(CCDF_{balance}^{P}(P_{0})\right)^{W}$$

$$= \left(1 - \left(1 - e^{-P_{0}/\sigma_{\overline{K}}^{2}}\right)^{n_{T}K}\right)^{W},$$
(19)

where W is the number of candidates that are assumed to be independent. Recall that, by definition, the CCDF value is always smaller than one (cf. (9)). Therefore, the CCDF value in (19) is smaller than that in (17); that is,

$$\operatorname{CCDF}_{\operatorname{balance+reduced}}^{P}(P_0) \leq \operatorname{CCDF}_{\operatorname{balance}}^{P}(P_0).$$
 (20)

International Journal of Antennas and Propagation

4.2. Power Efficiency of HPAs. We now analyze the power efficiency (PE) of high-power amplifiers (HPAs). The drain efficiency of HPAs, which is defined as a ratio between the power drawn from the DC source  $P_{dc}$  and the average output power  $P_{out}$ , is considered in this paper. Denote  $P_{in}^i$  and  $P_{out}^i$  to be the average input and output powers of the HPA for the *i*th antenna, respectively. Recall that all HPAs are assumed to have a unity gain; that is,  $P_{out}^i = P_{in}^i$ ,  $\forall i = 1, 2, ..., n_T$ . Hence, the instantaneous overall power efficiency of HPAs in the MIMO-OFDM system can be expressed as [16]

$$\eta^{\rm PE} = \frac{\sum_{i=1}^{n_T} P_{\rm out}^i}{n_T P_{\rm dc}} = \frac{\sum_{i=1}^{n_T} P_{\rm in}^i}{n_T P_{\rm dc}} = \alpha \frac{n_D \sigma^2}{n_T P_{\rm dc}}$$

$$= \frac{n_D \sigma^2 P_{\rm sat}}{n_T P_{\rm dc}} \frac{1}{P} = \frac{n_D \sigma^2}{2n_T} \frac{1}{P}.$$
(21)

In the above manipulations, we have used the fact that  $\sum_{i=1}^{n_T} P_{in}^i = \alpha n_D \sigma^2$  and  $P_{dc} = 2P_{sat}$  for class-A HPAs, regardless of the average powers of the input time-domain signals. Denoting CCDF<sup>PE</sup> ( $\eta_0^{PE}$ ) = Pr( $\eta_0^{PE} > \eta_0^{PE}$ ) to be the CCDF of the power efficiency, we obtain the following result with respect to  $\eta_0^{PE}$ .

**Theorem 2.** In per-subcarrier antenna selection MIMO-OFDM systems with linear scaling, the probability of achieving high instantaneous overall power efficiency of HPAs is the largest when all transmit antennas have the same number of allocated data symbols; that is,

$$CCDF_{balance}^{PE}\left(\eta_{0}^{PE}\right) \geq CCDF_{imbalance}^{PE}\left(\eta_{0}^{PE}\right),$$
 (22)

where

$$CCDF_{balance}^{PE}\left(\eta_{0}^{PE}\right) = \left(1 - e^{-1/2\eta_{0}^{PE}}\right)^{n_{T}K},$$

$$CCDF_{imbalance}^{PE}\left(\eta_{0}^{PE}\right) = \prod_{i=1}^{n_{T}} \left(1 - e^{-n_{D}K/2n_{T}K_{i}\eta_{0}^{PE}}\right)^{K}.$$

$$(23)$$

Proof. From (18) and (21), it is readily to obtain (22) and (23).

With respect to the average value of the overall power efficiency, from (21), we can express

$$\overline{\eta}^{\text{PE}} = E\left\{\eta^{\text{PE}}\right\} = \frac{n_D \sigma^2}{2n_T} E\left\{\frac{1}{P}\right\} = \frac{n_D \sigma^2}{2n_T} \int \frac{1}{x} p(x) \, dx, \quad (24)$$

where p(x) is the pdf (probability distribution function) of the peak power. In the system with a balance constraint (i.e., only use Step 1), the pdf of the peak power can be calculated as

$$p_{\text{balance}}(x) = \frac{d}{dx} \text{CDF}_{\text{balance}}^{P}(x) = \frac{d}{dx} \left(1 - e^{-x/\sigma_{\overline{K}}^{2}}\right)^{n_{T}K}$$
$$= \frac{n_{T}K}{\sigma_{\overline{K}}^{2}} e^{-x/\sigma_{\overline{K}}^{2}} \left(1 - e^{-x/\sigma_{\overline{K}}^{2}}\right)^{n_{T}K-1}.$$
(25)

Hence,

$$\overline{\eta}_{\text{balance}}^{\text{PE}} = \frac{n_D \sigma^2}{2n_T} \int \frac{1}{x} p_{\text{balance}}(x) dx$$
$$= \frac{n_T K}{2} \int_{\sigma_K^2}^{\overline{K} \sigma_{\overline{K}}^2} \frac{1}{x} e^{-x/\sigma_{\overline{K}}^2} \left(1 - e^{-x/\sigma_{\overline{K}}^2}\right)^{n_T K - 1} dx \qquad (26)$$
$$= \frac{n_T K}{2} \int_{1}^{\overline{K}} \frac{1}{x} e^{-x} (1 - e^{-x})^{n_T K - 1} dx.$$

Similarly, the power efficiency of HPAs in the conventional system is

$$\overline{\eta}_{\text{imbalance}}^{\text{PE}} = \frac{n_D \sigma^2}{2n_T} \int \frac{1}{x} p_{\text{imbalance}}(x) \, dx$$

$$= \frac{n_D K \sigma^2}{2n_T} \int_{\sigma_{\text{max}}^2}^{K_{\text{max}} \sigma_{\text{max}}^2} \frac{1}{x} \left\{ \sum_{j=1}^{n_T} \frac{e^{-x/\sigma_{K_j}^2}}{\left(1 - e^{-x/\sigma_{K_j}^2}\right) \sigma_{K_j}^2} \right\}$$

$$\times \prod_{i=1}^{n_T} \left(1 - e^{-x/\sigma_{K_i}^2}\right)^K dx,$$
(27)

where  $\sigma_{\max}^2 = \max\{\sigma_{K_1}^2, \dots, \sigma_{K_{n_T}}^2\}$ , and  $K_{\max} = \max\{K_1, \dots, K_{n_T}\}$ .

When the peak-power reduction algorithm is also implemented (i.e., the system employs both Steps 1 and 2), it is readily from (20) and (21) that

$$\operatorname{CCDF}_{\operatorname{balance+reduced}}^{\operatorname{PE}}\left(\eta_{0}^{\operatorname{PE}}\right) \geq \operatorname{CCDF}_{\operatorname{balance}}^{\operatorname{PE}}\left(\eta_{0}^{\operatorname{PE}}\right), \quad (28)$$

where

$$\operatorname{CCDF}_{\text{balance+reduced}}^{\text{PE}}\left(\eta_{0}^{\text{PE}}\right) = 1 - \left(1 - \left(1 - \left(1 - e^{-1/2\eta_{0}^{\text{PE}}}\right)^{n_{T}K}\right)^{W}\right). \tag{29}$$

Also, the average power efficiency can now be calculated as

$$\overline{\eta}_{\text{balance+reduced}}^{\text{PE}} = \frac{n_D \sigma^2}{2n_T} \int \frac{1}{x} p_{\text{balance+reduced}}(x) dx$$
$$= \frac{n_T K W}{2} \int_1^{\overline{K}} \frac{1}{x} e^{-x} (1 - e^{-x})^{n_T K - 1}$$
$$\times \left(1 - (1 - e^{-x})^{n_T K}\right)^{W-1} dx.$$
(30)

It can be seen from (30) that the smaller the peak power is reduced (i.e., the larger the number of candidates W is used), the higher the power efficiency could be achieved. Note that although the integrals in (26), (27), and (30) have no closed-form solutions, they can be evaluated numerically. Also, the average power efficiencies in (26), (27), and (30) are taken with respect to the input data. In other words, they are considered as instantaneous power efficiencies with respect to the channel distribution. Consequently, the power efficiency in the systems is obtained by averaging these values over the fading channel distribution.

#### 5. Analyses of Capacity and Energy Efficiency

It has been shown in (22) and (28) that the proposed system could achieve a better power efficiency of HPAs than its counterpart. Thus, when the power  $P_{dc}$  is fixed, it is intuitive that an increased average power efficiency results in an increased average transmit power and, in turn, leads to an increase in the achievable rate. Moreover, an increase in the data rate under a constant consumption power will translate into an improvement in energy efficiency. The achieved capacity and energy efficiency are now investigated in this section.

5.1. System Capacity. We begin by rewriting the mutual information in (7) with respect to the average SNR value of  $\overline{\rho} = \overline{P}_t / \sigma_n^2$ , where  $\overline{P}_t = \overline{\alpha} n_D \sigma^2 = \overline{\eta}^{\text{PE}} n_T P_{\text{dc}}$  (cf. (21)). The ergodic capacity is then calculated by averaging the mutual information over the fading channel distribution; that is,  $C(\overline{\rho}) = E_{\text{H}} \{I(\overline{\rho}, \underline{\mathbf{H}})\}$ . From (6) and (7), the mutual information in the proposed and conventional systems can be expressed, respectively, as

$$I\left(\overline{\rho}_{\text{proposed}}, \underline{\mathbf{H}}\right) = \frac{1}{K} \sum_{k=0}^{K-1} \log_2\left(\det\left(\mathbf{I}_{n_R} + \frac{\overline{\rho}_{\text{proposed}}}{n_D} \underline{\mathbf{H}}_k \underline{\mathbf{H}}_k^H\right)\right),$$
(31)

$$I\left(\overline{\rho}_{\text{imbalance}}, \underline{\mathbf{H}}\right) = \frac{1}{K} \sum_{k=0}^{K-1} \log_2\left(\det\left(\mathbf{I}_{n_R} + \frac{\overline{\rho}_{\text{imbalance}}}{n_D} \underline{\mathbf{H}}_k \underline{\mathbf{H}}_k^H\right)\right),$$
(32)

where  $\overline{\rho}_{\text{proposed}} = \overline{\eta}_{\text{proposed}}^{\text{PE}} n_T P_{\text{dc}} / \sigma_n^2$  and  $\overline{\rho}_{\text{imbalance}} = \overline{\eta}_{\text{imbalance}}^{\text{PE}} n_T P_{\text{dc}} / \sigma_n^2$ . Here,  $\overline{\eta}_{\text{proposed}}^{\text{PE}} = \overline{\eta}_{\text{balance+reduced}}^{\text{PE}}$  if the peak-power reduction algorithm is implemented; otherwise,  $\overline{\eta}_{\text{proposed}}^{\text{PE}} = \overline{\eta}_{\text{balance}}^{\text{PE}}$ . Also,  $\overline{\mathbf{H}}_k$  in (31) denotes the effective channel matrix on the *k*th subcarrier in the proposed system, which is obtained when solving the problem in (8). This channel matrix is generally different from the effective channel matrix is generally different. The difference in the mutual information between the two systems can be now calculated as

$$\Delta I = I\left(\overline{\rho}_{\text{proposed}}, \overline{\underline{\mathbf{H}}}\right) - I\left(\overline{\rho}_{\text{imbalance}}, \underline{\underline{\mathbf{H}}}\right) = \frac{1}{K} \sum_{k=0}^{K-1} \Delta I_k, \quad (33)$$

where

$$\Delta I_{k} = \log_{2} \left( \det \left( \mathbf{I}_{n_{R}} + \frac{\overline{\rho}_{\text{proposed}}}{n_{D}} \overline{\mathbf{H}}_{k} \overline{\mathbf{H}}_{k}^{H} \right) \right) - \log_{2} \left( \det \left( \mathbf{I}_{n_{R}} + \frac{\overline{\rho}_{\text{imbalance}}}{n_{D}} \underline{\mathbf{H}}_{k} \underline{\mathbf{H}}_{k}^{H} \right) \right).$$
(34)

For analytical simplicity, we focus on the high-SNR regime. At the high SNR, the mutual information at the *k*th subcarrier can be approximated as [25]

$$I_k = \log_2\left(\det\left(\frac{\overline{\rho}}{n_D}\Omega_k\right)\right),\tag{35}$$

where

$$\Omega_{k} = \Omega\left(\underline{\mathbf{H}}_{k}\right) = \begin{cases} \underline{\mathbf{H}}_{k}\underline{\mathbf{H}}_{k}^{H}, & n_{R} \leq n_{D} \\ \underline{\mathbf{H}}_{k}^{H}\underline{\mathbf{H}}_{k}, & n_{R} > n_{D}. \end{cases}$$
(36)

Thus, the difference in the mutual information can be rewritten as

$$\Delta I = \frac{1}{K} \sum_{k=0}^{K-1} \left( \log_2 \left( \det \left( \frac{\overline{\rho}_{\text{proposed}}}{n_D} \overline{\Omega}_k \right) \right) -\log_2 \left( \det \left( \frac{\overline{\rho}_{\text{imbalance}}}{n_D} \Omega_k \right) \right) \right)$$
(37)
$$= p \log_2 \frac{\overline{\eta}_{\text{proposed}}^{\text{PE}}}{\overline{\eta}_{\text{imbalance}}^{\text{PE}}} - \frac{1}{K} \sum_{k=0}^{K-1} \Delta_k := T_1 + T_2,$$

where  $p = \min(n_D, n_R)$ ,  $\overline{\Omega}_k = \Omega(\overline{\underline{\mathbf{H}}}_k)$ , and

$$\Delta_{k} = \log_{2}\left(\det\left(\mathbf{\Omega}_{k}\right)\right) - \log_{2}\left(\det\left(\overline{\mathbf{\Omega}}_{k}\right)\right)$$
(38)

is the loss in the mutual information associated with the *k*th subcarrier due to the constrained allocation. Note that if both systems have the same selected antenna subset at the *k*th subcarrier, then  $\Delta_k = 0$ ; otherwise,  $\Delta_k > 0$ . Thus, the total loss in the mutual information  $\Delta = (1/K) \sum_{k=0}^{K-1} \Delta_k > 0$ .

We have some important observations with respect to the value of  $\Delta I$  in (37).

(i) The change in mutual information  $\Delta I$  comes from two sources. The first source  $T_1$  is a benefit in mutual information due to the improvement in the power efficiency of HPAs. The second source  $T_2$  is a penalty that incurs because the chosen effective channel matrices in the proposed system are different from the ones in the conventional system.

(ii) For each channel realization, the matrix  $\mathbf{H}_{k}$  is fixed and the first term  $T_1$  in (37) is a constant. Thus, the value of  $\Delta I$  depends on how the effective channel matrix  $\underline{\mathbf{H}}_k$  is selected in the constrained selection scheme. From this observation, it is clear that, to make the value  $\Delta I$  become as positive as possible, the constrained selection method should result in the cost penalty  $\Delta$  as small as possible. We note that the formulated optimization in (8) could achieve the minimum possible value of the total cost. Hence, it is expected that the constrained selection scheme based on linear optimization will guarantee the maximum achievable value of  $\Delta I$ . In addition, to have an insight into the cost penalty, we derive the upper bound of the expected value of the cost penalty in Appendix B. Based on the obtained bound, it is observed that, for fixed values of  $n_T$  and  $n_D$ , the cost penalty becomes smaller with an increasing value of  $q = \max(n_D, n_R)$ .

(iii) As  $\Delta > 0$ , the upper bound of the capacity improvement can be given as

$$\Delta C = E_{\mathbf{H}} \{\Delta I\} \le p \log_2 E_{\mathbf{H}} \left\{ \frac{\overline{\eta}_{\text{proposed}}^{\text{PE}}}{\overline{\eta}_{\text{imbalance}}^{\text{PE}}} \right\}.$$
(39)

In (39), we have used Jensen's inequality of  $E\{\log(x)\} \leq \log(E\{x\})$  as  $\log(x)$  is a concave function. Based on this bound, we could estimate the maximum improvement in capacity that could be realized in the proposed system compared to its counterpart.

It is now necessary to evaluate the change in capacity; that is,  $\Delta C$ . We note that although the distribution of the mutual information at high SNRs can be well approximated by a Gaussian distribution [25], it is still challenging to perform a mathematical evaluation of  $\Delta C$  from a statistical viewpoint. This is mainly due to the fact that the two terms in (37) are complicated, dependent random variables. Thus, we perform a numerical evaluation of  $\Delta C$  instead. Figure 4 plots the empirical CCDF of  $T_1$ , CDF of  $T_2$ , and CCDF of  $\Delta I$ . In the figure, "W = 1" stands for the case in which only Step 1 in Section 3.1 is implemented. The results are obtained in the systems with  $n_T = 4$ ,  $n_D = 2$ ,  $n_R = 2$ , K = 128, and are averaged over 10<sup>3</sup> channel realizations. Details about other simulation parameters are described in Section 6. The numerical results confirm that  $T_1 > 0$  and  $T_2 < 0$ . Moreover, as shown in Figure 4(c),  $\Delta I$  is always positive when the peakpower reduction algorithm is implemented (i.e., W=4 and W = 8). For the case of W = 1, the probability of  $\Delta I$  being positive is significant. Therefore, the proposed system attains a better ergodic capacity than that in the conventional system. The achieved capacities in the considered systems will be provided in Section 6.

5.2. Energy Efficiency. In this subsection, we further examine the efficacy of the proposed system from an energyefficiency (EE) perspective. Normalized energy efficiency (bits/Hz/Joule) in MIMO-OFDM systems can be defined as [3, 26]

$$\eta^{\rm EE} = \frac{C\left(\overline{\rho}\right)}{P_{\rm total}},\tag{40}$$

where  $C(\overline{\rho})$  is the achievable rate and the total power consumption per-subchannel is  $P_{\text{total}} = n_T P_{\text{dc}} + n_T P_{\text{RF}} + P_{\text{sp}}$ , where  $P_{\text{RF}}$  is the RF power consumption in each transmit branch excluding the associated HPA, and  $P_{\text{sp}}$  is the baseband processing power consumption. It can be seen from (40) that given a fixed value of  $P_{\text{total}}$ , a comparison of energy efficiency achieved in the two systems is based on the capacity comparison that has been analyzed in Section 5.1. We are now interested in evaluating a useful metric of energy efficiencyspectral efficiency performance. Recall that the average SNR  $\overline{\rho}$  is given as  $\overline{\rho} = \overline{P}_t / \sigma_n^2 = \overline{\eta}^{\text{PE}} n_T P_{\text{dc}} / \sigma_n^2$ . Thus, we can rewrite the energy efficiency in (40) as a function of  $P_{\text{dc}}$  as

$$\eta^{\text{EE}}\left(P_{\text{dc}}\right) = \frac{C\left(\overline{\eta}^{\text{PE}} n_T P_{\text{dc}} / \sigma_n^2\right)}{n_T P_{\text{dc}} + n_T P_{\text{RF}} + P_{\text{sp}}}.$$
(41)

TABLE 2: Simulation parameters.

Parameter	Value
Bandwidth	528 MHz
FFT size	128
Number of samples in zero-padded suffix	37
Modulation scheme	4-QAM
IEEE 802.15.3a channel model	CM1

The energy efficiency of the proposed and conventional systems can now be, respectively, expressed as

$$\eta_{\text{proposed}}^{\text{EE}} \left( P_{\text{dc}} \right) = \frac{C_{\text{proposed}} \left( \overline{\eta}_{\text{proposed}}^{\text{PE}} n_T P_{\text{dc}} / \sigma_n^2 \right)}{n_T P_{\text{dc}} + n_T P_{\text{RF}} + P_{\text{sp}}}$$

$$= \frac{E_{\text{H}} \left\{ I \left( \overline{\eta}_{\text{proposed}}^{\text{PE}} n_T P_{\text{dc}} / \sigma_n^2, \overline{\underline{\mathbf{H}}} \right) \right\}}{n_T P_{\text{dc}} + n_T P_{\text{RF}} + P_{\text{sp}}},$$

$$\eta_{\text{imbalance}}^{\text{EE}} \left( P_{\text{dc}} \right) = \frac{C_{\text{imbalance}} \left( \overline{\eta}_{\text{imbalance}}^{\text{PE}} n_T P_{\text{dc}} / \sigma_n^2 \right)}{n_T P_{\text{dc}} + n_T P_{\text{RF}} + P_{\text{sp}}},$$

$$= \frac{E_{\text{H}} \left\{ I \left( \overline{\eta}_{\text{imbalance}}^{\text{PE}} n_T P_{\text{dc}} / \sigma_n^2, \underline{\underline{\mathbf{H}}} \right) \right\}}{n_T P_{\text{dc}} + n_T P_{\text{RF}} + P_{\text{sp}}}.$$

$$(43)$$

Similarly to the case of capacity, we compare the energy efficiency achieved in the two systems by means of numerical results in the next section. Note that the calculation of energy efficiency in the proposed system (i.e., (42)) has assumed that a reduction in spectral efficiency due to the side information as well as additional processing power required for the peak-power reduction algorithm is negligible. In fact, a reduction in spectral efficiency is very small. For example, in a system with 16-QAM, FFT size of 128,  $n_D = 2$ , and W = 4 (i.e., 2 bits are needed for side information), a spectral efficiency loss is 2 bits/(128 × 4 × 2 + 2) bits = 0.19%. Also, it was shown in [27] that the additional power cost when implementing SLM schemes is minuscule. Thus, the proposed peak-power reduction algorithm in Section 3.2, which is a SLM-based scheme, requires a small additional power cost.

#### 6. Numerical Results

In this section, we provide numerical results to validate the analyses mentioned in the previous sections as well as demonstrate the effectiveness of the proposed allocation scheme over its counterpart. A MIMO-OFDM system with  $n_T = 4$ ,  $n_D = 2$ , and  $n_R = 2$  is considered in our simulations. The system parameters are listed in Table 2. These parameters are chosen based on the legacy WiMedia MB-OFDM UWB (Multiband-OFDM UWB) standard [28]. Also, the channel CMI, defined in the IEEE 802.15.3a channel model [29], is based on a measurement of a line-of-sight scenario where the distance between the transmitter and the receiver is up to 4 m. Moreover, the multipath gains are modeled as independent log normally distributed random variables. We assume that perfect channel state information is available at the receiver. Also, the feedback link has no delay and is error-free.



FIGURE 4: Statistical distributions (note:  $T_2$  is independent of W).

6.1. Evaluations of Peak-Power Distribution. In Figure 5, we plot the CCDFs of the peak power of time-domain signals. The analytical curves are based on (16), (17), and (19). Meanwhile, the simulation curves are empirical CCDF values. The simulation result confirms that a system with the proposed allocation scheme offers a better CCDF performance than its counterpart. As expected, the occurrence of high peak power is significantly reduced when the peak-power reduction algorithm is implemented. Also, it can be seen that the improvement associated with this algorithm is reduced with increasing W. In other words, a very large value of W, while requiring higher complexity in terms on the number of IFFT operations, results in a marginal improvement. Thus, it is reasonable to choose a relatively small value for W (e.g., W = 4). It is also worth noting that the analytical curves are relatively close to the simulation curves. The small gaps exist due to the fact that the assumption of independent samples  $|s_i(n)|$  to obtain (13) does not strictly hold as we have  $\sum_{n=0}^{K-1} |s_i(n)|^2 = \sigma^2 K_i$  by Parseval's relation [24].

6.2. Evaluations of Power Efficiency of HPAs. Figure 6 compares the CCDFs of the power efficiency achieved in the proposed and conventional systems. It can be seen that the probability of power efficiency being large highly likely occurs in the proposed system, compared to its counterpart. Also, the simulation results agree well with the analytical results derived in (23) and (29). In Table 3, we compare the average power efficiencies. Here, the analytical values are obtained according to (26), (27), and (30). Meanwhile, the simulation values are empirical values based on the original definition of the drain efficiency in (21). Also, these values are averaged over the fading channel realizations. It can be seen that the derived expressions approximate well the achieved power efficiencies. Table 3 also provides relative improvements of the power efficiencies achieved in the proposed system over the conventional system. Here, only  $\overline{\eta}^{\rm PE}$  (Simulation) values are used for calculating these improvements. It is clear that the proposed system could achieve a significant improvement in terms of average power efficiency.

6.3. Evaluations of Capacity and Energy Efficiency. Figure 7 shows the system capacity in Mbps (i.e., the normalized value in (6) is scaled up with the system bandwidth) versus the SNR value of  $\sigma^2/\sigma_n^2$ . It is clear that a system with the proposed allocation scheme achieves a higher capacity than its counterpart. This agrees with the analysis in Section 5.1 that the change in capacity  $\Delta C$  is positive. In Figure 8, we plot the energy efficiency (Mbits/Joule) versus spectral efficiency (Mbps/Hz). This figure is obtained based on (42) and (43) when varying  $P_{\rm dc}$ . Other parameters are  $P_{\rm RF} = 100$  mW,  $P_{\rm sp} = 10$  mW, and

TABLE 3: A comparison of average power efficiencies.

	Imbalance	Proposed $(W = 1)$	Proposed $(W = 4)$	Proposed $(W = 8)$
$\overline{\eta}^{\mathrm{PE}}$ (Simulation)	0.0697	0.0768	0.0894	0.0942
$\overline{\eta}^{ ext{PE}}$ (Analysis)	0.0691	0.0757	0.0892	0.0943
Improvement $\frac{\overline{\eta}_{\text{proposed}}^{\text{PE}} - \overline{\eta}_{\text{imbalance}}^{\text{PE}}}{\overline{\eta}_{\text{imbalance}}^{\text{PE}}}$	_	10.19%	28.26%	35.15%



FIGURE 5: Comparison of CCDFs of the peak powers.



FIGURE 6: Comparison of CCDFs of the power efficiencies.



FIGURE 7: Comparison of the ergodic capacities.

 $\sigma^2/\sigma_n^2 = 15$  dB. As expected, the improvement in the power efficiency of HPAs results in an improved energy efficiency. In addition, it can be observed that there exists an energy efficiency-spectral efficiency tradeoff in the systems. This tradeoff clearly needs to be taken into consideration when designing energy-efficient per-subcarrier antenna selection based OFDM wireless systems.

#### 7. Conclusions

In this paper, a per-subcarrier antenna subset selection MIMO-OFDM system with linear scaling has been investigated from an energy-efficiency perspective. We have shown that an imbalance allocation of data subcarriers associated with the conventional selection scheme affects the power efficiency of HPAs, as well as the energy-efficiency of the whole system. To deliver the maximum overall power efficiency, we have proposed the two-step strategy, consisting of equal allocation data subcarriers across antennas and peak-power reduction. It has been proved from the power-efficiency viewpoint that the proposed allocation scheme outperforms the conventional scheme. We have also derived the expressions for measuring the average power efficiency. Moreover, the improvements in terms of capacity and energy efficiency



FIGURE 8: Energy efficiency versus spectral efficiency.

resulting from the improved power efficiency have been analyzed. The analytical results are validated by simulation results. The simulation result also shows that the system with the proposed allocation scheme could achieve a better efficiency efficiency-spectral energy tradeoff, compared to its counterpart.

#### Appendices

#### A. Proof of Theorem 1

Let us consider a function  $f(v) = 1 - e^{-P_0/v}$ ,  $0 < v \le \sigma_{\max}^2$ , where  $\sigma_{\max}^2 = \max\{\sigma_{K_1}^2, \sigma_{K_2}^2, \dots, \sigma_{K_{n_T}}^2\}$ . The second derivative of this function is

$$f''(v) = \frac{2P_0v - P_0^2}{v^4}e^{-P_0/v}.$$
 (A.1)

From a HPAs' viewpoint, it is of interest to consider the situation when the peak power across antennas  $P_0$  is large. Thus, we consider the scenarios of  $P_0 > 2\sigma_{\max}^2$ , where  $\sigma_{\max}^2$  is the maximum average power across antennas. Under these situations, it is clear that  $f''(v) < 0, \forall v \in (0, \sigma_{\max}^2]$ . Hence, the function f(v) is concave. By applying Jensen's inequality, we obtain the following inequality:

$$\frac{1}{n_T} \sum_{i=1}^{n_T} f\left(\sigma_{K_i}^2\right) \le f\left(\frac{1}{n_T} \sum_{i=1}^{n_T} \sigma_{K_i}^2\right)$$

$$= f\left(\frac{1}{n_T} \sum_{i=1}^{n_T} \sigma_{\overline{K}}^2\right) = f\left(\sigma_{\overline{K}}^2\right),$$
(A.2)

where the equality comes from the fact that  $\sum_{i=1}^{n_T} \sigma_{K_i}^2 = \sum_{i=1}^{n_T} \sigma_{K_i}^2 = n_D \sigma^2$ . Expression (A.2) can be rewritten as

$$\sum_{i=1}^{n_T} \left( 1 - e^{-P_0/\sigma_{K_i}^2} \right) \le n_T \left( 1 - e^{-P_0/\sigma_{\overline{K}}^2} \right), \tag{A.3}$$

with equality if and only if  $\sigma_{K_i}^2 = \sigma_{\overline{K}}^2$ ,  $\forall i = 1, ..., n_T$ .

On the other hand, by applying the arithmetic-geometric mean inequality, and note that  $(1 - e^{-x}) > 0$ ,  $\forall x > 0$ , we have

$$\prod_{i=1}^{n_T} \left( 1 - e^{-P_0/\sigma_{K_i}^2} \right) \le \left( \frac{1}{n_T} \sum_{i=1}^{n_T} \left( 1 - e^{-P_0/\sigma_{K_i}^2} \right) \right)^{n_T}.$$
 (A.4)

Note that the equality in (A.4) holds if and only if  $\sigma_{K_i}^2 = \sigma_{\overline{K}}^2$ ,  $\forall i = 1, ..., n_T$ .

Combining (A.3) and (A.4) results in

$$\prod_{i=1}^{n_T} \left( 1 - e^{-P_0/\sigma_{K_i}^2} \right) \le \left( 1 - e^{-P_0/\sigma_{K}^2} \right)^{n_T}, \tag{A.5}$$

with equality if and only if  $\sigma_{K_i}^2 = \sigma_{\overline{K}}^2$ ,  $\forall i = 1, ..., n_T$ . Thus, we get the following desired inequality:

$$1 - \prod_{i=1}^{n_T} \left( 1 - e^{-P_0/\sigma_{K_i}^2} \right)^K \ge 1 - \left( 1 - e^{-P_0/\sigma_{K}^2} \right)^{n_T K}, \quad (A.6)$$

or

$$\operatorname{CCDF}_{\operatorname{imbalance}}^{P}(P_{0}) \ge \operatorname{CCDF}_{\operatorname{balance}}^{P}(P_{0}).$$
 (A.7)

This completes the proof.

## **B. Upper Bound of an Expected** Value of Cost Penalty

In this appendix, we derive an upper bound of the expected value of  $\Delta_k$ . It can be seen from (5) that, among all possible matrices  $\underline{\mathbf{H}}_k$ , the matrix  $\underline{\mathbf{H}}_k$  with the highest value of  $\log_2(\det(\mathbf{\Omega}_k)), \mathbf{\Omega}_k = \Omega(\underline{\mathbf{H}}_k)$ , will be selected as the effective channel matrix for the *k*th subcarrier in the conventional scheme. Meanwhile, in the proposed scheme, due to the balance constraint, the effective channel matrix associated with the *k*th subcarrier is not necessarily the one with the highest  $\log_2(\det(\mathbf{\Omega}_k))$ ; that is,  $\log_2(\det(\overline{\mathbf{\Omega}}_k)) \leq \log_2(\det(\mathbf{\Omega}_k))$ . Thus, the expected value of  $\Delta_k$  can be computed by using order statistics. In particular, an upper bound on the expected difference of two-order statistics, the  $\Gamma$ th and  $\gamma$ th,  $1 \leq \gamma < \Gamma$ , is given by [30]

$$E \{\Delta_k\} = E \{\log_2 \left(\det \left(\Omega_k\right)\right)\} - E \left\{\log_2 \left(\det \left(\overline{\Omega}_k\right)\right)\right\}$$
$$\leq \sigma_{\rm C} \sqrt{\frac{\Gamma\left(\gamma+1\right)}{\gamma}},$$
(B.1)

where  $\sigma_C^2$  is the variance of  $\log_2(\det(\Omega_k))$  that is assumed to be the same for all possible matrices  $\underline{\mathbf{H}}_k$ .

On the other hand, suppose that the entries of the  $n_R \times n_T$ matrix  $\mathbf{H}_k$  are *i.i.d.* complex Gaussian random variables with zero mean and unit variance; then for any effective channel matrix  $\underline{\mathbf{H}}_k$ ,  $\mathbf{\Omega}_k$  is a complex Wishart matrix. It follows from [25] that the variance of  $\log_2(\det(\mathbf{\Omega}_k))$  can be expressed as

$$\sigma_C^2 = \left[\log_2(e)\right]^2 \sum_{m=1}^p \psi'(q-m+1), \qquad (B.2)$$

where  $p = \min(n_D, n_R)$ ,  $q = \max(n_D, n_R)$ , and  $\psi'(x) = \sum_{\varphi=1}^{\infty} 1/(\varphi + x - 1)^2$  is the first derivative of the digamma function. By approximating  $\psi'(x) \approx 1/x$  [25], the simpler expression for  $\sigma_C^2$  in (B.2) is given as

$$\sigma_C^2 = \left[\log_2\left(e\right)\right]^2 \sum_{m=1}^p \frac{1}{q-m+1}.$$
 (B.3)

Substitute (B.3) into (B.1), we finally arrive at

$$E\left\{\Delta_{k}\right\} \leq \log_{2}\left(e\right)\sqrt{\left(\sum_{m=1}^{p}\frac{1}{q-m+1}\right) \times \frac{\Gamma\left(\gamma+1\right)}{\gamma}}.$$
 (B.4)

#### **Conflict of Interests**

The authors declare that there is no conflict of interests regarding the publication of this paper.

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