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Research Article

Sparse Recovery for Bistatic MIMO Radar Imaging in the Presence of Array Gain Uncertainties

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A sparse recovery based transmit-receive angle imaging scheme is proposed for bistatic multiple-input multiple-output (MIMO) radar. The redundancy of the transmit and receive angles in the same range cell is exploited to construct the sparse model. The imaging is then performed by compressive sensing method with consideration of both the transmit and receive array gain uncertainties. An additional constraint is imposed on the inverse of the transmit and receive array gain errors matrices to make the optimization problem of the CS solvable. The image of the targets can be reconstructed using small number of snapshots in the case of large array gain uncertainties. Simulation results confirm the effectiveness of the proposed scheme.

1. Introduction

Multiple-input multiple-output (MIMO) radar has multiple transmit channels and multiple receive channels, and the transmit channels can be separated by waveforms or time or frequencies or polarizations at each receiver. So, the transmit aperture can be exploited completely by processing receive data [1–3]. Most of the advantages of the MIMO radar come from increasing the number of channels. Two main classes of MIMO radar have been proposed, with widely separated antennas [1] and with colocated antennas [2]. The first class utilizes the different scattering properties of a target from sufficiently spaced antennas to improve the performance of the systems. The second class allows the improvement of the radar performances by coherent processing of the multiple channels.

Bistatic MIMO radar scheme has been proposed in [3], where a two-dimensional radar imaging method based on the Capon method is developed. Bistatic MIMO radar has the particular advantage of being able to obtain the target angles with respect to both the transmit and the receive arrays by processing the receive data [3–7]. So, the range information of the target is redundant in this case and the time synchronization of the bistatic radar is relaxed. Nevertheless, the errors of both the transmit array and the receive array will degrade the performances of these techniques. Many works have been done to estimate the array errors and correct the transmit array and receive array simultaneously in bistatic MIMO radar [8, 9]. However, these methods need large number of snapshots to estimate the covariance matrix and some well-calibrated elements.

Compressive sensing (CS) has received considerable attention recently and has been applied to source localization by exploiting the spatial sparsity of the sources [10]. The CS can work even in the case of single snapshot. A CS based multitarget detection method for bistatic MIMO radar is presented in [11]. However, the range cell is not considered in this paper. Furthermore, they assume that there are no array errors in the systems. In this paper, the redundancy of the transmit and receive angles in the same range cell is exploited to construct the sparse model. CS based method is presented to image transmit-receive angle image in the presence of unknown array gain errors for bistatic MIMO radar. The image can be recovered well in the case of small snapshots and large array gain uncertainties.

This paper is organized as follows. The sparse signal model of bistatic MIMO radar with uncertain array gain is presented in Section 2. In Section 3, CS based algorithm is applied to estimate the transmit angle and receive angle of targets in the presence of array gain errors. The effectiveness of the method will be confirmed by simulations in Section 4. Finally, Section 5 concludes the paper.

2. Sparse Model of Bistatic MIMO Radar

The configuration of the bistatic MIMO radar used in this paper is illustrated in Figure 1. An M-transmit/N-receive (M T/N R) antenna configuration is considered, and both transmitter and receiver are uniform linear array (ULA). Let the signal transmitted by M-transmitters at every pulse period be $\mathbf{S} \in \mathbf{C}^{M\times L}$, where L is the number of the codes in one pulse period. Assume that the target is at angles (θ_t, θ_r), where θ_t is the angle of the target with respect to the transmit array (i.e., DOD) and θ_r is the angle with respect to the receive array (DOA). λ denotes the carrier wavelength. In the case of P pixels at location $(\boldsymbol{\theta}_t, \boldsymbol{\theta}_r)$, the received signal during the qth pulse period can be expressed as

$$\mathbf{Y}_q = \mathbf{A}_{ur}\mathbf{D}_q\mathbf{A}_{ut}^T\mathbf{S} + \mathbf{E}_q, \quad q = 1, 2, \ldots, Q, \tag{1}$$

where $(\cdot)^T$ denote transpose operator. $\mathbf{A}_{ur} = \boldsymbol{\Gamma}_r\mathbf{A}_r$ and $\mathbf{A}_{ut} = \boldsymbol{\Gamma}_t\mathbf{A}_t$ are the unknown gain steering matrices of the receive and transmit array, respectively. $\boldsymbol{\Gamma}_t = \mathrm{diag}[\rho_{t1}, \ldots, \rho_{tM}]$ and $\boldsymbol{\Gamma}_r = \mathrm{diag}[\rho_{r1}, \ldots, \rho_{rN}]$ are the diagonal matrices with array gain errors at diagonal elements. $\mathbf{A}_r = [\mathbf{a}_{rp}]_{N\times P}$ and $\mathbf{A}_t = [\mathbf{a}_{tp}]_{M\times P}$ are the receive and transmit steering matrices of P targets, respectively, where $\mathbf{a}_{rp} = [1\ e^{j(2\pi/\lambda)d_r\sin\theta_{rp}}\ e^{j(2\pi/\lambda)2d_r\sin\theta_{rp}}\ \cdots\ e^{j(2\pi/\lambda)(N-1)d_r\sin\theta_{rp}}]^T$ and $\mathbf{a}_{tp} = [1\ e^{j(2\pi/\lambda)d_t\sin\theta_{tp}}\ e^{j(2\pi/\lambda)2d_t\sin\theta_{tp}}\ \cdots\ e^{j(2\pi/\lambda)(N-1)d_t\sin\theta_{tp}}]^T$. d_t and d_r are the ideal interelement space at the transmitter and receiver. $\mathbf{D}_q = \mathrm{diag}(d_1, \ldots, d_P)$ is a diagonal matrix composed of target reflection coefficients for the qth pulse period. The noise vector $\mathbf{E}_q$ is assumed to be independent, zero-mean complex Gaussian distribution with $\mathbf{E}_q \sim N^c(0, \sigma_n^2\mathbf{I}_N)$.

We divide the whole area of interest in some discrete set of angular positions [10]. Let the two-dimensional grid consist of the dictionary of all potential angular position pairs $\Omega = \{(\overline{\theta}_k, \overline{\theta}_l) : (k,l) \in \{1, \ldots, G\} \times \{1, \ldots, G\}\}$. Then we construct the matrices composed of steering vectors corresponding to each potential source location as its columns: $\boldsymbol{\Phi}_t = [\mathbf{a}_t(\overline{\theta}_1), \ldots, \mathbf{a}_t(\overline{\theta}_G)]$ and $\boldsymbol{\Phi}_r = [\mathbf{a}_r(\overline{\theta}_1), \ldots, \mathbf{a}_r(\overline{\theta}_G)]$. Let $\mathbf{X}_q \in \mathbb{C}^{G\times G}$ be the matrix of reflection coefficients of the targets at G^2 possible grid point of interest during qth pulse period. Assume that the transmit waveforms are orthogonal to each other; that is, $\mathbf{S}\mathbf{S}^H = \mathbf{I}$. Then the receive signals in (1) after being matched by transmit waveforms can be rewritten as

$$\mathbf{Y}_q = \boldsymbol{\Gamma}_r\boldsymbol{\Phi}_r\mathbf{X}_q\boldsymbol{\Phi}_t^T\boldsymbol{\Gamma}_t^T + \overline{\mathbf{E}}_q, \quad q = 1, 2, \ldots, Q. \tag{2}$$

$\mathbf{X}_q[k,l]$ is nonzero only if there is pixel of the target at (θ_k, θ_l). Fortunately, we can recover the image range by range. It can be observed in Figure 1 that the grid points which are in the same range cell should be distributed on the surface of an ellipse with the focuses on receivers and transmitters, respectively. So, only surface of the ellipse has the pixels of the target and any other grid points in the Ω are zeros when we process the data of one range cell. It is clear that $\mathbf{X}_q$ is a sparse matrix in this case. The pixels in the same range cell are virtually sparse because of the redundancy of the transmit and receive angles. This implies that we can recover the scene by sparse recovery method even though the actual scene is not sparse.

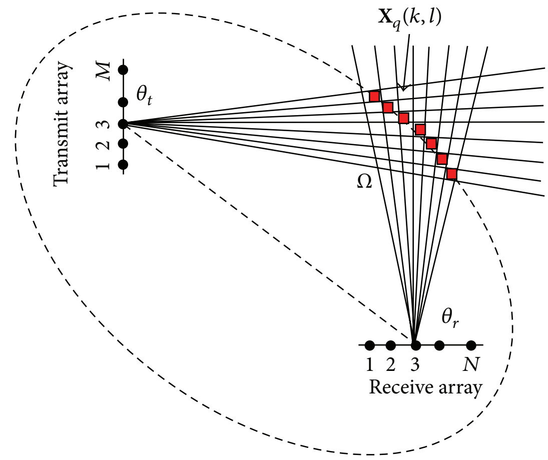


Figure 1: Bistatic MIMO radar configuration.

3. CS Based Sparse Imaging with Array Gain Uncertainties

In this section, we develop the CS based sparse imaging method in the presence of the array gain uncertainties for bistatic MIMO radar imaging.

3.1. Problem Formulation. The radar imaging is an inverse scattering problem. The spatial map of reflectivity can be reconstructed from measurements of scattered electronic fields. To transform our problem into the standard framework of the sparse recovery, we first rewrite (2) as

$$\begin{aligned}\mathbf{y}_q &= \mathrm{vec}\left(\mathbf{Y}_q\right) = [(\boldsymbol{\Gamma}_t\boldsymbol{\Phi}_t) \otimes (\boldsymbol{\Gamma}_r\boldsymbol{\Phi}_r)]\,\mathrm{vec}\left(\mathbf{X}_q\right) + \overline{\mathbf{e}}_q \\ &= \left(\boldsymbol{\Gamma}_t \otimes \boldsymbol{\Gamma}_r\right)\left(\boldsymbol{\Phi}_t \otimes \boldsymbol{\Phi}_r\right)\mathbf{x}_q + \overline{\mathbf{e}}_q \\ &= \boldsymbol{\Gamma}\boldsymbol{\Phi}\mathbf{x}_q + \overline{\mathbf{e}}_q,\end{aligned} \tag{3}$$

where $\otimes$ denote Kroneck product; $\boldsymbol{\Gamma} = \boldsymbol{\Gamma}_t \otimes \boldsymbol{\Gamma}_r$ and $\boldsymbol{\Phi} = \boldsymbol{\Phi}_t \otimes \boldsymbol{\Phi}_r$; $\mathbf{x}_q = \mathrm{vec}(\mathbf{X}_q)$ and $\overline{\mathbf{e}}_q = \mathrm{vec}(\overline{\mathbf{E}}_q)$.

When Q pulse periods are transmitted, (3) can be expressed as follows:

$$\mathbf{Y} = \boldsymbol{\Gamma}\boldsymbol{\Phi}\mathbf{X} + \overline{\mathbf{E}}, \tag{4}$$

where $\mathbf{Y} = [\mathbf{y}_1, \ldots, \mathbf{y}_Q]$ and $\mathbf{X} = [\mathbf{x}_1, \ldots, \mathbf{x}_Q]$. $\overline{\mathbf{E}}$ is the noise matrix composed by $\overline{\mathbf{e}}_q$; that is, $\overline{\mathbf{E}} = [\overline{e}_1, \overline{e}_2, \ldots, \overline{e}_Q]$. Here, what we need to do is to recover matrix $\mathbf{X}$ from the given data $\mathbf{Y}$.

CS can be used to efficiently reconstruct a signal with a sparse representation. For a given observation matrix $\mathbf{Y}$ and a sensing matrix $\boldsymbol{\Phi}$, $\mathbf{Y} = \boldsymbol{\Phi}\mathbf{X}$. The recovery process is formulated as an l_1-optimization problem; that is,

$$\begin{aligned}\min \quad & \|\mathbf{X}\|_1, \\ \text{s.t.} \quad & \mathbf{y} = \boldsymbol{\Phi}\mathbf{X}.\end{aligned} \tag{5}$$

However, the CS method in (5) cannot resolve the the problem in model (4) directly as there is an additional unknown array gain error matrix $\mathbf{\Gamma}$. The optimization problem will lead to a trivial solution if we add $\mathbf{\Gamma}$ directly in (5) without further constraint on $\mathbf{\Gamma}$. Also, the Gaussian noise $\overline{\mathbf{E}}$ is not considered in the optimization problem in (5), which will degrade the performance of the recovery. What we are interested in is to construct an optimization problem that considers both the array gain uncertainties $\mathbf{\Gamma}$ and the noise $\overline{\mathbf{E}}$. We will achieve the imaging recovery of bistatic MIMO radar with array gain uncertainties by the help of the idea from [12] later.

3.2. Direct CS Method. The problem of recovering the sparse $\mathbf{X}$ from the measurement data $\mathbf{Y}$ is commonly known as multiple measurement vector (MMV) problem in CS [13]. Many sparse recovery methods of this problem have considered effect of the noise. Considering the effect of the noise, the MMV problem can be formulated as

$$\left(\widehat{\mathbf{X}}\right) = \arg\min\left(\|\mathbf{X}\|_1 + \frac{\mu}{2}\|\mathbf{Y} - \mathbf{\Phi}\mathbf{X}\|_F^2\right), \tag{6}$$

where μ is a balance constant related to the noise.

We rewrite model (4) as

$$\mathbf{Y} = \mathbf{\Phi}\mathbf{X} + (\mathbf{\Gamma} - \mathbf{I})\,\mathbf{\Phi}\mathbf{X} + \overline{\mathbf{E}}, \tag{7}$$

where $(\mathbf{\Gamma}-\mathbf{I})\mathbf{\Phi}\mathbf{X}$ is the error from the array gain uncertainties which can be combined with the noise $\overline{\mathbf{E}}$. So, (7) can be expressed as

$$\mathbf{Y} = \mathbf{\Phi}\mathbf{X} + \widetilde{\mathbf{E}}, \tag{8}$$

where $\widetilde{\mathbf{E}} = (\mathbf{\Gamma} - \mathbf{I})\mathbf{\Phi}\mathbf{X} + \overline{\mathbf{E}}$. The revised model (8) can be resolved directly by using the optimization problem of CS in (6).

As was known to all, CS methods are sensitive to the noise. So the performance of imaging recovery will degrade when the noise is large even though (6) considers the noise. So, this direct CS method can only be used well with small array gain uncertainties. We will evaluate the performance of this method in Section 4.

3.3. CS Method with Constraint of the Array Gain Uncertain. As the method above regards the array gain uncertainties as noise, the performance will degrade with large array gain errors. In fact, we can regard both the image matrix $\mathbf{X}$ and the array gain uncertain $\mathbf{\Gamma}$ as the estimated value. Considering the CS method, it seems natural to consider the following optimization problem:

$$\left(\widehat{\mathbf{X}}, \mathbf{\Gamma}\right) = \arg\min\left(\|\mathbf{X}\|_1 + \frac{\mu}{2}\|\mathbf{Y} - \mathbf{\Gamma}\mathbf{\Phi}\mathbf{X}\|_F^2\right). \tag{9}$$

However, the optimization problem in (10) will lead a trivial solution without further constraint on $\mathbf{\Gamma}$ or $\mathbf{X}$ [12]. In order to construct a solvable optimization problem of the CS, the constraint on trace of $\mathbf{\Gamma}$ should be considered.

Considering the noise reduction, we construct the following optimization problem:

$$\begin{gathered}\left(\widehat{\mathbf{X}}, \widehat{\mathbf{\Gamma}}\right) = \arg\min\left(\|\mathbf{X}\|_1 + \frac{\mu}{2}\left\|\mathbf{\Gamma}^{-1}\mathbf{Y} - \mathbf{\Phi}\mathbf{X}\right\|_F^2\right) \\ \text{s.t.}\quad \operatorname{tr}\left(\mathbf{\Gamma}^{-1}\right) = MN,\end{gathered} \tag{10}$$

where μ is a balance coefficient which is selected according to the noise level. The trace of the unknown gain matrix can be derived as

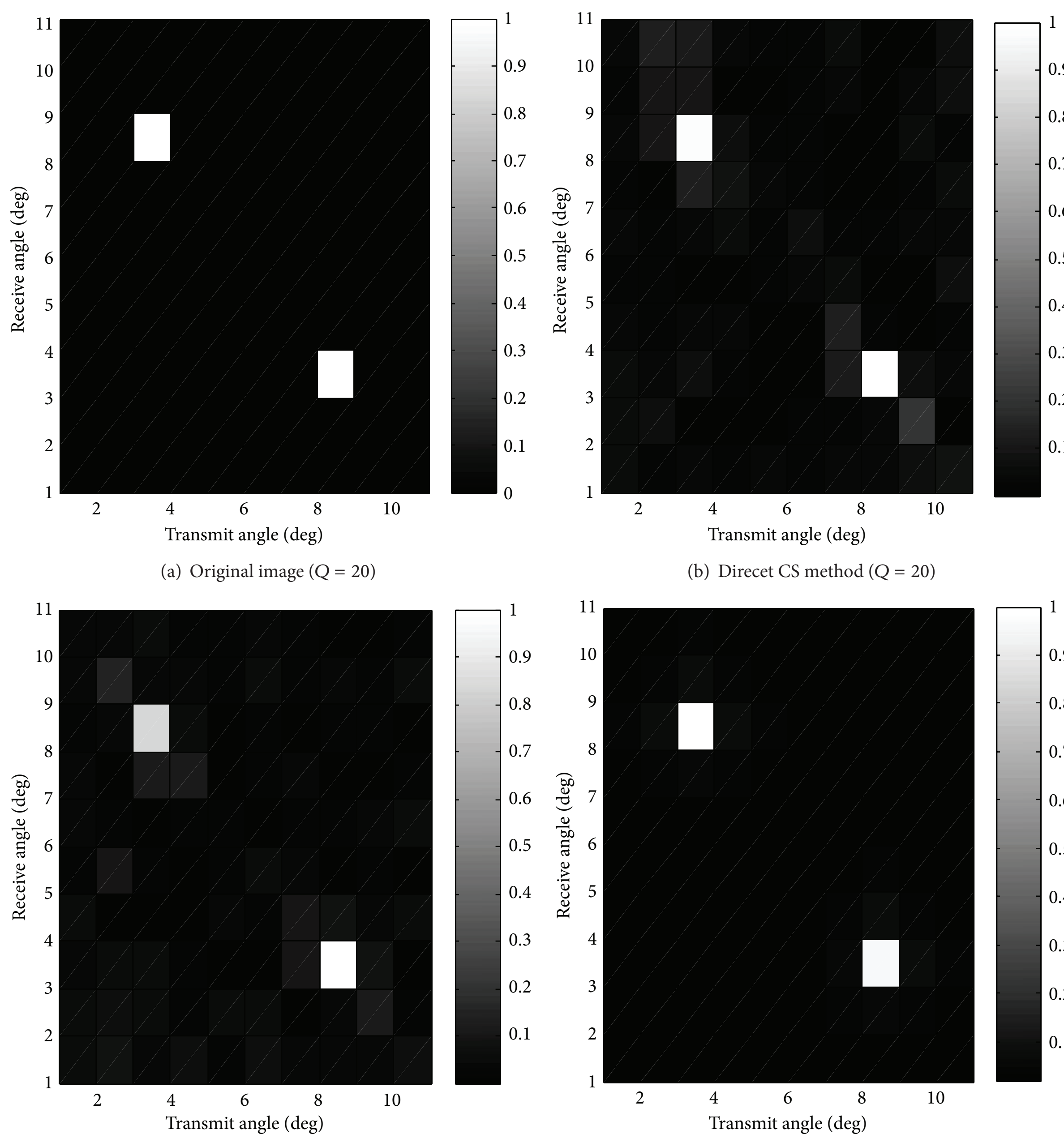
$$\operatorname{tr}\left(\mathbf{\Gamma}^{-1}\right) = \operatorname{tr}\left(\mathbf{\Gamma}_t^{-1} \otimes \mathbf{\Gamma}_r^{-1}\right) = \operatorname{tr}\left(\mathbf{\Gamma}_t^{-1}\right)\operatorname{tr}\left(\mathbf{\Gamma}_r^{-1}\right). \tag{11}$$

The estimate of $\mathbf{X}$ can be obtained by resolving the optimization problem (11) and image of bistatic MIMO radar is then reconstructed.

4. Simulation Results

In this section, we evaluate the performance of the proposed bistatic MIMO radar sparse imaging methods and compare them with the robust Capon beamforming method (RCB) [14]. We consider bistatic MIMO radar with 20 transmit elements and 20 receive elements. Both the transmit and the receive antennas are uniform linear array with half-wavelength space between adjacent elements. The radar will be scanned across a transmit angular region range from 1° to 10° and a receive angular from 1° to 10°. We place two targets in the scene. Assume that two targets are located at angles $[\theta_{t1}, \theta_{r1}] = [8^\circ, 3^\circ]$ and $[\theta_{t2}, \theta_{r2}] = [3^\circ, 8^\circ]$. The number of snapshots is 20 for the sparse recovery methods and 500 for the RCB. The transmit array and receive array gain uncertainties are generated by $\mathbf{\Gamma}_t = \operatorname{diag}\{\exp[N(0, \sigma_t^2)]\}$ and $\mathbf{\Gamma}_r = \operatorname{diag}\{\exp[N(0, \sigma_r^2)]\}$, where σ_t and σ_r are the parameter governing the array gain. $N(0, \sigma_t^2)$ denotes the Gaussian distribution. We select the balance coefficients $\mu = 1$ in both the direct CS and the constraint CS methods.

Figure 2 shows the results of the image recovery using the proposed method with small array gain uncertainties; that is, $\sigma_t = \sigma_r = 0.1$. It can be observed that both the proposed methods and RCB method can recover the image. The direct CS and constraint CS obtain almost equal performance and the performance of RCB is better than the one of proposed method. However, RCB needs very large amount of samples to enable the algorithm to work. The results of the image recovery with large array gain uncertainties are plotted in Figure 3. It is shown that the recovery performance of constraint CS method is better than that of direct CS. The reason had been discussed in Section 3.2. The performance of both of the CS methods is better than that of RCB method even though the RCB method uses 500 samples compared to 20 samples of the CS method. It implies that the direct CS method is suitable to imaging recovery for bistatic MIMO radar with large array gain errors in the case of small samples.



(a) Original image (Q = 20)

(b) Direcet CS method (Q = 20)

(c) CS with array gain uncertain constraint (Q = 20)

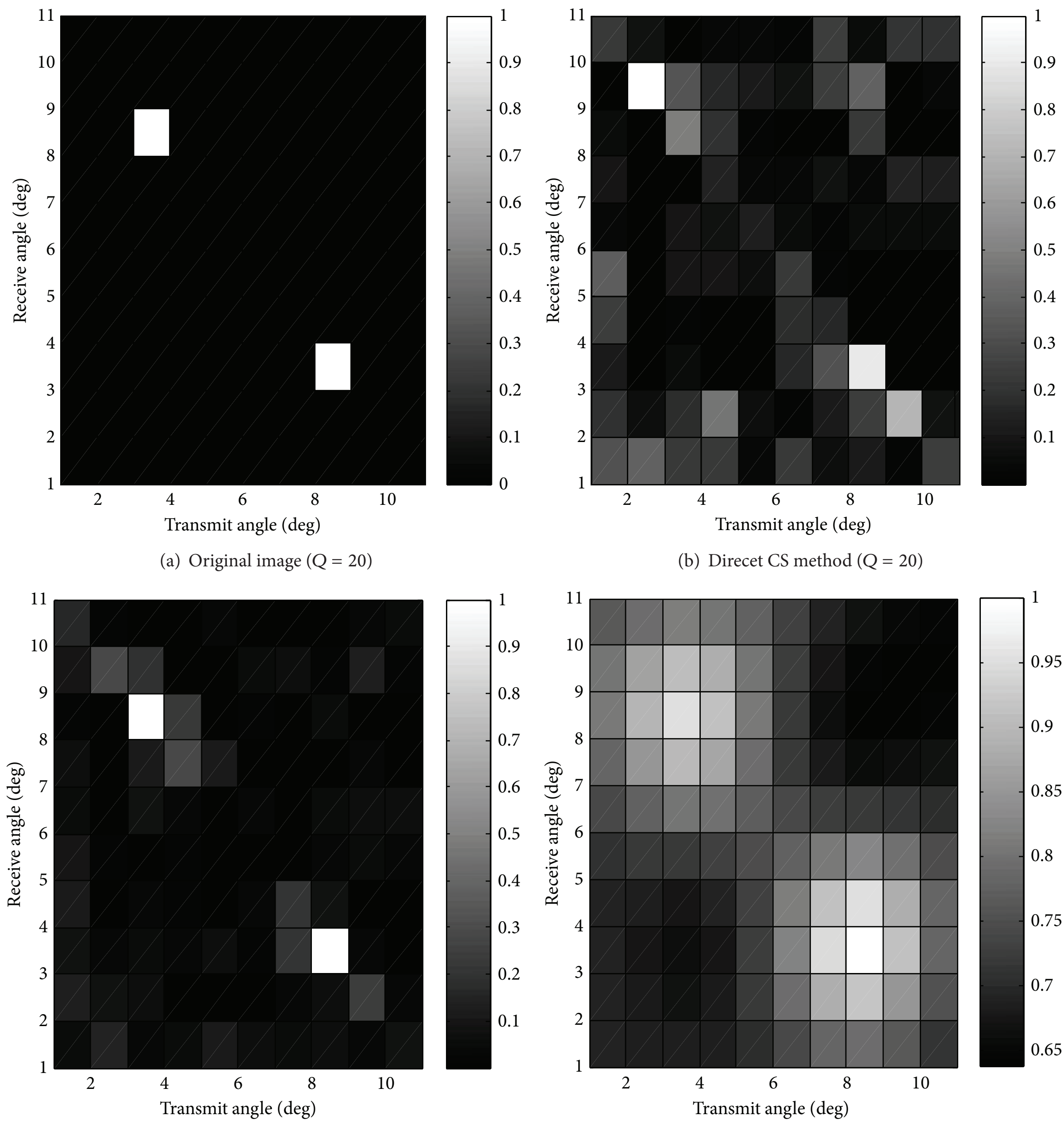
(d) Robust Capon method (Q = 500)

Figure 2: The performance of the proposed method with small array gain uncertain ($M = N = 20$, $[\theta_{t1}, \theta_{r1}] = [8^{\circ}, 3^{\circ}]$, $[\theta_{t1}, \theta_{r1}] = [3^{\circ}, 8^{\circ}]$, $Q = 20$, SNR = 10 dB, $\sigma_t = \sigma_r = 0.1$).

We defined the performance recovery coefficient (RPC) γ to evaluate the performance of the imaging. The RPC is defined as

$$\gamma = \frac{\left| x_1' x_2 \right|}{\left\| x_1 \right\|_2 \left\| x_2 \right\|_2}, \tag{12}$$

where x_1 represents the estimated target coefficient and x_2 represents the true target coefficient. RPC describes the similarity of the true image and the recovering one. Figure 4 plots the variation of the RPC of the CS with array gain uncertain constraint, direct CS, and the RCB method with array gain errors. It is shown that the performance of RCB is better than that of the CS with array gain uncertain constraint and direct CS methods in small array gain error case. The performance of direct CS is better than that of CS with array gain uncertain constraint in the case of small errors. However, the performance of the CS with array gain uncertain constraint method is stable in all of array errors. When the array gain errors are large, the performance of



(a) Original image (Q = 20)

(b) Direcet CS method (Q = 20)

(c) CS with array gain uncertain constraint (Q = 20)

(d) Robust Capon method (Q = 500)

Figure 3: The performance of the proposed method with large array gain uncertain ($M = N = 20$, $[\theta_{t1}, \theta_{r1}] = [8^{\circ}, 3^{\circ}]$, $[\theta_{t1}, \theta_{r1}] = [3^{\circ}, 8^{\circ}]$, $Q = 20$, SNR = 10 dB, $\sigma_t = \sigma_r = 0.8$).

the error constraint CS is the best. The results confirm that the CS with array gain uncertain constraint method is suitable to imaging recovery for bistatic MIMO radar with large array gain errors in the case of small samples.

5. Conclusions

Sparse recovery based transmit-receive angle imaging scheme is proposed for bistatic MIMO with array gain uncertainties in this paper. The redundancy of the transmit and receive angles in the same range cell is exploited to construct the sparse model. CS based algorithm with consideration of both transmit and receive array gain errors is presented for image recovery. Simulation results show that the transmit-receive angle image can be recovered well in bistatic MIMO radar with small number of snapshots in the case of large array gain errors by using sparse recovery based method. Further works should be done to develop sparse recovery based imaging method for bistatic MIMO radar when both array gain and phase errors exist.

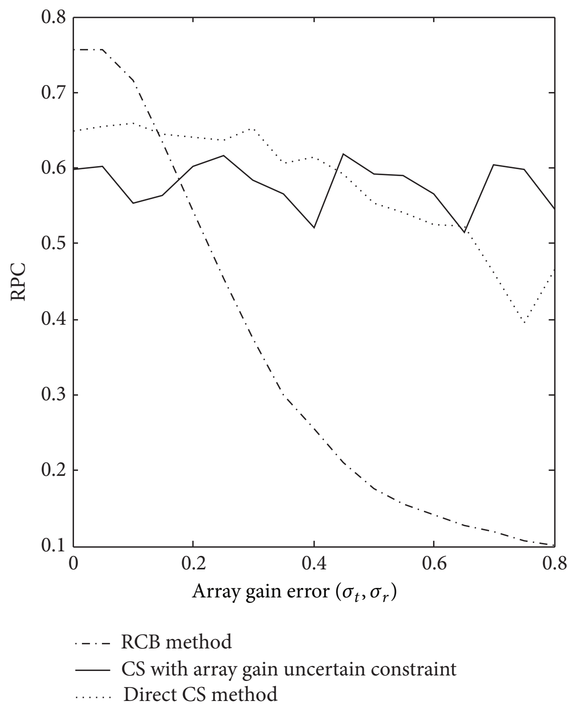


Figure 4: Comparison of the performance of sparse recovery method with the RCB.

Conflict of Interests

The authors declare that they have no competing interests.

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