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Research Article

Robust Stability for Nonlinear Systems with Time-Varying Delay and Uncertainties via the H_∞ Quasi-Sliding Mode Control

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This paper considers the problem of the robust stability for the nonlinear system with time-varying delay and parameters uncertainties. Based on the H_∞ theorem, Lyapunov-Krasovskii theory, and linear matrix inequality (LMI) optimization technique, the H_∞ quasi-sliding mode controller and switching function are developed such that the nonlinear system is asymptotically stable in the quasi-sliding mode and satisfies the disturbance attenuation (H_∞-norm performance). The effectiveness and accuracy of the proposed methods are shown in numerical simulations.

1. Introduction

In general, in all engineering systems, such as communication system, electronics, neural networks, and control systems, disturbance usually emerges in many kinds of situation and then affects the performances of systems. It deserves to be mentioned that, in some special cases, disturbance can reform the instant condition to improve system performances. For examples, the proper local noise power can increase the signal-to-noise ratio of a noisy bistable system and optimize the stochastic resonance of coupled systems, such as small-world and scale-free neuronal networks. [1–3]. However, disturbances such as uncertainty [4–6], impulse interference [7], and nonautonomous effects [8] are usually the unstable and destructive elements to force the main system to make poor performances and abrupt changes. That is why restraining disturbance is always the first and serious study in system control. Recently, the H_∞ controlled method is usually used to suppress disturbances in many investigations of the robust control. Depending on its effective utilization, the disturbance can be restrained with the disturbance attenuation gain in the purposed controller [9–14]. In this paper, we consider a nonlinear system with the nonautonomous noise, time-varying delay, and perturbation of the uncertainty. Based on the H_∞ theorem, a H_∞ quasi-sliding mode controller is designed to guarantee the stability of the main nonlinear system.

In these two decades, sliding mode control (SMC) has been a useful and distinctive robust control strategy for many kinds of engineer systems. Depending on the proposed switching surface and discontinuous controller, the trajectories of dynamic systems can be guided to the fixed sliding manifold. The proposed performance on request can be satisfied. In general, there are two main advantages of SMC which are the reducing order of dynamics from the purposed switching functions and robustness of restraining system uncertainties. Many studies have been conducted on SMC [15–20]. However, the chattering phenomenon is a serious problem which is needed to overcome. In the real application, the chattering phenomenon may cause electronic circuits to be superheated and broken. In order to solve this problem, quasi-sliding mode control method is studied recently [21–25]. Based on the quasi-sliding mode controller, when the trajectories of dynamic systems converge to the sliding manifold, the trajectories can be bounded in the settled region along the sliding surface and the impulse and chattering phenomenon can be avoided.

On the other hand, in the past, most of papers set the control gain in advance to achieve the sufficient condition of the stability. It is not a suitable and accurate way to define the parameters of the system although the parameters are gotten by trial and error. Therefore, in this paper, we use the linear matrix inequality (LMI) theorem to optimize the quasi-sliding mode control gain. By using the computer software MATLAB, quasi-sliding mode controller gain can be found. Based on the H_∞ theorem, Lyapunov-Krasovskii theory, and LMI optimization technique, the H_∞ quasi-sliding mode control and switching function can be designed such that the resulting nonlinear system is asymptotically stable in the quasi-sliding mode and satisfies the disturbance attenuation (H_∞-norm performance).

Throughout this paper, I denotes the identity matrix of appropriate dimensions. For a real matrix A, we denote the transpose by A^T and spectral norm by $\|A\|$. Considering $Q = Q^T > 0$ ($Q = Q^T < 0$), Q is a symmetric positive (negative) definite matrix. The notation * in symmetric block matrices or long matrix expressions throughout the paper represents an ellipsis for terms that are induced by symmetry; for example, $\begin{bmatrix} B & C \\ * & D \end{bmatrix} = \begin{bmatrix} B & C \\ C^T & D \end{bmatrix}$. For a vector x, $\|x(t)\|$ means the Euclidean vector norm at time t, while $\|x\|_2 := \sqrt{\int_0^\infty \|x(t)\|^2 dt}$. If $\|x\|_2 < \infty$, then $x(t) \in L_2[0,\infty)$, where $L_2[0,\infty)$ stands for the space of square integral functions on $[0,\infty)$.

This paper is organized as follows. In Section 2, H_∞ theorem, LMI optimization techniques, and the Lyapunov-Krasovskii stability theorem are used to derive the proposed controller, switching function, and corresponding parameters such that the nonlinear system is asymptotically stable in the quasi-sliding mode. Then, the central discussion of this paper is illustrated by a numerical example in Section 3. Finally, some conclusions are presented in Section 4.

2. Nonlinear System Description and Proposed Controller Design

In this paper, we consider the stability of a nonlinear uncertain neutral system with time-varying delayed state and disturbance. At first, we consider the system without the uncertain segment. Then, a simple extension of nonlinear systems with uncertain part is considered at the next narration. As mentioned above, we firstly consider the nonlinear system (1) without uncertain part as follows:

$$\begin{aligned} \dot{x}(t) &= A_1 x(t) + A_2 x(t-h(t)) \\ &\quad + Bu(t) + Cf(x(t)) + Dw(t) \\ x(t) &= \phi(t), \quad t \in [-h_M, 0], \end{aligned} \tag{1}$$

where $0 \le h(t) \le h_M$, $\dot{h}(t) \le h_D < 1$, $x(t) \in R^n$ is the nonlinear system state, $f(x(t)) \in R^m$ is a continuous nonlinear function vector, $w(t) \in L_2[0,\infty)$ is the noise perturbation input, $u(t) \in R^m$ is a control input vector, and the matrices A_1, A_2, B, C, and D are some known constant matrices with appropriate dimensions. The initial vector ϕ is a differentiable function from $[-h_M, 0]$ to R^n.

Before presenting the main result, we need the following lemma and definition.

Lemma 1. *Let M, N, and $F(t)$ be real matrices of appropriate dimensions with $\|F(t)\| \le 1$ and any scalar; then*

$$\begin{aligned} M^T F(t) N + N^T F^T(t) M &< \varepsilon^{-1} \cdot M^T M \\ &\quad + \varepsilon^{-1} \cdot (\varepsilon N)^T (\varepsilon N) < 0. \end{aligned} \tag{2}$$

Lemma 2 (Schur complement of [26]). *For a given matrix $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$ with $S_{11} = S_{11}^T$, $S_{22} = S_{22}^T$, the following conditions are equivalent:*

(1) $S < 0$,

(2) $S_{22} < 0$, $S_{11} - S_{12} S_{22}^{-1} S_{21} < 0$.

Definition 3. H_∞ control problem addressed in this paper can be formulated as finding a stabilizing controller, such that the following conditions are held.

Under the control input $u(t)$, the states of the nonlinear system (1) are asymptotically stable when $w(t) = 0$.

Under the zero initial conditions, the performance index $J = \int_0^\infty [x^T(t)x(t) - \gamma^2 \cdot w^T(t)w(t)]dt \le 0$ ($\|x\|_2 \le \gamma \|w\|_2$) holds for any nonzero $w \in L_2$ and the H_∞ performance $\gamma > 0$.

This paper aims at proposing the H_∞ quasi-sliding mode controller and switching function to not only asymptotically stabilize the nonlinear system but also guarantee a prescribed performance of noise perturbation attenuation $r > 0$. The first step is to select an appropriate switching surface which is shown as (3) to ensure the asymptotical stability of the sliding motion on the sliding manifold:

$$\begin{aligned} S(t) &= Gx(t) + r(t), \\ \dot{r}(t) &= GBr(t) - GA_1 x(t) \\ &\quad - GA_2 x(t-h(t)) - GBKx(t) - GBKx(t-h(t)), \end{aligned} \tag{3}$$

where $S(t) \in R^m$ is the switching surface, K is the quasi-sliding mode controller gain and G is chosen such that GB is nonsingular and $GD = 0$. When the system is placed on the switching surface, it satisfies the following equations:

$$S(t) = 0, \qquad \dot{S}(t) = 0. \tag{4}$$

Then, the equivalent controller u_{eq} in the sliding manifold can be obtained by solving for $\dot{S}(t) = 0$:

$$\begin{aligned} \dot{S}(t) &= GA_1 x(t) + GA_2 x(t-h(t)) + GBu_{eq}(t) \\ &\quad + GCf(x(t)) + GDw(t) + GBr(t) - GA_1 x(t) \\ &\quad - GA_2 x(t-h(t)) - GBKx(t) - GBKx(t-h(t)). \end{aligned} \tag{5}$$

From $r(t) = -Gx(t)$, as $S(t) = 0$,

$$\begin{aligned}\dot{S}(t) &= GA_1x(t) + GA_2x(t-h(t)) + GBu_{eq}(t) \\ &\quad + GCf(x(t)) + GDw(t) - GBGx(t) - GA_1x(t) \\ &\quad - GA_2x(t-h(t)) - GBKx(t) - GBKx(t-h(t)) = 0.\end{aligned} \tag{6}$$

Since GB is nonsingular, the equivalent controller $u_{eq}(t)$ in the sliding mode is given by

$$\begin{aligned}u_{eq}(t) &= Kx(t) + Kx(t-h(t)) \\ &\quad - (GB)^{-1}GCf(x(t)) + Gx(t).\end{aligned} \tag{7}$$

Therefore, substituting $u_{eq}(t)$ into (1), the nonlinear dynamics in the quasi-sliding mode can be obtained:

$$\begin{aligned}\dot{x}(t) &= (A_1 + BK + BG)x(t) \\ &\quad + (A_2 + BK)x(t-h(t)) + Dw(t).\end{aligned} \tag{8}$$

Then, the second step is to design the proposed H_∞ quasi-sliding mode controller. In the following, Theorem 4 will derive that the trajectories of the nonlinear dynamics system converge to the quasi-sliding manifold based on the quasi-sliding mode controller $u(t)$.

Theorem 4. *Consider nonlinear dynamics system* (1) *with the switching function; the trajectories of the nonlinear dynamics system converge to the quasi-sliding manifold if the controller $u(t)$ is given by*

$$\begin{aligned}u(t) &= Kx(t) + Kx(t-h(t)) + Gx(t) \\ &\quad - (GB)^{-1}\left(GCf(x(t)) + \frac{S(t)}{(\|S(t)\| + \delta_Q)}\right).\end{aligned} \tag{9}$$

Proof. Define the Lyapunov function:

$$V_s(t) = \frac{1}{2}S^T(t)S(t). \tag{10}$$

Taking the derivative of $V_s(t)$ and introducing (3), one has

$$\begin{aligned}\dot{V}_s(t) &= S^T(t)\dot{S}(t) \\ &= S^T(t)\left[GA_1x(t) + GA_2x(t-h(t)) + GCf(x(t))\right. \\ &\quad + GBu(t) + GDw(t) - GBGx(t) \\ &\quad - GA_1x(t) - GA_2x(t-h(t)) \\ &\quad \left.- GBKx(t) - GBKx(t-h(t))\right] \\ &= -S^T(t)\left(\frac{S(t)}{(\|S(t)\| + \delta_Q)}\right) \\ &= -\frac{\|S(t)\|^2}{(\|S(t)\| + \delta_Q)} \\ &= -\left(\|S(t)\| - \frac{(\|S(t)\|^2 + \|S(t)\|\delta_Q - \|S(t)\|^2)}{(\|S(t)\| + \delta_Q)}\right) \\ &= -\left(\|S(t)\| - \frac{\|S(t)\|\delta_Q}{(\|S(t)\| + \delta_Q)}\right) \\ &< -(\|S(t)\| - \delta_Q).\end{aligned} \tag{11}$$

From the above proof, we can obtain that $\|S(t)\| > \delta_Q$ is stable which satisfies the reached condition of quasi-sliding mode control. This completes the proof. □

In order to solve the H_∞ quasi-sliding mode controller gain K in the switching surface and optimized performance of noise perturbation attenuation $\gamma > 0$, we use the LMI formulation to look for them such that the nonlinear dynamic system (1) is asymptotically stable in the quasi-sliding mode.

Theorem 5. *Consider nonlinear dynamics system* (1), *if there are a constant $\gamma > 0$, positive definite symmetric matrix $\overline{P}_1$ and $\overline{P}_2$, and matrix $\overline{K}$ such that the following LMI condition holds:*

$$\Sigma_2 < 0, \tag{12}$$

where $\widehat{\Sigma}_{12} = A_2\overline{P}_1 + B\overline{K}$, $\widehat{\Sigma}_{13} = D$, $\widehat{\Sigma}_{14} = \overline{P}_1$, $\widehat{\Sigma}_{22} = -(1-h_D)\overline{P}_2$, $\widehat{\Sigma}_{33} = -\gamma^2 \cdot I$, $\widehat{\Sigma}_{44} = I$, and $\widehat{\Sigma}_{11} = \overline{P}_1A_1^T + A_1\overline{P}_1 + \overline{K}^TB^T + B\overline{K} + \overline{P}_1G^TB^T + BG\overline{P}_1 + \overline{P}_2$. Then, the nonlinear dynamic system (1) *is asymptotically stable in the quasi-sliding mode with noise perturbation attenuation $\gamma > 0$ and quasi-sliding mode controller with its gain $K = \overline{K}\overline{P}_1^{-1}$.*

Proof. Define Lyapunov function:

$$V(t) = x^T(t)P_1x(t) + \int_{t-h(t)}^{t} x^T(\tau)P_2x(\tau)\,d\tau. \tag{13}$$

Taking the derivative of $V(t)$ and introducing (8), one has

$$\begin{aligned}\dot{V}(t) &= x^T(t)\left(A_1^TP_1 + P_1A_1 + K^TB^TP_1\right. \\ &\quad \left.+P_1BK + G^TB^TP_1 + P_1BG\right)x(t) \\ &\quad + x^T(t)(P_1A_2 + P_1BK)x(t-h(t)) \\ &\quad + x^T(t-h(t))\left(A_1^TP_1 + K^TB^TP_1\right)x(t) \\ &\quad + 2x^T(t)P_1Dw(t) + x^T(t)P_2x(t) \\ &\quad - (1-\dot{h}(t)) \cdot x^T(t-h(t))P_2x(t-h(t)).\end{aligned} \tag{14}$$

Define a function $J(t)$ as follows:

$$J(t) = \dot{V}(t) + x^T(t)x(t) - \gamma^2 w^T(t)w(t) \le \begin{bmatrix} x(t) \\ x(t-h(t)) \\ w(t) \end{bmatrix}^T \Sigma_1 \begin{bmatrix} x(t) \\ x(t-h(t)) \\ w(t) \end{bmatrix}, \tag{15}$$

where $\Sigma_1 = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} \\ * & \Sigma_{22} & 0 \\ * & * & \Sigma_{33} \end{bmatrix}$, $\Sigma_{11} = A_1^T P_1 + P_1 A_1 + K^T B^T P_1 + P_1 BK + G^T B^T P_1 + P_1 BG + P_2 + I$, $\Sigma_{12} = P_1 A_2 + PBK$, $\Sigma_{13} = P_1 D$, $\Sigma_{22} = -(1-h_D)P_2$, and $\Sigma_{33} = -\gamma^2 \cdot I$.

Pre-multiplying and post-multiplying the Σ_1 by $\Phi = \operatorname{diag}\lfloor P_1^{-1} \;\; P_1^{-1} \;\; I \rfloor$, defining $\overline{P}_1 = P_1^{-1}$, $\overline{P}_2 = \overline{P}_1 P_2 \overline{P}_1$, and $\overline{K} = K P_1^{-1}$, we can obtain

$$\Sigma_2 = \begin{bmatrix} \widehat{\Sigma}_{11} & \widehat{\Sigma}_{12} & \widehat{\Sigma}_{13} & \widehat{\Sigma}_{14} \\ * & \widehat{\Sigma}_{22} & 0 & 0 \\ * & * & \widehat{\Sigma}_{33} & 0 \\ * & * & * & \widehat{\Sigma}_{44} \end{bmatrix}, \tag{16}$$

where $\widehat{\Sigma}_{11} = \overline{P}_1 A_1^T + A_1 \overline{P}_1 + \overline{K}^T B^T + B\overline{K} + \overline{P}_1 G^T B^T + BG\overline{P}_1 + \overline{P}_2$, $\widehat{\Sigma}_{12} = A_2 \overline{P}_1 + B\overline{K}$, $\widehat{\Sigma}_{13} = D$, $\widehat{\Sigma}_{14} = \overline{P}_1$, $\widehat{\Sigma}_{22} = -(1-h_D)\overline{P}_2$, $\widehat{\Sigma}_{33} = -\gamma^2 \cdot I$, and $\widehat{\Sigma}_{44} = -I$.

If matrix (16) is a negative definite matrix and $w(t) = 0$, there exists a $\rho > 0$ such that

$$\dot{V}(t)\big|_{w(t)=0} \le -\rho \cdot \|x(t)\|^2 \tag{17}$$

which guarantees the stability of the system (1). Therefore, by the Lyapunov-Krasovskii stability theorem, the nonlinear system with $w(t) = 0$ is asymptotically stable. Furthermore, combining the condition $\Sigma_2 < 0$ with (15) from 0 to ∞, obviously

$$V(x_\infty) - V(\phi) + \|x(t)\|_2^2 - \gamma^2 \cdot \|w(t)\|_2^2 \le 0. \tag{18}$$

In zero initial condition ($\phi = 0$), it can be denoted by

$$V(\phi) = 0, \qquad V(x_\infty) \ge 0, \qquad \|x(t)\|_2 \le \gamma^2 \cdot \|w(t)\|_2, \quad \forall w(t) \in L_2[0,\infty). \tag{19}$$

Based on Definition 3, the nonlinear dynamic system is stabilized with noise perturbation attenuation $\gamma > 0$ by the quasi-sliding mode control gain given by $K = \overline{K}\overline{P}_1^{-1}$. This completes the proof. □

At last, we consider the nonlinear system (1) with uncertain part as the following form:

$$\begin{aligned} \dot{x}(t) &= A_1(t)x(t) + A_2(t)x(t-h(t)) + B(t)u(t) \\ &\quad + C(t)f(x(t)) + D(t)w(t), \\ x(t) &= \phi(t), \quad t \in [-h_M, 0], \end{aligned} \tag{20}$$

where $A_1(t) = A_1 + \Delta A_1(t)$, $A_2(t) = A_2 + \Delta A_2(t)$, $B(t) = B + \Delta B(t)$, $C(t) = C + \Delta C(t)$, and $D(t) = D + \Delta D(t)$. $\Delta A_1(t)$, $\Delta A_2(t)$, $\Delta B(t)$, $\Delta C(t)$, and $\Delta D(t)$ are some time-varying continuous perturbed matrices and satisfy

$$\begin{aligned} &[\Delta A_1(t) \;\; \Delta A_2(t) \;\; \Delta B(t) \;\; \Delta C(t) \;\; \Delta D(t)] \\ &\quad = M \cdot F(t) \cdot [N_1 \;\; N_2 \;\; N_3 \;\; N_4 \;\; N_5], \end{aligned} \tag{21}$$

where M and N_i, $i \in \{1 \sim 5\}$, are some given constant matrices and $F(t)$ is unknown continuous function satisfying

$$F^T(t) \cdot F(t) \le I, \quad \forall t \ge 0. \tag{22}$$

The following result is obtained from Theorem 5. This will be a simple extension of Theorem 5.

Theorem 6. *Consider nonlinear dynamics system with the uncertainty part* (20)*, if there are a constant $\gamma > 0$, positive definite symmetric matrix $\overline{P}_1$ and $\overline{P}_2$, and matrix $\overline{K}$ such that the following LMI condition holds:*

$$\widehat{\Sigma} = \begin{bmatrix} \widehat{\Sigma}_{11} & \widehat{\Sigma}_{12} & \widehat{\Sigma}_{13} & \widehat{\Sigma}_{14} & \widehat{\Sigma}_{15} & \widehat{\Sigma}_{16} \\ * & \widehat{\Sigma}_{22} & 0 & 0 & 0 & \widehat{\Sigma}_{26} \\ * & * & \widehat{\Sigma}_{33} & 0 & 0 & \widehat{\Sigma}_{36} \\ * & * & * & \overline{\Sigma}_{44} & 0 & 0 \\ * & * & * & * & \widehat{\Sigma}_{55} & 0 \\ * & * & * & * & * & \widehat{\Sigma}_{66} \end{bmatrix} < 0, \tag{23}$$

where $\widehat{\Sigma}_{15} = \varepsilon M^T$, $\widehat{\Sigma}_{16} = \overline{P}_1 N_1^T + N_1 \overline{P}_1 + \overline{K}^T N_3^T + N_3 \overline{K} + \overline{P}_1 G^T N_3^T + N_3 G \overline{P}_1$, $\widehat{\Sigma}_{26} = N_2 \overline{P}_1 + N_3 \overline{K}$, $\widehat{\Sigma}_{36} = N_5$, and $\widehat{\Sigma}_{55} = \widehat{\Sigma}_{66} = -\varepsilon \cdot I$.

Then, the nonlinear dynamic system with the uncertainty part (20) *is asymptotically stable in the quasi-sliding mode with noise perturbation attenuation $\gamma > 0$ by quasi-sliding mode controller with its gain $K = \overline{K}\,\overline{P}^{-1}$.*

Proof. From condition (16) and nonlinear dynamic system with the uncertainty part (20), we can obtain the following condition to guarantee the H_∞ quasi-sliding mode control:

$$\Sigma_3 = \Sigma_2 + \Pi^T F(t)\Omega + \Omega^T F(t)\Pi < 0, \tag{24}$$

where

$$\begin{aligned} \Omega &= \left[\overline{P}_1 N_1^T + N_1 \overline{P}_1 + \overline{K}^T N_3^T + N_3 \overline{K} + \overline{P}_1 G^T N_3^T + N_3 G \overline{P}_1 \;\; N_2 \overline{P}_1 + N_3 \overline{K} \;\; N_5\right], \\ \Pi &= [M \;\; 0 \;\; 0]. \end{aligned} \tag{25}$$

By Lemmas 1 and 2, the condition $\Sigma_3 < 0$ in (24) is equivalent to $\widehat{\Sigma} < 0$ in (23). By similar demonstration of Theorem 5, we can complete this proof. □

3. Illustrative Simulations

Consider the following nonlinear system with the nonautonomous noise, time-varying delay, and parameters uncertainty described as follows:

$$\begin{aligned}\dot{x}(t) = &\left(\begin{bmatrix}-2 & 1 & 0.2\\ 0.6 & -5 & -0.1\\ 0 & 0.3 & 0.1\end{bmatrix} + (MF(t)N_1)\right)x(t)\\ &+\left(\begin{bmatrix}-0.1 & 0 & -0.1\\ 0.4 & -0.2 & 0\\ 0.1 & 0.5 & -0.1\end{bmatrix} + (MF(t)N_2)\right)x(t-h(t))\\ &+\left(\begin{bmatrix}0\\0\\2\end{bmatrix} + (MF(t)N_3)\right)u(t)\\ &+\left(\begin{bmatrix}1\\0\\0\end{bmatrix} + (MF(t)N_4)\right)f(x(t))\\ &+\left(\begin{bmatrix}0\\1\\0\end{bmatrix} + (MF(t)N_5)\right)w(t),\end{aligned} \tag{26}$$

where

$$\begin{gathered} f(x(t)) = x_1(t)x_2(t), \qquad G = [1 \;\; 0 \;\; -1],\\ \delta_Q = 0.01, \qquad h_M = 0.2, \qquad M = 0.1,\\ N_1 = \begin{bmatrix}0.1 & 0.1 & 0.1\\ 0.5 & 0.1 & 0\\ 0.3 & 0.1 & 0.1\end{bmatrix}, \qquad N_2 = \begin{bmatrix}0 & 0.1 & 0\\ 0 & 0.2 & 0\\ 0.1 & 0.01 & 0\end{bmatrix},\\ N_3 = [0.2 \;\; 0.1 \;\; 0.1]^T, \qquad N_4 = [0.5 \;\; 0.1 \;\; 0.1]^T,\\ N_5 = [0.01 \;\; 0.02 \;\; 0.1]^T. \end{gathered} \tag{27}$$

$F(t) = \sin(t)$ is given and $w(t)$ is the Gauss white noise which is shown in Figure 1.

Based on Theorem 6 with $\varepsilon = 0.1$ and disturbance attenuation $\gamma = 0.4$, the appropriate answer can be solved as

$$\begin{gathered} \overline{P}_1 = \begin{bmatrix}0.2726 & -0.0917 & 0.0654\\ -0.0917 & 2.3626 & -0.4097\\ 0.0654 & -0.4097 & 1.7972\end{bmatrix},\\ \overline{P}_2 = \begin{bmatrix}0.1794 & -0.5512 & -0.2904\\ -0.5512 & 3.2315 & 0.1598\\ -0.2904 & 0.1598 & 1.1451\end{bmatrix},\\ \overline{K} = [0.1128 \;\; -1.2885 \;\; 0.1663]. \end{gathered} \tag{28}$$

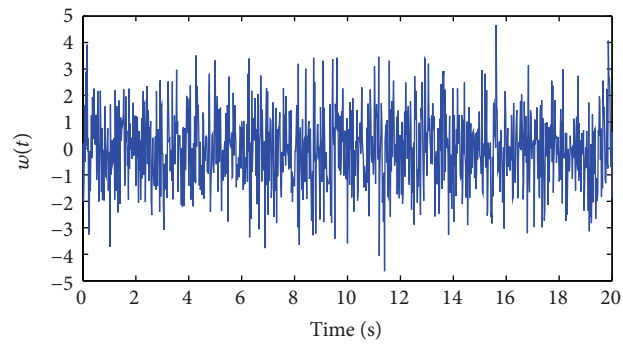

Figure 1: Time responses of Gauss white noise $w(t)$.

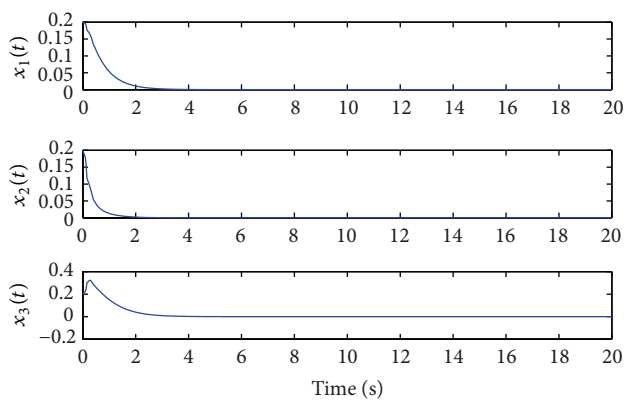

Figure 2: Time responses of states of the controlled nonlinear system without disturbances.

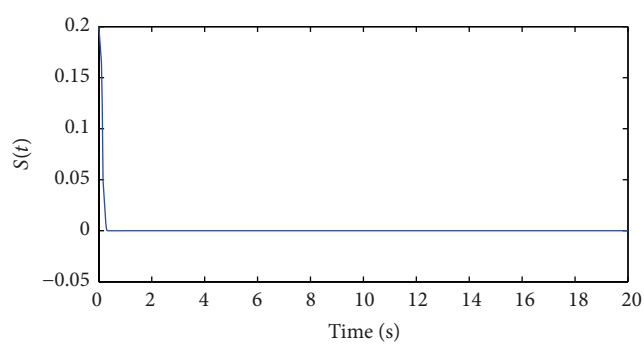

Figure 3: Time responses of switching surface $S(t)$ without disturbances.

Then, a quasi-sliding mode control gain can be derived as

$$K = \overline{K}\overline{P}_1^{-1} = [0.2409 \;\; -0.5430 \;\; -0.0400]. \tag{29}$$

Based on the proposed controller, the asymptotical stability for the nonlinear system without the disturbances on the sliding manifold is shown in Figure 2, and the relative switching sliding surface is shown in Figure 3. Oppositely, under effects of disturbances, the asymptotical stability for the nonlinear system on the sliding manifold is shown in Figure 4, and the switching sliding surface is shown in Figure 5. They show that disturbances can be restrained by the main controller

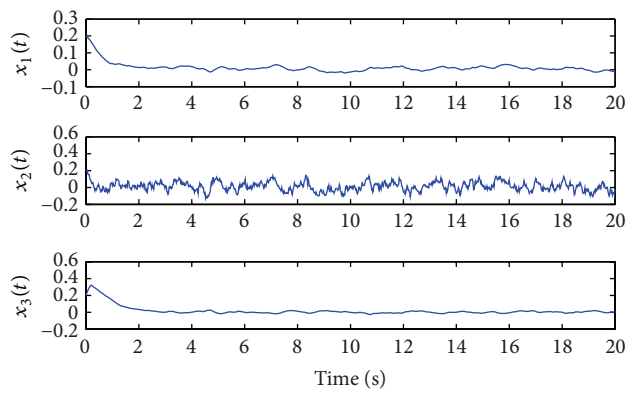

Figure 4: Time responses of states of the controlled nonlinear system under effects of disturbances.

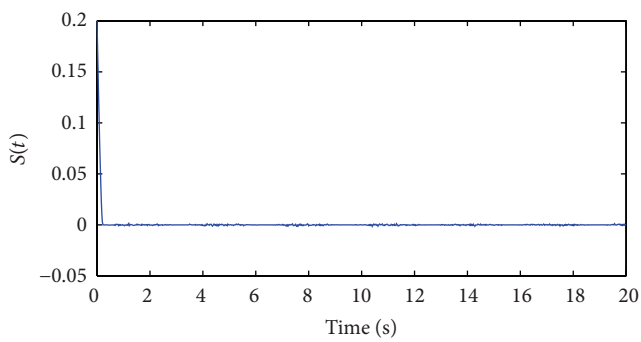

Figure 5: Time responses of switching surface $S(t)$ under effects of disturbances.

with noise perturbation attenuation γ. According to the above simulation, the asymptotical stability of the nonlinear system with the nonautonomous noise, time-varying delay, and parameters uncertainties on the sliding manifold is guaranteed by the H_∞ quasi-sliding mode controller with noise perturbation attenuation γ.

4. Conclusions

This study investigated the robust stability of the nonlinear system with time-varying delay and parameters uncertainties. Based on the H_∞ theorem, Lyapunov-Krasovskii theory, and LMI optimization technique, the asymptotical stability of the nonlinear system could be guaranteed by the purposed H_∞ quasi-sliding mode controller and switching surface in the quasi-sliding mode and the disturbance attenuation can be also satisfied (H_∞-norm performance). Numerical simulations displayed the feasibility and usefulness of the central discussion.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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