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Research Article

Effects of Medium Characteristics on Laser RCS of Airplane with E-Wave Polarization

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Plane wave incidence should be postulated to have an authentic target detection. Practically, the plane wave is incapable usually of keeping its power in the far field especially when propagating through an inhomogeneous medium. Consequently, we assume an incident beam wave with a finite width around the target. In this work, we calculate numerically a laser radar cross section (LRCS) of conducting targets having smooth cross sections with inflection points such as airplane in random media. Effects of fluctuations intensity of random media on the LRCS performance are studied in this paper. E-wave polarization (E-wave incidence) is considered while the mean target size is approximately twice the wavelength.

1. Introduction

Methods to formulate scattered waves were conducted over decades [1–3]. Yet, these methods require long processing time and several memory resources. These restrictions would, therefore, lessen the usefulness of such methods particularly when we consider objects detection applications in real time. A method has been presented in the previous years to solve the scattering problem numerically [4–6]. In this method, the problem is solved as a boundary value problem where the current is calculated on the outer surface of the target. As a result, waves intensity and accordingly target detection coefficients are accurate in a forward scattering environment such as turbulence. Effects of targets configuration, random media strength, and the polarization on the LRCS and the backscattering enhancement for a beam wave incidence were studied in several publications (e.g., [7–9]). Our results proved to be in excellent agreement with those assuming a cylinder with circular cross section in free space in [10]. Lately, this method was verified using FDFD method showing a fair agreement with an accuracy below 5% error rate for objects in random media and even less error in the free space [11].

Approximate solutions for beam wave functions were obtained and research on laser radar was pursued as in [12, 13]. Most of the proposed techniques have limitations on targets configuration as being restricted to be point targets or simple canonical cross sections. Here, LRCS is calculated for targets of analytical cross sections with inflection points in random medium such as turbulence. Targets applications include civil and military airplane.

Propagation of wide-pulse in sparse medium such as atmospheric and other aerosols was studied [14, 15]. Medium dispersion likely causes an unwanted signal distortion and retrogradation [16]. The work in this paper is to probe the frequency range where the proper selection of beam width may result in a minimal effect of random medium parameters on the LRCS. Consequently, we will be able to find a mechanism that maximizes the accuracy of radar detection in a random medium particularly in the far field. Correspondingly, we present a study on random media parameters and analyze their performance to examine their effects on the scattered waves. These parameters include both medium fluctuations intensity implemented by the medium correlation function and the spatial coherence length (SCL) of waves around the target.

Having the radius of the scatterer much smaller than the size of the wavelength assumed in [17] is not the general case particularly in the high frequency band. As a result, a target of a large size compared to the wavelength is a crucial issue

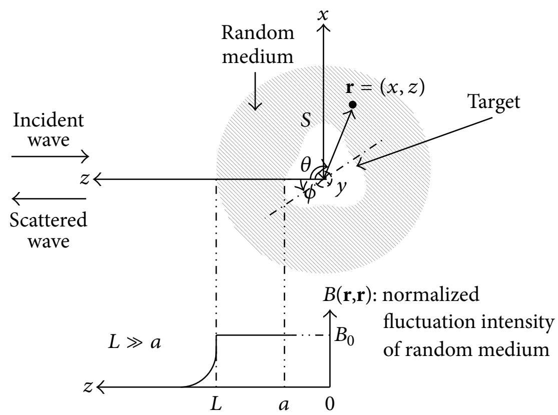

Figure 1: Geometry of the problem of wave scattering from a conducting cylinder in a random medium.

for the frequency spectrum of waves propagating in highly fluctuating media that in turn affects the electromagnetic waves detection capabilities. Putting this in mind, our work assumes a target size approximately twice the wavelength to suit the remote sensing using high frequency range. On the other hand, the case where the beam width is smaller than the target dimensions is considered to meet the needs of radar applications. LRCS is calculated using our prescribed method for convex illumination region of partially convex targets.

Applications for our study consider conducting targets such as airplane where models have different contour complexity and shapes. We deal with the scattering problem two-dimensionally considering the horizontal polarization (E-wave incidence). The time factor $\exp(-iwt)$ is assumed and suppressed in the following section.

2. Formulation

Geometry of the problem is shown in Figure 1. A random medium is assumed as a sphere of radius L around a target of the mean size $a \ll L$ and also to be described by the dielectric constant $\varepsilon(\mathbf{r})$, the magnetic permeability μ, and the electric conductivity ν. For simplicity $\varepsilon(\mathbf{r})$ is expressed as

$$\varepsilon(\mathbf{r}) = \varepsilon_0 [1 + \delta\varepsilon(\mathbf{r})], \tag{1}$$

where ε_0 is assumed to be constant and equal to free space permittivity and $\delta\varepsilon(\mathbf{r})$ is a random function with

$$\begin{gathered} \langle \delta\varepsilon(\mathbf{r}) \rangle = 0, \\ \left\langle \delta\varepsilon(\mathbf{r})\,\delta\varepsilon(\mathbf{r}') \right\rangle = B(\mathbf{r}, \mathbf{r}'), \end{gathered} \tag{2}$$

$$\begin{gathered} B(\mathbf{r}, \mathbf{r}) \ll 1, \\ kl(\mathbf{r}) \gg 1. \end{gathered} \tag{3}$$

Here, the angular brackets denote the ensemble average and $B(\mathbf{r}, \mathbf{r})$, $l(\mathbf{r})$ are the local intensity and local scale-size of the random medium fluctuation [18, 19], respectively, and $k = \omega\sqrt{\varepsilon_0\mu_0}$ is the wavenumber in free space. Also μ and ν are assumed to be constants; $\mu = \mu_0$, $\nu = 0$. For practical turbulent media condition (3) may be satisfied. Therefore, we can assume the forward scattering approximation and the scalar approximation [20]. Consider the case where a directly incident beam wave is produced by a line source $f(\mathbf{r}')$ along the y axis. Here, let us designate the incident wave by $u_{\text{in}}(\mathbf{r})$, the scattered wave by $u_s(\mathbf{r})$, and the total wave by $u(\mathbf{r}) = u_{\text{in}}(\mathbf{r}) + u_s(\mathbf{r})$. The target is assumed to be a conducting cylinder of which cross section is expressed by

$$r = a[1 - \delta\cos 3(\theta - \phi)], \tag{4}$$

where ϕ is the rotation index and δ is the concavity index. We can deal with this scattering problem two-dimensionally under condition (3); therefore, we represent $\mathbf{r}$ as $\mathbf{r} = (x, z)$. Assuming a horizontal polarization of incident waves (E-wave incidence), we can impose the Dirichlet boundary condition for wave field $u(\mathbf{r})$ on the cylinder surface S. That is, $u(\mathbf{r}) = \mathbf{0}$, where $u(\mathbf{r})$ represents E_y.

Using the current generator Y_E and Green's function in random medium $G(\mathbf{r} \mid \mathbf{r}')$, we can express the surface current wave as

$$J(\mathbf{r}_2) = \int_S Y_E(\mathbf{r}_2 \mid \mathbf{r}_1)\, u_{\text{in}}(\mathbf{r}_1 \mid \mathbf{r}_t)\, d\mathbf{r}_1, \tag{5}$$

where $\mathbf{r}_t$ represents the source point location and it is assumed as $\mathbf{r}_t = (0, z)$ in Section 3. The first order of $\langle G(\mathbf{r} \mid \mathbf{r}') \rangle$ is M_{10} and expressed as

$$M_{10}(z) = \langle G(\mathbf{r} \mid z) \rangle = G_0(\mathbf{r} \mid z)\, e^{-\alpha(z)z}, \tag{6}$$

where $G_0(\mathbf{r} \mid z)$ is Green's function in free space [20] and α is the coherence attenuation index [21] at the far field in the z direction. We consider $u_{\text{in}}(\mathbf{r}_1 \mid \mathbf{r}_t)$, whose dimension coefficient is understood, to be represented as [20]

$$u_{\text{in}}(\mathbf{r}_1 \mid \mathbf{r}_t) = G(\mathbf{r}_1 \mid \mathbf{r}_t)\exp\left[-\left(\frac{kx_1}{kW}\right)^2\right], \tag{7}$$

where W is the beam width. The beam expression is approximately useful only around the cylinder. Then, the scattered wave is given by

$$u_s(\mathbf{r}) = \int_S J_E(\mathbf{r}_2)\, G(\mathbf{r} \mid \mathbf{r}_2)\, d\mathbf{r}_2. \tag{8}$$

This can be represented by

$$\begin{aligned} u_s(\mathbf{r}) &= \int_S d\mathbf{r}_1 \int_S d\mathbf{r}_2 \left[G(\mathbf{r} \mid \mathbf{r}_2)\, Y_E(\mathbf{r}_2 \mid \mathbf{r}_1)\, u_{\text{in}}(\mathbf{r}_1 \mid \mathbf{r}_t)\right]. \end{aligned} \tag{9}$$

Here, Y_E is the operator that transforms incident waves into surface currents on S and depends only on the scattering body [4, 6]. The current generator can be expressed in terms of wave functions that satisfy Helmholtz equation and the radiation condition.

That is, the surface current is obtained as

$$\begin{aligned} &\int_S Y_E(\mathbf{r}_2 \mid \mathbf{r}_1)\, u_{\text{in}}(\mathbf{r}_1 \mid \mathbf{r}_t)\, d\mathbf{r}_1 \\ &\quad \simeq \mathbf{\Phi}_M^*(\mathbf{r}_2)\, A_E^{-1} \int_S \langle\!\langle \mathbf{\Phi}_M^T(\mathbf{r}_1), u_{\text{in}}(\mathbf{r}_1 \mid \mathbf{r}_t) \rangle\!\rangle\, d\mathbf{r}_1, \end{aligned} \tag{10}$$

where

$$\int_S \langle\langle \Phi_M^T(\mathbf{r}_1), u_{\text{in}}(\mathbf{r}_1 \mid \mathbf{r}_t)\rangle\rangle\, d\mathbf{r}_1 \equiv \int_S \left[\phi_m(\mathbf{r}_1)\frac{\partial u_{\text{in}}(\mathbf{r}_1 \mid \mathbf{r}_t)}{\partial n} - \frac{\partial \phi_m(\mathbf{r}_1)}{\partial n} u_{\text{in}}(\mathbf{r}_1 \mid \mathbf{r}_t)\right] d\mathbf{r}_1. \tag{11}$$

The above equation is sometimes called "reaction," as named by Rumsey [22]. In (10), the basis functions $\mathbf{\Phi}_M$ are called the modal functions and constitute the complete set of wave functions satisfying the Helmholtz equation in free space and the radiation condition; $\mathbf{\Phi}_M = [\phi_{-N}, \phi_{-N+1}, \ldots, \phi_m, \ldots, \phi_N]$, Φ_M^* and Φ_M^T denote the complex conjugate and the transposed vectors of Φ_M, respectively, $M = 2N + 1$ is the total mode number, $\phi_m(\mathbf{r}) = H_m^{(1)}(kr)\exp(im\theta)$, and A_E is a positive definite Hermitian matrix given by

$$A_E = \begin{pmatrix} (\phi_{-N}, \phi_{-N}) & \cdots & (\phi_{-N}, \phi_N) \\ \vdots & \ddots & \vdots \\ (\phi_N, \phi_{-N}) & \cdots & (\phi_N, \phi_N) \end{pmatrix}, \tag{12}$$

in which its m, n element is the inner product of ϕ_m and ϕ_n:

$$(\phi_m, \phi_n) \equiv \int_S \phi_m(\mathbf{r})\phi_n^*(\mathbf{r})\, d\mathbf{r}. \tag{13}$$

Y_E is proved to converge in the sense of mean on the true operator when $M \to \infty$.

Therefore, the average intensity of backscattering wave for E-wave incidence is given by

$$\begin{aligned}\left\langle |u_s(\mathbf{r})|^2 \right\rangle &= \int_S d\mathbf{r}_{01} \int_S d\mathbf{r}_{02} \int_S d\mathbf{r}_1' \\ &\cdot \int_S d\mathbf{r}_2' Y_E(\mathbf{r}_{01} \mid \mathbf{r}_1') Y_E^*(\mathbf{r}_{02} \mid \mathbf{r}_2') \\ &\cdot \exp\left[-\left(\frac{kx_1'}{kW}\right)^2\right] \exp\left[-\left(\frac{kx_2'}{kW}\right)^2\right] \\ &\cdot \left\langle G(\mathbf{r} \mid \mathbf{r}_{01}) G(\mathbf{r} \mid \mathbf{r}_{02}) G^*(\mathbf{r} \mid \mathbf{r}_1') G^*(\mathbf{r} \mid \mathbf{r}_2') \right\rangle.\end{aligned} \tag{14}$$

Here, we calculate the current generated at $\mathbf{r}_{01}$ as a result of the waves incidence at $\mathbf{r}_1'$ on the object surface and similarly at the point $\mathbf{r}_{02}$ as a result of the waves incidence at $\mathbf{r}_2'$. In our representation of $\langle |u_s(\mathbf{r})|^2 \rangle$, we use an approximate solution for the fourth moment of Green's function in random medium M_{22} [5]:

$$\begin{aligned} M_{22} &= \left\langle G(\mathbf{r} \mid \mathbf{r}_1') G(\mathbf{r} \mid \mathbf{r}_{01}) G^*(\mathbf{r} \mid \mathbf{r}_2') G^*(\mathbf{r} \mid \mathbf{r}_{02}) \right\rangle \\ &\simeq \left\langle G(\mathbf{r} \mid \mathbf{r}_1') G^*(\mathbf{r} \mid \mathbf{r}_2') \right\rangle \left\langle G(\mathbf{r} \mid \mathbf{r}_{01}) G^*(\mathbf{r} \mid \mathbf{r}_{02}) \right\rangle \\ &+ \left\langle G(\mathbf{r} \mid \mathbf{r}_1') G^*(\mathbf{r} \mid \mathbf{r}_{02}) \right\rangle \\ &\cdot \left\langle G(\mathbf{r} \mid \mathbf{r}_{01}) G^*(\mathbf{r} \mid \mathbf{r}_2') \right\rangle. \end{aligned} \tag{15}$$

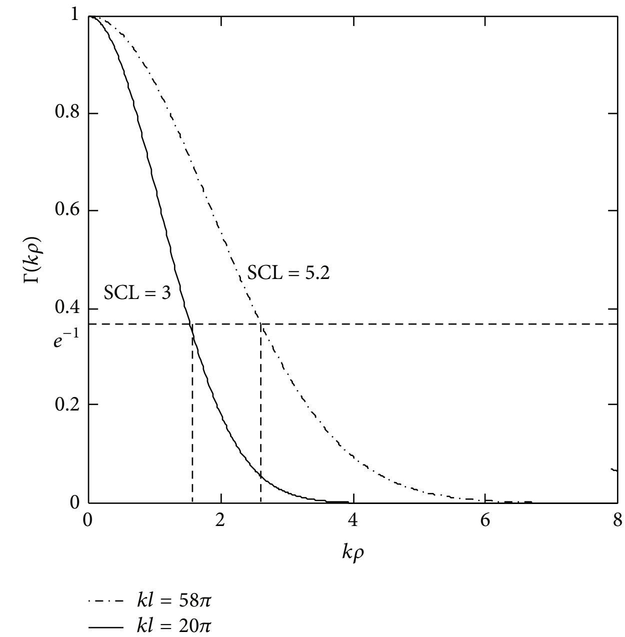

Figure 2: The degree of spatial coherence of an incident wave about the cylinder.

We can obtain LRCS σ_b using (14):

$$\sigma_b = \left\langle |u_s(\mathbf{r})|^2 \right\rangle \cdot k(4\pi z)^2. \tag{16}$$

3. Numerical Results

Although the incident wave becomes sufficiently incoherent, we should pay attention to the spatial coherence length (SCL) of incident waves around the target. In [6], the degree of spatial coherence is defined as

$$\Gamma(\rho, z) = \frac{\langle G(\mathbf{r}_1 \mid \mathbf{r}_t) G^*(\mathbf{r}_2 \mid \mathbf{r}_t)\rangle}{\left\langle |G(\mathbf{r}_0 \mid \mathbf{r}_t)|^2 \right\rangle}, \tag{17}$$

where $\mathbf{r}_1 = (\rho, 0)$, $\mathbf{r}_2 = (-\rho, 0)$, $\mathbf{r}_0 = (0, 0)$, and $\mathbf{r}_t = (0, z)$. In this study, we consider different random medium intensity $B(\mathbf{r}, \mathbf{r})$ defined in (2) and it is assumed to be constant B_0 as shown in Figure 1. The condition of the coherence attenuation index α defined in [4] holds with all values of B_0, L, and kl considered in this paper. The SCL is defined as the $2k\rho$ at which $|\Gamma| = e^{-1} \simeq 0.37$. Figure 2 shows a relation between SCL and kl in this case and SCL, accordingly, is equal to 3 and 5.2. We use both B_0 and SCL to represent the random medium effects on the LRCS. We consider a variety of values for these parameters to investigate the impact of media with different inhomogeneous fluctuations strength.

In the far field, as a result of waves propagation through random medium, beam waves may lose some of its concentration by diffraction so we assume here a limited kW in our numerical results.

3.1. Radar Cross Section RCS. Wings of airplane have various slopes with fuselages according to their shapes and models.

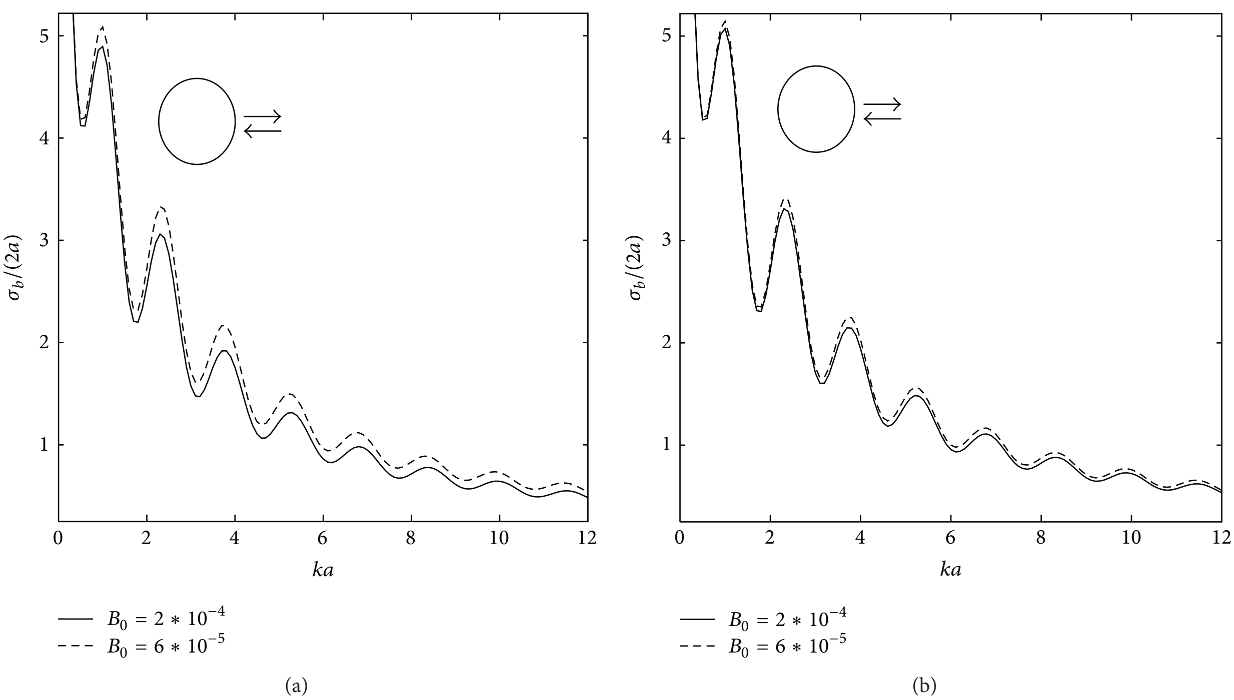

FIGURE 3: LRCS versus target size for $\delta = 0$ and $kW = 1.5$ where the target is located in random medium with (a) SCL = 3, (b) SCL = 5.2.

As a result, the intensity of scattered waves is different with the angle of waves incidence on such models and also with the random medium coherence function SCL and the medium fluctuations intensity B_0. We conduct results in Figures 3 to 6 presenting effects of these parameters on the LRCS. In other words, when waves are scattered from fuselage only, we postulate $\delta = 0$ where the cross section has a circular contour. On the other hand, waves scattering from spots in the neighborhood of the fuselage attached to the wings would assume other values for δ such as Q-170 and X-47A airplane ($\delta \simeq 0.09$).

At relatively small ka, LRCS σ_b suffers from oscillating behavior due to the contributions from the illumination region particularly in addition to the stationary and inflection points. The scattered waves are sometimes in-phase so they add up and sometimes out-of-phase so they cancel out depending on the scattered rays directions that result in such substantial oscillating behavior [8]. As ka increases, LRCS gradually lessens as a result of the lack in the surface current generated when ka gets greater than kW.

When ka is comparable with kW, LRCS behaves almost indifferently with B_0. This behavior is more obvious with wider SCL since waves are correlated enough around the target that B_0 does not alter the LRCS. In this range of ka, the target illumination region is almost covered by incident waves and, therefore, the scattered waves are relatively correlated and that is why B_0 has a less effect on the LRCS. As ka gets greater than kW, LRCS changes with B_0 particularly with shorter SCL but yet slightly with longer SCL. In this range, the target is not fully covered by incident waves so scattered waves, particularly scattering from unilluminated portion of the target surface, are randomly back to the observation point propagating through the medium and then B_0 has a larger influence on the LRCS. It should be noted that the scattering intensity, and therefore σ_b, is decreased for strong correlation function B_0 through the propagation due to the strong absorbance of the medium and this is in a complete agreement with [23].

The similarity, to some extent, of the LRCS behavior with δ is obvious regardless of the range of both ka and B_0 in Figures 3 and 4.

As shown in Figure 2, the scale size kl of medium particles relates to the SCL. With a small SCL, waves transverse suffers from relatively sharp transitions on the particles edges. Consequently, waves scatter in a wide spectrum of medium particles leading to having a higher effect of B_0 on (14) as shown in Figures 3(a) and 4(a), while with having a wider SCL or alternatively big kl in Figures 3(b) and 4(b) particles would have smoother surface and, therefore, waves transverse straightly through the particles in the forward propagation direction reducing the scattering in other directions. As a result, waves do not penetrate that much through medium particles rather than the forward direction and accordingly the correlation intensity B_0 would have a little effect on the scattering intensity defined in (14). To confirm such behavior, we reduce the SCL width in Figure 5 and compare it with Figure 3. We can notice that the effect of B_0 is increased on the LRCS in accordance.

Effect of the kW on the LRCS is considered in Figure 6. Greater kW would result in major effects of B_0 on the LRCS and vice versa. This is owing to having a more uncorrelated amount of waves in a wider range of the medium within kW.

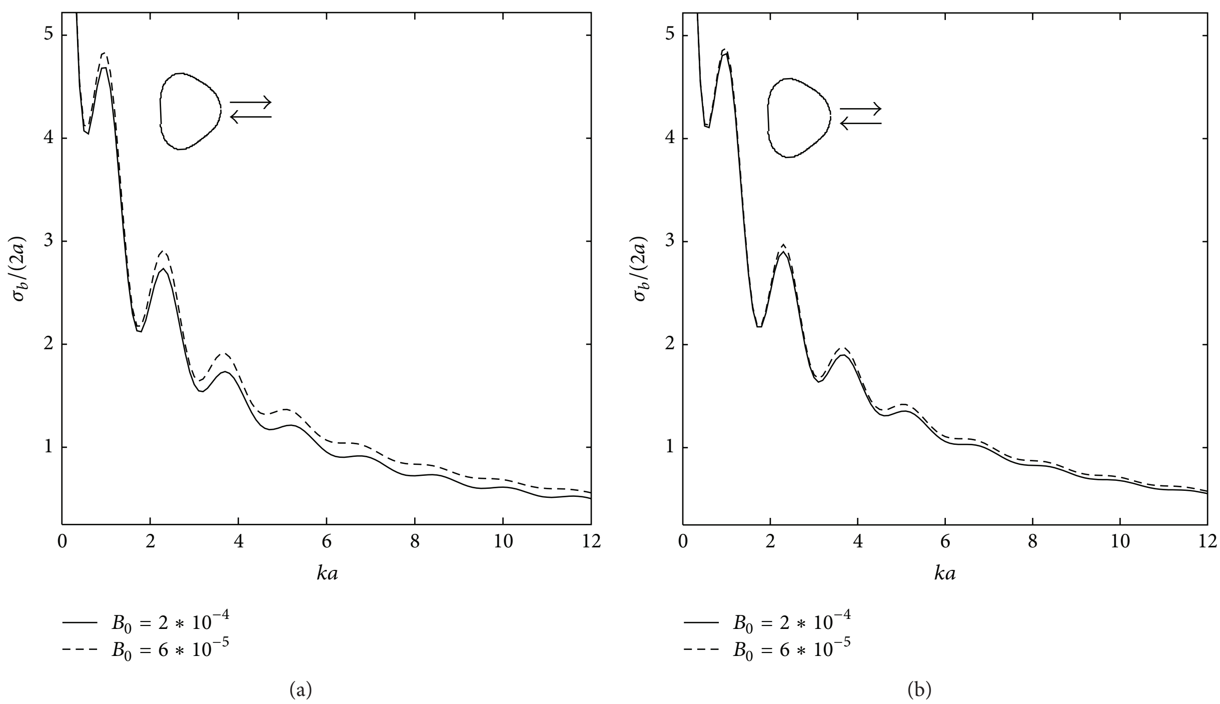

FIGURE 4: LRCS versus target size for $\delta = 0.09$ and $kW = 1.5$ where the target is located in random medium with (a) SCL = 3, (b) SCL = 5.2.

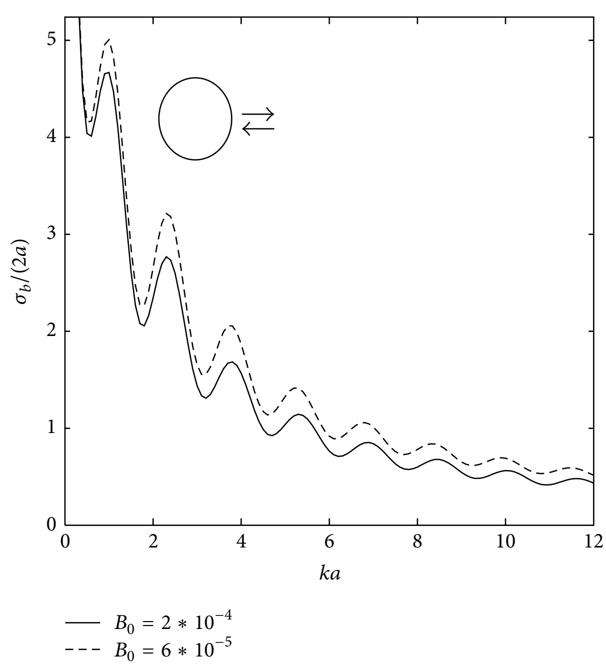

FIGURE 5: LRCS versus target size for $\delta = 0$ and $kW = 1.5$ where the target is located in random medium with SCL = 2.2.

Propagation of such uncorrelated waves would return scattering waves intensity that is in turn obviously different from the medium fluctuations density B_0. This affirms that, to have an authentic radar detection, beam width should be narrow enough so the severity of the medium heterogeneity would have a minimal impact.

4. Conclusion

In this work, the random medium parameters, object configuration parameters, and beam wave size that influences the behavior of waves scattering from conducting targets with partially convex contour such as airplane were discussed.

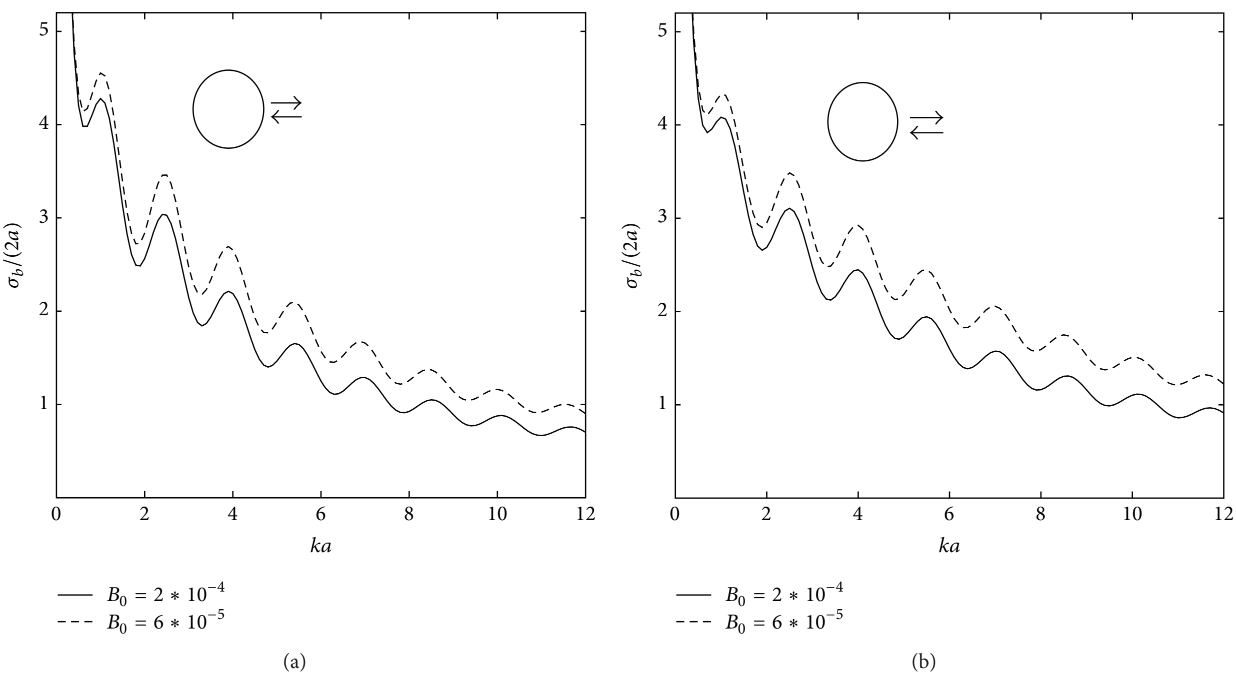

FIGURE 6: LRCS versus target size for $\delta = 0$ and SCL = 2.2 in random medium where (a) $kW = 2$, (b) $kW = 2.5$.

In doing this, laser RCS (LRCS) of partially convex targets is calculated and analyzed numerically. Medium parameters include the spatial coherence length (SCL) around the target and the fluctuations intensity B_0. LRCS does not change much with B_0 when target size is comparable with the beam width; however, B_0 has an obvious effect on the LRCS when airplane mean size is greater than the beam width particularly with shorter length of SCL. Also it should be noted that beam size around the object should be limited enough to reduce the effect of the intensity of the medium fluctuations. On the other hand, and with a wider SCL or alternatively greater scale size kl, intensity of waves scattering by medium particles lessens and the amount of waves correlation increases. Accordingly, this lets B_0 have a limited impact on the LRCS.

As a result, to have an accurate radar detection, it is recommended to have a beam width of incident waves narrow in a way to avoid the medium randomness effects. This would be quite essential mainly in the high frequency band needed in the far field propagation. Complexity of the airplane shape has a slight effect on the scattering waves propagating in media having different B_0.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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