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Research Article

Time-Domain Analytical Expression for Near Fields of Arbitrarily Oriented Electric Dipole and Its Application

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The near fields of electric dipole are commonly used in wide-band analysis of complex electromagnetic problems. In this paper, we propose new near field time-domain expressions for electric dipole. The analytical expressions for the frequency-domain of arbitrarily oriented electric dipole are given at first; next we give the time-domain expressions by time-frequency transformation. The proposed expressions are used in hybrid TDIE/DGTD method for analysis of circular antenna with radome. The accuracy of the proposed algorithm is verified by numerical examples.

1. Introduction

The electric dipole is an important unit in electromagnetism; its analytical expressions of the radiation field have been described in a number of works, for example, the expressions of special oriented electric dipole [1–3] and the expressions of arbitrarily oriented electric dipole by using the far-field approximation [4]. But the time-domain expressions of arbitrarily oriented electric dipole are necessary to analyze the time-domain near fields problems. The near field is governed by several type fields, the relationship between **E** and **H** becomes very complex, and special care must be taken when dealing with near fields problems.

The hybrid method takes advantage of several methods and is often used to analyze multiscale electromagnetic problems [5–7], like the shortwave antenna in complex environment [8]. In this study, the analytical time-domain expressions of arbitrarily oriented electric dipole are proposed and used in hybrid TDIE/DGTD method for analysis of circular antenna with radome. The numerical results verify the validity of our algorithm.

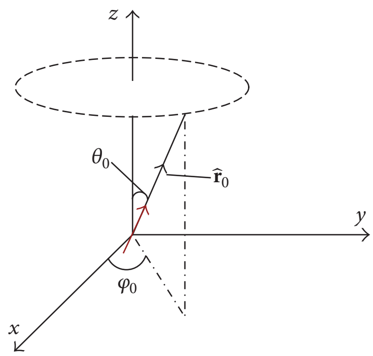
2. The Frequency-Domain Near Fields for an Arbitrarily Oriented Electric Dipole

Suppose an electric dipole which along $\hat{\mathbf{r}}_0$ is located at original point; the polar angle and the azimuthal angle of $\hat{\mathbf{r}}_0$ are θ_0 and φ_0, as Figure 1(a) shows. The observation point is located at P, and the coordinates of point P are r, θ, and φ, respectively, as Figure 1(b) shows.

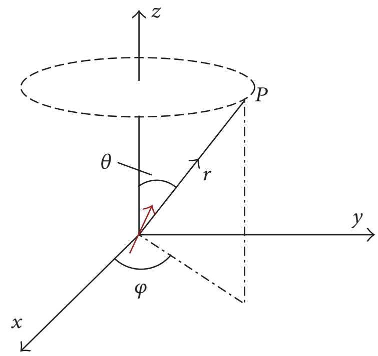
The magnetic vector potential produced by the electric current in an infinite medium can be written as

$$\mathbf{A}(r) = \mu \iiint_v \mathbf{J}(\mathbf{r}') \frac{\exp\left(ik\left|\mathbf{r}-\mathbf{r}'\right|\right)}{4\pi R} dv', \qquad (1)$$

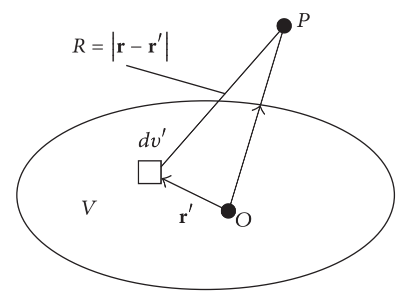
where μ, $\mathbf{r}'$, $\mathbf{r}$, R, and k are the permeability, the position vector of the integrating point, the position vector of the observation point $\mathbf{P}$, the distance from the integrating point to the observation point, and the wave number, respectively, as Figure 1(c) shows.



(a) An electric dipole at the original point



(b) The observation point (r, θ, φ)



(c) Integrating point and observation point

FIGURE 1: The arbitrarily oriented electric dipole.

A short current source $\hat{\mathbf{r}}_0 Il$ can be seen as an electric dipole; considering the characteristic of the $\delta(\mathbf{r}')$ function, we have

$$\mathbf{A}(r) = \mu \iiint_v \hat{\mathbf{r}}_0 Il\delta(\mathbf{r}') \frac{\exp(ik|\mathbf{r} - \mathbf{r}'|)}{4\pi R} dv' = \hat{\mathbf{r}}_0 Il\mu \frac{\exp(ikr)}{4\pi r}, \tag{2}$$

where $\hat{\mathbf{r}}_0$ is defined as

$$\hat{\mathbf{r}}_0 = \hat{\mathbf{x}} \sin\theta_0 \cos\varphi_0 + \hat{\mathbf{y}} \sin\theta_0 \sin\varphi_0 + \hat{\mathbf{z}} \cos\theta_0, \tag{3}$$

where the $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, and $\hat{\mathbf{z}}$ are the unit vector of the rectangular coordinate system. Magnetic field can be evaluated as

$$\begin{aligned}\mathbf{H} &= \frac{1}{\mu}\nabla \times \mathbf{A} = \frac{1}{\mu}\nabla \times \left[\hat{\mathbf{r}}_0 Il\mu \frac{\exp(ikr)}{4\pi r}\right] = \frac{1}{\mu}\nabla \\ &\times \left[(\hat{\mathbf{x}} \sin\theta_0 \cos\varphi_0 + \hat{\mathbf{y}} \sin\theta_0 \sin\varphi_0 + \hat{\mathbf{z}} \cos\theta_0) Il\mu \right. \\ &\left. \cdot \frac{\exp(ikr)}{4\pi r}\right] = \frac{Il}{4\pi}\left[\sin\theta_0 \cos\varphi_0 \nabla \times \left(\hat{\mathbf{x}}\frac{\exp(ikr)}{r}\right)\right. \\ &+ \sin\theta_0 \sin\varphi_0 \nabla \times \left(\hat{\mathbf{y}}\frac{\exp(ikr)}{r}\right) + \cos\theta_0 \nabla \\ &\left.\times \left(\hat{\mathbf{z}}\frac{\exp(ikr)}{r}\right)\right],\end{aligned} \tag{4}$$

$$\begin{aligned}\nabla \times \mathbf{A} &= \hat{\mathbf{x}}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{\mathbf{y}}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) \\ &+ \hat{\mathbf{z}}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\end{aligned} \tag{5}$$

which implies curl elements of (4) can be rewritten as

$$\begin{aligned}&\nabla \times \left(\hat{\mathbf{x}}\frac{\exp(ikr)}{r}\right) \\ &\quad = (\hat{\mathbf{y}} \cos\theta - \hat{\mathbf{z}} \sin\theta \sin\varphi)\left(\frac{ik}{r} - \frac{1}{r^2}\right)\exp(ikr), \\ &\nabla \times \left(\hat{\mathbf{y}}\frac{\exp(ikr)}{r}\right) \\ &\quad = (\hat{\mathbf{z}} \sin\theta \cos\varphi - \hat{\mathbf{x}} \cos\theta)\left(\frac{ik}{r} - \frac{1}{r^2}\right)\exp(ikr), \\ &\nabla \times \left(\hat{\mathbf{z}}\frac{\exp(ikr)}{r}\right) \\ &\quad = (\hat{\mathbf{x}} \sin\theta \sin\varphi - \hat{\mathbf{y}} \sin\theta \cos\varphi)\left(\frac{ik}{r} - \frac{1}{r^2}\right)\exp(ikr).\end{aligned} \tag{6}$$

Substituting (6) into (2), we get expression of magnetic field for the frequency-domain of an arbitrarily oriented electric dipole

$$\begin{aligned}\mathbf{H} = \frac{ikIl\exp(ikr)}{4\pi r}&[\hat{\mathbf{x}}(\cos\theta_0 \sin\theta \sin\varphi \\ &- \sin\theta_0 \sin\varphi_0 \cos\theta) + \hat{\mathbf{y}}(\sin\theta_0 \cos\varphi_0 \cos\theta \\ &- \cos\theta_0 \sin\theta \cos\varphi) + \hat{\mathbf{z}}(\sin\theta_0 \sin\varphi_0 \sin\theta \cos\varphi \\ &- \sin\theta_0 \cos\varphi_0 \sin\theta \sin\varphi)]\left(1 + \frac{i}{kr}\right).\end{aligned} \tag{7}$$

The expression of electric field is

$$\begin{aligned}\mathbf{E} &= \frac{-i}{\omega\varepsilon}\nabla \times \mathbf{H} = \frac{-i}{\omega\mu\varepsilon}\nabla \times (\nabla \times \mathbf{A}) \\ &= \frac{-i}{\omega\mu\varepsilon}\left[\nabla^2\mathbf{A} - \nabla(\nabla \cdot \mathbf{A})\right] = \frac{i}{\omega\mu\varepsilon}\left[k^2\mathbf{A} + \nabla(\nabla \cdot \mathbf{A})\right];\end{aligned} \tag{8}$$

substituting (2) into (8), we have

$$\begin{aligned}\mathbf{E} &= \frac{i}{\omega\mu\varepsilon}\left[k^2\hat{\mathbf{r}}_0 Il\mu\frac{\exp(ikr)}{4\pi r} + \nabla\left(\nabla\cdot\hat{\mathbf{r}}_0 Il\mu\frac{\exp(ikr)}{4\pi r}\right)\right] \\ &= \frac{iIl\mu}{4\pi\omega\mu\varepsilon}\left[k^2\hat{\mathbf{r}}_0\frac{\exp(ikr)}{r} + \nabla\left(\nabla\cdot\hat{\mathbf{r}}_0\frac{\exp(ikr)}{r}\right)\right],\end{aligned} \tag{9}$$

where

$$\begin{aligned}&\nabla\left(\nabla\cdot\hat{\mathbf{r}}_0\frac{\exp(ikr)}{r}\right) \\ &\quad = \sin\theta_0\cos\varphi_0\nabla\left(\nabla\cdot\hat{\mathbf{x}}\frac{\exp(ikr)}{r}\right) \\ &\quad + \sin\theta_0\sin\varphi_0\nabla\left(\nabla\cdot\hat{\mathbf{y}}\frac{\exp(ikr)}{r}\right) \\ &\quad + \cos\theta_0\nabla\left(\nabla\cdot\hat{\mathbf{z}}\frac{\exp(ikr)}{r}\right).\end{aligned} \tag{10}$$

3. The Time-Domain Near Fields for an Arbitrarily Oriented Electric Dipole

An electric dipole consists of a positive charge $+q$ and an negative charge $-q$, distance l apart. The dipole moment $\mathbf{p} = q\mathbf{l}$, so that electric current can be represented as

$$I\mathbf{l} = -i\omega\mathbf{p}. \tag{11}$$

Substituting (11) into (7),

$$\begin{aligned}\mathbf{H}(\mathbf{r},\omega) &= \frac{\omega^2 p\exp(ikr)}{4\pi cr}\left(1+\frac{ic}{\omega r}\right) \\ &\cdot[\hat{\mathbf{x}}(\cos\theta_0\sin\theta\sin\varphi - \sin\theta_0\sin\varphi_0\cos\theta) \\ &+ \hat{\mathbf{y}}(\sin\theta_0\cos\varphi_0\cos\theta - \cos\theta_0\sin\theta\cos\varphi) \\ &+ \hat{\mathbf{z}}(\sin\theta_0\sin\varphi_0\sin\theta\cos\varphi \\ &- \sin\theta_0\cos\varphi_0\sin\theta\sin\varphi)].\end{aligned} \tag{12}$$

The time-frequency conversion relation and the flourier transformation are

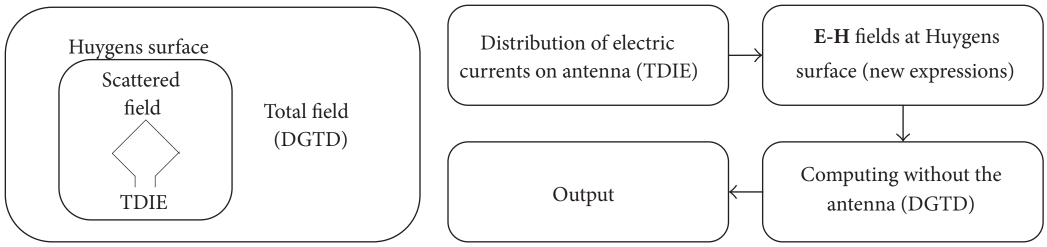
$$\begin{aligned}&-i\omega \longrightarrow \frac{\partial}{\partial t}, \\ &p(t) = \frac{1}{2\pi}\int p(\omega)\exp(-i\omega t)\,d\omega, \\ &p\left(t-\frac{r}{c}\right) = \frac{1}{2\pi}\int p(\omega)\exp(ikr)\exp(-i\omega t)\,d\omega.\end{aligned} \tag{13}$$

The time-domain expression of (12) can be obtained by (13)

$$\begin{aligned}\mathbf{H}(\mathbf{r},t) &= -\frac{1}{4\pi cr}\left(\frac{\partial^2}{\partial t^2}+\frac{c}{r}\frac{\partial}{\partial t}\right)p\left(t-\frac{r}{c}\right) \\ &\cdot[\hat{\mathbf{x}}(\cos\theta_0\sin\theta\sin\varphi - \sin\theta_0\sin\varphi_0\cos\theta) \\ &+ \hat{\mathbf{y}}(\sin\theta_0\cos\varphi_0\cos\theta - \cos\theta_0\sin\theta\cos\varphi) \\ &+ \hat{\mathbf{z}}(\sin\theta_0\sin\varphi_0\sin\theta\cos\varphi \\ &- \sin\theta_0\cos\varphi_0\sin\theta\sin\varphi)].\end{aligned} \tag{14}$$

The time-domain expression of electric field can be obtained in the same way

$$\begin{aligned}E_x &= \left\{\sin\theta_0\cos\varphi_0\left[A_1\cdot\sin^2\theta\cos^2\varphi - A_2\right.\right. \\ &\left.\cdot\left(\cos^2\theta\cos^2\varphi + \sin\theta\sin^2\varphi\right) - \frac{\partial^2}{\partial t^2}\right] + \sin\theta_0\sin\varphi_0 \\ &\cdot\left[A_1\cdot\sin^2\theta\sin\varphi\cos\varphi - A_2\right. \\ &\left.\cdot\sin\varphi\cos\varphi\left(\cos^2\theta - \sin\theta\right)\right] + \cos\theta_0\left[A_1\right. \\ &\left.\left.\cdot\sin\theta\cos\theta\cos\varphi + A_2\cdot\sin\theta\cos\theta\cos\varphi\right]\right\}\frac{\mu}{4\pi r} \\ &\cdot p\left(t-\frac{r}{c}\right), \\ E_y &= \left\{\sin\theta_0\cos\varphi_0\left[A_1\cdot\sin^2\theta\cos\varphi\sin\varphi - A_2\right.\right. \\ &\left.\cdot\sin\varphi\cos\varphi\left(\cos^2\theta - \sin\theta\right)\right] + \sin\theta_0\sin\varphi_0\left[A_1\right. \\ &\cdot\sin^2\theta\sin^2\varphi - A_2\cdot\left(\cos^2\theta\sin^2\varphi + \sin\theta\cos^2\varphi\right) \\ &\left.-\frac{\partial^2}{\partial t^2}\right] + \cos\theta_0\left[A_1\cdot\sin\theta\cos\theta\sin\varphi + A_2\right. \\ &\left.\left.\cdot\sin\theta\cos\theta\sin\varphi\right]\right\}\frac{\mu}{4\pi r}p\left(t-\frac{r}{c}\right), \\ E_z &= \left\{\sin\theta_0\cos\varphi_0\left[A_1\cdot\sin\theta\cos\theta\cos\varphi + A_2\right.\right. \\ &\left.\cdot\sin\theta\cos\theta\cos\varphi\right] + \sin\theta_0\sin\varphi_0\left[A_1\right. \\ &\left.\cdot\sin\theta\cos\theta\sin\varphi + A_2\cdot\sin\theta\cos\theta\sin\varphi\right] + \cos\theta_0 \\ &\left.\cdot\left[A_1\cdot\cos^2\theta - A_2\cdot\sin^2\theta - \frac{\partial^2}{\partial t^2}\right]\right\}\frac{\mu}{4\pi r}p\left(t-\frac{r}{c}\right), \\ A_1 &= \left(\frac{2c^2}{r^2}+\frac{2c}{r}\frac{\partial}{\partial t}+\frac{\partial^2}{\partial t^2}\right)\quad A_2 = \left(c\frac{\partial}{\partial t}+\frac{c^2}{r}\right).\end{aligned} \tag{15}$$



(a) Hybrid method setup (b) Computing procedures

Figure 2: The hybrid TDIE/DGTD method.

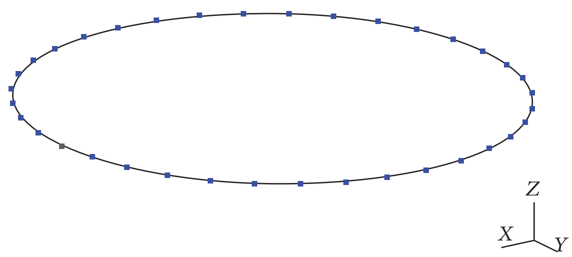


Figure 3: Circular antenna (divided into 35 straight lines).

4. The Hybrid Method Combining DGTD and TDIE for Wire Antenna-Dielectric Interaction

TDIE is widely used for analyzing thin-wire antenna (radiation on scattering) problems, but it is difficult to deal with interaction of the antenna and complexity media. The TDIE/FDTD method and the FDTD/FETD/TDIE method have been proposed for complex electromagnetic problems [9, 10]. Discontinuous Galerkin Time-Domain (DGTD) method [11–15] inherits from Finite Element Time-Domain (FETD) the advantage of unstructured grids without solving large linear systems. DGTD is more flexible than Finite Difference Time-Domain (FDTD) method in geometry modeling, but like FDTD, it also becomes resource-consuming dealing with thin-wire structures.

Gao et al. proposed the TDIE/DGTD method [16]; in our study, the currents on the antenna can be easily obtained by TDIE. These currents are used to calculate **E**-**H** fields on Huygens surface which yields the same radiating fields. The **E**-**H** fields are deduced from our new expressions, as shown in Figure 2.

4.1. A Circular Antenna. The antenna is parallel to the *XOY* plane, the radius of the antenna is 0.5 meter, the excitation wavelength is 0.5 meter, and the time harmonic current is excited at every line element (Figure 3). The normalized results of TDIE/DGTD with our new expressions are shown in Figures 4(a) and 4(b); a good agreement is observed between our algorithm (circle) and analytical solution (solid line) [17].

4.2. A Right Angle Antenna with Radome. We analyzed a right angle antenna located in the center of a dielectric ellipsoid-shell radome (semiprincipal axes $a = 1$ meter, $b = 1$ meter, and $c = 0.5$ meters); the thickness of the radome is 0.25 meters, the relative electric permittivity is 3.0, and the relative permeability is 1.0. The antenna is parallel to the plane *YOZ* (Figure 5); each arm of the antenna is 0.2 meters long. Antenna is excited by time harmonic current whose wavelength is 0.5 meters. The results of TDIE/DGTD and DGTD are shown for comparison in Figure 6; a good agreement is observed between two results.

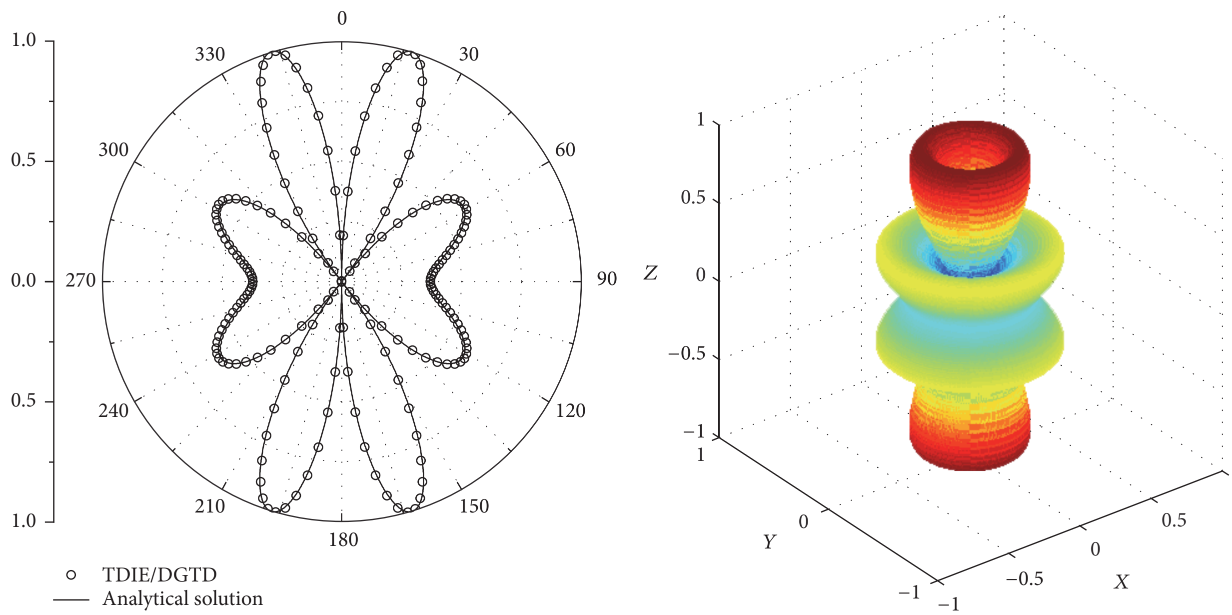
4.3. A Circular Antenna with Radome. In this example, the antenna in the example of Section 4.2 is replaced with the circular antenna in the example of Section 4.1; excitation parameters of antenna are the same as the example of Section 4.2 (Figure 7).

Figure 8(a) is the normalized radiation pattern of circular antenna with radome. It is obvious that the main lobe of the pattern with radome is more intensive than without radome, as Figure 8(a) shows.

Next, the antenna is excited by a voltage pulse source, as shown in Figure 9, the distribution of currents on antenna is derived from TDIE. The voltage source is defined as

$$V = \exp\left(\frac{t - t_0}{\tau}\right)^2 \quad (\tau = 2\,\text{ns},\ t_0 = 8\,\text{ns}). \tag{16}$$

Follow the steps in Figure 2(b); the snapshots in Figure 11 of the absolute value of electric field $|\mathbf{E}|$ are obtained from the hybrid TDIE/DGTD method. As shown in Figure 11(a), the electric field density is higher in the half-space $x > 0$, because the voltage source is located at the cross point of antenna and $+x$ axis. The reflection radiation caused by radome can be seen in Figures 11(b), 11(c), and 11(d). The current waveform of source point is shown in Figure 10; the current tends to a value that is not zero.



(a) The radiation pattern of *XOZ* plane

(b) Three-dimensional radiation pattern

Figure 4: The radiation pattern of a circular antenna.

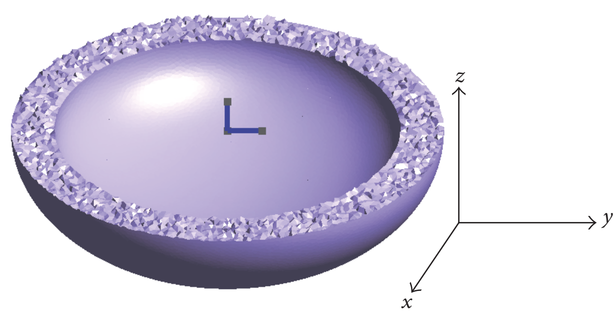


Figure 5: Sectional view grid.

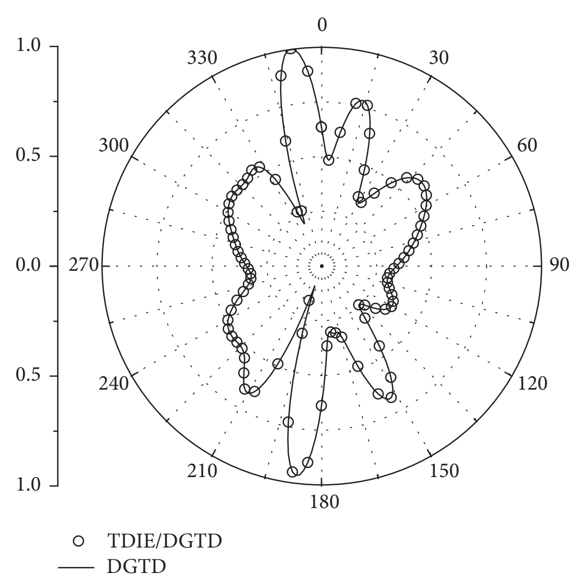


Figure 6: The radiation pattern of *YOZ* plane.

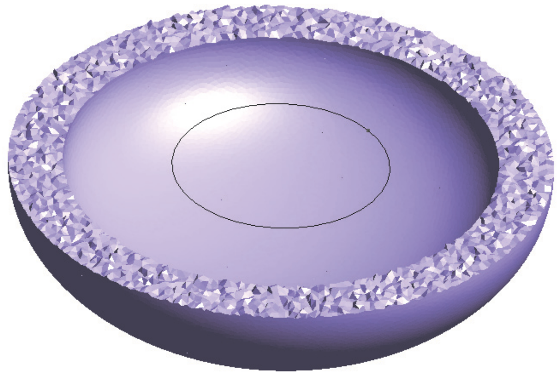
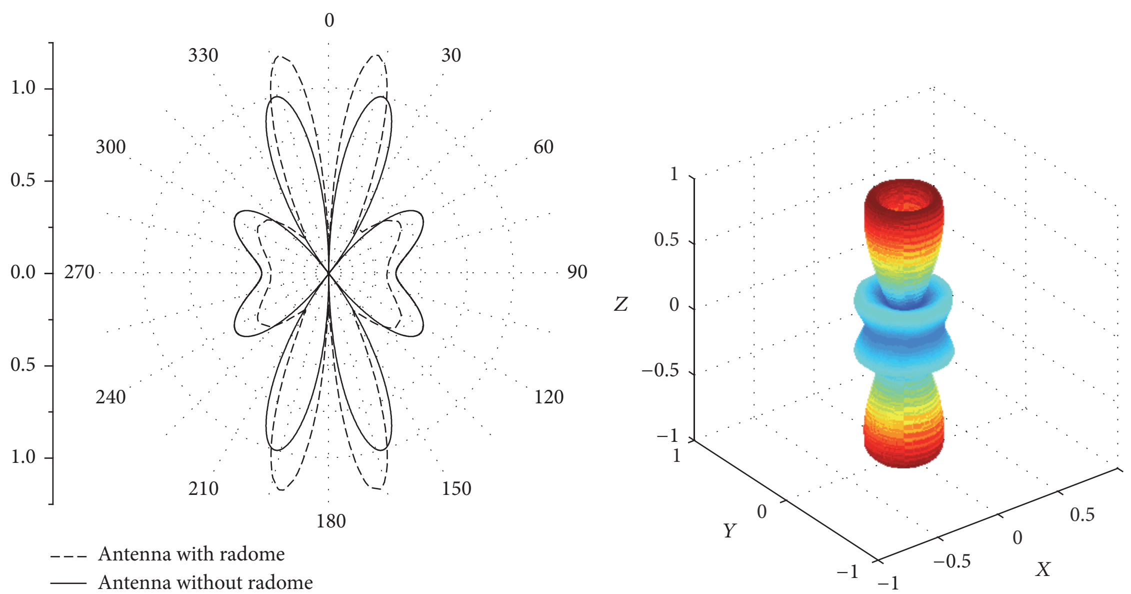


Figure 7: Sectional view grid.



(a) The radiation pattern of *XOZ* plane

(b) Three-dimensional radiation pattern

Figure 8: The radiation pattern of circular antenna with radome.

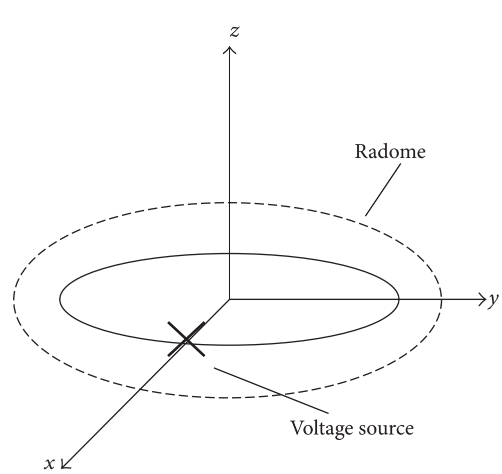


Figure 9: Voltage source.

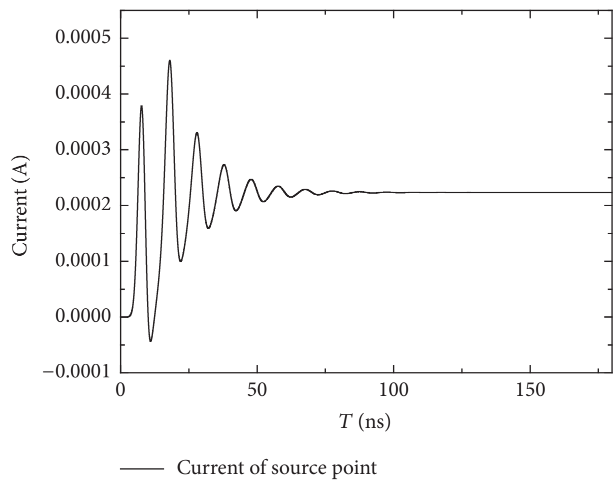


Figure 10: Current of source point.

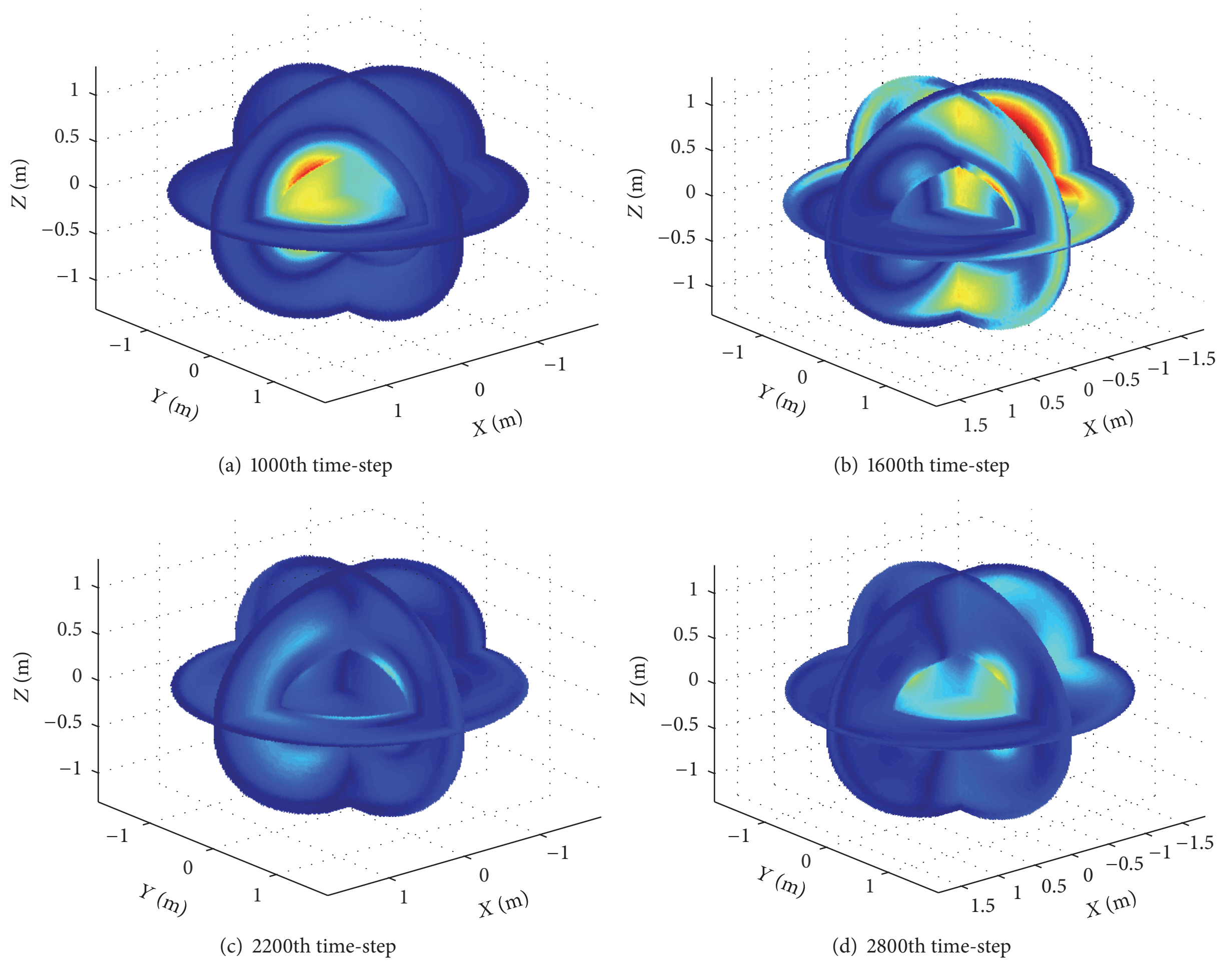


Figure 11: Snapshots of $|\mathbf{E}|$.

5. Conclusion

The analytical expressions of time-domain near fields of an arbitrarily oriented electric dipole are derived in this paper; then the expressions are used in hybrid TDIE/DGTD method for analysis of circular antenna with radome. Our study provides a new way to study the time-domain radiation problems of complex medium and structure.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

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