

Research Article

Joint High-Resolution Range and DOA Estimation via MUSIC Method Based on Virtual Two-Dimensional Spatial Smoothing for OFDM Radar

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For the purpose of target parameter estimation of the orthogonal frequency-division multiplexing (OFDM) radar, a high-resolution method of joint estimation on range and direction of arrival (DOA) based on OFDM array radar is proposed in this paper. It begins with the design and analysis of an echo model of OFDM array radar. Since there is no coupling between range and angle parameter estimation for a narrow-band signal, a method which exploits the data of one snapshot to estimate the range and angle of the target by means of multiple signal classification (MUSIC) based on virtual two-dimensional spatial smoothing in range and angle dimensions is devised. The proposed method is capable of joint estimating the range and DOA of the target in a high resolution under a single snapshot circumstance. Simulation experiments demonstrate the validity of the proposal.

1. Introduction

Recently, with the development of hardware technique for complicated waveform generator, it is possible to configure the radar waveform flexibly [1]. Multicarrier signal has been widely applied to radar [2, 3], because it can provide more information about targets. Orthogonal frequency-division multiplexing (OFDM) radar modulates traditional single-carrier transmitting signal to subcarrier signals one by one, and then, it transmits the superposed signal, which greatly improves the performance of radar system and attracts thousands of scholars' wide attention.

OFDM radar has the following advantages [4]: flexible waveform design, using orthogonal subcarrier signal as sub-channel which improves the utilization ratio of the spectrum, enhancing the ability of resisting the pulse noise and channel fading, with large time-bandwidth product, and easy to be processed digitally. Reference [5] firstly considers the subcarrier phase modulation into multicarrier structure of OFDM radar; the spectrum utilization ratio can be improved. In

[6], the concept of OFDM applied to the radar system is discussed in detail. Meanwhile, the framework of the OFDM radar system is proposed and the simulation analysis of each module is carried out. It has been proved that OFDM radar has strong abilities of antinoise and antijamming in [6]. The theory of Doppler signal processing for the OFDM radar signal is proposed in [7]. However, Doppler processing on receiver based on correlation function usually encounters the troublesome Doppler ambiguity. Tigrek et al. [8] and Garmatyuk et al. [9] make a feasible study about the influence of OFDM signal on radar imaging and establishment of the communication system network, which lays the foundation of the integration of radar and communication. Using OFDM radar signal to eliminate Doppler ambiguity is presented in [10], and the echoed signal model is established in vector expression. However, it does not take angle information of the target into consideration. Reference [11] utilizes the property that different carrier waveforms of OFDM radar have different responses to the target signal to improve the frequency diversity and solve the problem of

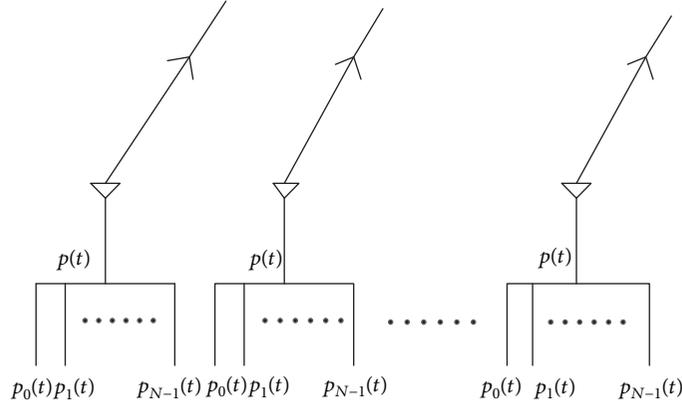


FIGURE 1: Allocation of transmitting antennas.

multipath reflection. Reference [12] proposes an algorithm of joint estimation on direction of departure (DOD) and direction of angle (DOA) based on multiple-input multiple-output (MIMO) radar. The complexity of the algorithm is decreased by the dimension reduction, and it is suitable to estimate DOD and DOA under the condition of a single snapshot and multiple targets. A DOA estimation method based on subspace is put forward in [13], providing a fundamental for time-varying DOA estimation. References [14, 15] transform the DOA model based on sparse representation of covariance matrix into sparse representation model of a single vector, and the effect of imprecise estimation on noise power is mitigated by the dimension reduction. In [16], a method which determines the range and angle of target is presented, by sending double pulses based on traditional frequency diverse array radar.

In this paper, aiming at target parameter estimation in the OFDM array system, the characteristic of received signal is analyzed and a high resolution method of joint estimation on the range and DOA of the target in OFDM array radar is proposed. Under the narrow-band signal condition, since there is no coupling between the range and angle of the target in the echoed signal of the OFDM radar, the target information is included in the range and angle steering vector, respectively, which can be utilized to jointly estimate the range and angle of signal source. In single snapshot case, a joint high resolution estimation method for the range and the angle of signal source can be obtained by means of the multiple signal classification (MUSIC) algorithm and virtual two-dimensional spatial smoothing technique. In order to evaluate the performance of the proposed method in this paper, the root-mean-square errors (RMSE) of the estimates of the range and angle parameters are examined. Numerical simulation experiments are designed to prove the effectiveness of the proposed method.

2. Echoed Signal Model of OFDM Radar

Taking linear array as an example, it is assumed that the number of array elements is M , the distance between two elements is d , the propagation speed of electromagnetic wave is c , and signal bandwidth is B . Considering phased array radar and MIMO (multiple-input and multiple-

output) radar system, there are P far-field narrow-band signals in space. If the longest time which signal spends on traversing array aperture is much shorter than reciprocal of signal bandwidth, $((M-1)d)/c \ll 1/B$; the signal can be called "narrow-band signal."

Under narrow-band signal condition, the time delay between different array elements can be expressed as a formula about phase difference of signal. The response of different array elements to the same signal is to have different phase shifts.

Consider a uniform linear array (ULA) including M elements, where each of them transmits OFDM waveform. There are N subcarrier channels in the OFDM waveform, and the transmitting signal can be expressed as

$$p(t) = \sum_{n=0}^{N-1} A_n \exp \{j2\pi(f_c + n\Delta f)t\} \exp \{j\varphi_n\}, \quad (1)$$

where A_n is amplitude, φ_n is the initial phase of the n th subcarrier, Δf denotes the frequency interval of two adjacent subcarriers, and f_c is the central frequency. Furthermore, in order to ensure the orthogonality between subcarriers, Δf needs to meet with $T = 1/\Delta f$, where T is the radar pulse duration. Figure 1 shows the allocation of transmitting antennas and transmitting signals on each element. All the antennas transmit the same signal at the same time. And the signal of each subcarrier channel can be expressed as

$$p_n(t) = A_n \exp \{j2\pi(f_c + n\Delta f)t\} \exp \{j\varphi_n\}, \quad n = 0 \cdots N-1. \quad (2)$$

Under narrow-band circumstance and for arbitrary target signal in space, the downconverted echoes of the OFDM array radar can be written as

$$\mathbf{s}(t) = \left\{ \sum_{n=0}^{N-1} \exp \left\{ j2\pi n\Delta f \left(t \left(1 - \frac{2v}{c} \right) - \frac{2R}{c} \right) - j2\pi f_c \frac{2R}{c} - j2\pi f_c \frac{2v}{c} \right\} \exp \{j\varphi_n\} \right\} \mathbf{a}^T(\theta), \quad (3)$$

where R is the target distance, v is the target radial velocity, c is the speed of light, d is the element interval of ULA, θ denotes DOA of the single point target, and M stands for the number of array elements. Note that t is sampled discretely, i.e., $t = nT/N = n/N\Delta f$, where n denotes the sampling index. And $\mathbf{a}(\theta)$ is steering vector,

$$\mathbf{a}(\theta) = \left[1, \exp\left(\frac{j2\pi d \sin \theta}{\lambda}\right), \dots, \exp\left(\frac{j2\pi \sin \theta(M-1)}{\lambda}\right) \right]^T, \quad (4)$$

where λ is wave length.

The signal model in the discretization form can be written as

$$\mathbf{s} = \psi \mathbf{\Gamma} \mathbf{\beta} \mathbf{A} \mathbf{\phi} \mathbf{a}^T(\theta), \quad (5)$$

where \mathbf{s} denotes a $N \times M$ data matrix of discreted echoed signal, and

$$\psi = \exp\left\{-j2\pi f_c \frac{2R}{c}\right\}, \quad (6)$$

$$\mathbf{\Gamma} = \text{diag}\{1, \gamma, \gamma^2, \dots, \gamma^{N-1}\}, \quad (7)$$

$$\gamma = \exp\left\{-j2\pi f_c \frac{2v}{c} \frac{1}{N\Delta f}\right\}, \quad (8)$$

$$\mathbf{\beta} = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & \beta & \beta^2 & & \beta^{N-1} \\ 1 & \beta^2 & \beta^4 & & \beta^{2(N-1)} \\ \dots & & & \ddots & \vdots \\ 1 & \beta^{N-1} & \beta^{2(N-1)} & \dots & \beta^{(N-1)^2} \end{bmatrix}, \quad (9)$$

$$\beta = \exp\left\{j2\pi\left(1 - \frac{2v}{c}\right)\frac{1}{N}\right\}, \quad (10)$$

$$\mathbf{A} = \text{diag}\{1, \alpha, \alpha^2, \dots, \alpha^{N-1}\}, \quad (11)$$

$$\alpha = \exp\left\{-j2\pi\Delta f \frac{2R}{c}\right\}, \quad (12)$$

$$\mathbf{\phi}^T = [\exp\{j\phi_0\}, \exp\{j\phi_1\}, \dots, \exp\{j\phi_{N-1}\}], \quad (13)$$

$$\mathbf{a}(\theta) = \left[1, \exp\left(\frac{j2\pi d \sin \theta}{\lambda}\right), \dots, \exp\left(\frac{j2\pi \sin \theta(M-1)}{\lambda}\right) \right]^T. \quad (14)$$

Assuming $2v/c \ll 1/N$, and $\mathbf{\beta}$ is with the same dimension as the inverse discrete Fourier transform (IDFT) matrix, where $(\cdot)^T$ stands for the matrix transpose. The useful information about range and angle is contained in $\mathbf{a}(\theta)$, $\mathbf{\Gamma}$, and \mathbf{A} . In order to obtain information about range and angle, the signal phase $\mathbf{\phi}$ needs to be compensated. Firstly, $\mathbf{\beta}$ should be compensated by Fourier transform matrix \mathbf{F} ; then, next step is phase compensation [10].

The signal phase $\mathbf{\phi}$ can be compensated by \mathbf{P} , where $\mathbf{P} = \text{diag}\{\exp\{-j\phi_0\}, \exp\{-j\phi_1\}, \dots, \exp\{-j\phi_{N-1}\}\}$ gives the diagonal matrix. After the compensation, the expression of the echoed signal can be written according to (3) as

$$\mathbf{P}\mathbf{F}\mathbf{s} = \psi \mathbf{P}\mathbf{F}\mathbf{\Gamma}\mathbf{\beta}\mathbf{A}\mathbf{\phi}\mathbf{a}^T(\theta). \quad (15)$$

When the product of matrix \mathbf{F} and $\mathbf{\Gamma}$ meets (15), it will cause cyclic shift in each row of \mathbf{F} . In order to compensate for the cyclic shift, Doppler compensation should be completed before the phase compensation.

$$\frac{2v}{c}f_c = \varepsilon\Delta f, \quad (16)$$

where ε is an integer. Each column of the echoed signal in (5) is sequentially processed with Doppler compensation, $\mathbf{\beta}$ compensation, and phase compensation. Then, each column of the data that has been compensated is rearranged as a $MN \times N$ data matrix. Therefore, the space-domain steering vector $\mathbf{a}(\theta, R)$ of OFDM radar echo signal includes direction steering vector $\mathbf{a}(\theta)$ in (14) and distance steering vector $\mathbf{a}(R)$, that is,

$$\mathbf{a}(\theta, R) = \mathbf{a}(\theta) \otimes \mathbf{a}(R), \quad (17)$$

where \otimes denotes Kronecker product and $\mathbf{a}(R)$ can be expressed as

$$\mathbf{a}(R) = \left[1, \exp\left(-j2\pi\frac{2R}{c}\Delta f\right), \dots, \exp\left(-j2\pi\frac{2R}{c}(N-1)\Delta f\right) \right]^T. \quad (18)$$

From (17), it can be noted that there is no coupling between the distance and angle parameter estimation for OFDM array radar echo signal under the condition of narrow-band signal, and therefore, the joint estimation on the range and DOA can be carried out.

3. A Method of Joint Estimation on Range and Angle in High Resolution by Virtual Two-Dimensional Spatial Smoothing

It is well known that the Fourier-based beamforming method applies the Fourier transform to the echoed signal to estimate DOA. However, because Fourier transform is limited by Rayleigh limitation, the performance of the beamforming is with high side lobes and limited resolution. To this end, the high-resolution method based on the second-order statistical properties can overcome this limitation. The MUSIC (multiple signal classification) can make an asymptotic and unbiased angle estimation of the target in theory. Under a certain SNR (signal-noise ratio) threshold, the performance of MUSIC is closed to ML (maximum likelihood) estimation. It utilizes the orthogonality between signal subspace and noise subspace to construct spatial spectral function and searches array manifold vector which is orthogonal to noise subspace for the DOA estimation.

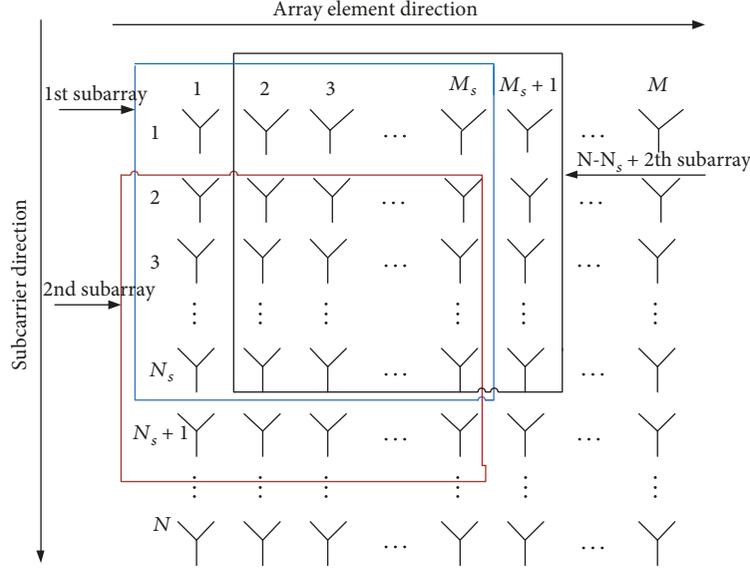


FIGURE 2: Virtual two-dimensional spatial smoothing method.

Under the condition of narrow-band signal, the distance and angle parameter estimation of target echoed signal in OFDM array radar have no coupling relationship. Therefore, a high-resolution estimation of range and angle can be realized by exploiting the data obtained from a single snapshot. Since the traditional MUSIC algorithm needs many snapshots to estimate covariance matrix, a novel method that uses a virtual two-dimensional spatial smoothing method to sample a single snapshot data is proposed in this paper. It is capable of estimating the covariance matrix by using sampled data which has been smoothed in two-dimensional space.

3.1. The Sampling Method of Virtual Two-Dimensional Spatial Smoothing. Consider the compensated data of one single snapshot. After match filtering and signal separation processing, the data is rearranged into a $N \times M$ matrix, and then smooth the matrix spatially by column, and finally divide it into $(N - N_s + 1) \times (M - M_s + 1)$ subarrays, and the dimension of a subarray is $N_s \times M_s$, where N_s denotes the row number of subarrays and M_s is the column number of subarrays. Then, rearrange all the subarrays into a $N_s M_s \times (N - N_s + 1)(M - M_s + 1)$ matrix. Figure 2 shows the virtual two-dimensional spatial smoothing method.

The first subarray is

$$\mathbf{X}_1(t) = \mathbf{a}_{N_s \times M_s}(\theta, R) \mathbf{S}(t) + \mathbf{N}_1(t), \quad (19)$$

and the steering vector is $\mathbf{a}_{N_s \times M_s}(\theta, R) = \mathbf{a}_{M_s}(\theta) \otimes \mathbf{a}_{N_s}(R)$.

The second subarray is

$$\mathbf{X}_2(t) = \mathbf{a}_{N_s \times M_s}(\theta, R) D \mathbf{S}(t) + \mathbf{N}_2(t), \quad (20)$$

where $D = \exp(j2\pi d \sin \theta / \lambda)$ is the rotation factor.

The $N - N_s + 1$ th subarray is

$$\mathbf{X}_{N-N_s+1}(t) = \mathbf{a}_{N_s \times M_s}(\theta, R) D^{N-N_s} \mathbf{S}(t) + \mathbf{N}_{N-N_s+1}. \quad (21)$$

The $(N - N_s + 1) \times 1 + 1$ th subarray

$$\mathbf{X}_{N-N_s+2}(t) = \mathbf{a}_{N_s \times M_s}(\theta, R) L \mathbf{S}(t) + \mathbf{N}_{N-N_s+2}(t), \quad (22)$$

where $L = \exp(-j2\pi(2R/c)\Delta f)$ is the rotation factor.

The $(N - N_s + 1) \times (m - 1) + n$ th subarray is

$$\mathbf{X}_{(N-N_s+1) \times (m-1) + n}(t) = \mathbf{a}_{N_s \times M_s}(\theta, R) D^{n-1} L^{m-1} \mathbf{S}(t) + \mathbf{N}_{(N-N_s+1) \times (m-1) + n}(t), \quad (23)$$

where $1 \leq m \leq M - M_s + 1$, $1 \leq n \leq N - N_s + 1$, and $D^{n-1} L^{m-1}$ is the combined rotational factor,

$$D^{n-1} L^{m-1} \mathbf{S}(t) = \mathbf{S}(t) \exp \left(j \left(\frac{2\pi \sin \theta (n-1)}{\lambda - 2\pi(2R/c)(m-1)\Delta f} \right) \right) = \mathbf{S}'(t). \quad (24)$$

Therefore, it is equivalent to translate each subarray spatially through above steps. The rest rotation factors can be combined into signal envelope $\mathbf{S}(t)$. The data of each subarray can be regarded as sampling data $\mathbf{X}(t)$ which has been sampled $(N - N_s + 1) \times (M - M_s + 1)$ times, that is,

$$\begin{aligned} \mathbf{X}(t) &= \left[\mathbf{X}_1(t), \mathbf{X}_2(t), \mathbf{X}_{N-N_s+1}(t), \dots, \right. \\ &\quad \left. \mathbf{X}_{(N-N_s+1) \times (m-1) + n}(t), \dots, \right. \\ &\quad \left. \mathbf{X}_{(N-N_s+1) \times (M-M_s+1)}(t) \right] \\ &= \mathbf{a}_{M_s}(\theta) \otimes \mathbf{a}_{N_s}(R) \mathbf{S}'(t) + \mathbf{N}(t) \\ &= \mathbf{a}_{N_s \times M_s}(\theta, R) \mathbf{S}'(t) + \mathbf{N}(t). \end{aligned} \quad (25)$$

It is reasonable to estimate covariance matrix by $\mathbf{X}(t)$ which has been two-dimensional spatial smoothed, and it is suitable to realize the joint estimation of range and angle in high resolution.

3.2. Procedures of Estimating Range and Angle by a MUSIC Algorithm. In this section, we can estimate the range and angle of the target by applying the MUSIC algorithm into $\mathbf{X}(t)$, which has been smoothed in two-dimensional space in Section 3.1. The detailed processing procedures of the proposed method are given as follows:

- (1) The covariance matrix $\hat{\mathbf{R}}$ can be estimated from $\mathbf{X}(t)$, and

$$\hat{\mathbf{R}} = \frac{1}{(N - N_s + 1) \times (M - M_s + 1)} \mathbf{X}(t) \mathbf{X}(t)^H. \quad (26)$$

- (2) Perform eigenvalue decomposition to covariance matrix $\hat{\mathbf{R}}$
- (3) Sort eigenvalues and determine the number of signal sources P , then extract eigenvectors corresponding to $N_s \times M_s - P$ small eigenvalues to form the noise subspace $\mathbf{N}_{N_s \times M_s}^{N_s \times M_s - P}$
- (4) Project two-dimensional search vector $\mathbf{a}_{N_s \times M_s}(\theta, R)$ onto noise subspace $\mathbf{N}_{N_s \times M_s}^{N_s \times M_s - P}$, and calculate spectral peak

$$S(\theta, R) = \frac{1}{\sum_{i=P+1}^{N_s \times M_s} |\mathbf{a}_{N_s \times M_s}^H(\theta, R) \mathbf{v}_i|^2}. \quad (27)$$

- (5) The position (θ, R) of the spectral peak is angle and range of the target

3.3. Performance Analysis of Joint Estimation on Range and Angle in High Resolution. In order to test the performance of joint estimation on range and angle in high resolution, this section proposes Cramer-Rao bound (CRB) of parameter estimation on range and angle in OFDM radar.

Consider that estimating distance and angle parameters of a single point source in the condition of Gaussian white noise. Assume that the value of signal-to-noise ratio is SNR, and Fisher information matrix corresponding to angle θ and distance R can be written as

$$\mathbf{J} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}, \quad (28)$$

TABLE 1: Simulation parameters of joint estimation on range and angle in high resolution.

Parameter	Value
Center frequency (f_c)	1 GHz
Channel number (M)	10
Sub-carrier number (N)	10
Array element distance (d)	$\lambda/2$
Frequency interval (Δf)	100 kHz
Monte Carlo times (K)	100
Pulse duration time (T_g)	$1/\Delta f$

then

$$J_{11} = 2 \operatorname{Re} \left[\left(\frac{\partial \mathbf{a}_{N_s \times M_s}(\theta, R)}{\partial \theta} \right)^H \left(\frac{\partial \mathbf{a}_{N_s \times M_s}(\theta, R)}{\partial \theta} \right) \right] = 2 \operatorname{Re} [\dot{\mathbf{a}}_\theta^H \dot{\mathbf{a}}_\theta], \quad (29)$$

where $\dot{\mathbf{a}}_\theta = (\partial \mathbf{a}_{M_s}(\theta)) / \partial \theta \otimes \mathbf{a}_{N_s}(R)$ and $\dot{\mathbf{a}}_R = \mathbf{a}_{M_s}(\theta) \otimes (\partial \mathbf{a}_{N_s}(R)) / \partial R$. Similarly, there are

$$\begin{aligned} J_{12} &= 2 \operatorname{Re} [\dot{\mathbf{a}}_\theta^H \dot{\mathbf{a}}_R], \\ J_{21} &= 2 \operatorname{Re} [\dot{\mathbf{a}}_R^H \dot{\mathbf{a}}_\theta], \\ J_{22} &= 2 \operatorname{Re} [\dot{\mathbf{a}}_R^H \dot{\mathbf{a}}_R], \end{aligned} \quad (30)$$

then $\operatorname{CRB} = (\operatorname{SNR} \times \mathbf{J})^{-1}$.

4. Results and Discussion

The basic parameters of OFDM array radar and target parameters information used in the numerical simulation are shown in Table 1.

4.1. Performance Comparison of Range and Angle Estimation with Multiple Targets. Aiming to verify the effectiveness of range and angle estimation method in OFDM radar, Figure 3 shows the results of target parameter estimation, respectively, using two-dimensional (range-angle) common beamforming and two-dimensional MUSIC algorithm. In simulation experiments, assume that there are three noncoherent point targets, and the angle is, respectively, 14° , 41° , and -18° . The distance is, respectively, 30,550 m, 30,200 m, and 29,700 m, and SNR of three point targets is 10 dB. Other parameters are shown in Table 1. Note that the experiment is carried out in the case of Gaussian white noise.

From Figure 3, since the common beamforming is limited by the aperture of the physical antenna, the resultant beam width is wide and the ability of target resolution is low. Because the target data is sampled for only once, the noise has a great influence on side lobe and target parameter estimation is effected badly by SNR. On the contrary, the MUSIC algorithm utilizes virtual two-dimensional smoothing technique. Although it loses certain array aperture and distance ‘‘aperture,’’ it has a good performance of noise

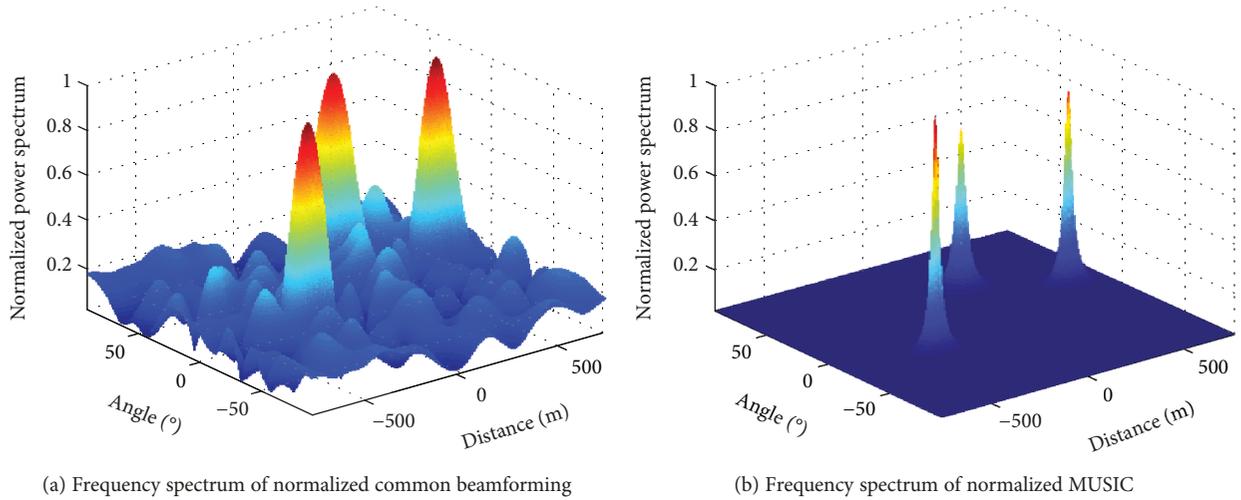


FIGURE 3: Spectrum comparison of two methods after normalized.

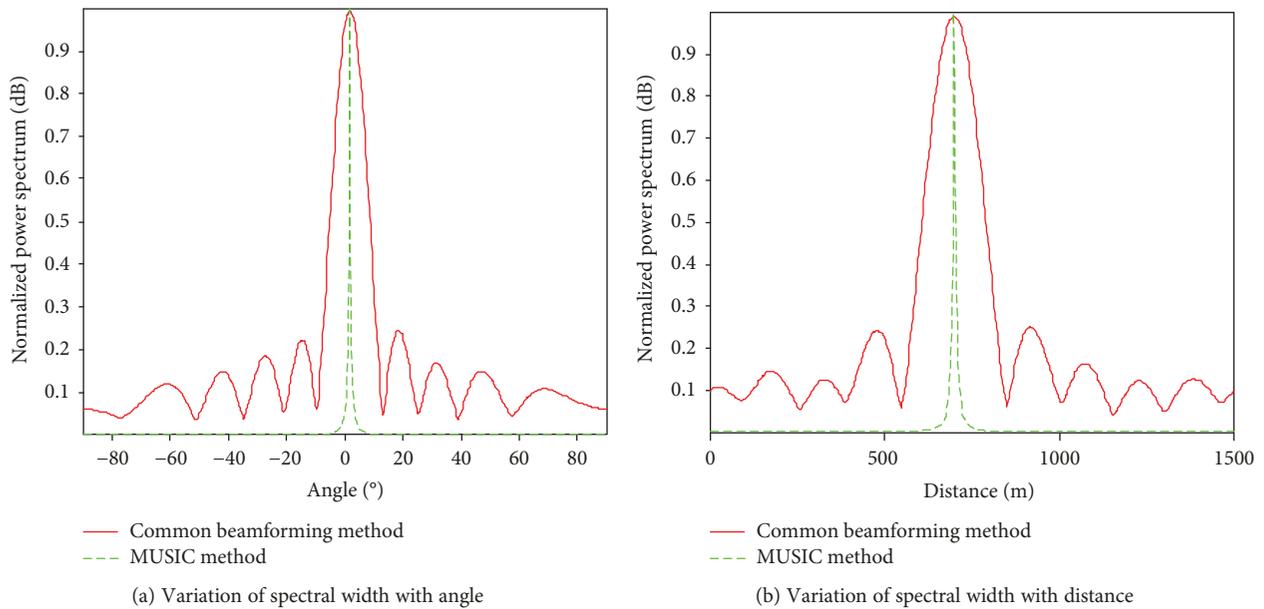


FIGURE 4: Spectrum comparison of two methods after normalized.

statistics. According to the results of power spectrum estimation, the side lobe is very low and SNR effect on target parameter estimation is deduced. Additionally, the MUSIC algorithm can break the physical limitation of antenna aperture, and its resolution is higher than the common beamforming method. Therefore, MUSIC has a better ability of resolution in the estimation of range and angle.

4.2. Comparison about Spectrum Width of a Single Point Target with a Single Snapshot. For quantitative description of target parameter estimation accuracy, Figure 4 shows the comparing results of using the two-dimensional common beamforming method and virtual two-dimensional smoothing spectrum estimation in a high-resolution method, respectively, to estimate the range and angle in OFDM radar. Simulation parameters are shown in Table 1.

According to simulation experiments, whether in angle domain or in range domain, the spectrum peaks of the MUSIC algorithm are sharper than that of the common beamforming method. The spectrum peaks of the MUSIC algorithm almost have no side lobes. However, the side lobes are high by using the common beamforming method. The results not only demonstrate the effectiveness of analysis about echoed signal properties of OFDM radar but also show that in a single snapshot case, the signal space that is estimated by the sampled data, which is obtained from a virtual two-dimensional smoothing method, is accurate. It is successful to break the limitation that the performance of the MUSIC algorithm depends on a large number of snapshots. Although a virtual two-dimensional smoothing method loses a part of apertures, the resolution ability of target parameters can still reach 5 times more than that of the two-dimensional common beamforming method, as shown in Figure 4.

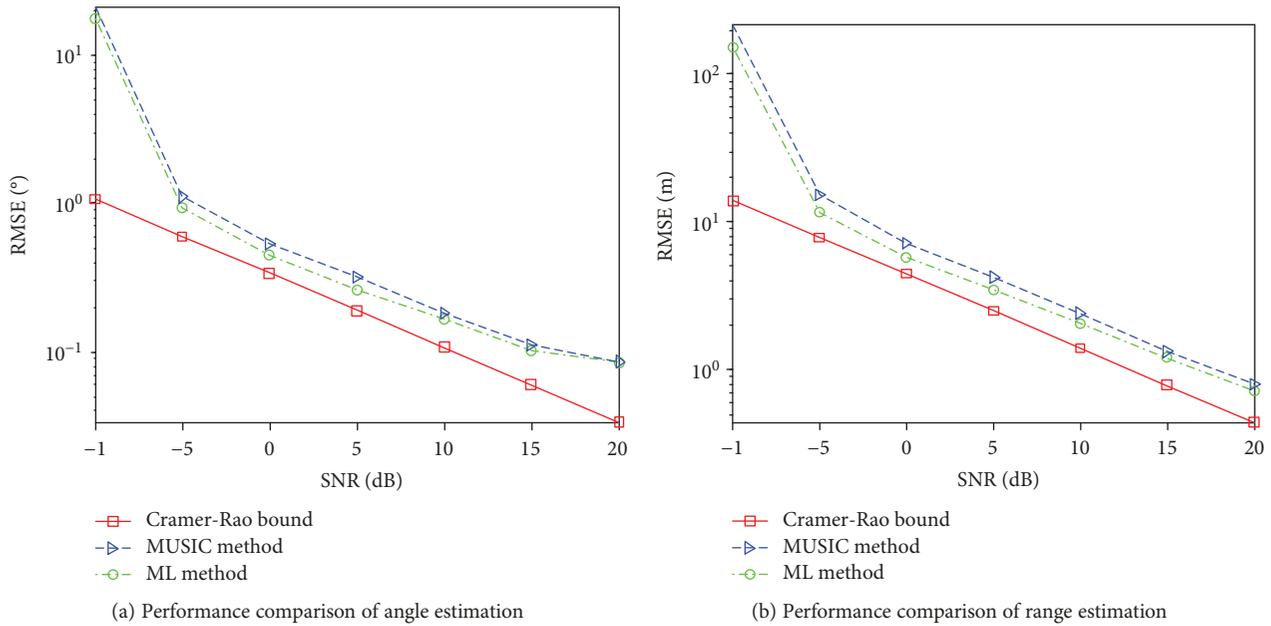


FIGURE 5: Performance comparison of range and angle joint estimation.

4.3. *The Performance Analysis of Joint Estimation on Range and Angle in High Resolution of a Single Point Target with a Single Snapshot.* Monte Carlo experiments are carried out to estimate angle θ and distance R . It is obvious that RMSE of angle θ is $\text{RMSE}(\theta) = \sqrt{1/K \sum_{m=1}^K (\theta_m^\wedge - \theta)^2}$, and RMSE of distance R is $\text{RMSE}(R) = \sqrt{1/K \sum_{m=1}^K (R_m^\wedge - R)^2}$, where K is the number of Monte Carlo experiments, θ_m^\wedge and R_m^\wedge are, respectively, the estimated angle and range value of m th Monte Carlo experiment, θ and R are, respectively, the true angle and range of the target, and RMSE functions of θ and R vary with SNR.

Exploiting the simulation parameters in Table 1 to simulate how the RMSEs, which are estimated by using a high-resolution method of range and angle, varies with SNR. Then, compare the above simulation performance with the results when using the ML method to estimate RMSE and the outcomes that how Cramer-Rao bound varies with SNR, respectively. Figure 5 shows the comparison results.

In a Gaussian white noise condition, the ML method is the most optimal estimation algorithm. It is found from Figure 5 that the performance of MUSIC is close to the ML method. That is to say, although the MUSIC algorithm is suboptimal in a Gaussian white noise case, its estimation accuracy loss can be negligible. Also, the MUSIC algorithm has a high resolution for the range and angle estimates. It is seen that both the performances of two algorithms are awful when SNR is below -5 dB. The accuracy of two algorithms is close to Cramer-Rao bound when SNR is from -5 dB to 15 dB. Because with a low SNR, the estimation performance of two methods is influenced greatly by SNR, that is to say, the application of two methods needs a certain SNR threshold; the two algorithms are not close to Cramer-Rao bound when SNR is above 15 dB for the reason that there is inherent

error caused by angle and distance increment. Therefore, under the condition of a certain SNR value, as long as the step size in search is small enough, the MUSIC algorithm can make an accurate estimation of distance and angle.

4.4. *Analysis of Speed and Computational Complexity for MUSIC.* The first step of MUSIC is to compute covariance matrix $\hat{\mathbf{R}}$. According to Section 3.1, the dimension of $\mathbf{X}(t)$ is $N_s \times 1$ so that the computational complexity of this step is $O(N_s^2)$. The second step is to perform eigenvalue decomposition. The computational complexity of eigenvalue decomposition is $O(N_s^3)$ [17]. The third step is to sort the eigenvalues which have been obtained from the second step, and the average computational complexity is $O(N_s M_s)$ when utilizing bubble sort [18]. The fourth step is to calculate spectral peaks and search. According to formula (27) above, the computational complexity of every spectral peak searching is $O(N_s M_s^2)$. Aiming to two-dimensional searching of θ, R and to meet the demand of real-time processing when it is put into practice, the proper computing choices include FPGA (field programmable gate array) and GPU (graphics processing units) for their parallel computing ability. The real-time processing speed depends on the processor type and the range of spectral peak searching [19, 20].

5. Conclusion

In this paper, aiming at the problem of radar target parameter estimation in the OFDM array system, the property of received signal has been analyzed and a method about high-resolution joint estimation on range and angle in OFDM radar is proposed. Under narrow-band signal condition, there is no coupling between the range and angle of a target in the echoed signal. The information about the range

and angle is included, respectively, in its steering vector, which can be used to jointly estimate range and angle of the target. Under the single snapshot condition, combining the MUSIC algorithm with a virtual two-dimensional spatial smoothing method can jointly estimate range and angle in high resolution. In the background of the Gaussian white noise, the accuracy of the proposed method is close to that of the common beamforming method. Although the high-resolution method which utilizes a virtual two-dimensional smoothing method loses a part of aperture under the condition of a single snapshot, it still has high resolution in distance and angle. In the future, further investigation on the joint high-resolution estimation of OFDM radar based on sparse recovery will be performed.

Abbreviations

OFDM:	Orthogonal frequency-division multiplexing
DOA:	Direction of arrival
MUSIC:	Multiple signal classification
DOD:	Direction of departure
RMSE:	Root-mean-square errors
ULA:	Uniform linear array
IDFT:	Inverse discrete Fourier transform
SNR:	Signal-noise ratio
ML:	Maximum likelihood
CRB:	Cramer-Rao bound
FPGA:	Field programmable gate Array
GPU:	Graphics processing units.

Data Availability

We have presented some persuasive simulation results as figures based on MATLAB. Requests for access to these detailed data which used to support the findings of this study should be made to Ying-Hui Quan with email addresses yhquan@mail.xidian.edu.cn and quanyinghui@126.com.

Conflicts of Interest

The authors declare that they have no competing interests.

Authors' Contributions

Rui Zhang, Ying-Hui Quan, and Sheng-Qi Zhu designed the algorithm scheme. Rui Zhang and Ying-Hui Quan performed the experiments and analyzed the experiment results. Lei Yang, Ya-chao Li, and Meng-Dao Xing contributed to the manuscript drafting and critical revision. All authors read and approved the final manuscript.

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References

- [1] R. Calderbank, S. Howard, and B. Moran, "Waveform diversity in radar signal processing," *IEEE Signal Processing Magazine*, vol. 26, no. 1, pp. 32–41, 2009.
- [2] N. Levanon, "Multifrequency complementary phase-coded radar signal," *IEE Proceedings - Radar, Sonar and Navigation*, vol. 147, no. 6, pp. 276–284, 2000.
- [3] G. Lellouch, P. Tran, R. Pribic, and P. van Genderen, "OFDM waveforms for frequency agility and opportunities for Doppler processing in radar," in *2008 IEEE Radar Conference*, pp. 1–6, Rome, Italy, May 2008.
- [4] Y. Liu, G. Liao, J. Xu, Z. Yang, and Y. Zhang, "Adaptive OFDM integrated radar and communications waveform design based on information theory," *IEEE Communications Letters*, vol. 21, no. 10, pp. 2174–2177, 2017.
- [5] B. J. Donnet and I. D. Longstaff, "Combining MIMO radar with OFDM communications," in *2006 European Radar Conference*, pp. 37–40, Manchester, UK, September 2006.
- [6] N. N. S. S. R. K. Prasad, V. Shameem, U. B. Desai, and S. N. Merchant, "Improvement in target detection performance of pulse coded Doppler radar based on multicarrier modulation with fast Fourier transform (FFT)," *IEE Proceedings - Radar, Sonar and Navigation*, vol. 151, no. 1, pp. 11–17, 2004.
- [7] R. F. Tigrek, W. J. A. D. Heij, and P. V. Genderen, "Multi-carrier radar waveform schemes for range and Doppler processing," in *2009 IEEE Radar Conference*, pp. 1–5, Pasadena, CA, USA, May 2009.
- [8] R. F. Tigrek, W. J. A. D. Heij, and P. V. Genderen, "Solving Doppler ambiguity by Doppler sensitive pulse compression using multi-carrier waveform," in *2008 European Radar Conference*, pp. 72–75, Amsterdam, Netherlands, October 2008.
- [9] D. Garmatyuk, J. Schuerger, Y. T. Morton, K. Binns, M. Durbin, and J. Kimani, "Feasibility study of a multi-carrier dual-use imaging radar and communication system," in *2007 European Radar Conference*, pp. 194–197, Munich, Germany, October 2007.
- [10] R. F. Tigrek, W. J. A. D. Heij, and P. Van Genderen, "OFDM signals as the radar waveform to solve Doppler ambiguity," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 48, no. 1, pp. 130–143, 2012.
- [11] S. Sen and A. Nehorai, "Adaptive OFDM radar for target detection in multipath scenarios," *IEEE Transactions on Signal Processing*, vol. 59, no. 1, pp. 78–90, 2011.
- [12] Y. Jia, X. Zhong, Y. Guo, and W. Huo, "DOA and DOD estimation based on bistatic MIMO radar with co-prime array," in *2017 IEEE Radar Conference (RadarConf)*, Seattle, WA, USA, May 2017.
- [13] C. Kusuma and G. Radha, "Robust direction of arrival estimation using subspace methods," *International Journal of Computer Applications*, vol. 159, no. 1, pp. 24–29, 2017.
- [14] Z. Q. He, Q. H. Liu, L. N. Jin, and S. Ouyang, "Low complexity method for DOA estimation using array covariance matrix sparse representation," *Electronics Letters*, vol. 49, no. 3, pp. 228–230, 2013.
- [15] T. Qian, J. Z. Xiang, and W. Cui, "Fast covariance matrix sparse representation for DOA estimation based on dynamic dictionary," in *2016 IEEE 13th International Conference on Signal Processing (ICSP)*, pp. 138–143, Chengdu, China, November 2016.

- [16] W.-Q. Wang and H. Shao, "Range-angle localization of targets by a double-pulse frequency diverse array radar," *IEEE Journal of Selected Topics in Signal Processing*, vol. 8, no. 1, pp. 106–114, 2014.
- [17] Y. Quan, Y. Li, X. Gao, and M. Xing, "FPGA implementation of real-time compressive sensing with partial Fourier dictionary," *International Journal of Antennas and Propagation*, vol. 2016, no. 2, Article ID 1671687, 12 pages, 2016.
- [18] O. Astrachan, "Bubble sort: an archaeological algorithmic analysis," in *SIGCSE '03 Proceedings of the 34th SIGCSE technical symposium on Computer science education*, pp. 1–5, ACM, New York, NY USA, February 2003.
- [19] Z. Zou, W. Hongyuan, and Y. Guowen, "An improved MUSIC algorithm implemented with high-speed parallel optimization for FPGA," in *2006 7th International Symposium on Antennas, Propagation & EM Theory*, pp. 1–4, Guilin, China, October 2006.
- [20] M. Kim, K. Ichige, and H. Arai, "Real-time smart antenna system incorporating FPGA-based fast DOA estimator," in *IEEE 60th Vehicular Technology Conference, 2004. VTC2004-Fall. 2004*, pp. 160–164, Los Angeles, CA, 2004.



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