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Research Article

Uncertainty Quantification for the Transient Response of Human Equivalent Antenna Using the Stochastic Collocation Approach

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The paper deals with the uncertainty quantification of the transient axial current induced along the human body exposed to electromagnetic pulse radiation. The body is modeled as a straight wire antenna whose length and radius exhibit random nature. The uncertainty is propagated to the output transient current by means of the stochastic collocation method. The stochastic approach is entirely nonintrusive and serves as a wrapper around the deterministic code. The numerical deterministic model is based on the time domain Hallen integral equation solved by means of the Galerkin-Bubnov indirect boundary element method (GB-IBEM). The stochastic moments, i.e., the mean and the variance of the transient current, are calculated. Confidence margins are obtained for the whole duration of the transient response as well as for the maximal current value. The presented approach enables the estimation of the probability for the induced current to exceed the basic restrictions prescribed by regulatory bodies. The sensitivity analysis of the input parameters indicates to which extent the variation of the input parameter set influences the output values which is particularly interesting for the design of the human equivalent antenna.

1. Introduction

Axial current distribution has been one of the quantities of interest not only to quantify human exposure to low-frequency fields by determining the current density/induced fields but also to compute specific absorption (SA) for the case of human exposure to transient radiation [1]. Thus, these quantities are the first step in quantifying the effects of EM exposure. On the other hand, biological effects depend on the frequency of the EM field. Due to the absence of resonance effects at low frequencies (LF), the thermal effects are negligible while the nonthermal effects could possibly have severe effects on membrane cells [2, 3]. On the contrary, in a high-frequency (HF) range, where the body dimensions are comparable to external field wavelength and resonances become significant, thermal effects are dominant [4, 5]. In any case, there is no way to directly measure the induced electric fields in humans and related biological effects; hence, the use of reliable computational models is mandatory.

However, computational models used in EM dosimetry, simplified or anatomically realistic ones, are subjected to variation of input parameters' values. The morphology (dimensions), the tissue conductivity and relative permittivity, and other constants related to the model description are often partially or even entirely unknown. In the past decade, some efforts have been made to provide the means to include the parameter variability into the model and propagate it to the output value of interest. A term "stochastic dosimetry" has been coined under the idea that by using only average values, the computational models are rough approximation of the real scenarios in EM dosimetry [6]. Some examples of stochastic dosimetry simulations are presented in [7–10]. Such an approach becomes even more important when it comes to international and national guidelines and standards, respectively, which ought to account for the stochastic nature of the input variables thus providing certain expected values for the restrictions along with the confidence margins and worst-case predictions [11, 12].

Generally, the Monte Carlo simulation (MCS) method is considered to be the most reliable and robust stochastic method [13]. However, relatively slow convergence makes this somewhat unattractive and sometimes inconvenient even for the validation of results with respect to other methods. Among various alternative methods reported in the literature, the generalized polynomial chaos expansion (gPCE) emerged as the most often used approach in the stochastic computational electromagnetics (SCEM). This technique for solving stochastic equations is based on spectral discretization, and it comprises the stochastic Galerkin method (SGM) and stochastic collocation method (SCM) [13]. The main difference between the two methods is their intrusive/nonintrusive approach to computational models. The intrusive nature of SGM implies a more demanding implementation since the development of new codes is required. On the other hand, the nonintrusiveness of SCM enables the use of previously validated deterministic models as black boxes. Still, both approaches exhibit fast convergence and high accuracy under different conditions. The analysis regarding the applications in computational electromagnetics may be found elsewhere, e.g., in [14].

The properties of the field induced in the human body due to EM exposure could be studied by means of a simple but rather useful cylindrical model of the human body [1]. This model has been widely used for LF ranges. First, King and Sandler proposed the parasitic-antenna model of the human body exposed to extremely low-frequency (ELF) and very-low-frequency (VLF) sources providing some closed-form expressions for the induced current [15]. An overview of some numerical methods for human exposure from ELF to a microwave region is reported in [16]. Furthermore, for the case of ELF range, the loaded thick-wire model of human body based on Pocklington's integrodifferential equation in FD has been proposed by Poljak and Rashed [17]. The model is based on the solution of Pocklington's equation via the Galerkin-Bubnov boundary element method (GB-BEM).

Although FD techniques enable the use of simpler formulations and therefore the numerical treatment is less complicated, when the human body is exposed to transient radiation, a direct time domain approach offers a better insight into the physical behavior of the phenomena [18]. One possible approach would be to use an indirect approach, i.e., to implement the FD-based models along with the IFFT algorithm. However, the thick-wire model from [17] suffers from some numerical instabilities when used for TD response, and moreover, coupling of such approach with nonintrusive and sampling-based stochastic methods would imply a significant burden on computational resources.

A human equivalent thin-wire antenna model has been proposed for experimental dosimetry [19], and it is valid in the frequency region from 50 Hz to 110 MHz. Therefore, TD simulation of the body based on the human equivalent antenna model is proposed in [20]. The formulation is based on Hallen's type of integral equation numerically treated via the GB-IBEM. This deterministic model has been validated providing the stable numerical results.

This paper presents a step forward towards the deterministic-stochastic coupling of TD Hallen's equation GB-IBEM solver, for the unknown current induced along the antenna, and the SCM, respectively. The SCM treats the deterministic part as a black box, thus preserving the functionalities of validated codes from [20]. The novel stochastic approach treats the dimensions of the antenna, namely, its length and radius, as random variables (RV) with predefined stochastic distributions. The aim is to obtain the expected value of the induced current and provide the estimation of confidence margins. The results will also show which of the two parameters have a higher impact on the output value and to which extent. The modeling carried out in the presented stochastic-deterministic approach may be useful in designing such antennas like in Gandhi and Aslan's patent in [19]. The present work could be regarded as a follow-up to the stochastic-deterministic coupling of the FD thick-wire model and the SCM approach for the LF exposure assessment reported in [21]. Though such a canonical representation can be regarded as a rather simplified deterministic-stochastic model of the body for a transient plane wave exposure, it still ensures a rapid estimation of the phenomenon. Moreover, this cylindrical model could be considered as a starting point to establish a more realistic and eventually anatomically based direct time domain model. To the best of our knowledge, even oversimplified models of this kind are rather rare in the relevant literature if they are available at all.

This paper is organized as follows. Section 1 introduces the basic concept of the time domain model of the human equivalent antenna. Section 2 outlines the fundamentals of the stochastic collocation method. Numerical examples are presented in Section 3 while some concluding remarks and considerations related to the future work are given in Section 4.

2. Time Domain Model of Human Equivalent Antenna

As it has been shown by King and Sandler in [15], the perpendicular component of the incident electric field tangential to the body can be assumed constant along the body. This field component induces the axial current not forming a closed loop but ending on charge density induced on the surface of the body. Provided that the human body is represented by a cylindrical antenna, the first task of the electromagnetic dosimetry is to calculate the current distribution induced along the antenna. Once the axial current is known, it is possible to calculate the electric field, the power density, and the specific absorption (SA) induced in the body. Furthermore, the macroscopic average electric field can be subsequently used for the calculation of the corresponding local electric fields induced in different organs [15].

The TD model of the human equivalent antenna used in this work has been proposed in [20]. The antenna with radius a and length L representing the human body is assumed to be insulated in the free space. The arms are positioned close to the body. All body organs are considered to behave as good conductors for a given frequency range. As the impact of the conductivity variations on the results is negligible [20], the antenna is considered to be perfectly conducting (PEC). The cylindrical antenna of length L and radius a representing the human body is presented in Figure 1.

The formulation is based on the Hallen space-time integral equation [20]:

$$\int_0^L \frac{I\left(z', t - R/c\right)}{4\pi R} dz' = F_0\left(t - \frac{z}{c}\right) + F_L\left(t - \frac{L-z}{c}\right) + \frac{1}{2Z_0}\int E_z^{\mathrm{inc}}\left(z', t - \frac{|z-z'|}{c}\right) dz', \tag{1}$$

where $I(z', t - R/c)$ is the unknown space-time dependent current to be determined for a given incident electric field E_z^{inc}, c is the light velocity, and Z_0 is the free-space wave impedance. Multiple reflections of the current wave from the wire free ends are taken into account via unknown time signals $F_0(t)$ and $F_L(t)$. A detailed derivation of (1) could be found elsewhere, e.g., in [20].

In the past two decades, some studies have provided evidence of greater effect on biological systems when exposed to pulsed signals such as radar or mobile radio telephones than to nonpulsed signals [22]. In this paper, the excitation field is given as the standard double-exponential EMP waveform [20]:

$$E_z^{\mathrm{inc}}(t) = E_0\left(e^{-\alpha t} - e^{-\beta t}\right), \tag{2}$$

where $E_0 = 1$ kVm^{-1}, $\alpha = 4 * 10^6$ s^{-1}, and $\beta = 4.76 * 10^8$ s^{-1}. The EMP is chosen in such a way that its frequency range corresponds to the frequency range of the human equivalent antenna presented in [19].

The transient current induced in the human body due to a particular EMP excitation is obtained by solving TD Hallen's integral equation (1) via the TD version of Galerkin-Bubnov indirect boundary element method (GB-IBEM). The mathematical details concerning the method of solution are given in [20].

3. Stochastic Collocation Method

3.1. Uncertainty Quantification. The process of quantifying the uncertainty in the output value of interest due to the uncertain nature of input parameters is usually referred to as the uncertainty quantification (UQ). The aim is to calculate the stochastic moments, namely, the expected value and variance, and provide confidence margins and probability density function.

The fundamental principle of the stochastic collocation method lies in the polynomial approximation of the considered output Y in the stochastic space that consists of N previously selected collocation points [23]:

$$\widehat{Y}(X) = \sum_{i=1}^{N} L_i(X) \bullet Y^{(i)}, \tag{3}$$

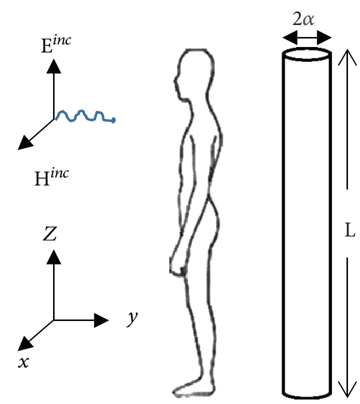

Figure 1: The cylindrical model of the body exposed to transient radiation.

where $L_i(X)$ are basis functions and $Y^{(i)}$ is the output realization for the ith input point $X^{(i)}$. The stochastic space is d dimensional as there are d random input variables, each prescribed with the corresponding probability density functions (pdf). The input variables are defined as vector $X = [x_1, \ldots, x_d]$; therefore, $X^{(i)} = [x_1^{(i)}, \ldots, x_d^{(i)}]$ is the ith input point. In this work, input variables are considered independent, an assumption valid for majority of applications in the computational electromagnetics (CEM).

The simplest way to interpolate in the multivariate dimension space is to use the tensor product in each random dimension and this approach is valid for $d < 5$. Thus, the basis function $L_i(X)$ from equation (3) is constructed as follows [23]:

$$L_k(X) = l_1^{m_1} \otimes \cdots \otimes l_d^{m_d}, \quad k = 1, \ldots N, \tag{4}$$

where l_i, $i = 1, \ldots, d$, is the one-dimensional Lagrange polynomial and the maximal value of m_j equals to the number of points in the corresponding stochastic dimension. Successful applications of a fully tensorised SCM model can be found in [23–25].

For the univariate case and m collocation points, the Lagrange basis function is given as follows:

$$l_i(x) = \prod_{k=0, k\neq i}^{m} \frac{x - x_k}{x_i - x_k}, \tag{5}$$

with the property $l_i(x_j) = \delta_{ij}$, where δ_{ij} denotes the Kronecker symbol. Other types of basis functions are possible, e.g., piecewise multilinear basis functions as in [26].

The expression in equation (3) can be used as a surrogate for the given mathematical model such as the one in equation (1). Namely, the expression may be used to estimate the probability density function since the MC sampling of this equation is more practical than the MC sampling of the original mathematical model.

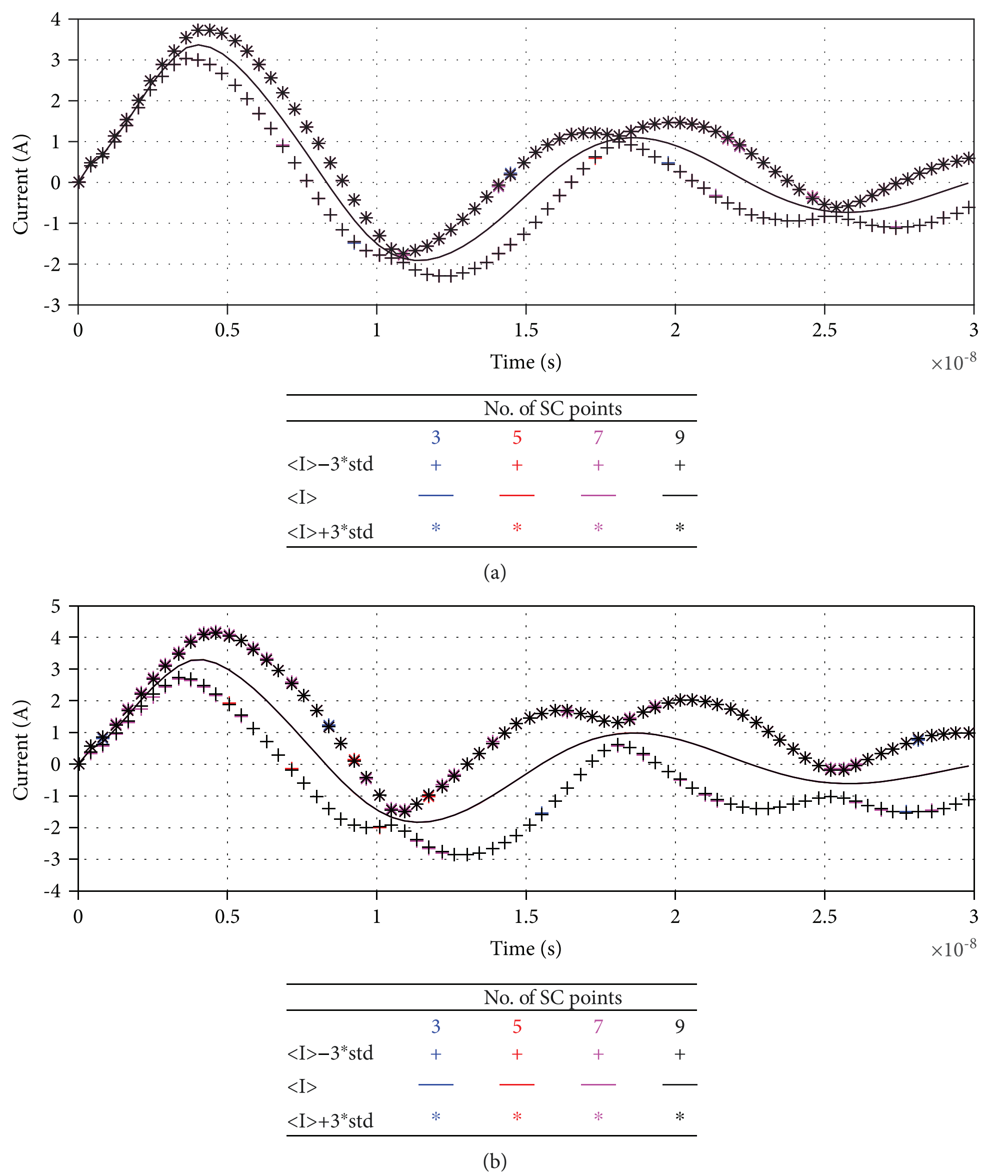

Figure 2: The stochastic transient current induced in the human body exposed to the double-exponential EMP. The observation point is the middle point of the antenna. The stochastic mean is denoted with $<I>$ and std stands for standard deviation. (a) The coefficient of variation CF = 5.71%; (b) CF = 12%.

Furthermore, the expressions for the stochastic moments are derived following their definitions from the statistics. Thus, the expectation (mean) of $\widehat{Y}(X)$ is computed as [13]

$$\begin{aligned}\mu\left(\widehat{Y}(X)\right) &= \int_{-\infty}^{+\infty} \widehat{Y}(\boldsymbol{X}) p(\boldsymbol{X}) d\boldsymbol{X} \\ &= \int_{-\infty}^{+\infty} \sum_{i=1}^{N} L_i(X) \bullet Y^{(i)} p(X) dX \\ &= \sum_{i=1}^{N} Y^{(i)} \bullet w_i,\end{aligned} \tag{6}$$

where $p(X)$ represents the joint probability density function of random variables in X and w_i are weights that can be precomputed as follows:

$$w_i = \int_{-\infty}^{+\infty} L_i(X) p(X) dX. \tag{7}$$

Accordingly, the variance is given as follows:

$$V\left(\widehat{Y}(X)\right) = \sum_{i=1}^{N} \left(\widehat{Y}^{(i)} - \mu\left(\widehat{Y}(X)\right)\right)^2 \bullet w_i. \tag{8}$$

It is worth noting that to calculate the stochastic moments, it is not necessary to build the polynomial form equation (3) which further reduces the computational burden.

The choice of the interpolation points $X^{(i)}$ in equation (3) can follow several approaches. In this work, the choice of

Table 1: The estimate of the confidence intervals based on quantile computation.

Quantiles	#1 CF = 5.71%	#2 CF = 12%
10.0%, 90%	[3.23, 3.54] A	[3.03, 3.65] A
5.00%, 95%	[3.20, 3.57] A	[2.96, 3.71] A
1.00%, 99%	[3.15, 3.61] A	[2.87, 3.80] A

points corresponds to the Gauss quadrature rule related to the probability distribution of random inputs, e.g., Legendre polynomials for a uniform distribution and Hermite polynomials for a Gaussian distribution [13]. Other formulas may be used to generate the abscissas for the interpolation in equation (3) such as the Clenshaw-Curtis formula with nonequidistant abscissas given as zeros of the extreme points of Chebyshev polynomials or equidistant points [26, 27]. These sets of points exhibit a nested fashion unlike Gauss points which are desirable in certain applications such as sparse grid interpolation [26, 27].

3.2. Sensitivity Analysis. The definition of sensitivity analysis (SA) according to [28] defines it as the study of how uncertainty in the output of a model (numerical or otherwise) can be apportioned to different sources of uncertainty in the model input. The variance-based SA presented in this work follows the "one at a time" principle for calculating sensitivity indices. The impact factors are thus calculated in the following way:

$$I_i = \frac{V(Y\,;X_i)}{V(Y)}, \quad i = 1, \ldots, d, \tag{9}$$

where I_i stands for the influence of a single input parameter's uncertainty.

4. Numerical Results

The TD human equivalent antenna model outlined in Section 1 is coupled with the SCM method presented in Section 2. The random dimension is $d = 2$ with the antenna length and antenna radius modeled as random variables (RVs). Both RVs are prescribed with uniform distributions whose range is defined as follows:

$$\langle x_i \rangle \pm \langle x_i \rangle \cdot \mathrm{CF}, \quad i = 1, 2, \tag{10}$$

where $\langle x_i \rangle$ is the mean value of the input parameter and CF is the coefficient of variation. The mean value of the antenna length and radius is $L = 1.75$ m and $a = 5$ cm, respectively. The coefficient of variation CF is chosen as 5.71% for the first test case and 12% in the second case.

Since this model approximates the antenna as perfectly conducting, the results are given in terms of the induced current instead of the induced electric field which would be appropriate according to the latest updates in international standards and guidelines [11, 12]. However, the results are still valid and can be readily interpreted.

First, to test the convergence of the stochastic approach, the full tensor model was built with 3×3, 5×5, 7×7, and 9×9 collocation points in the case of CF = 5.71% and 12%. The results presented in Figure 2 exhibit a satisfactory convergence rate of the method. The change in the CF value does not deteriorate the convergence of the SCM. The chosen observation point is the middle point of the antenna which corresponds to the body waist. Similar results are obtained for other space coordinates. The size of confidence margins doubles when CF is changed from 5.71% to 12%.

The estimation of confidence intervals (CI) for the maximal possible value of current based on quantile estimation is given in Table 1. The results are obtained as MC simulation of the surrogate model from equation (3) with 10^6 samples. The shape of the corresponding probability density function is given in Figure 3 for both test cases. The maximal possible value of the current appears in the early time of response (within 5 ns), and it is within the recommended intervals.

The presented stochastic-deterministic model is simple in terms of stochastic dimension, since there are only two random input variables. However, the information on the influence of the input parameters to the output value deserves some merit. The impact factors I_1 and I_2 for the antenna length and radius, respectively, are calculated according to equation (9), and the results are depicted in Figure 4. It is worth noting that, although the impact of the antenna length, i.e., body height, is overwhelming throughout the simulation period, the antenna radius exhibits much higher influence in the early time behavior of the transient response. This is exactly the time period in which the current reaches its maximal possible value; hence, the radius is the parameter that has very high impact to maximum current. The mutual interaction between the two parameters is obviously very weak, as the summation of I_1 and I_2 adds up to 1 almost everywhere.

5. Concluding Remarks

A stochastic perspective for the exposure of human body to electromagnetic pulse radiation is presented. The human body is modeled as a straight thin-wire antenna whose length and radius are considered to be random variables with associated uniform distribution. A nonintrusive stochastic collocation method is used to propagate the uncertainty to the output transient current. The mean value along with the confidence margins is presented for the current values, and it is shown that the obtained values are inside the recommended ranges for the presented exposure scenario. The probability density function of the maximal value of the transient current is presented. The impact of each input variable is calculated, and it is shown that the radius of the antenna has much higher impact on the maximal current value than the length, while for the rest of the impulse duration, the antenna length is more significant. The results may be taken into account when designing the human equivalent antenna that is used to replace the real body in experimental dosimetry

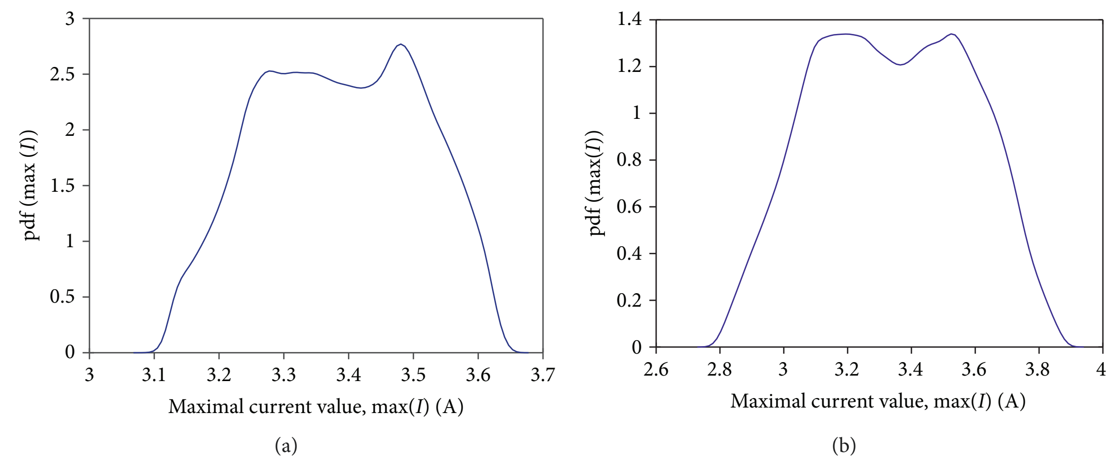

Figure 3: The probability density function of the maximal possible current value in the case of CF = 5.71% (a) and CF = 12% (b). The results are based on the MC sampling of the surrogate model from equation (3) with 106 samples.

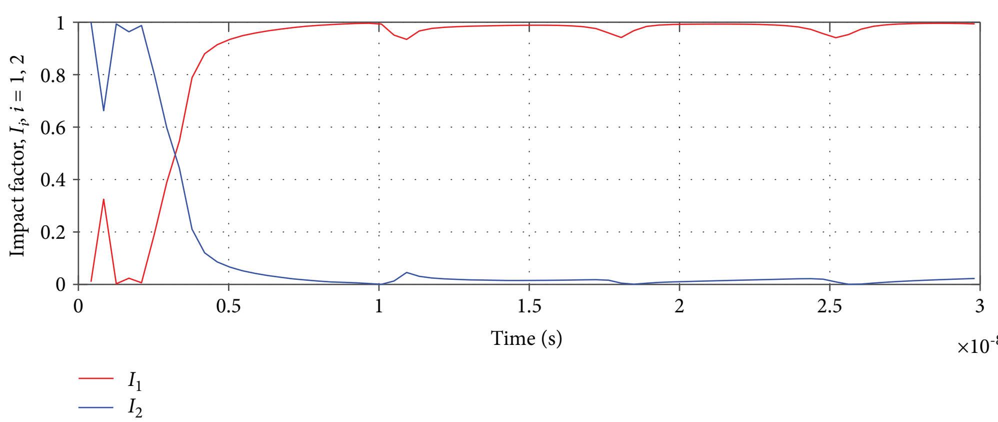

Figure 4: The impact factor of the antenna parameters: I_1 represents the influence of the antenna length and I_2 represents the influence of the antenna radius.

measurements. Also, the stochastic-deterministic approach offers a new perspective providing a deeper understanding of the influence of input parameters to the output values.

A first step to enhance the time domain cylindrical model is to include the body permittivity and conductivity into a deterministic and stochastic analysis, respectively. Furthermore, a future work would likely deal with anatomically realistic time domain body representations which were already developed by the authors in the frequency domain.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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