

Research Article

Determining the Effective Electromagnetic Parameters of Photonic Crystal by Phase Unwrapping and Denoising Method

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The electromagnetic parameters of the dispersion material and metamaterial are vital in the engineering. The phase unwrapping method is proposed to deal with the phase ambiguity of the transmission and reflection method in electromagnetic (EM) parameters extraction. The computed results demonstrate that the proposed method can give the correct effective parameters. In dealing with scattering parameters with noise, the wavelet transform method is utilized to remove the noise added to the scattering parameters. The simulated results show that the correct material parameters can be obtained by wavelet denoising method. Finally, the proposed method is used to extract the parameters of the photonic crystal. The effective parameter gives a different aspect in explanation to the function for the photonic crystal.

1. Introduction

The study of metamaterials with subwavelength basic unit has been a vibrant research topic in the field of THz frequency band. Structuring the metamaterials on a subwavelength scale makes it possible to create electromagnetic media with properties not found in natural materials, while still allowing describing them as effectively continuous media with constitutive parameters such as the electric permittivity and the magnetic permeability. A basic tool in the study of metamaterials is the so-called retrieval method, i.e., the extraction of effective medium parameters corresponding to a metamaterial with given microscopic structure. The effective material parameters are important because they give us the insight between the microscopic response of metamaterials and the macroscopic homogeneous media assumed in many proposed applications. The material parameter extraction methods have recently attracted attention in the literature due to the grown interest towards metamaterials and the need to characterize the electromagnetic properties of the man-made materials.

The subwavelength scale allows the heterogeneous material to be considered as a homogenized effective medium,

whereas local resonances lead to new phenomena of the effective medium parameters that are rarely or never observed in nature. The existence of resonance poses a considerable challenge to conventional effective medium theories [1].

Among the different extraction techniques, there exists a class of methods that are based on measurements (or numerical simulations) of the reflection and transmission coefficients of a planar material sample. The classical Nicolson-Ross-Weir (NRW) technique is part of this class, as well as many other more recent methods [2–11]. The intrinsic problem of the NRW technique relates to the electrical thickness of the material sample. Namely, the phase of the electromagnetic wave is periodic with a period of 2π , which causes ambiguity in the extracted results.

The phase ambiguity is a key problem for retrieving the electromagnetic parameters with transmission and reflection method. In order to overcome the phase ambiguity, the method that requires the imaginary part of the refraction index larger than zero is presented using physical principle, which can give the uniquely sign of the real part of the index [2]. In [3], the double negative material is analyzed by the transfer matrix method, in which the data of various

system lengths L is collected to calculate the refraction index from the linear fits of nkL and nkL versus the system length L . A general homogenization theory is developed to state the difference between the equivalent parameters and the effective constitutive parameters of periodic metamaterials [4]. Considering the relationship between frequency and phase of the transmitted coefficient, the successful phase summing is presented [5]. Since the constitutive parameters of a slab are hard to obtain nearby the total transmission frequency, the cycle shift operator to the periodic multilayer is proposed [6].

In what follows a novel way to overcome the phase ambiguity related to the extracted material parameters is proposed and the performance of the proposed technique is studied with examples. The phase unwrapping method serves to extract the scattering parameters with the sweep frequency. Upon the request of the phase difference of adjacent frequencies, the phase ambiguity can be resolved. Considering the effect of noise in the scattering parameters, the wavelet denoising method is involved with the data preprocessing. The results indicate that the proposed method can give the accurate permittivity and permeability of the dispersion materials. Finally, this method is employed to analyze the photonic crystal which shows resonance in the S11 and S21. The extracted results have many peaks in the permittivity and permeability, which are related to the photonic crystal resonance structure. The effective parameters can help us to understand the photonic crystal in physics and the design in engineering.

2. Method

In order to retrieve the effective permittivity and permeability of a metamaterial slab, we need to characterize it as an effective homogeneous slab. In this case, we can retrieve the permittivity and permeability from the reflection S11 and transmission S21 data. A typical method is to extract the impedance Z and refraction index n from scattering parameters, where Z and n are determined independently. The reflection coefficient at the interface of a semi-infinite material slab and the phase factor of the wave propagating through the measured material slab with thickness d is used in the early time. The difficulty with the aforementioned extraction technique relates to the periodicity of the phase factor of the wave propagating through the measured material slab. A method to overcome the phase ambiguity is proposed in [5]; the phase difference between two neighbor frequency points is used to select the correct branch at a given frequency point. However, the phase is obtained by summing all the phase differences before the computed frequency, which may induce errors when the phase difference jumps over 360° . In this paper an improvement is proposed, the phase is unwrapped, which makes the data more robust in the parameter extraction procedure. The proposed technique can be used for material parameter extraction without worrying about whether the solution branch is correct in the measured frequency band.

The first step in the algorithm is to obtain the impedance Z and transmission coefficient T from the scattering parameters S11 and S21.

$$Z = \sqrt{\frac{(1 + S_{11})^2 - S_{21}^2}{(1 - S_{11})^2 - S_{21}^2}} \quad (1)$$

$$T = \exp(-\gamma d) = \frac{1 - S_{11}^2 + S_{21}^2}{2S_{21}} + \frac{2S_{11}}{(Z - Z^{-1})S_{21}} \quad (2)$$

Starting from the phase factor appearing in formula (3)

$$T = \exp(\gamma d) = |T| \exp(j\phi) \quad (3)$$

rewrite the natural logarithm of the expression in formula (3) as follows.

$$\log(T) = \log(|T|) + j\phi \quad (4)$$

In order to obtain the corrected branch, the first frequency should be lower enough to insure the slab is electrically thin. The phase at each frequency point is obtained by using the phase unwrap technique, which can be expressed as

$$\phi = \phi_0 + \sum_{k=1}^N (\phi_k - \phi_{k-1}) \quad (5)$$

where $\phi_k = \phi(f_k)$ and

$$\phi_k = \phi_k - m \cdot 2\pi \quad (6)$$

where the integer m is determined from formula (7) as follows.

$$m = \left\lceil \frac{\Delta\phi_k}{2\pi} \right\rceil, \quad \Delta\phi_k = \phi_k - \phi_{k-1} \quad (7)$$

The square bracket in formula (7) is to take the integer. The improved method can overcome the ambiguity when the phase difference is larger than 360° , which is possible in some metamaterials. The cycle shift operator to the periodic multilayer in [6] can also remove the ambiguous phase, but sudden changes near the resonance frequency may cause the cycle shift operator calculation accumulates a lot of errors. Compared to that, the proposed method in this paper can overcome the phase ambiguity in an efficient way while averting the accumulated errors.

3. Removing Noise from the Scattering Parameters by Using Wavelet Method

Generally, the extracted parameters are influenced by the noise which may be caused by measurement or simulation. Based on the fact that noise and distortion are the factors that limit the accuracy of the extracted parameters, it is necessary to remove the disturbances before the extraction. Noise is defined as the unwanted signal that interferes with the parameters extraction from the measurement. Here wavelets denoising method is employed, which can increase

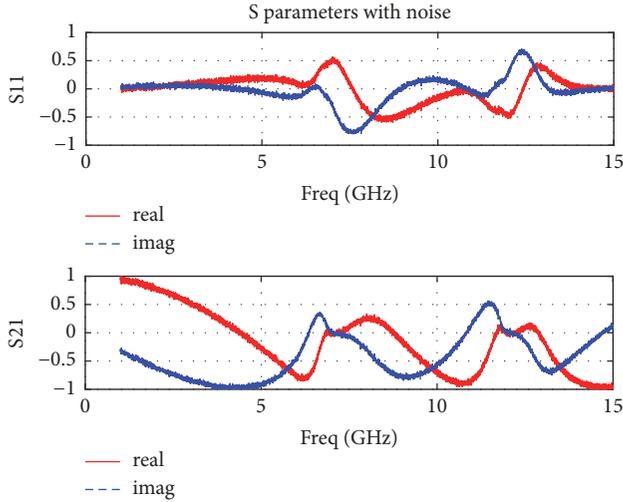


FIGURE 1: The Scattering parameters S11 and S21 with noise.

the accuracy of the extracted parameters. For the scattering parameters S11 and S21, both the real parts and imaginary parts can be regarded as one-dimensional signal and the fast random variation can be considered as noise. Wavelets are characterized by scale and position and are useful in analyzing variations in signals in terms of scale and position. Because of the fact that the wavelet size can vary, it has advantages over the classical signal processing transformations to simultaneously process time and frequency data. The vanishing moments of the wavelet basis can be used as a selection critic. Having p vanishing moment means that wavelet coefficients for p -th order polynomial will be zero. That is, any polynomial signal up to order p -1st can be represented completely in scaling space. In theory, more vanishing moments mean that the scaling function can represent more signals that are complex accurately; p is also called the accuracy of the wavelet. Wavelets that resemble the signal or its properties yield better signal, which can be another selection critic. A new wavelet denoising method is presented for the wavelet basis and level selection [7]. We will use the method to remove the noise from signals of S11 and S21 with noise.

As a validation, a dispersion material is utilized to verify the proposed method on the basis of [5]. Considering a dispersion material slab with thickness $d = 12.5$ mm, the permittivity and permeability are

$$\epsilon_r(\omega) = 1 - \frac{A^2}{\omega^2 - \omega_{0,\epsilon}^2 - j\Gamma\omega} \quad (8)$$

$$\mu_r(\omega) = 1 - \frac{A^2}{\omega^2 - \omega_{0,\mu}^2 - j\Gamma\omega} \quad (9)$$

respectively, where $A = 2\pi \times 4.3$ GHz, $\Gamma = 2$ GHz, $\omega_{0,\mu} = 2\pi \times 7$ GHz, and $\omega_{0,\epsilon} = 2\pi \times 12$ GHz.

The scattering parameters S11 and S21 are computed as Figure 1. The Gaussian noise with mean zero and standard variation 0.025 is added to the S11 and S21 signals.

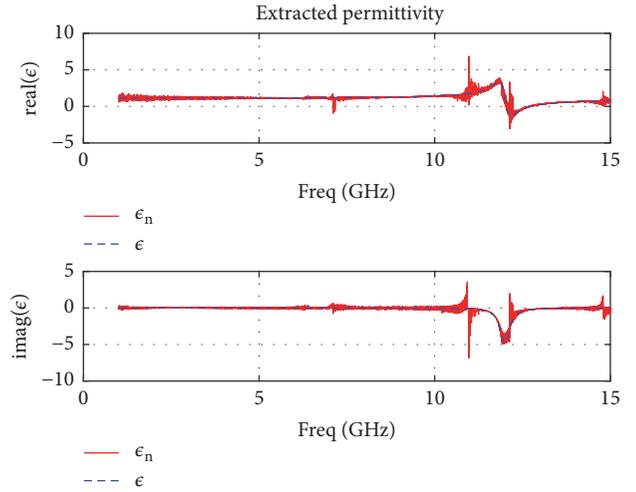


FIGURE 2: The extracted permittivity from the noised S11 and S21 data.

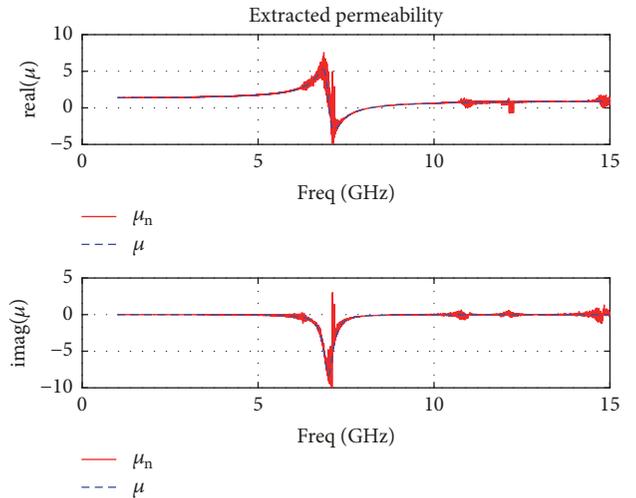


FIGURE 3: The extracted permeability from the noised S11 and S21 data.

The extracted permittivity and permeability results are presented in Figures 2 and 3, where ϵ_n and μ_n mean the extracted permittivity and permeability with noise; ϵ and μ mean the extracted permittivity and permeability without noise. From the figures, it is seen that the extracted parameters are affected by the noise and some large errors occurred at several frequencies.

Wavelet shrinkage methods provide effective signal denoising with minimum computational complexity. In the wavelet domain, the signal is coherent and has concentrated “energy” residing in just a few high magnitude coefficients, whereas incoherent noise is represented by a large number of coefficients with small magnitude. This sparsity of wavelet coefficients representing the signal is exploited by wavelet shrinkage methods to separate noise from signal coefficients. In the denoising by wavelet method, the selection of wavelet basis and decomposition level is very crucial. In general,

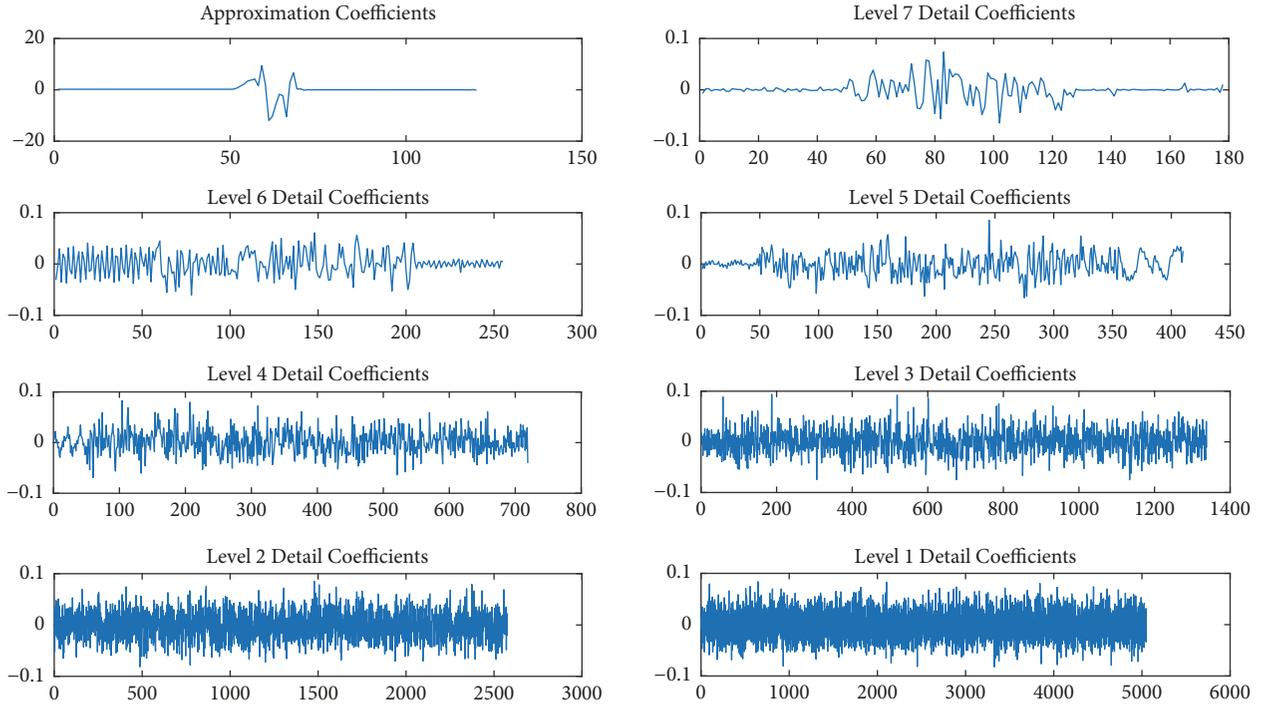


FIGURE 4: The wavelet coefficient distribution, the approximation and detail component.

standard wavelets that resemble the signal or its properties yield better signal and noise separation as well as sparsity. The Meyer wavelet here is more suitable to the problem, so the Meyer wavelet basis is selected. The decomposition level is selected as 7 from the wavelet decomposition as shown in Figures 4 and 5. From Figure 4, it is seen that the added noise is distributed in the detail coefficients and can be removed by setting the threshold. From Figure 5, the detail coefficient of levels 8 and 9 is very sparse; it can be deduced that the decomposition level 7 is enough to remove the noise of S11 and S21. To obtain the decomposition levels for noise thresholding, the first step is to calculate the “peak-to-sum ratio” of the detail components in wavelet transform.

$$S_j = \frac{\max(|w_j|)}{\sum_{i=1}^{N_j} |w_{j,i}|} \quad (10)$$

This equation is outlined in [7], and for any specific question the decomposition level must be selected based on it separately. The peak-to-sum ratio reflects the sparsity of a detail component and allows the identification of noise present in a detail component. A large peak-to-sum ratio implies that the signal possesses only a few large coefficient values, whereas a small peak-to-sum ratio reveals that the noise possesses a large number of small coefficient values, as shown in Figure 6.

The S11 and S21 signals with noise removed are shown in Figure 7. Comparing with Figure 1, we can see that the S11 and S21 are much smoother. Most of the added noise is successfully removed by the wavelet denoising method, which would give more accurate extracted parameters.

The extracted and denoised permittivity and permeability results are shown in Figures 8 and 9, where ϵ_e and μ_e are

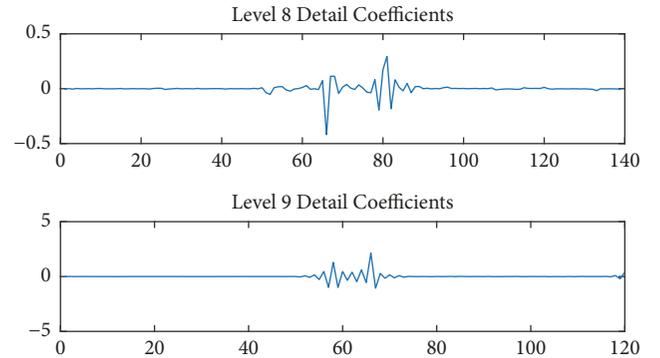


FIGURE 5: The wavelet coefficient distribution, the detail component for levels 8 and 9.

the extracted permittivity and permeability; ϵ_o and μ_o are the original permittivity and permeability. As can be observed in Figures 2, 3, 8, and 9, a significant improvement of the extracted parameters is obtained. From the curves of the scattering parameters, it is obvious that the added noise has fast fluctuation changes, which is reflected as details in the wavelet transformation. By the peak-to-sum ratio, the quantitative estimate of the effect of noise can be achieved. From Figure 6, the peak-to-sum ratio has a significant increasing after decomposition level 6, which means that the noise effect is lowered, so we can select the decomposition level as 7 in wavelet transform.

The results illustrate that the noise effects of the scattering parameters can be removed by using wavelet denoising method. Selection of the wavelet basis is based on the

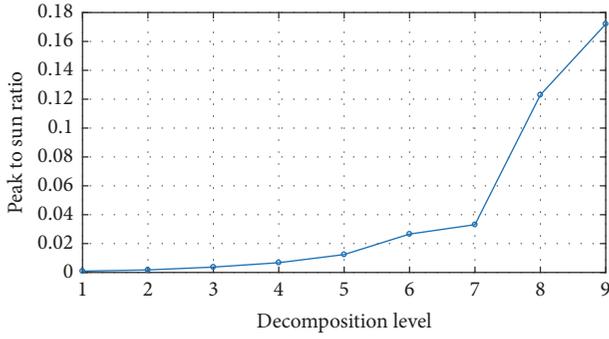


FIGURE 6: The peak-to-sum ratio of the wavelet detail coefficient for S11 real part.

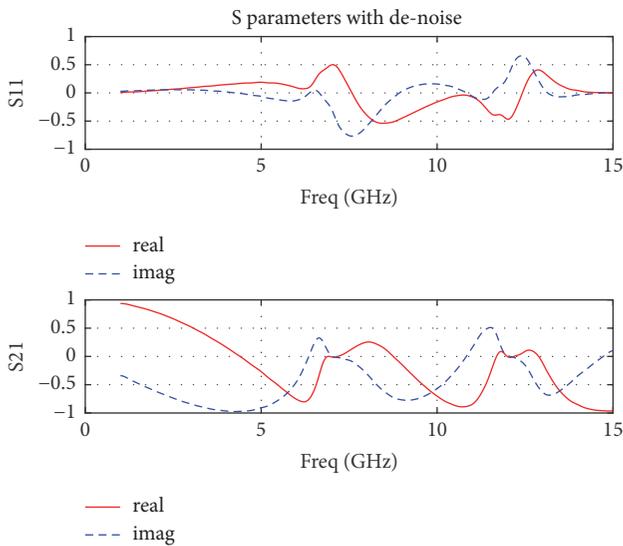


FIGURE 7: The S11 and S21 signals with noise removed.

resemblance between the signal and the wavelet basis. The decomposition level is determined by using the ratio of peak-to-sum for the detail coefficient of wavelet transform.

4. Effective Parameters Extraction of the Photonic Crystal Slab

Photonic crystals are comprised of periodic, dielectric structures. In its forbidden bands, the electromagnetic wave cannot travel through the photonic crystal. These disallowed bands of frequencies are called photonic band gaps. The fabrication of optical photonic crystals is quite complex. There are several methods to calculate the dispersion relation and thereby the range of the band gaps. Reference [8] presented a theory of the Fano resonance for optical resonators, based on temporal coupled-mode formalism. Here we consider this problem from the aspect of the effective medium.

Considering a photonic crystal as shown in Figure 10, the unit cell is perforated with cylinder. The period of photonic crystal is $50 \mu\text{m}$, the thickness of the slab is $25 \mu\text{m}$, and the

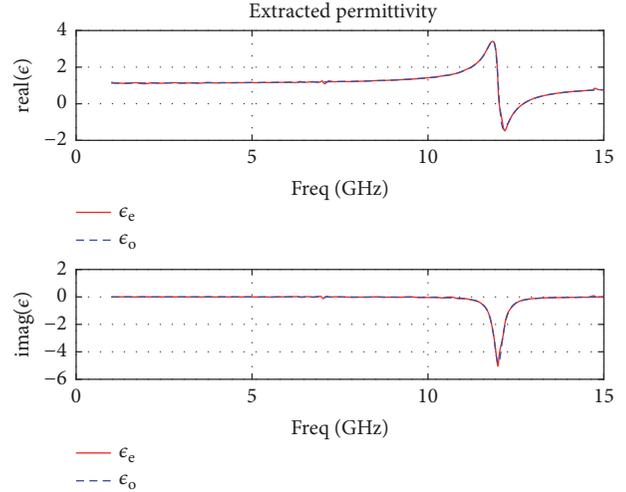


FIGURE 8: The comparison between the extracted permittivity and original permittivity.

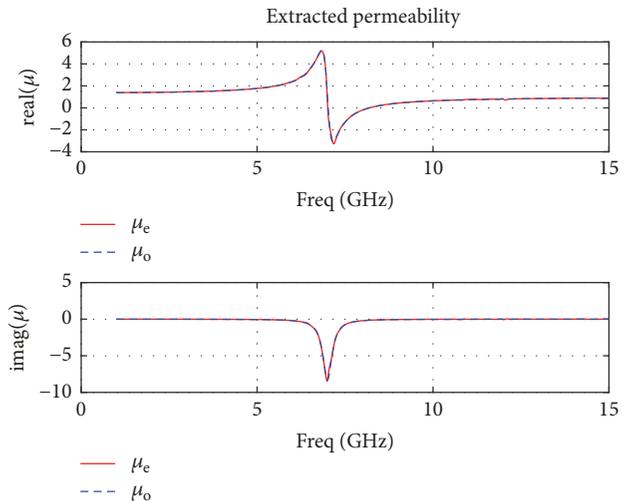


FIGURE 9: The comparison between the extracted permeability and original permeability.

radius of the perforated cylinder is $20 \mu\text{m}$. The permittivity of unit cell is 12 and the permeability is 1.

The numerical method FEM is used to compute the S11 and S21 parameters in the frequency band 1.4 THz to 2.8 THz, which is used to extract the effective permittivity and permeability of the photonic crystal slab. In the simulation, the incidence wave direction is normal to the slab surface. The reference planes of the two ports are selected as the slab top surface and bottom surface, respectively, which is important in extracting the correct effective permittivity and permeability. The wavelet denoising method is also utilized in the photonic crystal results, and the noise is also eliminated effectively by the method proposed in this paper. The simulated S11 and S21 parameters noise-added and denoised results are shown in Figure 11.

The extracted effective parameters of the photonic crystal are shown in Figure 12. It can be seen from the results

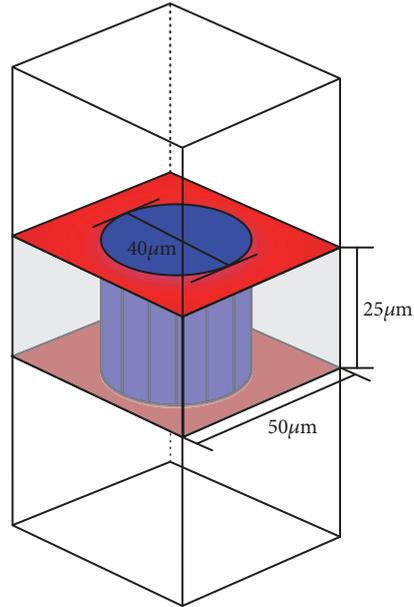


FIGURE 10: A unit cell of photonic crystal structure for numerical simulation.

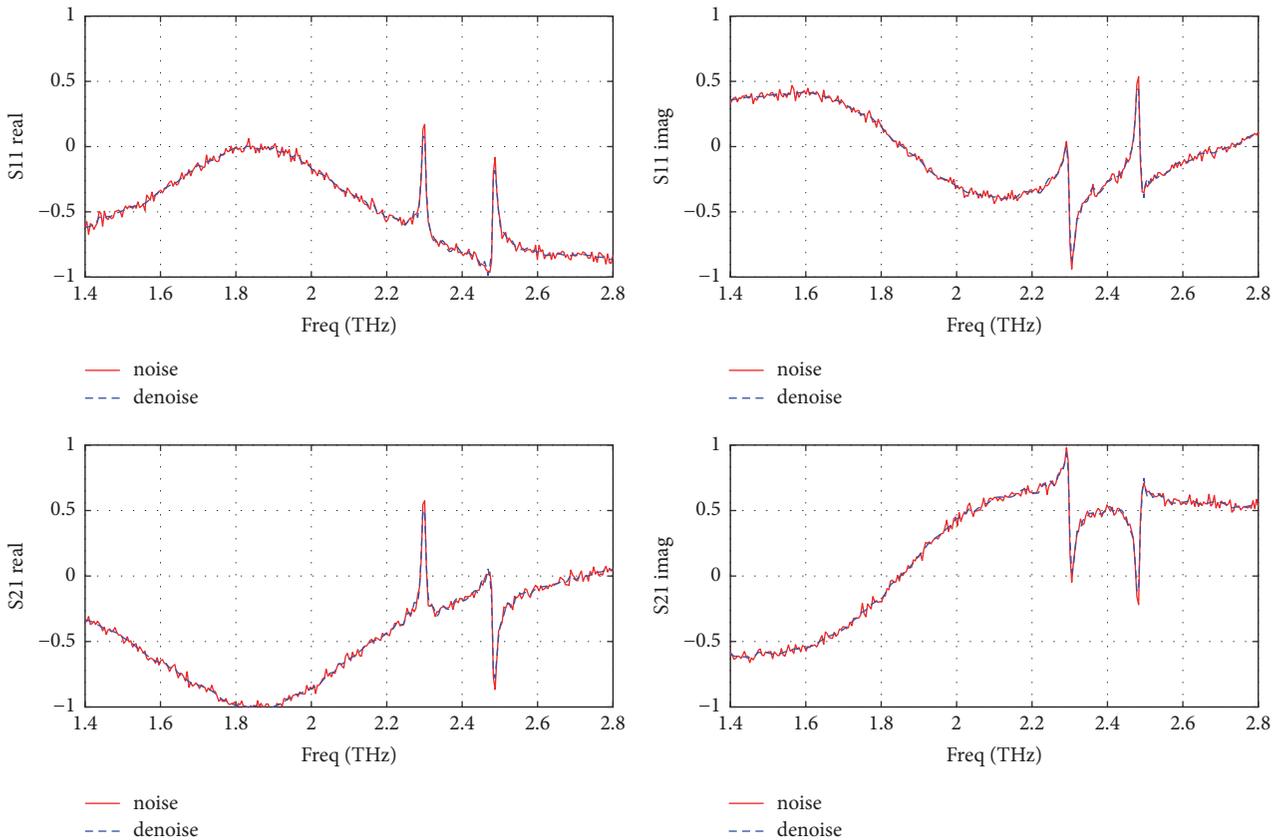


FIGURE 11: The S11 and S21 results of the photonic crystal.

that the resonance peaks appear in the Fabry slab resonance frequency and cavity resonance frequencies. Near the 1.9 THz, both permittivity and permeability have resonance peak, which can be attributed to the induced charge and current that are related to the changes of the electric field and

magnetic field. The peaks near the 2.35 THz and 2.5 THz are caused by the cavity resonance. Away from the peak, both the permittivity and permeability appeared as the average value of the slab material and the air in the cylinder. The results suggest that the permittivity and permeability can

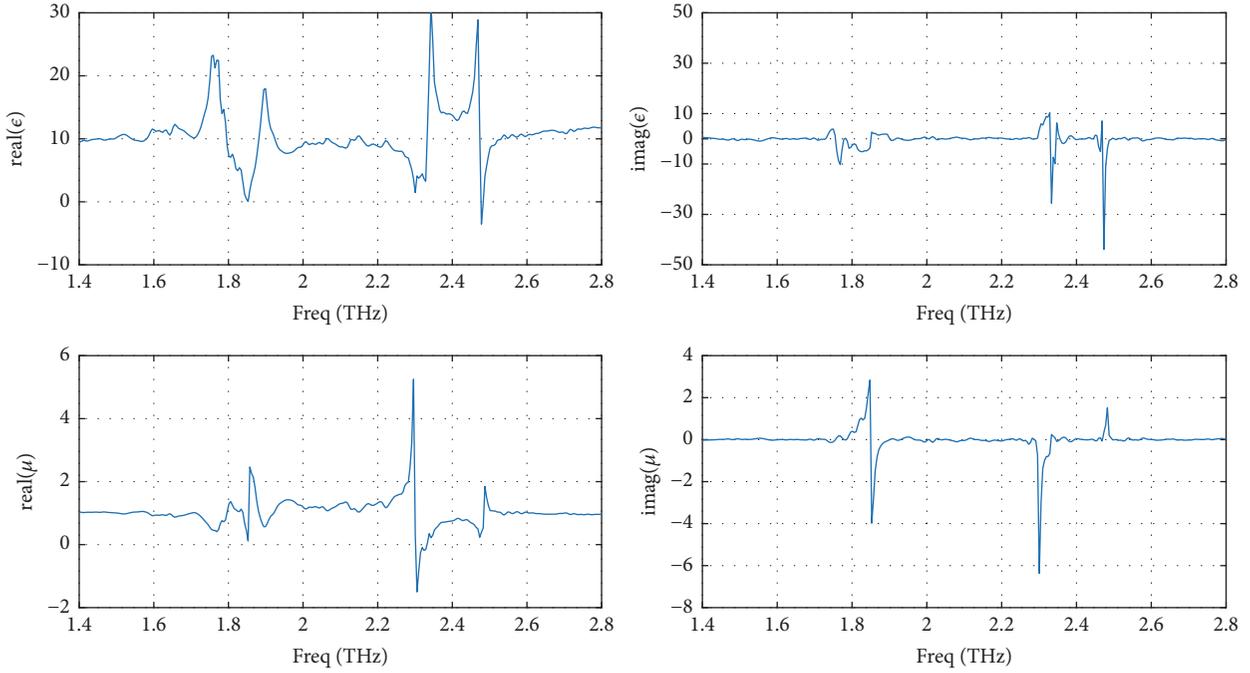


FIGURE 12: The effective permittivity and permeability for photonic crystal.

be obtained from the photonic crystal, which gives us more choices in the design.

As a validation, the S11 and S21 are computed from the effective homogeneous medium with the extracted permittivity and permeability shown in Figure 12. The S11 and S21 comparisons are made between the computed numerical results and effective medium results. The two results in Figure 13 show good agreements, which suggest the extracted parameters are correct.

The refraction index reflects the effect of the permittivity and permeability for the wave propagation phase, and the refraction index and impedance of the photonic crystal calculated from the scattering parameters are shown in Figures 14 and 15. It can be seen that the imaginary parts of the refraction index have negative values at the resonance frequencies, which seem to be against the energy conservation law. However, by combining the impedance results, the real part of the impedance is zero and the imaginary part of the impedance is negative at the resonance frequencies, which mean that the energy is stored and cannot propagate, just obeying the energy conservation law.

The impedance of the photonic crystal appears as average value, just like the mixture of the different materials far from the resonance position. In the resonance position, the real part of the impedance gives a lower or higher value. For the imaginary part of the impedance, there are large negative value or large positive value, which corresponds to the stored capacitive energy and inductive energy. Hence, in conclusion, only a small part of the energy is propagated along the propagation direction at the resonance frequency, and most of the energy is stored in the photonic crystal, which can also explain the negative imaginary value of the refraction index.

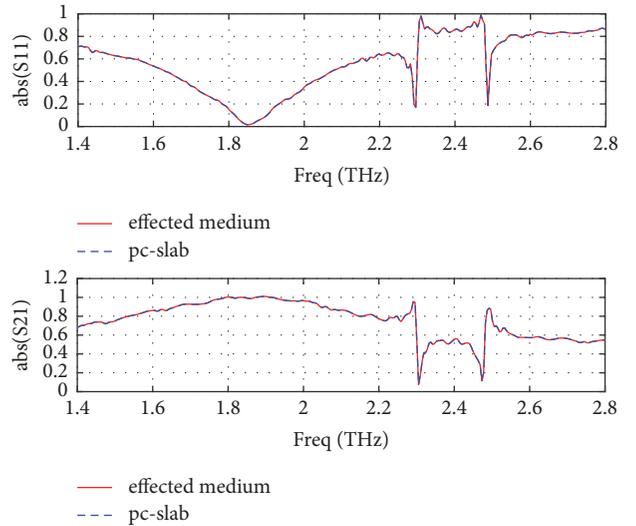


FIGURE 13: Scattering parameters comparison between the photonic crystal and effective medium.

The dispersion electromagnetic parameters such as permittivity and permeability abide by the Kramers-Kronig relation. The effective parameters of photonic crystal have the dispersion properties which may have negative properties in some frequencies; it is helpful to validate the Kramers-Kronig relation for the photonic crystal [12]. The Kramers-Kronig relation can be represented as follows:

$$\text{Re}(\epsilon(\omega)) = \epsilon_{\infty} + \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{\text{Im}(\epsilon(\omega'))}{\omega' - \omega} d\omega' \quad (11)$$

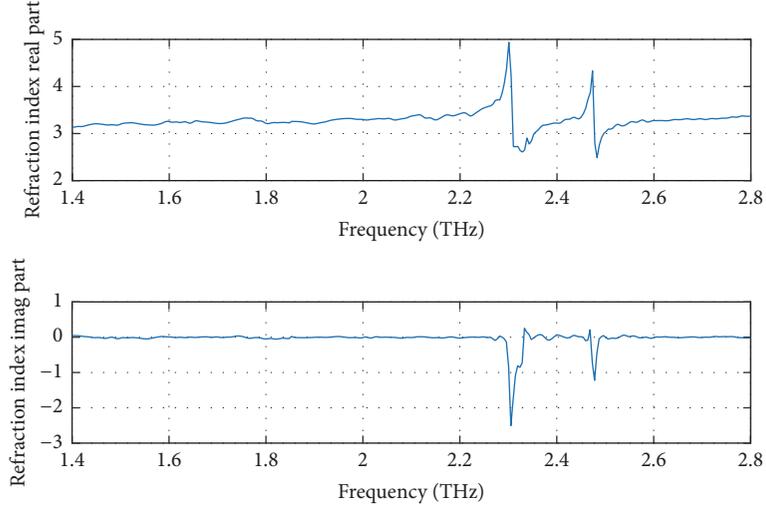


FIGURE 14: The refraction index calculated from the scattering parameters.

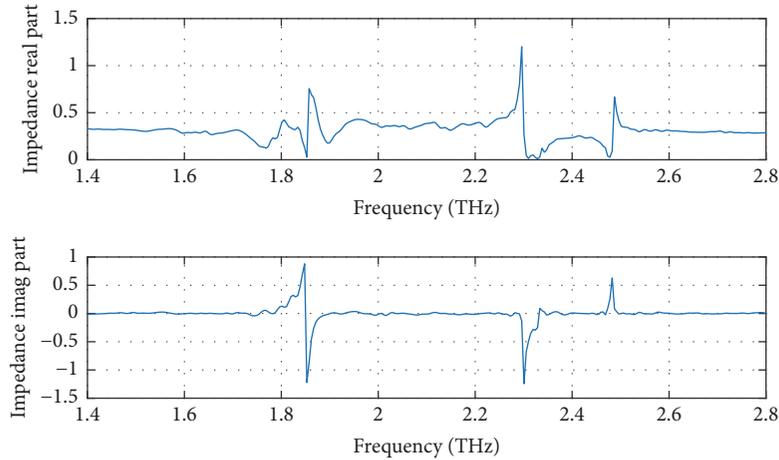


FIGURE 15: The normalized impedance calculated from the scattering parameters.

$$\text{Im}(\varepsilon(\omega)) = -\frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{\text{Re}(\varepsilon(\omega') - 1)}{\omega' - \omega} d\omega' \quad (12)$$

where P represents the principal value integral. Figures 16 and 17 give the results of the permittivity and permeability by using Kramers-Kronig relation, where KK means the Kramers-Kronig consequences and EE means the extracted effective results. Among them, the real part of the parameters is obtained from the imaginary part of the parameters and vice versa, which shows that both the permittivity and permeability obey the Kramers-Kronig relation. Because the permittivity and permeability are negative in some frequency, the validated results show that the Kramers-Kronig relation also holds in the negative parameter values.

5. Discussion and Conclusion

Our results indicate that the proposed technique is efficient when applied to material parameters extraction for realistic

material samples from the S-parameters. The phase ambiguity may cause the multivalued problem and give the wrong results in the material parameters extraction. The proposed phase unwrapping method can solve the phase ambiguity problem. Correctness of the effective parameters depends on the S11 and S21 parameters which are measured or computed by numerical method. The noise coming from the measurement and simulation may induce the errors. In order to address this issue, the wavelet denoising method is employed to eliminate the noise. The results demonstrate that the wavelet denoising method can improve the noised S11 and S21 signals significantly in parameters extraction. Finally, from the aspect of the effective medium, the effective homogeneous medium parameters of the photonic crystal slab are computed, which are validated by comparing the S parameter from numerical computing results and effective homogeneous medium parameter. The explanation for the effective homogeneous medium parameters is given, which can help us to understand the function of photonic crystal slab and the design in engineering.

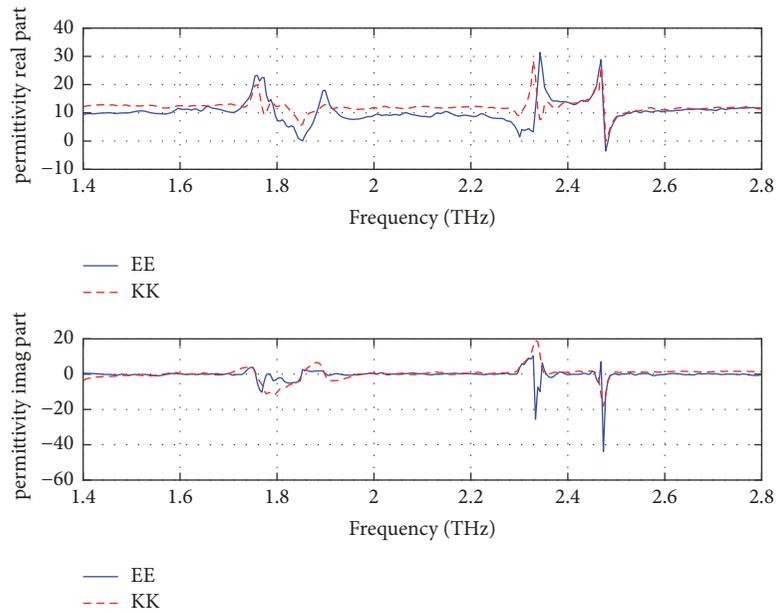


FIGURE 16: The comparison between the extracted permittivity and Kramers-Kronig results.

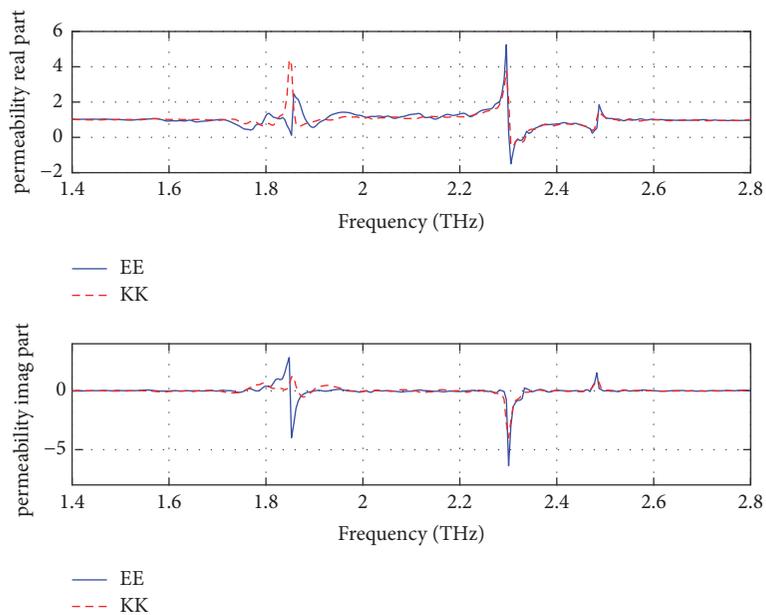


FIGURE 17: The comparison between the extracted permeability and Kramers-Kronig results.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

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