Research Article

Performance Evaluation of MPPM-Coded Wireless Optical MIMO System with Combined Effects over Correlated Fading Channel

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The performance of wireless optical MIMO system with multiple pulse position modulation (MPPM) over correlated fading channel is investigated. The combined effects of atmospheric attenuation, atmospheric turbulence, and pointing error are taken into consideration. The bit error rate (BER) and the ergodic channel capacity are analyzed by utilizing the Poisson counting model and the exponential correlation model. Moreover, their approximate expressions are derived. The simulation results demonstrate that the pointing error is the most prominent influence factor over weak correlated channel. The performance degradation caused by a high channel correlation coefficient is more than that of pointing error in strong correlated channel. Therefore, the use of pointing, acquisition, and tracking (PAT) system and reasonable arrangement of the number and spacing of antennas at the transceiver are the keys to improve system performance.

1. Introduction

Wireless optical communication (WOC) has the advantages of both optical fiber links and wireless communications which has attracted a lot of attention in recent years [1, 2]. One of the major challenges in WOC is channel fading caused by atmospheric attenuation and atmospheric turbulence [3]. Another major challenge is pointing error which cannot be neglected among the fading due to dynamic wind loads, thermal expansion, and internal vibrations [4]. To eliminate the fading caused by atmospheric channel and pointing error, a large fraction of research has focused on multiple-input multiple-output (MIMO) technology to improve the system performance [5]. The bit error rate (BER) performance and ergodic channel capacity of the WOC MIMO system with atmospheric attenuation and atmospheric turbulence have been studied in references [6, 7]. Furthermore, the influence of misalignment characterized by the pointing error on outage probability, channel capacity, and average BER has been studied in references [8–10].

However, the aforementioned investigations have neglected the spatial correlation of MIMO subchannels. In

practical WOC MIMO applications, the true benefits provided by multiple lasers may be limited by spatial correlation [11, 12]. The spatial correlation is quantified as a function of beam separation, turbulence strength, propagation distance, and receiver aperture for the multiple-input single-output (MISO) system in [13]. Moreover, the relationship of the correlation coefficient and receiving aperture size for the MIMO system has been analyzed in reference [14]. Thereafter, the BER and the outage probability of correlated gamma-gamma fading channel have been studied in reference [15]. The results indicate that the system performance is very sensitive to the presence of spatial correlation. Therefore, spatial correlation needs to be considered to analyze WOC MIMO performance.

On the contrary, background radiation is one of the major noise sources in WOC MIMO links, and the background radiation that impacts on BER and outage capacity of typical system has been considered in reference [16]. In addition, most of the researches utilize on-off keying (OOK) and pulse position modulation (PPM) as their modulation format. As is known, OOK has lower transmission efficiency and is difficult to satisfy the requirement of high-speed transmission [17, 18]. PPM has lower bandwidth efficiency

and higher synchronization requirement [19]. However, MPPM gets much attention due to its advantages of higher optical power utilization and background radiation tolerance than OOK and PPM in high-speed intensity modulation/direct detection (IM/DD) communications [20]. Therefore, it is necessary to quantify the performance of MPPM-coded WOC MIMO systems. The symbol error rate (SER) or the average BER performance of the MIMO system with the MPPM has been analyzed over turbulence fading channel in references [21–23]. But the spatial correlation and the background radiation are not considered in references [20–23].

Based upon the abovementioned analysis, it could be concluded that the factors that can make impacts on the WOC MIMO applications include atmospheric attenuation, atmospheric turbulence, pointing error, spatial correlation, background radiation, and modulation format. However, to the best of our knowledge, the performance of the MPPM-coded WOC MIMO system under combined effects over correlated fading channel has not been considered in previous studies. Concerning this issue, we focus on the BER performance and the ergodic channel capacity of the MPPM-coded WOC MIMO system over correlated log-normal channel when taking the background radiation and the combined effects into consideration. Also, their approximate expressions with low complexity are derived to achieve the performance bound of the WOC MIMO system in practical communication link. The rest of this paper is organized as follows: in Section 2, theoretical analysis is proposed. In Section 3, BER and ergodic channel capacity are analyzed, and their approximate expressions with combined effects over correlated fading channel are derived. Numerical simulation results are given in Section 4, where the Monte Carlo method is utilized to investigate the performance of our proposal. Finally, a conclusion is given in Section 5.

2. Theoretical Analysis

2.1. Correlated Channel. The channel coefficient under the combined effects of atmospheric attenuation, atmospheric turbulence, and pointing errors can be written as

$$h = h^1 h^a h^p = A_0 e^{\chi - 2R^2/(\omega^2 - \sigma L)}, \quad (1)$$

where $h^1 = \exp(-\sigma L)$ is the laser power attenuation with transmission distance L , σ is the attenuation coefficient, h^l can be considered as a constant scaling factor during a long period of time [24], $h^a = \exp(\chi)$ is the turbulence fading coefficient, and χ is a Gaussian random variable with mean μ_χ and variance σ_χ^2 . The variance $\sigma_\chi^2 = -2\mu_\chi$ can be rewritten as $\sigma_\chi^2 = 1.23C_n^2 k^{7/6} L^{11/6}$ [25]. C_n^2 is the refractive-index structure parameter, $k = 2\pi/\lambda$ is the wave number, and λ is the wavelength. $h^p \approx A_0 \exp(-2R^2/\omega^2)$ is the loss caused by zero-boresight misalignment [8], $A_0 = [\text{erf}(v)]^2$ is the fraction of the collected power at $R = 0$, and R is the radial displacement at the receiver plane. $\omega^2 = \omega_L^2 \sqrt{\pi} \text{erf}(v)/[2v \exp(-v^2)]$, $v = \sqrt{\pi}a/(\sqrt{2}\omega_L)$, ω_L is the beam waist at distance L , and a is the receiving aperture radius.

It should be noted that in practical WOC MIMO systems, there is a certain spatial correlation among subchannels. Therefore, the channel coefficient matrix of the correlated channel can be defined as

$$\mathbf{G} = \mathbf{R}_r \mathbf{H} \mathbf{R}_t = [g_{nm}]_{NM}, \quad (2)$$

where $\mathbf{H} = [\mathbf{h}_{nm}]_{NM}$ is the intensity fading coefficient matrix when the subchannels are independent, $\mathbf{R}_r$ is the $N \times N$ receiver correlation matrix, and $\mathbf{R}_t$ is the $M \times M$ transmitter correlation matrix. In order to make a simple description of spatial correlation, the exponential correlation model [26] can be used to define $\mathbf{R}_r$ and $\mathbf{R}_t$ as $\mathbf{R}_r = [\mathbf{r}_r^{[c-b]}]_{c,b=1,\dots,N}$ and $\mathbf{R}_t = [\mathbf{r}_t^{[c-b]}]_{c,b=1,\dots,M}$, where r_r ($0 \leq r_r \leq 1$) and r_t ($0 \leq r_t \leq 1$) are the correlation coefficients of receiver and transmitter, respectively.

When the transmitter and receiver are correlated ($r_r \neq 0$, $r_t \neq 0$), the channel coefficient matrix can then be expressed as

$$\mathbf{G} = [g_{nm}]_{NM} = [\mathbf{r}_r^{[c-b]}]_{c,b=1,\dots,N} \cdot \left[A_0 e^{\chi_{nm} - 2R_{nm}^2/(\omega^2 - \sigma L)} \right]_{NM} \cdot [\mathbf{r}_t^{[c-b]}]_{c,b=1,\dots,M}. \quad (3)$$

2.2. MIMO Channel. A WOC MIMO system is composed of M lasers diodes (LDs) and N photodetectors (PDs), as shown in Figure 1.

Assume the total transmitting power is E_s , then the average power of each laser is E_s/M . When using the Q-ary MPPM modulation format, a symbol period is divided into Q slots. Each symbol contains ζ “on” slots ($\zeta \leq Q$) and $Q - \zeta$ “off” slots. Therefore, there are $C_Q^\zeta = \binom{Q}{\zeta}$ number of

symbols available. As a result, a subset of size $\log_2 \left(\binom{Q}{\zeta} \right)$ can be selected for symbol constellation. The modulated signal that transmitted on the m -th ($m = 1, 2, \dots, M$) LD can be represented by $x_m = [0, 0, \dots, A_m, \dots, A_m, \dots, 0]$, where A_m denotes light intensity in an “on” slot and 0 denotes an “off” slot. Assume the mean of counted photons in “on” slots is $\lambda_{\text{on}|g_{nm}}$ and in “off” slots is λ_{off} , respectively. When the Poisson statistics model and the equal gain combination (EGC) method are utilized at the receiver end, then there are

$$\begin{cases} \lambda_{\text{on}|g_{nm}} = \frac{\eta E_s}{\epsilon f M} \sum_{n=1}^N \sum_{m=1}^M g_{nm} + \frac{\eta N E_b}{\epsilon f}, \\ \lambda_{\text{off}} = \frac{\eta N E_b}{\epsilon f}, \end{cases} \quad (4)$$

where η is the photoelectric conversion efficiency, f is the center frequency of optical carriers, ϵ is the Planck constant, E_b is the noise power at the PD, and g_{nm} is the channel coefficient from the m -th LD to the n -th PD.

It can be seen from equation (4) that the simplification of $\sum_{m=1}^M \sum_{n=1}^N g_{nm}$ is important for the effective calculation of photons at the receiver. When the subchannels are correlated, the assuming that variable D is equal to the $\sum_{m=1}^M \sum_{n=1}^N g_{nm}$ in equation (4) and D should be written as

FIGURE 1: WOC MIMO system with M LDs and N PDs.

$$\begin{aligned}
 D &= \sum_{m=1}^M \sum_{n=1}^N g_{nm} = \sum_{m=1}^M \sum_{n=1}^N (r_r^{|c-b|})_{c,b=1,2,\dots,N} \cdot \left[A_0 \exp\left(\chi_{nm} - \frac{2R_{nm}^2}{\omega^2 - \sigma L}\right) \right] \cdot (r_t^{|c-b|})_{c,b=1,2,\dots,M} \\
 &= A_0 \exp\left(-\frac{2R^2}{\omega^2}\right) \exp(-\sigma L) \sum_{m=1}^M \sum_{n=1}^N (r_r^{|c-b|})_{c,b=1,2,\dots,N} \cdot \exp\left[\chi_{nm} - \frac{2(R_{nm}^2 - R^2)}{\omega^2}\right] \cdot (r_t^{|c-b|})_{c,b=1,2,\dots,M} \\
 &= A_0 l_0 \exp(-\delta) \sum_{m=1}^M \sum_{n=1}^N (r_r^{|c-b|})_{c,b=1,2,\dots,N} \cdot \exp(K_{nm}) \cdot (r_t^{|c-b|})_{c,b=1,2,\dots,M},
 \end{aligned} \tag{5}$$

where $l_0 = \exp(-\sigma L)$, $\delta = 2R^2/\omega^2$ and its probability density function (PDF) is $f_\delta(\delta) = \gamma^2 e^{-\gamma^2 \delta}$, $\gamma = \omega/(2\sigma_s)$, and σ_s is the standard deviation of the jitter offset at the receiver. The jitter variance σ_s^2 is generally used to define the intensity of pointing error. $K_{nm} = \chi_{nm} - 2(R_{nm}^2 - R^2)/\omega^2$ follows normal distribution; therefore, its mean and variance can be written as $\mu_{K_{nm}} = \mu_{\chi_{nm}} - (2/\omega^2)\|\phi_m - \phi_n\|^2$, and $\sigma_{K_{nm}}^2 = \sigma_{\chi_{nm}}^2 + (16\sigma_s^2/\omega^4)(\phi_m - \phi_n)^T(\phi_m - \phi_n)$, respectively, where ϕ_n ($\phi_n \in \psi$, ψ is the coordinate set of all receiving apertures) denotes the central coordinate vector of the n -th received laser beam.

On the contrary, when K_{nm} multiplies $\mathbf{R}_t$ and $\mathbf{R}_r$, the result still follows the normal distribution. Therefore, variable D can be expressed as

$$D = A_0 l_0 \exp(-\delta) \sum_{k=1}^{NM} \exp(V_k), \tag{6}$$

where V_k follows normal distribution, and its mean and variance are $\mu_{V_k} = \mu_{K_{nm}} \cdot [(1 - r_t^M)/(1 - r_t)] \cdot [(1 - r_r^N)/(1 - r_r)]$ and $\sigma_{V_k}^2 = \sigma_{K_{nm}}^2 \cdot [(1 - r_t^{2M})/(1 - r_t^2)] \cdot [(1 - r_r^{2N})/(1 - r_r^2)]$, respectively.

The sum of log-normal variables can be approximated to another log-normal variable [27], that is, $\sum_{k=1}^{NM} \exp(V_k) = \exp(W)$. Therefore, variable D can be simplified as

$$D = A_0 l_0 \exp(W - \delta) = A_0 l_0 \exp(U), \tag{7}$$

where $U = W - \delta$, W is a normal distributed variable and their PDF can be written as $f_W(W) = (1/\sqrt{2\pi}\sigma_W) \exp(-(W - \mu_W)^2/2\sigma_W^2)$. The mean $\mu_W = 2\ln \alpha - 1/2 \ln \beta$ and the variance $\sigma_W^2 = \ln \beta - 2\ln \alpha$, where $\alpha = \sum_{k=1}^{NM} e^{\mu_{V_k} + (\sigma_{V_k}^2/2)}$, and $\beta = \sum_{k=1}^{NM} \exp(2\mu_{V_k} + 2\sigma_{V_k}^2) + 2\sum_{k=1}^{NM-1} \sum_{j=k+1}^{NM} \{\exp(\mu_{V_k} + \mu_{V_j}) \exp[1/2(\sigma_{V_k}^2 + \sigma_{V_j}^2 + 2r_{kj}\sigma_{V_k}\sigma_{V_j})]\}$.

As a result, $\lambda_{\text{on}|D}$ can be simplified as

$$\lambda_{\text{on}|D} = \frac{\eta E_s D}{\epsilon f M} + \frac{\eta N E_b}{\epsilon f} = \frac{\eta E_s A_0 l_0 \exp(U)}{\epsilon f M} + \frac{\eta N E_b}{\epsilon f}. \tag{8}$$

The probability distribution of U can be expressed as

$$f_U(U) = \int_0^\infty f_{U|\delta}(U|\delta) f_\delta(\delta) d\delta, \tag{9}$$

where

$$f_{U|\delta}(U|\delta) = \frac{1}{\sqrt{2\pi}\sigma_W} \exp\left\{-\frac{[U - (\mu_W - \delta)]^2}{2\sigma_W^2}\right\}. \tag{10}$$

Substituting $f_\delta(\delta)$ and equation (10) into equation (9), the result is

$$f_U(U) = \frac{\gamma^2}{2} \exp\left[\frac{\gamma^4 \sigma_W^2}{2} + \gamma^2(U - \mu_W)\right] \text{erfc}\left(\frac{U + \gamma^2 \sigma_W^2 - \mu_W}{\sqrt{2}\sigma_W}\right). \tag{11}$$

Therefore, the PDF of variable D can then be expressed as

$$\begin{aligned}
 f_D(D) &= \frac{1}{D} f_U(\ln D) = \frac{\gamma^2}{2D} \exp\left[\frac{\gamma^4 \sigma_W^2}{2} + \gamma^2(\ln D - \mu_W)\right] \\
 &\quad \cdot \text{erfc}\left(\frac{\ln D + \gamma^2 \sigma_W^2 - \mu_W}{\sqrt{2}\sigma_W}\right).
 \end{aligned} \tag{12}$$

It should be noted that the cumulative distribution of intensity attenuation coefficients is no longer a log-normal distribution under the combined effects. It follows a distribution composed of coefficients, complementary error function, and exponential function.

3. Performance Analysis

3.1. Maximum Likelihood (ML) Detection. Assume $Z = \{z_{nq}, n = 1, 2, \dots, N; q = 1, 2, \dots, Q\}$ is a set of received

observations, where z_{nq} denotes the observed photoelectrons of n -th PD in time slot q . When employing the EGC method, there is $z_q = \sum_{n=1}^N z_{nq}$. If one of the $\binom{Q}{\zeta}$ binary

patterns in the x_k is launched, then the value detected by ML can be written as

$$\hat{x}_k = \arg \max_{x_k} f(Z | x_k) = \arg \max_{x_k} \prod_{q \in Q_{\text{on}}^{(k)}} \frac{\exp(-\lambda_{\text{on}|g_{nm}})(\lambda_{\text{on}|g_{nm}})^{z_q}}{z_q!} \prod_{q \in Q_{\text{off}}^{(k)}} \frac{\exp(-\lambda_{\text{off}})(\lambda_{\text{off}})^{z_q}}{z_q!}, \quad (13)$$

where $Q_{\text{on}}^{(k)}$ and $Q_{\text{off}}^{(k)}$ denote the k -th “on” and “off” slot, respectively. $z_q!$, $\exp(-\lambda_{\text{on}|g_{nm}})$, and $\exp(-\lambda_{\text{off}})$ are constant

to x_k . Therefore, equation (13) can be simplified by taking the logarithm as

$$\begin{aligned} \hat{x}_k &= \arg \max_{x_k} \prod_{q \in Q_{\text{on}}^{(k)}} (\lambda_{\text{on}|g_{nm}})^{z_q} \prod_{q \in Q_{\text{off}}^{(k)}} (\lambda_{\text{off}})^{z_q} = \arg \max_{x_k} \left[\sum_{q \in Q_{\text{on}}^{(k)}} z_q \ln(\lambda_{\text{on}|g_{nm}}) + \sum_{q \in Q_{\text{off}}^{(k)}} z_q \ln(\lambda_{\text{off}}) \right] \\ &= \arg \max_{x_k} \left[\sum_{q \in Q_{\text{on}}^{(k)}} z_q \ln \left(\frac{\eta E_s}{\epsilon f M} \sum_{m=1}^M \sum_{n=1}^N g_{nm} + \frac{\eta N E_b}{\epsilon f} \right) + \sum_{q \in Q_{\text{off}}^{(k)}} z_q \ln \left(\frac{\eta N E_b}{\epsilon f} \right) \right]. \end{aligned} \quad (14)$$

Since $\sum_{m=1}^M \sum_{n=1}^N g_{nm}$ is simplified to D , equation (14) can be written as

$$\hat{x}_k = \arg \max_{x_k} \left[\sum_{q \in Q_{\text{on}}^{(k)}} z_q \ln(\lambda_{\text{on}|D}) + \sum_{q \in Q_{\text{off}}^{(k)}} z_q \ln(\lambda_{\text{off}}) \right] = \arg \max_{x_k} \left[\sum_{q \in Q_{\text{on}}^{(k)}} z_q \ln \left(\frac{\eta E_s D}{\epsilon f M} + \frac{\eta N E_b}{\epsilon f} \right) + \sum_{q \in Q_{\text{off}}^{(k)}} z_q \ln \left(\frac{\eta N E_b}{\epsilon f} \right) \right]. \quad (15)$$

The results and discussion may be presented separately, or in one combined section, and may optionally be divided into headed subsections.

3.2. Bit Error Rate Performance. Assume the signal passes through a slow fading channel, which means the channel fading coefficient is constant in a symbol period. Taking the background radiation into consideration, the possible errors fall into two categories: the definite error and the indefinite error.

3.2.1. Definite Error Probability. The definite error probability can be defined as that the number of photoelectrons detected in an “off” slot is higher than that in an “on” slot. Let $\{S_{\text{on},l}, l = 1, \dots, \zeta\}$ and $\{S_{\text{off},l}, l = 1, \dots, Q - \zeta\}$ denote the sum of photoelectrons count in “on” and “off” slots, respectively. The definite error probability can be expressed as

$$\begin{aligned} P_{\text{def}}|g_{nm} &= \sum_{u=0}^{\infty} P \left[\left(\left| \{l: S_{\text{on}|g_{nm},l} = u\} \right| = i \right) \cap \left(\left| \{l: S_{\text{on}|g_{nm},l} > u\} \right| = \zeta - i \right) \cap \left(\left| \{l: S_{\text{off},l} > u\} \right| = j \right) \cap \left(\left| \{l: S_{\text{off},l} \leq u\} \right| = Q - \zeta - j \right) \right] \\ &= \sum_{u=0}^{\infty} \sum_{i=1}^{\zeta} \binom{\zeta}{i} P(S_{\text{on}|g_{nm}} = u)^i P(S_{\text{on}|g_{nm}} > u)^{\zeta-i} \times \sum_{j=1}^{Q-\zeta} \binom{Q-\zeta}{j} P(S_{\text{off}} > u)^j P(S_{\text{off}} \leq u)^{Q-\zeta-j}. \end{aligned} \quad (16)$$

Since the received photoelectrons in “on” and “off” slots follow Poisson distribution, which can be described as

$$\begin{cases} P(S_{\text{on}} | g_{nm} = \mu) = \frac{\lambda_{\text{on}}^\mu | g_{nm} e^{-\lambda_{\text{on}} | g_{nm}}}{\mu!}, & \mu = 0, 1, 2, \dots, \\ P(S_{\text{on}} | g_{nm} \leq \mu) = \sum_{b=0}^{\mu} \frac{\lambda_{\text{on}}^b | g_{nm} e^{-\lambda_{\text{on}} | g_{nm}}}{b!}, & \mu = 0, 1, 2, \dots \end{cases} \quad (17)$$

Thus, equation (16) can be rewritten as

$$P_{\text{def}} | g_{nm} = \sum_{u=0}^{\infty} \sum_{i=1}^{\zeta} \binom{\zeta}{i} \left(\frac{\lambda_{\text{on}}^\mu | g_{nm} e^{-\lambda_{\text{on}} | g_{nm}}}{\mu!} \right)^i \left(1 - \sum_{b=0}^{\mu} \frac{\lambda_{\text{on}}^b | g_{nm} e^{-\lambda_{\text{on}} | g_{nm}}}{b!} \right)^{\zeta-i} \cdot \sum_{j=1}^{Q-\zeta} \binom{Q-\zeta}{j} \left(1 - \sum_{b=0}^{\mu} \frac{\lambda_{\text{off}}^b e^{-\lambda_{\text{off}}}}{b!} \right)^j \left(\sum_{b=0}^{\mu} \frac{\lambda_{\text{off}}^b e^{-\lambda_{\text{off}}}}{b!} \right)^{Q-\zeta-j} \quad (18)$$

3.2.2. Indefinite Error Probability. The indefinite error probability means that one or more of the “off” slots has the same photoelectrons count as the lowest-valued “on” slots.

As a result, the receiver is likely to make a wrong decision. The indefinite error probability can be expressed as

$$\begin{aligned} P_{\text{indef}} | g_{nm} &= \sum_{u=0}^{\infty} \sum_{i=1}^{\zeta} \binom{\zeta}{i} P(S_{\text{on}} | g_{nm} = \mu)^i P(S_{\text{on}} | g_{nm} > \mu)^{\zeta-i} \cdot \sum_{t=1}^{Q-\zeta} \frac{\binom{t+i}{i} - 1}{\binom{t+i}{i}} \binom{Q-\zeta}{t} P(S_{\text{off}} = \mu)^t P(S_{\text{off}} < \mu)^{Q-\zeta-t} \\ &= \sum_{u=0}^{\infty} \sum_{i=1}^{\zeta} \binom{\zeta}{i} \left(\frac{\lambda_{\text{on}}^\mu | g_{nm} e^{-\lambda_{\text{on}} | g_{nm}}}{\mu!} \right)^i \left(1 - \sum_{b=0}^{\mu} \frac{\lambda_{\text{on}}^b | g_{nm} e^{-\lambda_{\text{on}} | g_{nm}}}{b!} \right)^{\zeta-i} \cdot \sum_{t=1}^{Q-\zeta} \frac{\binom{t+i}{i} - 1}{\binom{t+i}{i}} \binom{Q-\zeta}{t} \left(\frac{\lambda_{\text{off}}^\mu e^{-\lambda_{\text{off}}}}{\mu!} \right)^t \\ &\quad \cdot \left(\sum_{b=0}^{\mu-1} \frac{\lambda_{\text{off}}^b e^{-\lambda_{\text{off}}}}{b!} \right)^{Q-\zeta-t}. \end{aligned} \quad (19)$$

As a result, the conditional probability of symbol error with background radiation and channel fading can be written as

$$\begin{aligned} P_s | g_{nm} &= P_{\text{def}} | g_{nm} + P_{\text{indef}} | g_{nm} = \sum_{u=0}^{\infty} \sum_{i=1}^{\zeta} \binom{\zeta}{i} \left(\frac{\lambda_{\text{on}}^\mu | g_{nm} e^{-\lambda_{\text{on}} | g_{nm}}}{\mu!} \right)^i \left(1 - \sum_{b=0}^{\mu} \frac{\lambda_{\text{on}}^b | g_{nm} e^{-\lambda_{\text{on}} | g_{nm}}}{b!} \right)^{\zeta-i} \\ &\quad \cdot \left[\sum_{j=1}^{Q-\zeta} \binom{Q-\zeta}{j} \left(1 - \sum_{b=0}^{\mu} \frac{\lambda_{\text{off}}^b e^{-\lambda_{\text{off}}}}{b!} \right)^j \left(\sum_{b=0}^{\mu} \frac{\lambda_{\text{off}}^b e^{-\lambda_{\text{off}}}}{b!} \right)^{Q-\zeta-j} + \sum_{t=1}^{Q-\zeta} \frac{\binom{t+i}{i} - 1}{\binom{t+i}{i}} \binom{Q-\zeta}{t} \left(\frac{\lambda_{\text{off}}^\mu e^{-\lambda_{\text{off}}}}{\mu!} \right)^t \left(\sum_{b=0}^{\mu-1} \frac{\lambda_{\text{off}}^b e^{-\lambda_{\text{off}}}}{b!} \right)^{Q-\zeta-t} \right]. \end{aligned} \quad (20)$$

Equation (20) shows the conditional probability of symbol error depends on the sum of channel fading

coefficients $\sum_{m=1}^M \sum_{n=1}^N g_{nm}$. Therefore, the theoretical probability of symbol error is

$$P_s = \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} P_{s|g_{nm}} f(g_{nm}) dg_{nm} \quad (21)$$

$$= \left[\sum_{j=1}^{Q-\zeta} \binom{Q-\zeta}{j} \left(1 - \sum_{b=0}^{\mu} \frac{\lambda_{\text{off}}^b e^{-\lambda_{\text{off}}}}{b!} \right)^j \left(\sum_{b=0}^{\mu} \frac{\lambda_{\text{off}}^b e^{-\lambda_{\text{off}}}}{b!} \right)^{Q-\zeta-j} + \sum_{t=1}^{Q-\zeta} \frac{\binom{t+i}{i}^{-1}}{\binom{t+i}{i}} \binom{Q-\zeta}{t} \left(\frac{\lambda_{\text{off}}^{\mu} e^{-\lambda_{\text{off}}}}{\mu!} \right)^t \left(\sum_{b=0}^{\mu-1} \frac{\lambda_{\text{off}}^b e^{-\lambda_{\text{off}}}}{b!} \right)^{Q-\zeta-t} \right]$$

$$\times \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} \sum_{u=0}^{\infty} \sum_{i=1}^{\zeta} \binom{\zeta}{i} \left(\frac{\lambda_{\text{on}}^{\mu} e^{-\lambda_{\text{on}}}}{\mu!} \right)^i \left(1 - \sum_{b=0}^{\mu} \frac{\lambda_{\text{on}}^b e^{-\lambda_{\text{on}}}}{b!} \right)^{\zeta-i} f(g_{nm}) dg_{11} \cdots dg_{nm}.$$

Because the sum of channel coefficients g_{nm} can be simplified to a variable D , the approximate probability of symbol error with background radiation can be written as

$$P_s = \int_{-\infty}^{\infty} P_{s|D} f(D) dD. \quad (22)$$

Considering that $P_b = [Q/2(Q-1)]P_s$, the approximate BER can be expressed as

$$P_b = \frac{Q}{2(Q-1)} \left[\sum_{j=1}^{Q-\zeta} \binom{Q-\zeta}{j} \left(1 - \sum_{b=0}^{\mu} \frac{\lambda_{\text{off}}^b e^{-\lambda_{\text{off}}}}{b!} \right)^j \left(\sum_{b=0}^{\mu} \frac{\lambda_{\text{off}}^b e^{-\lambda_{\text{off}}}}{b!} \right)^{Q-\zeta-j} + \sum_{t=1}^{Q-\zeta} \frac{\binom{t+i}{i}^{-1}}{\binom{t+i}{i}} \binom{Q-\zeta}{t} \left(\frac{\lambda_{\text{off}}^{\mu} e^{-\lambda_{\text{off}}}}{\mu!} \right)^t \left(\sum_{b=0}^{\mu-1} \frac{\lambda_{\text{off}}^b e^{-\lambda_{\text{off}}}}{b!} \right)^{Q-\zeta-t} \right]$$

$$\int_{-\infty}^{+\infty} \sum_{u=0}^{\infty} \sum_{i=1}^{\zeta} \binom{\zeta}{i} \left(\frac{\lambda_{\text{on}}^{\mu} e^{-\lambda_{\text{on}}}}{\mu!} \right)^i \left(1 - \sum_{b=0}^{\mu} \frac{\lambda_{\text{on}}^b e^{-\lambda_{\text{on}}}}{b!} \right)^{\zeta-i} \frac{\gamma^2}{2D} \exp\left(\frac{\gamma^4 \sigma_W^2}{2} + \gamma^2 (\ln D - \mu_W)\right) \text{erfc}\left(\frac{\ln D + \gamma^2 \sigma_W^2 - \mu_W}{\sqrt{2} \sigma_W}\right) dD. \quad (23)$$

Equation (23) reveals that the approximate BER upper bound related to the parameters of $Q, \zeta, M, N, E_s, E_b, \gamma$, and D . It should be noted that the parameters $l_0, r_t, r_r, \sigma_s^2$, and $\sigma_{\chi_{nm}}^2$ are factors which can influence D .

3.3. *Ergodic Channel Capacity.* According to equation (4), the channel capacity of the wireless optical MIMO system can be defined by

$$C = \max I(x; y), \quad (24)$$

where x and y denote the launched and received signal. The instantaneous channel capacity can be written as

$$C = \log_2 \binom{Q}{\zeta} - \sum_{i=1}^{\zeta} \sum_{d=0}^{\zeta-i} (-1)^d \binom{\zeta}{i} \binom{\zeta-i}{d} \log_2 \binom{Q-\zeta+i}{i} \exp\left(-\frac{\eta E_s (i+d) \sum_m \sum_n g_{nm}}{\epsilon f M}\right). \quad (25)$$

Considering the channel coefficient is a random variable. Therefore, the ergodic channel capacity is usually utilized to analyze the system capacity. As a result, the theoretical

MPPM-coded ergodic channel capacity of the WOC MIMO system can be expressed as

$$C_{\text{avg}} = \log_2 \binom{Q}{\zeta} - \underbrace{\int_0^\infty \dots \int_0^\infty}_{MN} \sum_{i=1}^{\zeta} \sum_{d=0}^{\zeta-i} (-1)^d \binom{\zeta}{i} \binom{\zeta-i}{d} \log_2 \binom{Q-\zeta+i}{i} \exp \left(-\frac{\eta E_s (i+d) \sum_m \sum_n g_{nm}}{\varepsilon f M} \right) f(g_{11}) \dots f(g_{NM}) \underbrace{dg_{11} \dots dg_{NM}}_{MN}. \quad (26)$$

It is complex to perform the $M \times N$ – fold integration. As mentioned in Section 2.2, the sum of $\sum_{m=1}^M \sum_{n=1}^N g_{nm}$ can

be approximated as variable D . Thus, equation (26) can be simplified as

$$C_{\text{avg}} = \log_2 \binom{Q}{\zeta} - \sum_{i=1}^{\zeta} \sum_{d=0}^{\zeta-i} (-1)^d \binom{\zeta}{i} \binom{\zeta-i}{d} \log_2 \binom{Q-\zeta+i}{i} \cdot \int_0^\infty \exp \left(-\frac{\eta E_s (i+d)D}{\varepsilon f M} \right) f(D) dD. \quad (27)$$

The approximation is much simpler than its former version since it only requires a single integration. Substituting equation (12) into equation (27), the

approximate ergodic channel capacity under combined effects can be further expressed as

$$C_{\text{avg}} = \log_2 \binom{Q}{\zeta} - \sum_{i=1}^{\zeta} \sum_{d=0}^{\zeta-i} (-1)^d \binom{\zeta}{i} \binom{\zeta-i}{d} \log_2 \binom{Q-\zeta+i}{i} \cdot \int_0^\infty \frac{\gamma^2}{2D} \exp \left(\frac{\gamma^4 \sigma_W^2}{2} + \gamma^2 (\ln D - \mu_W) + \frac{\eta E_s (i+d)D}{\varepsilon f M} \right) \text{erfc} \left(\frac{\ln D + \gamma^2 \sigma_W^2 - \mu_W}{\sqrt{2} \sigma_W} \right) dD. \quad (28)$$

Equation (28) is a simplification of equation (26). It is evident from equation (28) that the ergodic channel capacity is related to parameters as $Q, \zeta, \varepsilon, f, E_s, M$, and D , where $l_0, \sigma_{\chi_{nm}}^2, \sigma_s^2$ and r_r, r_t are factors which can influence D .

4. Numerical Investigation

In this section, the BER and the ergodic channel capacity of the WOC MIMO system with combined effects over correlated fading channel have been numerically investigated based on the foregoing analysis. Especially, the BER curves are obtained by Monte Carlo simulations, and the curves of ergodic channel capacity are based on the equations. Moreover, all the derived expressions are verified via simulations. Numerical simulations are carried out by MATLAB with parameters as $\eta = 0.5, Q = 5, \zeta = 2, \lambda = 1550 \text{ nm}, l_0 = 0.9, L = 1000 \text{ m}, E_b = -170 \text{ dBJ}, \sigma_s^2 = 0.1, C_n^2 = 2.23 \times 10^{-14}$, and $\sigma_{\chi_{nm}}^2 = 0.3$. The turbulence fading and misalignment fading are characterized by parameters $\sigma_{\chi_{nm}}^2$ and σ_s^2 .

Table 1 demonstrates the 5-ary MPPM symbol constellation with 2 active time slots. It is evident from the table that only 2 time slots need to be activated when sending 3 bit information during a symbol period.

Figure 2 shows the BER curves under combined effects, where CE denotes the condition of combined effects in

independent identically distributed (iid) channel, cor denotes the correlated channel, appo denotes the approximate value of the proposed method, and exact denotes the theoretical value. It is observed that the approximate BER performances are slightly worse than the corresponding theoretical value. Note that the computation complexity is nearly 75% lower than that of the theoretical one. Therefore, such an approximation is appropriate for practical application.

Figure 3 shows the approximate BER performance of a 2×2 WOC MIMO system, where Per denotes the ideal condition, AT denotes the condition only with atmospheric turbulence, and AT&AA denotes the condition with both the atmospheric turbulence and atmospheric attenuation. The result shows that the CE seriously impacts on the system performance. When $\text{BER} = 10^{-4}$, the required energy increases by about 24 dBJ compared with the Per case. It should be noted that the pointing error leads to the most significant impact on the BER performance among all the influencing factors. For example, when $\text{BER} = 10^{-4}$, the required energy for CE is 22.5 dBJ more than that of AT&AA. Therefore, the pointing error is the most prominent influence factor over independent channel.

Figure 4 demonstrates the approximate BER curves versus E_s under different correlation coefficients in a 2×2 WOC MIMO system. It can be observed that the higher the

TABLE 1: MPPM symbol constellation.

Bits	000	001	010	011
Modulation	$x_m = [0, 0, 0, A_m, A_m]$	$x_m = [0, 0, A_m, A_m, 0]$	$x_m = [A_m, A_m, 0, 0, 0]$	$x_m = [0, A_m, A_m, 0, 0]$
Bits	100	101	110	111
Modulation	$x_m = [0, A_m, 0, 0, A_m]$	$x_m = [0, 0, A_m, 0, A_m]$	$x_m = [A_m, 0, A_m, 0, 0]$	$x_m = [0, A_m, 0, A_m, 0]$

FIGURE 2: The exact and approximate of BER under combined effects.

FIGURE 3: BER of different conditions over independent channel in a 2×2 MIMO system.

correlation coefficient, the worse the BER performance. When $\text{BER} = 10^{-4}$ and the correlated coefficients are 0.3, 0.5, and 0.8, the required energy increases by about 10 dBJ, 15 dBJ, and 28 dBJ, respectively, compared with independent

channel. Particularly, the BER goes to its maximum when the correlation coefficient is 1, which can be regarded as the upper bound of the system BER. Moreover, it is evident that the strong channel correlation can seriously impact on the system BER performance. For example, when $\text{BER} = 10^{-4}$, the power loss caused by the combined effects is 24 dBJ, while the power loss caused by strong correlation ($r \geq 0.8$) is 28 dBJ. Therefore, high channel correlation coefficient-caused performance degradation is more than that of combined effects in strong correlated channel. According to Figures 3 and 4, the BER performance of the WOC MIMO system is seriously affected by pointing error over independent and weak correlated channel, while the spatial correlation is the most prominent influence factor over strong correlated channel.

Figure 5 shows the approximate BER performance under combined effects over correlated channel with different turbulence ($\sigma_{\chi_{mm}}^2$) and pointing error jitter variance (σ_s^2). It is evident from the plot that the increased $\sigma_{\chi_{mm}}^2$ and σ_s^2 lead to worse BER, where σ_s^2 plays a more prominent role in the process. For example, at $\text{BER} = 10^{-4}$, the required energy increases by about 0.1 dBJ and 10 dBJ, when $\sigma_{\chi_{mm}}^2$ and σ_s^2 increase from 0.1 to 0.3, respectively. Therefore, the increase of σ_s^2 has a greater impact on the BER performance of the WOC MIMO system than the increase of $\sigma_{\chi_{mm}}^2$.

Figure 6 shows the BER performance of PPM and MPPM formats in a 2×2 MIMO system. It is evident from the plot that the BER performance of $(Q = 5, \zeta = 2)$ MPPM is better than that of 4-PPM and 8-PPM formats over correlated channel. The reason for this is that, for activated pulse combination of MPPM, the redundancy resulting from the constellation causes stronger anti-interference ability. In addition, the spectrum efficiency of $(Q = 5, \zeta = 2)$ MPPM is 0.6 bits/s/Hz, which is 0.1 bits/s/Hz and 0.225 bits/s/Hz higher than that of 4-PPM and 8-PPM, respectively.

Figure 7 shows the ergodic channel capacity of PPM and MPPM formats in a 2×2 MIMO system. It is clear that the ergodic channel capacity of $(Q = 5, \zeta = 2)$ MPPM is 3.4 bits/channel, which is 1.4 bits/channel and 0.4 bits/channel higher than that of 4-PPM and 8-PPM, respectively. Comparing the BER, ergodic channel capacity, and spectrum efficiency of PPM and MPPM, it can be obtained that MPPM has advantages in the wireless optical MIMO communication system.

Figure 8 shows the theoretical and approximate ergodic channel capacity with different number of "on" slots. Both theoretical (equation (26)) and approximate (equation (28)) expressions are depicted in the figure. It is observed that the performance of approximation is slightly worse than the corresponding theoretical one. Note that the computation complexity of approximation is much lower. Therefore, the slight performance loss is acceptable for the approximation.

FIGURE 4: BER of different correlation coefficients in a 2×2 MIMO system.

FIGURE 6: BER performance of different modulation methods in a 2×2 MIMO system.

FIGURE 5: BER with combined effects over correlated channel for different atmospheric turbulence and pointing error jitter variance.

Figure 9 shows the approximate ergodic channel capacity of 2×2 and 4×4 WOC MIMO system over independent channel. It is evident from the plot that the CE will seriously impact on the ergodic channel capacity. For example, when $E_s = -190$ dB, the ergodic channel capacity

FIGURE 7: Ergodic channel capacity of different modulation methods in a 2×2 MIMO system.

reduced by 2.64 bits/channel compared with the Per case in the 4×4 system. It should be noted that the influence of pointing error is more significant than atmospheric attenuation and atmospheric turbulence. The reduction caused by pointing error is about 2.27 bits/channel.

Figure 10 shows the approximate ergodic channel capacity of a 4×4 WOC MIMO system under combined effects over correlated channel with different correlation coefficients. It can be observed that the ergodic channel capacity decreases with the increasing of correlation coefficient. For example, when the correlation coefficients are 0, 0.3, 0.5, and 0.8, the thresholds of capacity saturation are -167.5 dB, -162.5 dB, -155 dB, and

FIGURE 8: Theoretical and approximate ergodic channel capacity with different number of "on" slots.

-135 dBJ, respectively. Furthermore, when the ergodic channel capacity reaches its saturation, the power loss caused by the combined effects is 20 dBJ, while the power loss caused by the strong correlation is 22.5 dBJ, which is greater than that of combined effects. Therefore, it can be considered that strong correlation has more effect on ergodic channel capacity than combined effects.

Figure 11 shows the approximate ergodic channel capacity with different atmospheric turbulence and pointing error jitter variance over correlated channel. The ergodic channel capacity decreases with the increasing of $\sigma_{\chi_{nm}}^2$ and σ_s^2 , where σ_s^2 plays a more important role in the process. For example, at $E_s = -172.5$ dBJ, the ergodic channel capacity decreases by about 0.034 bits/channel and 0.467 bits/channel, when $\sigma_{\chi_{nm}}^2$ and σ_s^2 increase from 0.1 to 0.3, respectively. As a result, the increase of σ_s^2 has a greater impact on the ergodic channel capacity of the WOC MIMO system than the increase of $\sigma_{\chi_{nm}}^2$.

5. Conclusions

The MPPM-coded WOC MIMO system performance over correlated fading channel under combined effects of atmospheric attenuation, atmospheric turbulence, and pointing error is discussed. The approximate expressions of BER and ergodic channel capacity are proposed to reduce the computational complexity. The research results indicate that (1) the pointing error is the prominent influence factor in independent and weak correlated channel and (2) the performance degradation caused by high channel correlation coefficient is more than that of pointing error in strong correlated channel. Therefore, in practical applications, the number and spacing of lasers should be reasonably designed

FIGURE 9: Ergodic channel capacity with different conditions over independent channel.

FIGURE 10: Ergodic channel capacity under combined effects over correlated channel for different correlation coefficients.

to minimize the spatial correlation of subchannels. Additionally, the system performance can be further improved by

FIGURE 11: Ergodic channel capacity with combined effects over correlated channel for different atmospheric turbulence and pointing error jitter variance.

using the beam tracking systems to restrain the pointing error.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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