

Research Article

Approximate Calculation and Feature Analysis of Electric Field in Space by Thunderclouds

Lin Wang (), Haojiang Wan (), and Yazhou Chen ()

Army Engineering University, Shijiazhuang Campus, Key Laboratory on Electromagnetic Environmental Effects, Shijiazhuang 050003, China

Correspondence should be addressed to Yazhou Chen; chen_yazhou@sina.com

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The calculation of electric field in space excited by thunderclouds is an important basis for lightning warning and protection. In numerical calculation of the electromagnetic field, it is often necessary to perform multiple loop nesting calculations on several triple integrals, which consume a lot of computing resources. In order to shorten the calculation time and improve the calculation efficiency, the electric field excited by the charged thunderclouds in space is theoretically derived with the analytical method by the thundercloud cylindrical charge pile model and based on the electrostatic field theory. The complex integrand function is approximated, so that the analytic expression of electric field in space is obtained in this paper. Through simulation and comparison, it is found that the approximate solution and the exact solution are similar in size, the change trend is the same, and the approximate analytical expression can be used for the approximate calculation but also improves calculation efficiency. The calculation time has been shortened from tens of hours to less than one second by using different calculation methods, which is a difference of 7 orders of magnitude. With approximate analytical expression, the electric field excited by charge pile with typical structures in thunderclouds in space is calculated and the characteristics of that are analyzed in this paper. For lightning protection of mobile targets, approximate calculation is of great significance in shortening the lightning warning time and enhancing the protection effect.

1. Introduction

Lightning is a strong ultra-long-distance atmospheric discharge and a common geophysical phenomenon [1, 2] whose mechanism is complex, and the degree of harm to human life and production cause serious damage, so lightning protection is always an important research topic [3]. The research on the physical nature of lightning began in the Franklin era [4, 5]. The research on the physical nature of lightning began in the Franklin era. After more than two centuries of observational research, lightning protection has evolved from direct lightning protection to the use of multiple methods [6] to comprehensively protect direct lightning and lightning electromagnetic pulses [7–9]. The lightning pilot model [10], pilot discharge simulation [11], and other issues are still research hotspots in the fields of lightning physics and lightning protection.

Mastering the characteristics of the electric field distribution of thunderclouds is of great significance for in-depth study of the physical mechanism of lightning, conducting pilot discharge simulation, and carrying out lightning early warning and protection. There are two main methods for modeling thunderclouds. One is to start from the theory of thunderclouds. By establishing coupling equations, given the boundary value relationship, the structure of thundercloud can be obtained by numerical calculation [12–16]. The dynamic change process of thunderclouds can be obtained by this method, but it consumes huge computing resources and the analytical expressions cannot be obtained. The other is not to consider the mechanism of thunderclouds and use

mathematical and physical methods to obtain a simple and effective thundercloud structure based on the electric field observation data in practice [17–19].

However, this method still cannot get a more concise analytical expression, and complex calculations consume a lot of calculation time, and it is not suitable for occasions with high timeliness requirements, for example, the lightning early warning for a maneuvering target; if the calculation time is more than tens of hours, it is impossible to know whether the maneuvering target will be threatened by thunder and lightning and it is impossible to give an early warning of whether the maneuvering target needs to be protected.

In order to obtain a more concise analytical expression of the thundercloud electric field, shorten the calculation time, and facilitate the study of the lightning discharge law, this paper uses the thundercloud cylindrical charge stack model to simulate the charge carried by the thundercloud based on the electrostatic field theory [20]. It is a method to theoretically derive the electric field excited by thunderclouds in space. The approximate analytical expression of the electric field in space is obtained through approximate calculation, and the electric field excited by the typical thundercloud structure in space is calculated and analyzed by the approximate analytical expression. Compared with accurate calculation, approximate calculation has small error and short calculation time, which improves calculation efficiency and saves calculation resources. The approximate calculation method can be applied to the calculation of lightning electric field within a short distance, laying a solid foundation for rapid lightning early warning and effective protection.

2. Theoretical Derivation of the Approximate Calculation Method of Thundercloud Space Electric Field

This paper uses the classic thundercloud cylindrical charge pile mode [21, 22], Figure 1.

According to the electrostatic field theory, the electric charge carried by thunderclouds excites an electric field in space. Under the action of the electric field, induced charges appear on the surface of the Earth. The electric field excited by thundercloud charges and the electric field induced by the induced charges on the Earth's surface. First, the electric field excited by thundercloud charge in a cylindrical coordinate system is solved. Assuming that the volume charge density of a uniformly charged cylinder model is ρ , a point $P'(r', \varphi', z')$ in the cylinder is randomly selected as the coordinate of the charge element and the orientation is positive, the electric field $d\hat{E}$ generated by the charge element $\rho \, dv'$ at the observation point $P(r, \varphi, z)$ is

$$d\widehat{\mathbf{E}}(r,\varphi,z) = -\frac{\rho \, dv'}{4\pi\varepsilon_0 \widehat{R}^3} \widehat{\mathbf{R}}.$$
 (1)

Among them, $\hat{\mathbf{R}}$ is the vector diameter between the thundercloud charge element and the observation point,

$$\widehat{R} = \sqrt{r'2 + r^2 - 2r'r\cos(\varphi - \varphi') + (z' - z)^2}, \quad \varepsilon_0 \quad \text{is the}$$

vacuum permittivity, and the component of the electric field along the *z*-axis is $d\hat{\mathbf{E}}_z$:

$$d\widehat{\mathbf{E}}_{z}(r,\varphi,z) = -\frac{\rho \ d\nu'}{4\pi\varepsilon_{0}\widehat{R}^{2}} \frac{z'-z}{\widehat{R}} \mathbf{e}_{z}, \tag{2}$$

where \mathbf{e}_z is the unit vector in the z-direction.

For the volume of the entire cylinder, the electric field $\hat{\mathbf{E}}_z$ excited by the thundercloud at the observation point $P(r, \varphi, z)$ can be obtained as follows:

$$\widehat{\mathbf{E}}_{z}(r,\varphi,z) = \int_{V} d\widehat{\mathbf{E}}_{z}(r,\varphi,z)$$

$$= -\frac{\rho}{4\pi\varepsilon_{0}} \int_{h_{1}}^{h_{2}} \int_{0}^{2\pi} \int_{0}^{a} \frac{z'-z}{\widehat{R}^{3}} r' 0r' 0\varphi' 0z' \mathbf{e}_{z}.$$
(3)

According to the mirror image method, the effect of the ground-induced charge on the electric field in space can be replaced by the mirror image of the thundercloud relative to the ground, so that the boundary conditions can be satisfied without affecting the charge distribution in the space. Then, the electric field $\tilde{\mathbf{E}}_z$ excited by the thundercloud image at $P(r, \varphi, z)$ is

$$\widetilde{\mathbf{E}}_{z}(r,\varphi,z) = \int_{V} -\frac{\rho dv'}{4\pi\varepsilon_{0}\widetilde{R}^{2}} \frac{z'+z}{\widetilde{R}} \mathbf{e}_{z},$$

$$= -\frac{\rho}{4\pi\varepsilon_{0}} \int_{h_{1}}^{h_{2}} \int_{0}^{2\pi} \int_{0}^{a} \frac{z'+z}{\widetilde{R}^{3}} r' 0r' 0\varphi' 0z' \mathbf{e}_{z}.$$
(4)

Among them, \tilde{R} is the distance between the thundercloud image charge microelements and the observation point, $\tilde{R} = \sqrt{r'2 + r^2 - 2r'r \cos(\varphi - \varphi') + (z' + z)^2}$. Therefore, the total electric field \mathbf{E}_z excited by a charged

Therefore, the total electric field \mathbf{E}_z excited by a charged thundercloud at any point $P(r, \varphi, z)$ in space is

$$\mathbf{E}_{z}(r,\varphi,z) = \mathbf{E}_{z}(r,\varphi,z) + \mathbf{E}_{z}(r,\varphi,z),$$

$$= -\frac{\rho}{4\pi\varepsilon_{0}} \int_{h_{1}}^{h_{2}} \int_{0}^{2\pi} \int_{0}^{a} \left(\frac{z'-z}{\widehat{R}^{3}} + \frac{z'+z}{\widetilde{R}^{3}}\right) r' 0r' 0\varphi' 0z' \mathbf{e}_{z}.$$

(5)

The abovementioned formula cannot give a precise analytical expression; it can approximate the complex integrand; let $\widehat{R}' = \sqrt{r'^2 + r^2 + (z'-z)^2}$; then, $1/\widehat{R}^3$ can be expressed as

$$\frac{1}{\widehat{R}^{3}} = \frac{1}{\left[r'^{2} + r^{2} - 2r'r\cos(\varphi - \varphi') + (z' - z)^{2}\right]^{3/2}},$$

$$= \frac{1}{\left[\widehat{R}^{2} - 2r'r\cos(\varphi - \varphi')\right]^{3/2}},$$

$$= \frac{1}{\widehat{R'}^{3}} \frac{1}{\left[1 - 2r'r\cos(\varphi - \varphi')/\widehat{R'}^{2}\right]^{3/2}}.$$
(6)



FIGURE 1: Thundercloud cylindrical charge pile model.

Because $r'2 + r^2 - 2r'r \cos(\varphi - \varphi') > 0$, $(2r'r \cos(\varphi - \varphi')/\widehat{R'}^2) < 1$, it meets the condition of binomial expansion, so

we can expand $1/[1 - 2r'r \cos(\varphi - \varphi')/{R'}^2]^{3/2}$ as a binomial expansion, and we can get

$$\frac{1}{\left[1 - 2r'r\cos(\varphi - \varphi')/{R'}^2\right]^{3/2}} = 1 + \frac{3}{2} \left[\frac{2r'r\cos(\varphi - \varphi')}{{R'}^2}\right] + \frac{3 \cdot 5}{2!2^2} \left[\frac{2r'r\cos(\varphi - \varphi')}{{R'}^2}\right]^2 + \dots + \frac{(2n+1)!!}{n!2^n} \left[\frac{2r'r\cos(\varphi - \varphi')}{{R'}^2}\right]^n + \dots,$$
$$= \sum_{n=0}^{+\infty} \frac{(2n+1)!!}{n!2^n} \left[\frac{2r'r\cos(\varphi - \varphi')}{{R'}^2}\right]^n.$$
(7)

Substituting formulas (7) into (6), we can get

$$\frac{1}{\widehat{R}^{3}} = \frac{1}{\widehat{R'}^{3}} \sum_{n=0}^{+\infty} \frac{(2n+1)!!}{n!2^{n}} \left[\frac{2r'r \cos(\varphi - \varphi')}{\widehat{R'}^{2}} \right]^{n}.$$
 (8)

When *n* takes an odd number in the abovementioned formula, substituting the odd power term of $\cos(\varphi - \varphi t)$ into formula (3), the integral is all 0. Taking the first term of the binomial expansion to approximately replace $1/\hat{R}^3$, letting $1/\hat{R}^3 \approx 1/\hat{R}^3$, and substituting it into formula (3), we get

$$\widehat{\mathbf{E}}_{z}(r,\varphi,z) \approx -\frac{\rho}{4\pi\varepsilon_{0}} \int_{h_{1}}^{h_{2}} \int_{0}^{2\pi} \int_{0}^{a} \frac{z'-z}{R'^{3}} r' 0r' 0\varphi' 0z' \mathbf{e}_{z}$$

$$= -\frac{\rho}{4\pi\varepsilon_{0}} \int_{h_{1}}^{h_{2}} \int_{0}^{a} \int_{0}^{2\pi} \frac{r'(z'-z)}{\left[r'^{2}+r^{2}+(z'-z)\right]^{3/2}} 0\varphi' 0r' 0z' \mathbf{e}_{z}$$

$$= -\frac{\rho}{2\varepsilon_{0}} \left[\sqrt{r^{2}+(h_{2}-z)^{2}} - \sqrt{r^{2}+(h_{1}-z)^{2}} - \sqrt{a^{2}+r^{2}+(h_{2}-z)^{2}} + \sqrt{a^{2}+r^{2}+(h_{1}-z)^{2}} \right] \mathbf{e}_{z}.$$
(9)

In the same way, letting $\tilde{R'} = \sqrt{r'2 + r^2} + (z'+z)^2$, after approximate processing, letting $1/\tilde{R'} \approx q 1/\tilde{R}^3$, and substituting formula (4) into the integral, we can get

$$\widetilde{\mathbf{E}}_{z}(r,\varphi,z) \approx -\frac{\rho}{4\pi\varepsilon_{0}} \int_{h_{1}}^{h_{2}} \int_{0}^{2\pi} \int_{0}^{a} \frac{z'+z}{\tilde{R'}^{3}} r' 0r' 0\varphi' 0z' \mathbf{e}_{z},$$

$$= -\frac{\rho}{4\pi\varepsilon_{0}} \int_{h_{1}}^{h_{2}} \int_{0}^{a} \int_{0}^{2\pi} \frac{r(z'+z)}{\left[r^{2}+rr^{2}+(z'+z)^{2}\right]^{3/2}} 0\varphi' 0r' 0z' \mathbf{e}_{z},$$

$$= -\frac{\rho}{2\varepsilon_{0}} \left[\sqrt{r^{2}+(h_{2}+z)^{2}} - \sqrt{r^{2}+(h_{1}+z)^{2}} - \sqrt{a^{2}+r^{2}+(h_{2}+z)^{2}} + \sqrt{a^{2}+r^{2}+(h_{1}+z)^{2}} \right] \mathbf{e}_{z}.$$
(10)

Therefore, the approximate solution of the total electric field \mathbf{E}_z excited by a charged thundercloud at any point $P(r, \varphi, z)$ in space is

$$\mathbf{E}_{z}(r,\varphi,z) \approx -\frac{\rho}{2\varepsilon_{0}} \Big[\sqrt{r^{2} + (h_{2} - z)^{2}} - \sqrt{r^{2} + (h_{1} - z)^{2}} \\ - \sqrt{a^{2} + r^{2} + (h_{2} - z)^{2}} + \sqrt{a^{2} + r^{2} + (h_{1} - z)^{2}} + \sqrt{r^{2} + (h_{2} + z)^{2}} - \sqrt{r^{2} + (h_{1} + z)^{2}} - \sqrt{a^{2} + r^{2} + (h_{2} + z)^{2}} + \sqrt{a^{2} + r^{2} + (h_{1} + z)^{2}} \Big] \mathbf{e}_{z}.$$
(11)

3. Simulation Verification and Discussion

Formula (5) is an accurate solution of the electric field in thundercloud space. Because of the triple integration involved, it is impossible to obtain a more concise and accurate analytical expression, and the calculation time is longer. In order to improve the calculation efficiency, approximate processing is carried out to obtain formula (11), which is the approximate analytical expression of the electric field in thundercloud space.

In order to verify the correctness of the formula derivation and the rationality of the approximate processing, we take the volume charge density of the charged thundercloud as $\rho = 0.5 \text{ nC/m}^3$, the thundercloud radius as a = 3 km, and the height of the upper and lower boundary of the thundercloud from the ground as $h_2 = 8 \text{ km}$ and $h_1 = 3 \text{ km}$, radius $r = 0 \sim 5 \text{ km}$, and height $z = 0 \sim 3 \text{ km}$ under the thundercloud as the calculation area.

Figures 2 and 3 are the electric field distribution diagrams of a charged thundercloud in space, Figure 2 is a schematic diagram of the exact solution, and Figure 3 is a schematic diagram of the approximate solution. From the comparison of the two figures, it is found that the approximate solution and the exact solution are similar in size and the change trend is consistent.

The electric field distribution on the ground surface is widely used in theoretical calculations and experimental studies. Figure 4 shows the comparison between the approximate solution and the exact solution of the surface electric field distribution, and Figure 5 shows the relative error between the approximate solution and the exact solution of the surface electric field distribution. It can be seen from the two figures that, on the ground surface, the smaller the distance r between the observation point and the axis, the smaller the error between the approximate solution and the exact solution; when r = 0, the approximate solution and the exact solution are equal, and the error is 0; the greater the distance r from the z-axis, the greater the error between the approximate solution and the exact solution, but even at a distance of r = 5 km, the error between the approximate solution and the exact solution is less than 15%, indicating that the approximate solution of internal electric field is given in the following.

The electric field intensity directly below the thundercloud is widely used in pilot discharge simulation and lightning protection research. Figure 6 shows the comparison between the approximate solution and the exact solution when r = 0, and Figure 7 shows the relative error between the approximate solution and the exact solution when r = 0. It can be seen from the two figures that when the observation point is located on the *z*-axis directly below the thundercloud, the larger the *z* is, the closer the observation point is to the thundercloud and the greater the absolute value of the electric field intensity; the smaller the *z* is, the greater the observation point and the thundercloud are. Far, the smaller the absolute value of the electric field intensity, the electric field change trend conforms to the electrostatic





FIGURE 3: Approximate solution.



FIGURE 4: Comparison between approximate and accurate solutions while z = 0.

field distribution law. More importantly, at the distance of r = 0, no matter how *z* changes, the approximate solution and the exact solution are always equal; that is, when r = 0, the approximate analytical expression is transformed into an



FIGURE 5: The relative error between approximate and accurate solutions while z = 0.



--- Approximate solution

FIGURE 6: Comparison between approximate and accurate solutions while r = 0.



FIGURE 7: The relative error between approximate and accurate solutions while r = 0.

accurate analytical expression, which can be used for accurate electric field calculation.

Approximate processing not only simplifies theoretical derivation but also improves computational efficiency. MATLAB software is used to use the triple integral

TABLE 1: Comparison of calculation time between approximate and accurate solutions.

Electric field calculation method	Using approximate solution formula (11) to solve	Using exact solution formula (5) to solve
The calculation time (s)	0.012	231865.2



FIGURE 8: Thundercloud cylindrical four-layer charge stack model.

of formula (5) and the approximate analytical expression of formula (11) on a computer with a processor of Inter(R) Core(TM) i7-4500U CPU @ 1.8 GHz 2.4 GHz and a memory of 4 GB. The formula calculates the electric field of thunderclouds. It can be seen from Table 1 that, on a computer with the same configuration, it takes 64.407 hours to calculate with the exact solution; the calculation takes 0.012 seconds with the approximate solution. Using different calculation methods, the calculation time differs by 7 orders of magnitude. In electromagnetic field calculations, it is often necessary to perform complex loop nesting calculations on several triple integrals. Long-term calculations are not suitable for scenarios with high timeliness requirements, such as an important target encountering thunderstorms during maneuvering. If approximate processing and rapid calculation are not performed, it is impossible to quickly predict whether the target will be threatened by thunder and whether protection should be launched. Therefore, approximate calculation will play a huge role in the rapid warning of maneuvering targets.

In summary, the theoretical derivation of the approximate calculation method of the thundercloud space electric field in this paper is correct, the approximate treatment of the formula is reasonable, the advantage of approximate calculation is obvious, and approximate calculation is necessary in practical applications.

4. Calculation and Characteristic Analysis of the Spatial Electric Field of Thunder Clouds

The electric field generated by a typical four-layer cylindrical thundercloud in space is calculated in the following. Figure 8 shows the thundercloud cylindrical four-layer charge stack model, where *i* is the number of layers of the charge stack, ρ_i is the volume charge density, a_i is the thundercloud radius, and $h_{\text{U}i}$ and $h_{\text{L}i}$ are the height of the upper and lower boundary of the thundercloud, The parameters of the four-layer charge stack are shown in Table 2.

The electric field generated by the thundercloud fourlayer charge stack in space is simulated. Figures 9–12 are, respectively, the schematic diagrams of the electric field distribution on the longitudinal section of y = 0, y = 2000 m, y = 4000 m, and y = 6000 m; Figures 13–16 are, respectively, y = 0, y = 2000m, y = 4000 m, and y = 6000 m electric field contour diagrams on the longitudinal section.

It can be seen from Figure 9 that the spatial electric field on the y = 0 longitudinal section is symmetrically distributed with x = 0 as the axis, which is caused by the symmetrical distribution of the thundercloud four-layer charge stack with the z-axis as the center. The absolute value of the positive and negative peaks of the electric field in the thundercloud space decreases with the increase of the distance between the observation point and the thundercloud, which conforms to the characteristics of

The location of the charge pile i	Bulk charge density $\rho_i/nC \cdot m^{-3}$	The radius of the thundercloud a_i/m	The height of the upper boundary $h_{\text{U}i}/m$	The height of the lower boundary h_{Li}/m
Level 1	-0.65	5000	10000	9200
Level 2	0.65	5000	8900	6000
Level 3	-0.65	5000	5500	2600
Level 4	0.65	1500	2500	2000

TABLE 2: Four-layer charge stack parameters.



FIGURE 9: Electric field distribution on the longitudinal section while y = 0.



FIGURE 10: Electric field distribution on the longitudinal section while y = 2000 m.

the law of electrostatic field. In the electric field excited by the thundercloud four-layer cylindrical charge stack, two electric field positive peaks and one negative peak appear at the position between the first- and second-layer charge stacks and the third- and fourth-layer charge stacks. Between the second and third layer of charge piles, the three peaks appearing in the gap positions between the charge stacks are caused by the different polarities of charges carried by different charge stacks. It can be



FIGURE 11: Electric field distribution on the longitudinal section while y = 4000 m.



FIGURE 12: Electric field distribution on the longitudinal section while y = 6000 m.

predicted that when the observation point is farther from the thundercloud, that is, the value of y is larger, the peaks and troughs of the electric field are closer to a certain electric field value. When $y = \infty$, the spatial electric field distribution does not fluctuate and is a constant value because when $y = \infty$, the four-layer charge stack of the thundercloud can be regarded as a point charge at the observation point and the thundercloud has a single charge polarity and a fixed amount of charge.



FIGURE 13: Contour of potential distribution on the longitudinal section while y = 0.



FIGURE 14: Contour of potential distribution on the longitudinal section while y = 2000 m.



FIGURE 15: Contour of potential distribution on the longitudinal section while y = 4000 m.



FIGURE 16: Contour of potential distribution on the longitudinal section while y = 6000 m.

5. Conclusions

This paper proposes an approximate calculation method for thundercloud space electric field. This method can not only obtain concise analytical expressions but also shorten the calculation time and improve the calculation efficiency, laying a solid foundation for rapid lightning early warning and effective protection.

Approximate calculation is compared with accurate calculation result: similar size and consistent change trend; approximate calculation can be used for approximate calculation of electric field in a short range; when the observation point is on the central axis of thundercloud, the approximate analytical expression is transformed into an accurate analytical expression, which can be used for accurate calculation of electric field.

The approximate calculation method is used to calculate the electric field excited by a thundercloud of typical structure in space, and the electric field distribution characteristics are analyzed, which lays the foundation for the next step to study the law of lightning discharge and carry out pilot discharge simulation.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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