# Lowering the Sidelobe Level of a Two-Way Pattern in Shared Aperture Radar Arrays 

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#### Abstract

A study of lowering the peak SLL of shared aperture radar arrays is presented. A two-weight amplitude distribution for the elements of transmit and receive arrays is used. Imposing certain conditions, the relation of the number of elements of the arrays was found. One condition imposes the appearance of a minor lobe position of transmit or receive array pattern at a certain null of receive or transmit array pattern. A second condition imposes the equal sidelobe level of two consecutive minor lobes either near the main beam of the two-way array pattern or at certain positions of receive or transmit array pattern. The resulting peak SLL of the two-way radar array pattern depending on the conditions reaches from -47 dB up to less than -50 dB .


## 1. Introduction

Radar systems need to have antennas with narrow main beam and suppressed sidelobes. In [1-4] and the references therein, one can find recourses and design techniques of several radar antennas. A radar structure with excellent performance contains transmit and receive phased arrays at the same aperture. This structure, except for the performance, has significant advantages such as reduced bulkiness and lower manufacturing costs. In the international literature, [5-9], there are interesting design techniques of interleaving and/or tapered arrays where several analytical or numerical methods are given. The idea of Haupt [10-12] for synthesizing a receive array that places nulls of the receive pattern in the directions of the peak of the sidelobes of the transmit pattern was proved to be extremely useful and successful. In [13], Haupt's idea was extended by equating the level of the first two minor lobes of the two-way radiation pattern. Moreover, to reduce the SLL up to -45 dB , it was proposed in [13] to use two different amplitude weights for the elements of transmit and receive arrays.

In this paper, an effort is made to minimize the SLL by using the two-weight arrays and searching possible conditions for the two-way array pattern. The reduction of the

SLL, depending on the conditions, is better than that of [13] and goes from -47 dB up to less than -50 dB .

## 2. Formulation

Let us consider a transmit and a receive linear array of discrete elements along the $x$-axis with equal interelement distance $d$ (see Figure 1).

The transmit array has $N_{t}$ elements with array factor $\mathrm{AF}_{t}$ while the corresponding receive one has $N_{r}$ elements $\left(N_{r} \leq N_{t}\right)$ with array factor $\mathrm{AF}_{r}$.

The excitation of the above arrays combines the advantages of taper distribution and the simplicity of the feed network of uniform arrays [13]. The two amplitudes of the elements are $W_{1}$ and $W_{2}$ and their distribution is shown in Figure 1. The elements with the relative amplitude $W_{2}$ are in the middle, symmetric in both arrays. In the receive array, nonreceiving edge elements exist. The number of elements with a relative amplitude $W_{2}$ is $M$ in both arrays.

Let us suppose that $W_{1}=1$ and $W_{2}$ is expressed as

$$
\begin{equation*}
W_{2}=2+(\delta y), \quad(\delta y)>0 . \tag{1}
\end{equation*}
$$

The transmit and receive array factors are


Figure 1: Radar transmit and receive linear arrays.

$$
\begin{align*}
& \qquad \mathrm{AF}_{t}(\theta)=2 \frac{e^{-j\left(N_{t}-1 / 2\right) k d \sin \theta}}{N_{t}+(1+\delta y) M} \times\left\{\frac{\sin \left(\left(N_{t}+M / 4\right) k d \sin \theta\right) \cos \left(\left(N_{t}-M\right) / 4 k d \sin \theta\right)}{\sin ((1 / 2) k d \sin \theta)}+\frac{(\delta y) \sin (M / 2 k d \sin \theta)}{2 \sin ((1 / 2) k d \sin \theta)}\right\}, \\
& \qquad \mathrm{AF}_{r}(\theta)=2 \frac{e^{j\left(N_{r}-1 / 2\right) k d \sin \theta}}{N_{r}+(1+\delta y) M} \times\left\{\frac{\sin \left(\left(N_{r}+M\right) / 4\right) k d \sin \theta \cos \left(\left(N_{r}-M\right) / 4\right) k d \sin \theta}{\sin ((1 / 2) k d \sin \theta)}+\frac{(\delta y) \sin (M / 2) k d \sin \theta}{2 \sin ((1 / 2) k d \sin \theta)}\right\} \\
& \text { The two-way array factor AF is [11] } \\
& \qquad \mathrm{AF}=\mathrm{AF}_{t} \times \mathrm{AF}_{r} . \\
& \text { We start our study of AF for }(\delta y)=0 \text { where (2) and (3) } \quad \theta_{\mathrm{SLL}}^{t}=\left(\frac{2}{N_{t}+M}+\frac{1}{N_{t}-M}\right) \text {, } \\
& \text { are the same with }(11) \text { of }[13] . \\
& \text { For interelement distance } d=\lambda / 2 \text { where } k d=(2 \pi / \lambda) \times \quad \theta_{\mathrm{SL} 2}^{t}=\left(\frac{1}{N_{t}-M}+\frac{4}{N_{t}+M}\right) \text {, }  \tag{4}\\
& (\lambda / 2)=\pi \text { the nulls of } \mathrm{AF}_{t} \text { are at the angles }[13]
\end{align*}
$$

$$
\begin{align*}
& \theta_{i}^{t}=\sin ^{-1}\left(\frac{4 i}{N_{t}+M}\right), i=1,2,3, \ldots,  \tag{5}\\
& \theta_{l}^{t}=\sin ^{-1}\left[\frac{2(2 l-1)}{N_{t}-M}\right], \\
& l=1,2,3, \ldots
\end{align*}
$$

The position of nulls in ascending order depends on the ratio $b=\left(M / N_{t}\right)$. To be a null for a given $l$ of the second condition of (5) between the nulls for $i$ and $i+1$ of the first condition, it must be

$$
\begin{equation*}
\frac{4 i-2(2 l-1)}{4 i+2(2 l-1)}<b<\frac{4(i+1)-2(2 l-1)}{4(i+1)+2(2 l-1)} \tag{6}
\end{equation*}
$$

where always it is

$$
\begin{equation*}
2 i>2 l-1 \tag{7}
\end{equation*}
$$

Assuming that the peak of minor lobes is approximately in the middle between ascending nulls, we can find their position. For example, if $l=1$ and $i=1$, then $(2 / 6)<b<(6 / 10)$ and

The nulls of $\mathrm{AF}_{r}$ are [13]

$$
\begin{array}{ll}
\theta_{i}^{r}=\sin ^{-1}\left(\frac{4 i}{N_{r}+M}\right), & i=1,2,3, \ldots \\
\theta_{l}^{r}=\sin ^{-1}\left[\frac{2(2 l-1)}{N_{r}-M}\right], & l=1,2,3, \ldots \tag{9}
\end{array}
$$

The position of nulls of $\mathrm{AF}_{r}$ in ascending order follows similar conditions with these of $\mathrm{AF}_{t}$. We suppose that $N_{r}=a N_{t}$. Again, if a null for a given $l$ is between the nulls for $i$ and $i+1$, then

$$
\begin{equation*}
b \frac{4(i+1)+2(2 l-1)}{4(i+1)-2(2 l-1)}<a<b \frac{4 i+2(2 l-1)}{4 i-2(2 l-1)}, \quad 2 i>2 l-1 \tag{10}
\end{equation*}
$$

For $(2 / 6)<b<(6 / 10)$ and $(5 / 9)<a<1$, the peak of sidelobe positions is at the angles

$$
\begin{align*}
& \theta_{\mathrm{SL} 1}^{r}=\left(\frac{2}{N_{r}+M}+\frac{4}{N_{r}+M}\right), \\
& \theta_{\mathrm{SL} 2}^{r}=\left(\frac{4}{N_{r}+M}+\frac{6}{N_{r}+M}\right),  \tag{11}\\
& \theta_{\mathrm{SL} 3}^{r}=\left(\frac{6}{N_{r}+M}+\frac{8}{N_{r}+M}\right) .
\end{align*}
$$

From the numerical procedure and the examples given in [13], it appears that it could be useful to have a systematic and analytical search for the patterns of transmit and receive arrays. A case of interest is the one where the patterns have equal level of neighbor minor lobes. Equating the level of the lobes, a simple expression can be found in the following form:

$$
\begin{align*}
& \frac{\cos [(2 m-1 \pi / 2)(N-M / N+M)]}{2 m-1} \\
& \quad=\frac{\cos [(2 m+1 \pi / 2)(N-M / N+M)]}{2 m+1} . \tag{12}
\end{align*}
$$

Solving (12), we can see in Table 1 the relation between $N$ and $M$. All the values of $M / N$ are based on the first condition of nulls $\theta_{i}=\sin ^{-1}(4 i /(N+M)), \quad i=1,2,3, \ldots$ of (5). This condition starts from the $2^{\text {nd }}$ minor lobe.

To have the $1^{\text {st }}$ and $2^{\text {nd }}$ minor lobe levels equal, we use both conditions of (5) for $i=1$ and $l=1$. In this case, we have

$$
\begin{align*}
& \frac{\cos [\pi / 4(+M / N-M)] \times \cos [\pi / 4+(\pi / 2)(N-M / N+M)]}{2+(N+M / N-M)}= \\
& \frac{\sin [(\pi / 4)(N+M / N-M)] \times \cos [(\pi / 4)+\pi(N-M / N+M)]}{4+(N+M / N-M)} . \tag{13}
\end{align*}
$$

Solution of (13) gives $M / N=0.4857$.
It is obvious that $N$ could be the number of elements of either transmit or receive array.

In the following figures, the patterns of several arrays with equal neighbor minor lobe levels are given (see Figures 2-6).

As we will see next, arrays with neighbor minor lobes of equal level either near the main beam of AF or in certain positions of $\mathrm{AF}_{t}$ and/or $\mathrm{AF}_{r}$ will help design radars with low SLL.

## 3. Lowering the SLL: Procedures and Examples

3.1. Case 1: Equating Minor Lobes. One case to lower the SLL is to have equal the two neighbor minor lobes near the main beam of AF. This is similar to that made in [13] for uniformly excited arrays. Thus, using (2)-(4), we have the first condition that relates $N_{t}, N_{r}$ and $M$.

$$
\begin{equation*}
\left|\operatorname{AF}\left(\theta_{\mathrm{SL} 1}\right)\right|=\left|\mathrm{AF}\left(\theta_{\mathrm{SL} 2}\right)\right| \tag{14}
\end{equation*}
$$

where

$$
\begin{align*}
\operatorname{AF}\left(\theta_{\mathrm{SL} 1}\right)= & \frac{4}{(1+b)(a+b)} \times \frac{\cos (1+b / a+b)(\pi / 2) \cos ((1-b / 2) \pi(1 / 1+b+(1 / a+b)))}{\pi((1 / 1+b)+(1 / a+b))} \\
& \times \frac{\cos (a+b / 1+b(\pi / 2)) \cos ((a-b / 2) \pi(1 / 1+b+(1 / a+b)))}{\pi((1 / 1+b)+(1 / a+b))}, \tag{15}
\end{align*}
$$

and

$$
\begin{align*}
\operatorname{AF}\left(\theta_{\mathrm{SL} 2}\right)= & \frac{4}{(1+b)(a+b)} \times \frac{\sin (1+b(\pi / 4)((1 / 1-b)+(2 / a+b))) \cos (\pi / 4+((1-b / a+b)(\pi / 2)))}{(\pi / 2)((1 / 1-b)+(2 / a+b))} \\
& \times \frac{\cos (a+b / 1-b)(\pi / 4) \cos ((a-b / 4) \pi(1 / 1-b+(2 / a+b)))}{(\pi / 2)((1 / 1-b)+(2 / a+b))} \tag{16}
\end{align*}
$$

A second condition for $a$ and $b$ can be found by equating the level of two neighbor minor lobes of a single array. If, for example, we have equal $3^{\text {rd }}$ and $4^{\text {th }}$ minor lobes of the transmit array pattern, then

$$
\begin{equation*}
\frac{\cos [5 \pi / 2(1-b / 1+b)]}{5}=\frac{\cos [7 \pi / 2(1-b / 1+b)]}{7} \tag{17}
\end{equation*}
$$

Combining the two conditions, it is possible to find $N_{r}$ and $M$ versus $N_{t}$. For the solution of the above, the following are taken into account:
(1) The position of the peak of minor lobes is not exactly in the middle between the two neighbor nulls
(2) The integer number of array elements gives slightly different values than those given from the exact solutions of the two conditions

Thus, from (14) and (17), we approximately find that $a \sim(15 / 21)$ and $b \sim(11 / 21)$.

It is noticed that, in all examples that follow in this paper, the values of $a$ and $b$ will be given as the ratio of integer numbers.

Consider a 126 -element transmit linear array with $M=$ $(11 / 21) \times 126=66$ and a receive array with $(15 / 21)$ $\times 126=90$ elements. The two-way array pattern is given in Figure 7.

Figure 7 shows the enhanced performance of the twoway array pattern compared to the ones of [13]. We also

Table 1: Relation between $N$ and $M$.

| $m$ (equal the $m+1$ and $m+2$ side lobe) | $M / N$ | $M$ for $N=64$ |
| :--- | :---: | :---: |
| 1 | 0.3741 | 24 |
| 2 | 0.5385 | 34 |
| 3 | 0.6349 | 40 |
| 4 | 0.6965 | 44 |



Figure 2: The array factor for $N=70$, and $M=34$. The $1^{\text {st }}$ and $2^{\text {nd }}$ SLLs are approximately equal.


Figure 3: The array factor for $N=64$ and $M=24$. The $2^{\text {nd }}$ and $3^{\text {rd }}$ SLLs are approximately equal.


Figure 4: The array factor for $N=64$ and $M=34$. The $3^{\text {rd }}$ and $4^{\text {th }}$ SLLs are approximately equal.
notice that the next three minor lobes have approximately the same and lower level.

Another example for odd number of array elements is given next. The transmit array has 71 elements, and the receive one has 51 elements. Both of them have $M=37$. The two-way array pattern is given in Figure 8.

The SLL of the above two-way array pattern is -47.4 dB , and again the next three minor lobes are approximately equal with lower level.
3.2. Case 2: Equal Neighbor Receive Minor Lobes and Same Position of a Receive Null with a Transmit Minor Lobe. Searching for another solution, we start from the receive array, where the first condition has to do with the $4^{\text {th }}$ and $5^{\text {th }}$ minor lobes, which must have their peak levels approximately equal. For the second condition, it is desired to have the $4^{\text {th }}$ null of the receive array at the position of the $4^{\text {th }}$ minor lobe of the transmit one. The above two conditions give

$$
\begin{equation*}
\frac{\cos [(7 \pi / 2)(a-b / a+b)]}{7}=\frac{\cos [(9 \pi / 2)(a-b / a+b)]}{9}, \tag{18}
\end{equation*}
$$

$$
\begin{equation*}
\frac{14}{1+b}=\frac{12}{a+b} \tag{19}
\end{equation*}
$$

Thus, it is found that

$$
\begin{align*}
& a \approx \frac{19}{24}  \tag{20}\\
& b \approx \frac{12}{24}
\end{align*}
$$

As an example, consider a 48 -element transmit linear array with $M=24$ and a receive array with 38 elements. The two-way array pattern is presented in Figure 9.

The above pattern has an $\mathrm{SLL}=-49.18 \mathrm{~dB}$, which is much better than all the patterns given before.

We find similar results for other combinations of nulls and minor lobes. If the receive array is desired to have the $5^{\text {th }}$ and $6^{\text {th }}$ minor lobe levels approximately equal, then

$$
\begin{equation*}
\frac{\cos [(9 \pi / 2)(a-b / a+b)]}{9}=\frac{\cos [(11 \pi / 2)(a-b / a+b)]}{11} \tag{21}
\end{equation*}
$$

with the condition at the same time that the $1^{\text {st }}$ null of the receive array being at the position of the $1^{\text {st }}$ side lobe of the transmit array, and we have

$$
\begin{equation*}
\frac{4}{a+b} \approx\left(\frac{2}{1+b}+\frac{1}{1-b}\right) \tag{22}
\end{equation*}
$$

Approximate solution of (21) and (22) gives

$$
\begin{align*}
& a \approx \frac{23}{31}  \tag{23}\\
& b \approx \frac{16}{31}
\end{align*}
$$



Figure 5: The array factor for $N=64$ and $M=40$. The $4^{\text {th }}$ and $5^{\text {th }}$ SLLs are approximately equal.


Figure 6: The array factor for $N=64$ and $M=44$. The $5^{\text {th }}$ and $6^{\text {th }}$ SLLs are approximately equal.


Figure 7: The two-way array pattern for $N_{t}=126, M=66$ and $N_{r}=90$ elements. SLL $=-47 \mathrm{~dB}$.

An example for $N_{t}=62, N_{r}=46$ and $M=32$ is presented in Figure 10. The two-way array pattern has an SLL $=-48.17 \mathrm{~dB}$.
3.3. Case 3: Same Position of $2^{\text {nd }}$ and $3^{\text {rd }}$ Nulls and Same Position of a Transmit Null with a Receive Minor Lobe. Except of equal neighbor minor lobes, another interesting case is the one where the conditions that derive the $2^{\text {nd }}$ and $3^{\text {rd }}$ nulls of the pattern of an array give the same position.

The $2^{\text {nd }}$ null is coming from $l=1$ of (5), while the $3^{\text {rd }}$ one is coming from $i=2$ of (5). Making use of the above, we have

$$
\begin{equation*}
\frac{2}{N-M}=\frac{8}{N+M} . \tag{24}
\end{equation*}
$$

Solving (24), it is found that

$$
\begin{equation*}
M=0.6 \mathrm{~N} \tag{25}
\end{equation*}
$$

Based on (25), for an array with $N=75$, we have $M=0.6$, $N=45$. In this case, the array pattern is given in Figure 11.

Looking at the positions of the peak of the $2^{\text {nd }}$ and $3^{\text {rd }}$ minor lobes, we see that these are not in the middle of the neighbor nulls. Thus, for example, to use the above array as a receive one, the number of elements of the transmit array could be found by equating the position of a null of one array (receive or transmit) with the position of the minor lobe of the other (transmit or receive).

Let us suppose that the transmit array has its $4^{\text {th }}$ null at the position of the $2^{\text {nd }}$ minor lobe of the receive array. We look at the $2^{\text {nd }}$ minor lobe of Figure 11, which is at $5.05^{\circ}$. Thus, $N_{t}$ is found to be

$$
\begin{equation*}
\frac{12}{N_{t}+M}=5.05 \frac{\pi}{180} \longrightarrow N_{t} \simeq 91 \tag{26}
\end{equation*}
$$

The two-way array pattern for the above arrays has an SLL $=-49.4 \mathrm{~dB}$ and is presented in Figure 12.

Another example for even number of elements is given in Figure 13 (see Figures 14 and 15).

Using several arrays with similar characteristics, it was found that the suitable values for $a$ and $b$ are $a \sim(101 / 123)$ and $b \sim(61 / 123)$. The SLL is $<-49 \mathrm{~dB}$ for all cases.

Two more examples for such arrays are given below.
3.4. Case 4: $1^{\text {st }}$ Receive Minor Lobe at the Position of $2^{\text {nd }}$ Transmit Null and $2^{\text {nd }}$ and $3^{\text {rd }}$ Receive Nulls Approximately at the Same Position. We start from the condition that M/ $N_{r}=0.60$. In this case, the position of $1^{\text {st }}$ minor lobe of the pattern of the receive array must be derived exactly. If, for example, $N_{r}=26$, then $M \sim 16$. The peak of the $1^{\text {st }}$ minor lobe is at $\sim 7.2^{\circ}$. Equating this position with the $2^{\text {nd }}$ null of the transmit array pattern, we have

$$
\begin{equation*}
\frac{2}{N_{t}-M}=7.2 \frac{\pi}{180} \longrightarrow N_{t} \simeq 32 . \tag{27}
\end{equation*}
$$

The two-way array pattern for the array with $N_{t}=32$, $N_{r}=26$ and $M=16$ is presented in Figure 16. Using more numerical examples, we found that, for $a=26 / 32$ and $b=16 /$ 32, the patterns have always SLL $<-49.8 \mathrm{~dB}$.

After all the numerical procedures, Table 2 gives several cases of two-way array patterns with SLL $<-47 \mathrm{~dB}$.

From Table 2, the following steps for the design of transmit and receive arrays are proposed:
(1) Choice of the numbers $M$ and $N_{t}$ for the elements of transmit array.
(i) For even numbers, we have $M=2 m$ and $N_{t}=4 m$.


Figure 8: The two-way array pattern for $N_{t}=71, M=37$ and $N_{r}=51$ elements. $\mathrm{SLL}=-47.4 \mathrm{~dB}$.


Figure 9: The two-way array pattern for $N_{t}=48, M=24$ and $N_{r}=38$ elements. $\mathrm{SLL}=-49.18 \mathrm{~dB}$.


Figure 10: The two-way array pattern for $N_{t}=62, N_{r}=46$ and $M=32$. SLL $=-48.17 \mathrm{~dB}$.
(ii) For odd numbers, we have $M=2 m+1$ and $N_{t}=$ $2(2 m+1) \pm 1 \longrightarrow 4 m+3$ or $4 m+1$. Both values of $N_{t}$ must be checked.
(2) Derivation of $N_{r}$ by applying the appropriate condition. For example, in any of Cases $1-4$, the condition that contains $a$ as unknown is used.
3.5. Case 5: Two-Way Array Patterns with SLL <-50 dB. In the procedures given before, the SLLs of the two-way array patterns for $W_{2}=2$ were found to be close to -50 dB .


Figure 11: The array pattern for $N=75$, and $M=45$. 2nd and 3rd nulls are at the same position.


Figure 12: The two-way array pattern for $N_{t}=91, N_{r}=75$, and $M=45$. SLL $=-49.4 \mathrm{~dB}$.


Figure 13: The two-way array pattern for $N_{t}=44, N_{r}=36$, and $M=22$. SLL $\sim-49.7 \mathrm{~dB}$.

Looking at the certain minor lobe that gives the SLL, it could be interesting to search the possibility of slightly change $W_{2}$ and see the change of SLL.

Consider that $W_{1}=1$ and $W_{2}$, following expression (1), has $0<(\delta y) \ll 1$.

Since $(\delta y) \ll 1$, we suppose that the peak of a minor lobe of the two-way array pattern is approximately at the same angle $\theta=\theta_{0}$ for $W_{2}=2$ and for $W_{2}=2+(\delta y)$. To lower the SLL, we use the condition

$$
\begin{equation*}
\frac{\operatorname{AF}\left(\theta_{0}\right)}{\operatorname{AF}_{0}\left(\theta_{0}\right)}=\alpha<1 \tag{28}
\end{equation*}
$$



Figure 14: The two-way array pattern for $N_{t}=80, N_{r}=66, M=40$ and $\mathrm{SLL}=-49.7 \mathrm{~dB}$.


Figure 15: The two-way array pattern for $N_{t}=123, N_{r}=101$, and $M=61$. SLL $=-50.15 \mathrm{~dB}$.


Figure 16: The two-way array pattern for $N_{t}=32, N_{r}=26$, and $M=16 . \mathrm{SLL}=-49.82 \mathrm{~dB}$.


Figure 17: The two-way array pattern for $N_{t}=80, N_{r}=64$, and $M=40, W_{2}=2.1$. $\mathrm{SLL}=-51.05 \mathrm{~dB}$.


Figure 18: The two-way array pattern for $N_{t}=48, N_{r}=38$, and $M=24, W_{2}=2.1$. $\mathrm{SLL}=-50.53 \mathrm{~dB}$.


Figure 19: The two-way array pattern for $N_{t}=44, N_{r}=36$, and $M=22, W_{2}=2.055 . \mathrm{SLL}=-50.2 \mathrm{~dB}$.
$\mathrm{AF}_{0}$ is the two-way array factor for $W_{2}=2$, and AF is the one for $W_{2}=2+(\delta y)$. Taking into account the expressions

Table 2: The ratio of the number of array elements for small SLL of two-way array patterns.

| $a=\mathrm{N}_{r} / N_{t}$ | $b=M / N_{t}$ | SLL |
| :--- | :---: | :---: |
| $15 / 21$ | $11 / 21$ | $\sim-47.0 \mathrm{~dB}$ |
| $23 / 31$ | $16 / 31$ | $\sim-48.2 \mathrm{~dB}$ |
| $19 / 24$ | $12 / 24$ | $\sim-49.1 \mathrm{~dB}$ |
| $26 / 32$ | $16 / 32$ | $\sim-49.82 \mathrm{~dB}$ |
| $101 / 123$ | $61 / 123$ | $\sim-50.15 \mathrm{~dB}$ |

$\mathrm{AF}_{0}$ and AF , after some algebra, we have at $\Psi_{0}=k d \sin \theta_{0}$ that

$$
\begin{equation*}
\delta y=\frac{A}{B} \tag{29}
\end{equation*}
$$

where

$$
\begin{align*}
A & =2(1-\alpha)\left(N_{t}+M\right)\left(N_{r}+M\right) \\
B & =\alpha M\left(N_{t}+N_{r}+2 M\right)-\left[\left(N_{t}+M\right)\left(N_{r}+M\right) \sin \left(\frac{M}{2} \Psi_{0}\right)\right] \times  \tag{30}\\
& {\left[\frac{1}{\sin \left(\left(\left(N_{t}+M\right) / 4\right) \Psi_{0}\right) \cos \left(\left(\left(N_{t}-M\right) / 4\right) \Psi_{0}\right)}+\frac{1}{\sin \left(\left(\left(N_{r}+M\right) / 4\right) \Psi_{0}\right) \cos \left(\left(\left(N_{r}-M\right) / 4\right) \Psi_{0}\right)}\right] . }
\end{align*}
$$

Some examples of two-way array patterns are given below.

In a radar with $N_{t}=80, N r=64, M=40$ and $W_{2}=2.0$, the SLL is -49.7 dB . If it is desired to have SLL $<-50 \mathrm{~dB}$, by using (29) and (30), we find $\delta x \simeq 0.1 \longrightarrow W_{2}=2.1$. The SLL becomes -51.05 dB . This value shows that it is possible to improve the SLL by making a small change to the amplitude $W_{2}$. The two-way array pattern is shown in Figure 17.

The two-way array pattern for $N_{t}=48, N r=38, M=24$ and $W_{2}=2.1$ is given in Figure 18. It has SLL $=-50.5 \mathrm{~dB}$. This is compared with the array of Figure 9, where SLL $=-49.18 \mathrm{~dB}$. Finally, another case is shown in Figure 19 where we have the two-way array pattern for $N_{t}=44$,
$N r=36, \quad M=22$ and $W_{2}=2.055$. The pattern has $\mathrm{SLL}=-50.2 \mathrm{~dB}$. This is compared with the array of Figure 13 where SLL $=-49.7 \mathrm{~dB}$.

Following the procedures of classical texts [14,15], all calculations and presentations of the patterns were made by using the ORAMA computer tool [16].

It was noticed in $[11,13]$ that planar arrays can be designed with the same concept of linear arrays. In a planar array [11], the edge elements will be turned off for the receive array. Planar arrays provide more variables and offer higher directivity than linear ones. The same procedure as above can create equally sufficient SLL of the two-way array patterns.

## 4. Validation of the Results

It is well known that numerical and experimental technologies for radar arrays are equally important. Our study of lowering the peak SLL is a theoretical one. To validate our results, there are several choices. One choice is to provide experimental results and compare them with the theoretical ones. The other is to compare the method and results with other theoretical studies. Of course, a third choice is to combine both experiments and calculations. In our case, to throw some light on the validation of our results, we compared them with those given in [13]. The two-weight arrays with weights given (1) are a new idea. For the special case, where $(\delta y)=0$, as it is already mentioned, the results are the same. For example, the two-way patterns of $N_{t}=32$, $N_{r}=24, M=12$ and $N_{t}=80, N_{r}=60, M=30$ were again derived and compared. The results, as was expected, were the same. However, the new thing that is given here is that, instead of the above values, if we use arrays with the same number of elements of the transmitting arrays and modify the receiving ones, we can have better results. The two-way patterns of $N_{t}=32, N_{r}=26, M=16$ and $N_{t}=80, N_{r}=66$, $M=40$ offer a difference of the peak SLL than the above of [13], which is $<-4.5 \mathrm{~dB}$.

It would be an omission not to mention that, nowadays, antenna arrays become digital [17]. Analog-to-digital converters at each element of the arrays transform the beamforming from hardware-based techniques to software ones. In our case of lowering the peak SLL, signal-processing techniques can be implemented. To transmit/receive from/to a two-way radar array, a signal generator/receiver sends/ receives signals at/to each element. This approach needs processing systems with real-time calibration. It is obvious that the array technology changes fast. It is believed that, with digital technology and software techniques, our design will help have simpler two-way radar arrays with much better characteristics.

## 5. Conclusions

A procedure of lowering the peak SLL of a radar two-way array factor has been presented. A two-weight excitation for the elements of transmit and receive arrays was used. This combines the advantages of taper distribution and the simplicity of the feed network of uniform arrays [13]. The amplitude of the element excitation in the middle is symmetrical, and the same for transmit and receive arrays. The ratio of the number of elements of the arrays was found by applying a pair of conditions. One had to do with the appearance of a minor lobe position of transmit or receive array pattern at a certain null position of receive or transmit array pattern. A second condition imposed the equal sidelobe levels of two consecutive minor lobes either near the main beam of the two-way pattern or at certain positions of receive or transmit array pattern. The resulting peak SLL of the radar pattern was found to reach values from -47 dB up to less than -50 dB , which is sufficient for radar systems.

## Data Availability

The data used to support the findings of this study are included in the supplemental files and figure files.

## Conflicts of Interest

The author declares that there are no conflicts of interest.

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