Research Article

Design of a Novel Integral Sliding Mode-Based Composite Nonlinear Feedback Controller for Electrostatic MEMS Micromirror

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Received 15 December 2021; Accepted 4 March 2022; Published 31 March 2022

Academic Editor: Bin Zhang

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In this study, the precise tracking problem for electrostatic micromirror systems with disturbances and input saturation is investigated. Inspired by the composite nonlinear feedback (CNF)'s improvement of the transient performance and the sliding mode control's enhancement of the robustness, a novel integral sliding mode with reaching law (ISMRL)-based composite nonlinear feedback (CNF) controller is proposed. Then, the stability of the closed-loop system is proved based on Lyapunov theorem. Finally, numerical simulations are investigated to evaluate the effectiveness of the proposed scheme. It is shown that the closed-loop system with the proposed scheme has precise positioning and improved transient performance in presence of time-varying disturbances.

1. Introduction

Micro-electro-mechanical system (MEMS) micromirror has experienced enormous commercial success in applications such as optical switches [1, 2], biomedical imaging [3], and high-resolution displays [4]. Compared with electrothermal, electromagnetic, and piezoelectric actuation, the advantages of electrostatic actuation are fast response, simple electronic driving, and low power consumption [5]. However, the electrostatic MEMS micromirror suffers unsatisfied transient performance and pull-in instability under open loop control. Lots of efforts and control strategies have been introduced to tackle the problem. Classic strategies, such as PID controller [6, 7], H-infinity robust controller [8], sliding mode control [9], and adaptive control [10] schemes have been reported to improve its tracking performance and eliminate the effect of external disturbances. The aforementioned control strategies have been verified benefits of improving the positioning performance and extending the stable operational range of micromirrors. However, the transient response, which is essential for micromirror's application, is not directly considered. For instant, fast setting time and low overshoot of micromirror-based optical switches are required in order to reduce the insertion loss.

For the past few years, composite nonlinear feedback (CNF) control scheme has attracted many attentions as its meaningful improvements in transient behaviors [11]. This scheme was first studied for a class of second-order linear system [12]. Then, it was extended to partially linear composite system [13]. CNF control methods are also investigated for master/slave synchronization of nonlinear system [14], nonlinear time-delay systems [15], strict-feedback nonlinear systems [16], and under actuated systems [17]. The output tracking problem of time-varying references in descriptor systems is investigated using CNF control technique in [18]. Besides developments in theory, the CNF controller is proposed for hard-drive drive (HDD) servo system [19], spacecraft rendezvous systems [20], multi-quadrotor systems [21], robot manipulators [22], and autonomous vehicles [23]. However, when considering the inevitable external...
disturbances, the traditional CNF control reveals its lack of the ability to deal with it. Fortunately, in the field of control theory, sliding mode control is considered to be a solution for alleviating the effects of the parametric uncertainties and external disturbances [24–26]. As a result, the sliding mode control techniques combined with composite nonlinear feedback are proposed to improve system robustness in [27–38]. Recently, this scheme is also developed for more general class of linear and nonlinear systems with plant uncertainties [39].

Though sliding mode-based CNF controller has been fully studied and demonstrated the advantages of robustness, the chattering problem of sliding mode control is a serious situation which not only increases energy consumption but also leads to the instability. To avoid this, one solution is to introduce the second-order sliding mode control [40]. In [41], a super-twisting algorithm-based integral sliding mode control with composite nonlinear feedback control is proposed to eliminate the chattering effect for magnetic levitation system. Another effective solution to reduce the chattering is the sliding mode control based on the reaching law, which is first proposed in [42]. Recently, in [43–45], improved quick reaching law is proposed to speed up the response and reduce the chattering of a sliding mode control system simultaneously.

Motivated by the aforementioned problem through literature review, the main contribution of this research is that a novel integral sliding mode with reaching law-based composite nonlinear feedback (ISMRL-CNF) controller for angular control of an electrostatic MEMS micromirror is proposed. The CNF controller is designed to guarantee the system has fast dynamic performance and small overshoot. An integral sliding mode control with quick reaching law is designed to enhance the robustness, attenuate chattering, and achieve finite-time convergence of the sliding mode. Furthermore, the time-varying disturbances and input saturation are taken into account for controller design and stability analysis. Simulation study verifies that the closed-loop system with the proposed scheme has precise positioning and improved transient performance in presence of time-varying disturbances.

The rest of the paper is organized as follows. The dynamic model is described in Section 2. The design procedure of the proposed controller and stability analysis are developed in Section 3. Simulation study is given in Section 4. Finally, the conclusions are discussed.

2. The Simplified Dynamics of Electrostatic MEMS Micromirror

Figure 1 shows the schematic figure of a 2-degree-of-freedom electrostatic torsional MEMS micromirror. The studied micromirror consists of mirror plate, torsion bar, gimbal frame, and bottom and sidewall electrodes. The mirror plate is suspended by double frame structure and driven by electrostatic torque. When the driving voltage applied to the bottom and sidewall electrodes, the mirror plate is actuated about X-axis and Y-axis, respectively. The dynamic equations of the system are given as follows [5]:

\[
\begin{align*}
J_1 \ddot{x} + D_1 \dot{x} + K_1 x &= T_a, \\
J_2 \ddot{y} + D_2 \dot{y} + K_2 y &= T_b,
\end{align*}
\]

where \(\alpha\) and \(\beta\) represent the tilt angles of x-axis and y-axis, \(J_1\) and \(J_2\) denote the mass moment of inertias of the mirror plate and gimbal, respectively, \(D_1\) and \(D_2\) represent the damping coefficients, \(K_1\) and \(K_2\) represent the stiffness coefficients, and \(T_a\) and \(T_b\) are electrostatic torque. Introducing the parameter \(\tau = \sqrt{K_1^2 + J_1^2}, \) let \(x_1 = \alpha, x_2 = d\alpha/dt, x_3 = \beta,\) and \(x_4 = d\beta/dt.\) As a result, system (1) can be described as [5]

\[
\begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= -R_1 x_2 - \lambda_{\alpha \beta} x_1 + G_1 T_a, \\
\dot{x}_3 &= x_4, \\
\dot{x}_4 &= -R_2 x_4 - x_3 + G_2 T_b,
\end{align*}
\]

where the parameters are \(R_1 = 0.16, R_2 = 0.15, \lambda_{\alpha \beta} = 0.2251,\) \(G_1 = 3.0827 \times 10^6,\) and \(G_2 = 1.7894 \times 10^7.\) Considering input saturation and external disturbances, the system is rewritten as

\[
\begin{align*}
\dot{x} &= Ax + B s(\sigma) u + B d, \\
y &= C x,
\end{align*}
\]

where \(x \in \mathbb{R}^n\) is the state of micromirror system, \(u \in \mathbb{R}\) is the control input, \(y \in \mathbb{R}\) is the measurement output, And \(A, B, \) and \(C\) are constant appropriate dimensional matrices. The saturation function \(s(\sigma) : \mathbb{R} \rightarrow \mathbb{R}\) is defined as \(s(\sigma) = \text{sgn}(\sigma) \min\{u_{\text{max}}, |u|\}\) with the maximum of control input \(u_{\text{max}}\). The system uncertainties and disturbances \(d \in \mathbb{R}\) are bounded, and \(|d| \leq d_{\text{max}}\) with the maximum of disturbances \(d_{\text{max}}\). The following assumptions are satisfied for the investigated system (3) [19]: (1) \((A, B)\) is stabilizable, (2) \((A, B, C)\) is invertible, which means the system has no zeros at \(s = 0,\) and (3) \((A, C)\) is detectable.

3. The Proposed Integral Sliding Mode-Based Composite Nonlinear Feedback Controller Design

In this section, a novel integral sliding mode-based composite nonlinear feedback (ISMRL-CNF) controller is designed for the micromirror system with input saturation and disturbances. The objective is to ensure that the controlled output can track step command input precisely with enhanced transient performance and robustness, in presence of external disturbances. The proposed controller consists of a CNF control law and an integral sliding mode control with reaching law (ISMRL):

\[
u = u_N + u_{\text{is}},
\]

where the CNF control \(u_N\) is utilized to achieve good transient performance and the integral sliding mode control
with reaching law $u_r$ is designed to guarantee the system robustness under disturbances and reduce chattering.

The CNF controller consists of a linear control law and a nonlinear control law. The linear control law is presented to achieve fast response by using small damping ratio. The nonlinear control law is developed to change the damping ration in order to eliminate overshoot. The linear feedback control $u_L$ is designed as [12]

$$u_L = Fx + Gr,$$

(5)

where $F$ is chosen to guarantee that $(A + BF)$ an asymptotically stable matrix, $r$ is a step command input, and $G$ is scalar and calculated as

$$G = -[C(A + BF)^{-1}B]^{-1},$$

(6)

where $F$ and $G$ are defined since $(A, B, C)$ is assumed to have no invariant zeros at $s = 0$.

The nonlinear feedback law $u_{NL}$ is designed as

$$u_{NL} = \rho(y, r)B^T\hat{P}(x - x_c),$$

(7)

where $\rho(y, r)$ represents any monotonically increasing function. Different forms of $\rho(y, r)$ have been reported in previous works. A scaled nonlinear function is proposed to adapt the variation of tracking targets in [19] as

$$\rho(y, r) = -\bar{\beta}e^{-m(y - r)},$$

(8)

where $\bar{\beta}$ and $m$ are positive tunable parameters. Define $\hat{P}$ as a real positive definite symmetric matrix which can be solved from the Lyapunov equation:

$$(A + BF)^T\hat{P} + \hat{P}(A + BF) = -W.$$

(9)

With a given positive definite symmetric matrix $W$, consider that $\hat{P}$ always exists since $(A + BF)$ is defined to be asymptotically stable. $W$ can be chosen as

$$W = 10^{\theta} \cdot \hat{E},$$

(10)

where $\theta$ is a tunable parameter and $\hat{E}$ is an identity matrix. Then, the new steady-state value $x_c$ is computed as

$$x_c = -(A + BF)^{-1}BGr.$$

(11)

Finally, a CNF controller is formed by combining the linear feedback law (5) and the nonlinear feedback law (7) as follows:

$$u_N = u_L + u_{NL}$$

$$= Fx + Gr + \rho(y, r)B^T\hat{P}(x - x_c),$$

(12)

Theorem 1. For any $\delta \in (0, 1)$, choosing $c_\delta > 0$ to be the largest positive scalar and satisfying the conditions [27],

$$|Fx| \leq (1 - \delta)u_{\text{max}}, \forall x \in X_\delta := \{x| x^T P x \leq c_\delta \}. $$

(13)

The initial state $x_0$ and $r$ satisfy

$$x_0 - x_c \in X_\delta, |Hr| \leq \delta_1 u_{\text{max}},$$

(14)

where $H = [I - F(A + BF)^{-1}B]G$, $0 \leq \delta_1 < \delta$, and $|\delta - \delta_1| u_{\text{max}} = d_{\text{max}}$. Then, the control law (12) is capable of driving the system output $y$ to track the command input $r$ asymptotically for any nonpositive function $\rho(y, r)$.

Inspired by the robustness enhancement of the integral sliding mode control with reaching law, such a controller is designed and added with the CNF controller to make overall system robust. Taking into account the input saturation, the integral sliding surface is designed as [38]

$$s = B^T(x(t) - x_0 - \int_{0}^{t} A\tau + B(s + u - u_\text{is})(\tau)d\tau),$$

(15)

where $B^*$ is the pseudoinverse of $B$ and $u_\text{is}$ is sliding mode control. The reaching law approach is first proposed in [42] which is utilized to force the system state quickly arrives at the sliding mode surface in the whole approaching process [43]. A novel improved quick reaching law is designed as [44]

$$\dot{s} = -k_1(b^{(s)} - 1)\text{sgn}(s) - k_2\mid s\mid^n \text{sgn}(s),$$

(16)

where $0 < a < 1$, $k_1 > 0$, $k_2 > 0$, and $b = 1 + k_2/k_1$. When the system state is far away from the sliding surface, the change rate of the first term in (16) is larger than that of the power function. It speeds up the reaching rate in the case $|s| > 1$.

When the system state is near to the sliding surface, the second item in (16) can make the system approach the sliding surface with higher speed.

Finally, the proposed ISMRL-CNF scheme for system (3) is expressed as
\[ u = u_{\text{CNF}} + u_s = Fx + Gr + \rho(y, r)B^T P(x - x_c) - k_1\left(b_{|s|}^s - 1\right)\text{sgn}(s) - k_2|s|^a\text{sgn}(s). \]

**Remark 1.** In order to ensure the closed-loop system has a fast rise time, \( F \) is chosen such that the closed-loop poles of \( C(sI - A + BF)^{-1} \) have a dominant pair with a small ratio. \( F \) can be designed by using \( H_\infty \) optimization approach. In order to reduce the overshoot, \( \rho(y, r) \) is selected to gradually change the damping ratio of the closed-loop system. To obtain the control parameters \( F, m, \beta, \) and \( W \) properly, we have

\[
\min_{F, m, \beta, W} \int_0^\infty t|y - r|dt. \tag{18}
\]

The integrated time and absolute error (ITAE) is utilized as the performance criteria, and the minimization problem (18) is solved by the particle swarm optimization (PSO) algorithm.

\[
\dot{V}_1 = s\dot{s} = s(-k_1(b_{|s|} - 1)\text{sgn}(s) - k_2|s|^a\text{sgn}(s) + d)
= -k_1s(b_{|s|} - 1) - k_2|s|^{a+1} + s\, d \leq -k_1s(b_{|s|} - 1) - k_2|s|^{a+1} + |s|\, d \leq -k_2|s|^{a+1} - (k_1(b_{|s|} - 1) - d_{\text{max}})|s|, \tag{21}
\]

where \( |d| \leq d_{\text{max}} \). It can be noted that when \( k_1(b_{|s|} - 1) - d_{\text{max}} \geq 0 \) such that

\[
[s] \geq \log_2\left(\frac{d_{\text{max}} + k_1}{k_1}\right),
\]

then \( \dot{V}_1 \leq -k_2|s|^{a+1} \). Thus, the sliding mode variable \( s \) can converge to the finite-time convergence region \( |s| \leq \log_2(d_{\text{max}} + k_1/k_1) \) [44].

Take the control law into the system and let \( \tilde{x} = x - x_c \), and we have

\[
\dot{\tilde{x}} = (A + BF)\tilde{x} + B\psi, \tag{22}
\]

where \( \psi = \text{sat}(F\tilde{x} + Hr + u_{\text{NL}} + u_s) - F\tilde{x} - Hr + d \).

Define a Lyapunov function:

\[
V_2 = x^T\dot{P}x. \tag{23}
\]

Taking the derivative of \( V_2 \), we obtain

\[
\dot{V}_2 = \dot{x}^T P\dot{x} + \dot{x}^T P\dot{x} = \dot{x}^T (A + BF)^T P\dot{x} + \dot{x}^T (A + BF)P\dot{x} + 2\dot{x}^T PB\psi
= -\dot{x}^T Wx + 2\dot{x}^T PB\psi. \tag{24}
\]

Then, we have

\[
|F\tilde{x} + Hr + u_{\text{NL}}| \leq |F\tilde{x}| + |Hr| + |u_{\text{NL}}| \leq u_{\text{max}},
\]

When \( |F\tilde{x} + Hr + u_{\text{NL}} + u_s| \leq u_{\text{max}}, \) then

\[
3.1. \text{Demonstration of System Stability.} \text{ Taking the derivative of the sliding surface (15) along the trajectories of system yields}
\]

\[
\dot{s} = B^T\left(\dot{x} - (Ax + B(\text{sat}(u) - u_{\text{NL}}))\right)
= B^T \left( Ax + B\text{sat}(u) + B \left( u_{\text{NL}} \right) \right)
= B^T \left( Bu_{\text{NL}} - d \right)
= -k_1(b_{|s|} - 1)\text{sgn}(s) - k_2|s|^a\text{sgn}(s) + d. \tag{19}
\]

Defining a Lyapunov function \( V_1 = 1/2s^2 \) and taking the derivative of \( V_1 \) [44],

\[
\psi = F\tilde{x} + Hr + u_{\text{NL}} + u_s - F\tilde{x} - Hr + d, \tag{20}
\]

where \( \dot{s} \) would converge to zero in finite time; then, \( u_{\text{NL}} = -d \). Then, \( \psi = u_{\text{NL}} = \rho(y, r)B^TP\tilde{x} \). \( \rho(y, r) \) is a nonpositive function. So, \( V_2 \leq -\tilde{x}^T W\tilde{x} \).

When \( (F\tilde{x} + Hr + u_{\text{NL}} + u_s) > u_{\text{max}}, \) then

\[
u_{\text{NL}} < u_{\text{max}} - (F\tilde{x} + Hr + u_{\text{NL}}) < 0, \]

\[
\psi = u_{\text{max}} - (F\tilde{x} + Hr) + d \leq u_{\text{max}} - (F\tilde{x} + Hr + u_{\text{NL}}) \geq 0,
\]

where \( \rho(y, r) \) is a nonpositive function. So, it implies that \( \tilde{x}^T PB \leq 0 \). Then, \( V_2 \leq -\tilde{x}^T W\tilde{x} \).

When \( (F\tilde{x} + Hr + u_{\text{NL}} + u_s) < u_{\text{max}}, \) then

\[
u_{\text{NL}} \leq -u_{\text{max}} - (F\tilde{x} + Hr - u_{\text{NL}} \leq 0, \]

\[
\psi = -u_{\text{max}} - (F\tilde{x} + Hr) + d \leq u_{\text{max}} - (F\tilde{x} + Hr + u_{\text{NL}}) \leq 0,
\]

where \( \rho(y, r) \) is a nonpositive function. So, it implies that \( \tilde{x}^T PB \leq 0 \). Thus, \( V_2 \leq -\tilde{x}^T W\tilde{x} \). Therefore, we can summarize that \( V_2 \leq -\tilde{x}^T W\tilde{x} < 0 \).

4. **Simulation Results**

In this section, various simulations are conducted to test the performance of the proposed strategy. For simplicity, system (2) is consider as linear systems with disturbances:
where $u_a = 10^{6} T_a$ and $u_β = 10^{7} T_β$. The maximum inputs are $u_{α_{max}} = u_{β_{max}} = 10$. The control objective is to force the scan angle $α$ or $β$ and follow the reference trajectory $r$ precisely with fast response in presence of disturbances.

The parameters of CNF for system (29) are tuned by PSO as $F_a = [-40.3457\ -5.6301]$, $m_a = 62.9236$, $β_a = 49.4610$, and $θ_a = -0.1296$. Then, $G_a = 40.4187$, $W_a = \begin{bmatrix} 0.7420 & 0 \\ 0 & 0.7420 \end{bmatrix}$, and $P_a = \begin{bmatrix} 2.7124 & 0.0030 \\ 0.0030 & 0.0214 \end{bmatrix}$. The parameters of ISMRL are chosen as $k_{1α} = 1$, $k_{2α} = 1.6$, and $a_a = 0.05$. The parameters of CNF for system (30) are tuned by PSO as $F_β = [-40.5216\ -7.1381]$, $m_β = 4.5160$, $β_β = 35.0126$, and $θ_β = 0.4681$. Then, $G_α = 41.0804W_β = \begin{bmatrix} 2.9383 & 0 \\ 0 & 2.9383 \end{bmatrix}$ and $P_β = \begin{bmatrix} 8.7290 & 0.0200 \\ 0.0200 & 0.1152 \end{bmatrix}$. The parameters of ISMRL are chosen as $k_{1β} = 1.5$, $k_{2β} = 2.2$, and $a_β = 0.04$.

Figure 2 shows the MEMS micromirror along the $x$-axes tracking trajectories using the CNF, ISM-CNF, and proposed ISMRL-CNF controller, respectively. The target references for $α$ are set as $r = 2$; the disturbance $d_α = -\text{sgn}(\sin(0.3πt))$ is introduced when $t ≥ 3\ ms$. It can be noted that the proposed controller has better performance in comparison with the two other controller such as CNF and ISM-CNF. The actuation motion trajectory of the MEMS micromirror under the proposed ISMRL-CNF controller is consistent with the desired trajectory in presence of the disturbance. The closed-loop system has good transient performance such as very small overshoot and fast response. The control inputs of CNF, ISM-CNF, and ISMRL-CNF are shown in Figure 3; compared with ISM-CNF, the chattering problem is eliminated by using the proposed ISMRL-CNF.

Figure 4 shows the MEMS micromirror along the $y$-axes tracking trajectories using the CNF, ISM-CNF, and proposed ISMRL-CNF controller, respectively. The target references for $β$ are set as $r = 2.2$ and the disturbance $d_β = -1.5\text{sgn}(\sin(πt))$ is introduced when $t ≥ 3.5\ ms$. The results demonstrate that the CNF controller exhibits worst performance due to the disturbances. Compare with ISM-CNF, the proposed ISMRL-CNF controller can obtain a faster and more efficient performance in presence of the disturbances. The control inputs of CNF, ISM-CNF, and proposed controller

\[
\begin{align*}
\dot{x}_1 &= \begin{bmatrix} 0 & 1 \\ -0.2251 & -0.16 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ 3.0827 \end{bmatrix}(\text{sat}(u_a) + d_a), \\
y_1 &= \begin{bmatrix} 1 & 0 \end{bmatrix} x_1,
\end{align*}
\]

\begin{equation}
(29)
\end{equation}

\[
\begin{align*}
\dot{x}_3 &= \begin{bmatrix} 0 & 1 \\ -1 & -0.15 \end{bmatrix} x_3 + \begin{bmatrix} 0 \\ 1.7894 \end{bmatrix}(\text{sat}(u_β) + d_β), \\
y_2 &= \begin{bmatrix} 1 & 0 \end{bmatrix} x_3.
\end{align*}
\]

\begin{equation}
(30)
\end{equation}
are shown in Figure 5. It can be noted that the control input of ISMRL-CNF is more smoother than ISM-CNF.

To further verify the antidisturbances of the proposed controller, the time-varying disturbance is introduced. The comparison of angular $\alpha$ responses using CNF, ISM-CNF, and proposed ISMRL-CNF controller under disturbance $d = -1.5 \sin(1.2\pi t)$ is shown in Figure 6. It can be seen that the traditional CNF controller is not able to suppress the time-varying disturbance. Compared with ISM-CNF, the proposed ISMRL-CNF controller ensures the system has better response in presence of disturbance. The comparison of angular $\beta$ responses using CNF, ISM-CNF, and proposed ISMRL-CNF controller under disturbance $d = -1.4 \cos(1.3\pi t)$ is shown in Figure 7. The results demonstrate that the proposed controller ensures the system has better performance in presence of time-varying disturbance.
5. Conclusions

In this study, the precise tracking problem for electrostatic micromirror systems with disturbances and input saturation is investigated. Inspired by the composite nonlinear feedback (CNF)'s improvement of the transient performance and the sliding mode control's enhancement of the robustness, a novel integral sliding mode with reaching law (ISMRL)-based composite nonlinear feedback (CNF) controller is proposed. Then, the stability of the closed-loop system is guaranteed based on Lyapunov theorem. Numerical simulations verify the effectiveness of the proposed scheme. It is shown that the closed-loop system with the proposed scheme has precise positioning and improved transient performance even in presence of time-varying disturbances. It should be noted that the proposed controller needs the accurate model knowledge; as a result, more inclusive methods about the model uncertainty and model inaccuracy, combined with faster convergence rate and smaller chattering, will be inquired in future work.

Data Availability

All data, models, or code that support the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was supported by National Science Foundation of China, under Grants 61603093 and 61703142.

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