Hindawi
International Journal of Antennas and Propagation
Volume 2022, Article ID 7060674, 10 pages
https://doi.org/10.1155/2022/7060674

Research Article

Guiding and Leakage Dispersion Characteristic of TM_{01} Mode in a Circular Dielectric Rod

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Received 6 August 2022; Revised 25 September 2022; Accepted 28 September 2022; Published 17 October 2022

Academic Editor: Shobhit K. Patel

Open dielectric waveguide structures such as circular dielectric rods can be used as antennas by leaking energy in the transverse direction or as guiding structures with low loss in integrated circuits at different frequency ranges. It is essential to understand electromagnetic wave behavior in different frequency regions such as reactive mode region, leaky-wave or antenna mode region, and guided mode region. In this study, it is aimed to assimilate the electromagnetic phenomenon for open dielectric structure by showing the overlapping between the analytic solutions and the simulation results of the TM_{01} mode of the circular dielectric rod. The analytic solutions for leaky-wave modes have been obtained using the coefficients matrix of the system of characteristic equations of the structure and Davidenko's method. The field distributions and the scattering parameters have been obtained in CST microwave studio software. The outcomes obtained in the study presented the overlapping between the analytical results and the simulations. The simulation results show the leakage starts end of the reactive mode region where the electromagnetic energy reflects to the feed line, and the electromagnetic energy leaks at the leaky-wave/antenna mode region, and the leakage decreases at the guided mode region as much as it can be neglected while the frequency increases.

1. Introduction

In the last few decades, the way used to solve electromagnetic problems has changed with the increase in digital computing capabilities. Analytical solutions are no longer needed for many electromagnetic problems. Many practical problems with complex geometries having no analytical solutions in closed form can now be solved numerically. Transformer and motor problems in low-frequency or antenna, filter, oscillator, waveguide, and integrated circuit electromagnetic compatibility problems in high-frequency can be solved numerically, independently of the analytical solution, with digital calculation programs using numerical solution methods such as finite element, finite difference, or transmission line matrices. However, deriving the solution to an electromagnetic problem in an analytical way is still important to understand the basis or essence of electromagnetic phenomenon and the behavior of electromagnetic waves. In other words, the correct interpretation of numerical results and simulation should depend on knowing the essence of directed waves on certain structures [1]. On the other hand, digital computation and simulation programs that model electromagnetic behaviors corresponding to the results obtained by analytical methods make an important contribution to the assimilation of the physical system. In addition, the use of digital computational programs in the solution of structures having no analytical solution in closed form supplies the most useful way to comprehend the electromagnetic behavior of the structure. Based on the given concepts, this study aims to obtain simulations of transverse magnetic (TM) guided and leakage modes, obtained analytically in the previous studies, for the open circular dielectric rod waveguide, and to determine the relationship between the simulation results and the analytic modes.

Open dielectric waveguides having nonconducting boundaries are special structures that can guide the electromagnetic wave along the waveguide in an integrated circuit with low loss and/or radiate it transversely outside the waveguide as an antenna for different frequency regions.

Open dielectric waveguides direct the electromagnetic waves or carry electromagnetic energy using the phenomenon of total internal reflection as transmission lines. Since they do not have metallic wall losses, they have much lower attenuation than closed metallic waveguides, especially at high frequencies [2]. In a metallic waveguide, the total energy is contained within the waveguide whereas in open dielectric waveguides, they exist as the sum of the energy guided along the waveguide called guided wave modes, and the energy radiated in the transverse direction from the waveguide called radiation modes or leaky-wave modes. The electromagnetic waves radiating transversely from the waveguide are called leaky-waves in the literature. A representation of an open waveguide structure located between a source and a load, and the leaky-waves outward from the waveguide are given in Figure 1.

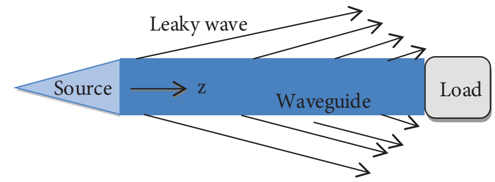


Figure 1: Open dielectric waveguide structure and leaky waves.

The wave leaking from an open waveguide structure depends on many factors such as operating frequency, physical geometry, or material filling the structure. Leakage wave structures have advantages such as simple geometric structure, cost-effectiveness, easy generation, high directivity, very narrow pattern bandwidth and are applicable in a wide frequency range from millimeter-wave frequency to optical frequency. Leaky waves in different geometric structures and the nature of leaky waves have attracted a lot of attention in the literature and have been studied both analytically and numerically [3–5].

Basically, there can exist transverse electric (TE) modes having no electric field in the direction of propagation, transverse magnetic (TM) modes having no magnetic field in the direction of propagation, and transverse electromagnetic (TEM) modes that do not have either one. In open waveguides, in addition to all these modes, there may also exist hybrid electromagnetic (HEM) modes with both electric field and magnetic field components in direction of the propagation because of their nonconducting wall. Analytical solutions for the respective modes for electromagnetic waveguides are obtained from the solutions of Maxwell's equations using the electromagnetic properties of the materials of the structure and the boundary conditions determined by the material-outdoor interfaces. As special solutions of Maxwell's equations, the eigenfunctions of the characteristic equations or the system of the characteristic equations correspond to a separate solution, and each is called a mode. These solutions are presented on frequency-propagation constant planes called Brillouin's diagrams. Brillouin's diagrams, which are easy to interpret and useful for engineers and physicists, show the transmission or cutoff characteristics of the structure by giving the waveguide's propagation constant values depending on the frequency. In this study, the dispersion characteristic of TM_{01} mode of the open circular dielectric rod is presented on Brillouin's diagram obtained from the analytical solution and the simulations obtained from the CST microwave studio software, and it will be shown how the analytical and numerical solutions overlap.

The characteristics of the modes in an open dielectric waveguide are more complex than those found in a metallic waveguide because the electromagnetic energy can propagate along the waveguide using the phenomenon of total internal reflection as in fiber optic cables and also radiate transversely from the waveguide depending on the wavelength [6, 7]. Open dielectric waveguides can be used as antennae by leaking electromagnetic waves in a certain frequency range due to their structure, and they are also used to guide electromagnetic waves in integrated circuits. Bulk metallic cavities with huge and not integrated microwave integrated circuits (MICs) were the first resonators in the early microwave systems. Additionally, the strip line resonators had weak temperature stability and a poor-quality factor. In the current microwave circuits, dielectric resonators, which are fundamentally open dielectric waveguides, can easily be integrated with MICs with low loss and thermal stability. The advantages of stability, cost, efficiency, dimension, mass, robustness, and ease of use make dielectric resonators the first choice in MICs. The open dielectric waveguides with low permittivity are used also for millimeter-wave communication. For example, ceramics with medium dielectric permittivity in the range of 25–50 is used for satellite communications and in cell phone base stations [8]. Various leaky-wave antennas with low cost, the capability of tunning a frequency spectrum, and high directivity can be performed by utilizing the electromagnetic properties of materials and mechanical flexibility of the materials. The graphene-based optical leaky-wave antennas can be given as one of the specific applications of leaky-wave antennas [9–11]. Open dielectric waveguides are widely used in optical communication to guide electromagnetic waves with low loss at high frequencies. Dielectric rod waveguides with cylindrical geometry are generally used as components in leakage wave antennas, low-loss waveguides, high-frequency filters, oscillators, integrated adapters, frequency tuners, amplifiers, and resonator circuits [12, 13]. Since open dielectric waveguide structures such as circular dielectric rods can be used as a guiding structure with low loss in integrated circuits or as an antenna by leaking energy in the transverse direction, it requires careful design in the applications. Because leakage is not desired in integrated circuits as the radiation efficiency is too important for an antenna application. As mentioned above, open waveguide structures have numerous applications in microwave circuits. In this study, instead of addressing a specific application, the simulation of electromagnetic behavior in the leakage and guidance regions obtained from the theoretical calculations is obtained for a simple open waveguide structure, the dielectric rod. Thus, the leakage and guiding electromagnetic behavior of the structure being simulated and the harmony between theory and simulation are revealed.

As explained above by giving literature studies, it is a critical point to determine the operating frequency region of an antenna or integrated circuit to be designed using an open waveguide structure such as a cylindrical dielectric rod. While the dielectric rod waveguide transmits the electromagnetic energy along the waveguide above the cutoff frequency, it radiates in a certain frequency range below the cutoff frequency and can be used as an antenna in this frequency region. The frequency region where electromagnetic energy radiates transversely from the waveguide can be determined analytically or numerically using simulation programs. In this study, the analytical results of the TM_{01} mode of the circular dielectric rod are obtained using the coefficients matrix of the system of characteristic equations of the structure and Davidenko's method. The field distributions for the frequency regions corresponding to the reactive mode, guided modes, and antenna modes have been obtained using CST microwave studio software. Thus, it will be possible to interpret the relationship between the analytically obtained dispersion curves and the field patterns obtained through the simulation program. As a result, the study helps engineers and physicists to match the analytic results and simulation and assimilate the electromagnetic phenomenon of an open dielectric structure in the different frequency region.

2. Dispersion Characteristic of Open Waveguide

Open waveguides having nonconducting boundaries can have HEM modes as well as TE and TM modes since all electric and magnetic fields can exist both inside and outside the waveguide. In these structures, the electromagnetic wave radiating transversely outside the waveguide is called a leaky wave. The propagation constant has complex values in mathematical description of the leaky waves. The complex-valued roots of the propagation constant obtained from the roots of the characteristic equation of the structure are used for the analytic analysis of the leakage behavior of an open waveguide. If the variation expression of the propagation constant is defined as $e^{-j\gamma z}$ with the direction of the wave being (z) and the propagation constant (γ), then the leakage and guiding regions are determined by interpreting pure real, pure imaginary, and complex values of the propagation constant in different frequency regions. The real part of the complex propagation constant is called the phase constant (β), as the imaginary part is called the attenuation constant (α). The normalized propagation constant is described below.

$$\overline{\gamma} = \frac{\beta - j\alpha}{k_0} = \overline{\beta} - j\overline{\alpha}. \tag{1}$$

Complex values of frequency dependent propagation constant obtained from analytical solutions of an open waveguide such as a dielectric rod indicate the usual physically behavior of electromagnetic waves such as transmitting along the waveguide in the propagation direction, radiating outward from the waveguide in the transverse direction, or reflecting from the waveguide to feed line. Thus, physical behaviors in different frequency regions are predicted depending on the values of the complex propagation constant with using only analytical solutions. In the study, it is proposed to present the matching between the analytical solutions and the simulation results. The general dispersion characteristic of lossless open dielectric waveguides surrounded by a vacuum is presented in Figure 2. The guided mode of the structure has no attenuation constant. They exist in the range, $\sqrt{\varepsilon_1\mu_1} < \overline{\beta} < \sqrt{\varepsilon_2\mu_2}$, ($\sqrt{\varepsilon_1\mu_1} = 1$ for vacuum). Here, ε and μ represent the relative dielectric and magnetic permeability of each medium, respectively. As seen in Figure 2, the reactive mode region, the antenna mode region and the guided mode region exist, respectively, as the frequency increases for lossless open dielectric waveguide.

The variation expression of the propagation constant approaches zero as the z value gets larger for the pure imaginary values of the complex propagation constant ($\overline{\beta} = 0$ and $\overline{\alpha} > 0$) when they are substituted into the expression. Basically, it means that the energy cannot propagate in the z direction and attenuates in the waveguide. It is correct for the case where the attenuation constant is very large ($\overline{\alpha} \gg$). The frequency region where the attenuation constant is always greater than the value of the phase constant ($\overline{\alpha} > \overline{\beta}$) is called the reactive mode region as shown in Figure 2. In addition, the very low-frequency region in the reactive mode region can be divided into a second subregion, called the nonphysical mode region, where the phase constant takes very large values ($\overline{\alpha} \gg$). However, this region physically corresponds to the reactive mode region where the electromagnetic energy cannot propagate along the waveguide and reflects to the feed line [14–16]. The electromagnetic energy propagates along the waveguide in the z direction when the complex propagation constant is pure real ($\overline{\beta} > 1$ and $\overline{\alpha} = 0$). This frequency region is called the guided mode region and exists on the frequency axes above the cutoff frequency for the lossless open dielectric waveguides. While the complex value propagation constants correspond essentially attenuated modes in which electromagnetic energy propagates by attenuating and is damped fast in the lossless closed waveguides, they are classified as leaky wave or antenna modes and reactive mode depending on the magnitudes of the attenuation constant and the phase constant in the lossless open waveguides. The frequency region where the attenuation constant is larger than the phase constant is accepted as the reactive mode region as seen in Figure 2. The frequency region where the complex propagation has a larger phase constant than the attenuation constants and both (normalized) are less than the unity below the cutoff frequency is called the leaky-wave mode or antenna mode region. It has been experimentally shown that electromagnetic energy radiates transversely outside the waveguide in this region with/for leaky-wave antenna applications [14, 15, 17–23]. In this study, it is fundamentally aimed to show the validity of this acceptance ($1 > \overline{\beta} > \overline{\alpha}$) for leaky-wave/antenna mode region with simulation results.

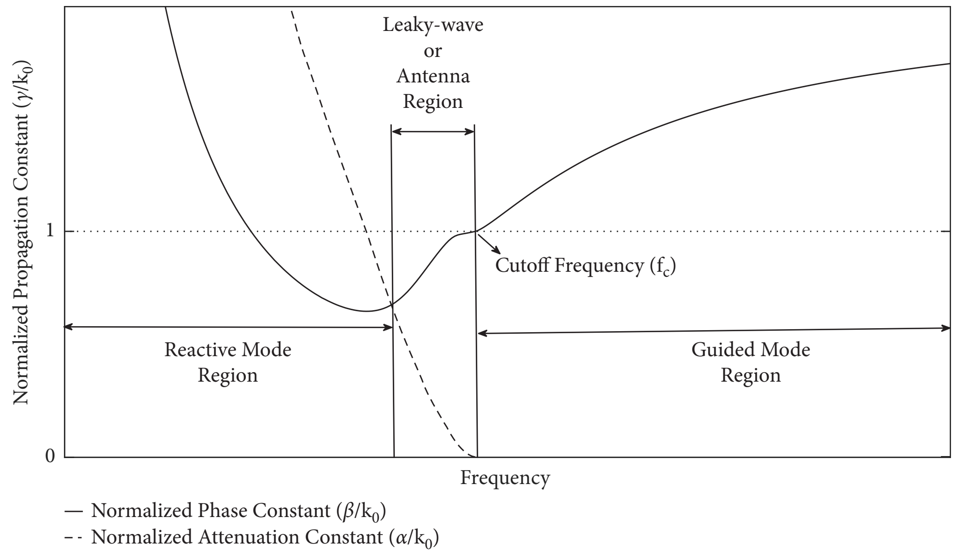


FIGURE 2: The general dispersion characteristic of open dielectric waveguides.

3. Dispersion Curves of Circular Dielectric Rod for TM_{01} Mode

The dispersion characteristics for circular dielectric rod waveguides are well known in the literature. In previous studies, TM and TE leaky-wave modes for a circular dielectric rod were obtained from Davidenko's method based on the closed function expression of the structure [24–26], and HEM leaky-wave modes were obtained from Davidenko's method based on the coefficients matrix of the system of characteristic equations [27, 28].Davidenko's method, requiring derivation of the dispersion equation with respect to the propagation constant, is insensitive to select initial guess and an efficient algorithm to find complex roots of multiple dimension systems of nonlinear algebraic equations. The approximation based on the coefficient matrix of the system of characteristic equations in references [27, 28] simplifies the derivation of the characteristic equation with respect to γ, required by Davidenko's method. Hereby, Davidenko's method can be used for the structures belonging to complex characteristic equations with numerous variables depending on the propagation constant. While only TE and TM modes could be obtained by using auxiliary mathematical software for derivations in previous studies, HEM modes of the circular dielectric rod were obtained from completely analytical expressions for the first time in the literature. Detailed analytical expressions for the approach based on Davidenko's method and the coefficient matrix of the system of characteristic equations are presented in references [27, 28].

In this section, the coefficient matrix of the system of characteristic equations of the circular dielectric rod and the derivation of the coe?cient matrix of the system of the characteristic equations with respect to γ due to the requirement of Davidenko's method is presented. The field expression for the lossless circular open waveguide is given below.

$$F(r,\varphi,z) = F(r)e^{j(\omega t-\gamma z-m\varphi)}. \quad (2)$$

where F represents a closed function. r, φ, and z are the cylindrical coordinate system components, j is the imaginary unit number, ω is the angular frequency, and m is the azimuthal variation. In the study, the complex propagation constant of a circular dielectric rod is computed by using Davidenko's method. If $f(x) = 0$ is the n-dimensional case of algebraic functions with n-unknowns, the formal expression of Davidenko's method for this is

$$\frac{dx}{dt} = -(\mathrm{J})^{-1}\mathrm{f}(\mathbf{x}), \quad (3)$$

where dt corresponds to the increment of dummy variable t, and $\boldsymbol{J}$ is the Jacobian matrix for n nonlinear algebraic equations set in n unknowns. The dispersion equation in the form of $F(\omega;\gamma) = 0$ is transformed into a system of two coupled first-order ODE's by Davidenko's method as below. The standard procedures and expressions for Davidenko's method are given in reference studies [24, 26–29].

$$\begin{aligned}\frac{d\alpha}{dt} &= -\frac{1}{\left|F_\gamma\right|^2}\left(\mathrm{Re}\left[F_\gamma\right]\mathrm{Re}[F] + \mathrm{Im}\left[F_\gamma\right]\mathrm{Im}[F]\right),\\ \frac{d\beta}{dt} &= \frac{1}{\left|F_\gamma\right|^2}\left(\mathrm{Im}\left[F_\gamma\right]\mathrm{Re}[F] - \mathrm{Re}\left[F_\gamma\right]\mathrm{Im}[F]\right).\end{aligned} \quad (4)$$

The derivative of the dispersion equation with respect to the propagation constant is necessary for Davidenko's method. The derivation of the implicit functions can be difficult especially if the function includes complex expressions. In the study, the coefficient matrix of the system of characteristic equations is used instead of the implicit function to simplify the derivation. The zeros of the implicit function and/or the determinant of the coefficient matrix of the system of characteristic equations correspond to the solution points.

$$F(\omega, \gamma) = \det(\mathbf{M}) = 0, \tag{5}$$

where $\boldsymbol{M}$ is the coefficients matrix of the system of characteristic equations. Equation (6) can be used to obtain the derivation of the coefficients matrix of the system of characteristic equations.

$$\frac{dF(\omega;\gamma)}{d\gamma} = F_\gamma = \frac{d\,\det(\mathbf{M})}{d\gamma} = tr\left(\operatorname{adj}(\mathbf{M})\frac{d\mathbf{M}}{d\gamma}\right), \tag{6}$$

where *adj* and tr are the adjugates of the matrix and the trace of the matrix, respectively. The derivation of the expression is obtained as below, if the definition of the inverse matrix is used.

$$F_\gamma = tr\left(\det(\mathbf{M})\mathbf{M}^{-1}\frac{d\mathbf{M}}{d\gamma}\right). \tag{7}$$

The derivation of a matrix with respect to a variable is obtained by the derivative of each element of the matrix with respect to the corresponding variable. So, Davidenko's expressions can be transformed into below expressions.

$$\begin{aligned}
\frac{d\alpha}{dt} &= -\frac{1}{\left|tr\left(\det(\mathbf{M})\mathbf{M}^{-1}d\mathbf{M}/d\gamma\right)\right|^2}\left\{\operatorname{Re}\left[tr\left(\det(\mathbf{M})\mathbf{M}^{-1}\frac{d\mathbf{M}}{d\gamma}\right)\right]\operatorname{Re}[\det(\mathbf{M})] + \operatorname{Im}\left[tr\left(\det(\mathbf{M})\mathbf{M}^{-1}\frac{d\mathbf{M}}{d\gamma}\right)\right]\operatorname{Im}[\det(\mathbf{M})]\right\} \\
\frac{d\beta}{dt} &= \frac{1}{\left|tr\left(\det(\mathbf{M})\mathbf{M}^{-1}d\mathbf{M}/d\gamma\right)\right|^2}\left\{\operatorname{Im}\left[tr\left(\det(\mathbf{M})\mathbf{M}^{-1}\frac{d\mathbf{M}}{d\gamma}\right)\right]\operatorname{Re}[\det(\mathbf{M})] - \operatorname{Re}\left[tr\left(\det(\mathbf{M})\mathbf{M}^{-1}\frac{d\mathbf{M}}{d\gamma}\right)\right]\operatorname{Im}[\det(\mathbf{M})]\right\}.
\end{aligned} \tag{8}$$

Consequently, it is possible to obtain easily the derivation of the dispersion expressions and to use Davidenkos method for the structures having complex analytic expressions. The coefficient matrix of the system of characteristic equations is defined for TM modes of the circular dielectric rod, as follows: $\begin{bmatrix} J & -H \\ -jb_1\varepsilon_r J' & jb_2 H' \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix} = 0$, where ε_r is the relative dielectric constant. $J = J_0(k_1 a)$, $H = H_0^{(2)}(k_2 a)$, and $b_n = \omega\varepsilon_0/k_n$ ve $n = 1, 2$. J_0 ve $H_0^{(2)}$ indicate the first kind Bessel function and the second kind Hankel function, respectively. a is the radius of the circular dielectric rod. The apostrophe above the Bessel and Hankel functions signi?es the derivative of the function. k_0, k_1, ve k_2 are defined $k_0 = \sqrt{\omega^2\varepsilon_0\mu_0}$, $k_1 = \sqrt{k_0^2\varepsilon_r - \gamma^2}$ and $k_2 = \sqrt{k_0^2 - \gamma^2}$, respectively. Derivation of the coef?cient matrix of the system of the characteristic equations with respect to γ due to requirement of Davidenko's method is obtained from the derivative of each component of the matrix. The components of the derivation matrix is given below.

$$M_{11} = -\frac{\gamma}{k_1}aJ' M_{12} = -\frac{\gamma}{k_2}aH' M_{21} = jb_1\varepsilon_r\frac{\gamma}{k_1}\left[\frac{1}{k_1^2}J_1 - \frac{a}{k_1}J_1'\right] M_{22} = -jb_2\frac{\gamma}{k_2}\left[\frac{1}{k_2^2}H_1 - \frac{a}{k_2}H_1'\right], \tag{10}$$

where $J_1 = J_1(k_1 a)$ and $H_1 = H_1^{(2)}(k_2 a)$. The numerical solutions can be calculated when the presented equations above substitute in the equations given in [27, 28] with. The leakage dispersion characteristics of TM, TE, and HEM modes of the circular dielectric rod with $a = 10$ mm and $\varepsilon_r = 4$ were presented in [27]. The aim of the study is to show the correspondence between analytic results and the simulations executed on CST microwave studio software. The leaky-wave characteristic of the TM_{01} obtained from the analytic solutions is given in Figure 3.

The cutoff frequency where the guided modes begin and exist at higher frequencies is 6.63 GHz for TM_{01} modes of the structure. The leaky-wave or antenna mode region where the electromagnetic energy propagates transversely outside the open dielectric and propagation direction (z) as a surface wave is between 2.31 GHz and 6.63 GHz. The region below 2.31 GHz is the reactive mode region where the electromagnetic energy reflects to the feed line. The numeric results obtained from the coefficient matrix of the system of characteristic equations and Davidenko's method in the study are compatible with the results presented in previous studies[24–26]. The sample visuals obtained from CST microwave studio software for the circular dielectric rod surrounded with vacuum has been added to the figure. The detailed simulation results are presented in the next section.

4. Simulation Results of the Circular Dielectric Rod

The circular dielectric rod has been investigated analytically and numerically in numerous studies for a century. Additionally, the leaky-wave concept previously discussed for applications of the leaky-wave antenna has been dealt with to be classified the modes in the leaky-wave region and guided

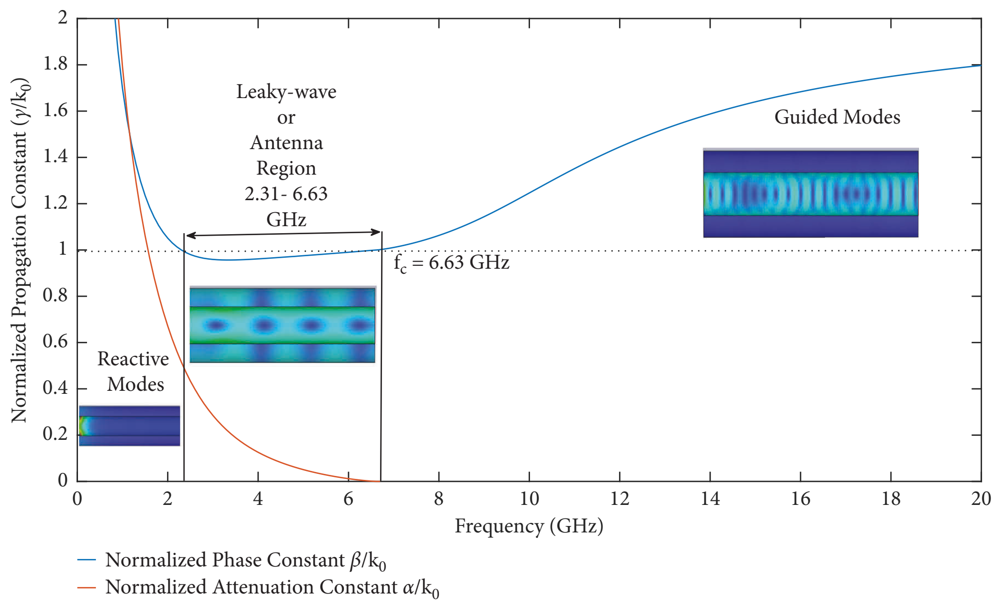


FIGURE 3: Leaky-wave characteristic of TM_{01} mode of the circular dielectric rod with $a = 10$ mm and $\varepsilon_r = 4$.

wave region after progressing the optic communication and capability of open dielectric structure using both an antenna and waveguide with the low loss [1, 12–18]. The boundaries of the leaky-wave or antenna region especially the transition point from the reactive mode region to the guided mode region was discussed in the studies. Because it was not agreed commonly while the cutoff frequency point was universally accepted as the transition point from the leaky-wave region to the guided mode region. Oliner and Lee firstly identified a physically unimportant region having a complex propagation constant with a high antennation constant except a leaky-wave region having a complex propagation constant with lower attenuation constant than the phase constant previously defined as the radiation region [17]. In the later studies, the radiation region was divided into two regions as the leaky-wave/antenna mode region where the electromagnetic energy leaks transversely outside the waveguide, and the reactive mode region where the energy reflects the feed line [14, 15]. After these studies, the leaky-wave or antenna mode region has been commonly accepted as the region where the phase constant is higher than the attenuation constant and both are less than unity $(1 > \overline{\beta} > \overline{\alpha})$. Additionally, in the studies, necessary of the mode classification for antenna applications was shown by presenting lower efficiency of a leaky-wave antenna outside leaky-wave/antenna mode region. In this section, the scattering parameters and electric field distributions at different frequencies in reactive modes, leaky-wave/antenna modes, and guided modes regions have been obtained from CST Microwave Studio software for the vacuum surrounded circular dielectric rod whose dispersion curve was presented in the previous section. The vicinity of the transition point between the region has been investigated in light of the literature discussion. The vacuum-enclosed circular dielectric rod with a relative dielectric constant of 4- and 10-mm radius constituted in the software is as shown in Figure 4.

Two waveguide ports were connected to both edges of the dielectric rod. The port one is used as the excitation port or feed line, and the port second is used as the receiver or load port. The metallic closed and/or open waveguide structures support numerous modes. TM, TE, TEM, and hybrid modes, HE and EH, can exist in the open waveguide structures while only TM, TE, and TEM modes exist in the closed waveguide structure. However, in the study, the TM_{01} mode is limited to investigate the correlation of the analytic results and simulation results. There is the infinite possibility to excite any port. There is no agreement between the analytic aspect and the simulation aspect describing which mode in simulation software corresponds to the exact analytic mode. Therefore, the first port was excited for five different modes in the simulation due to tracing the mode corresponding to TM_{01}. The structure was simulated in time domain solver in CST microwave studio software. The scattering parameters of the first simulation mode match the TM_{01} mode as seen in Figure 5.

The scattering parameters for the first two simulation modes coincide with each other. The situation is acceptable because, as known, the cutoff frequency of TM_{01} and TE_{01} is the same. The first simulation mode matched with TM_{01} has been approved as the corresponding mode for investigating mode, TM_{01}. The cutoff frequency of the guided modes is obtained analytically as 6.63 GHz for a circular dielectric rod with $a = 10$ mm and $\varepsilon_r = 4$. The transmission coefficient and the reflection coefficient are −1.37 dB and −13.33 dB at the cutoff frequency. While the frequency is increasing in the antenna mode region, the transmission coefficient increases from −34.46 dB at the beginning of the antenna region to −1.37 dB at the cutoff frequency as the reflection coefficient decreases. The scattering parameters show that the electromagnetic energy propagates mostly inside the dielectric at higher frequencies than that cutoff frequency. To observe the field distribution in the reactive mode region, the antenna

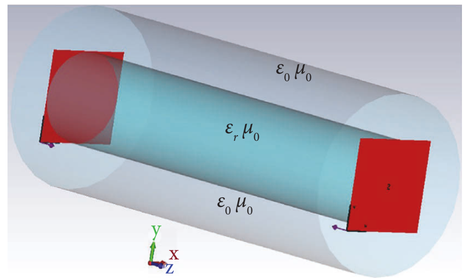


Figure 4: Perspective view of the circular dielectric rod surrounded with vacuum in CST.

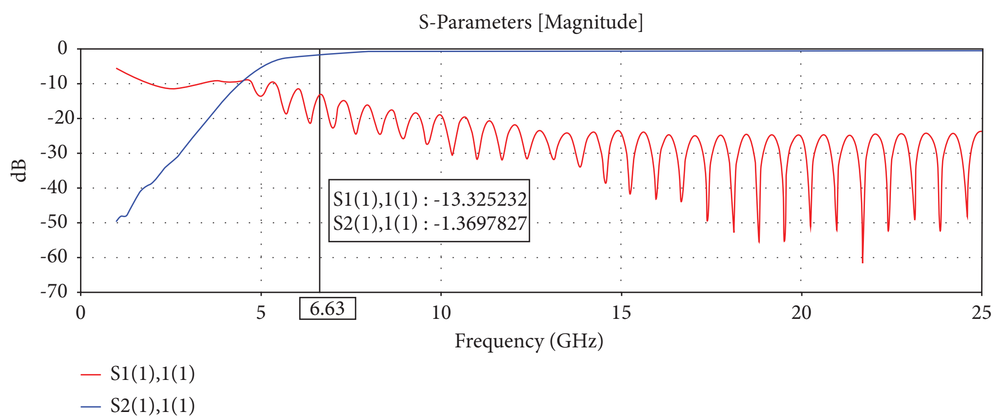
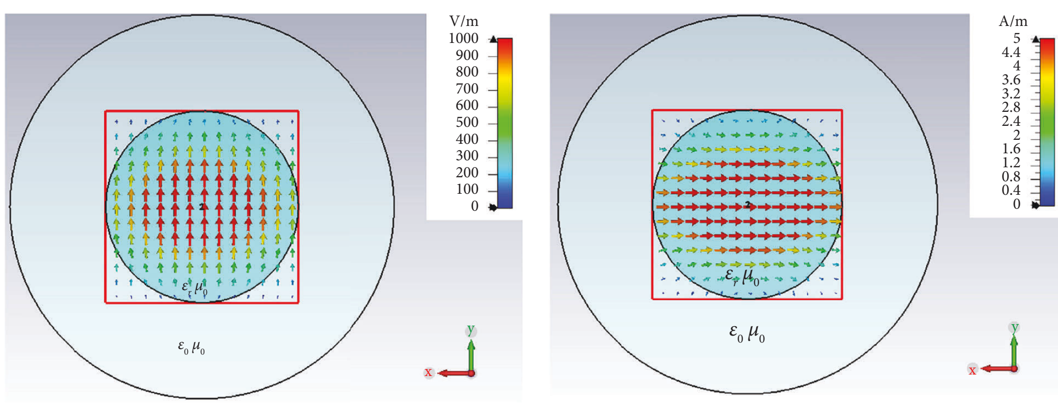


Figure 5: Refection coefficient and transmission coefficient for a circular dielectric rod with a = 10 mm and $\varepsilon_r = 4$.



(a) (b)

Figure 6: (a) Electric field distribution and (b) magnetic field distribution of the first simulation mode.

mode or leaky-wave region, and guided mode region, the structure has been simulated at different frequencies. The arrow presentation of the electric field distribution and the magnetic field distribution for the first simulation mode corresponding to TM_{01} is given in Figure 6.

The field distribution patterns obtained from CST microwave studio software have been presented in three grubs as reactive mode region, antenna mode region, and guided mode region. The scattering parameters and the electric field patterns in the reactive mode region (frequencies lower than 2.31 GHz) where the attenuation constant is very high against the phase constant and the electromagnetic energy reflects the excitation port are given in Table 1 for the frequencies 1 GHz and 2 GHz. All electric field patterns given in the study were scaled with 1000 V/m. Therefore, the E-field scale is seen only for 1 GHz.

The electromagnetic energy cannot propagate inside or outside of the dielectric as seen in the E-Field distribution belonging to the reactive mode region. The energy reflects the feed line, so the reflection coefficient is less than −35 dB as seen in Figure 5. As seen E-Field distribution for 1 GHz and 2 GHz in second column in Table 1, electromagnetic energy reflects the feed line instead of transforming into the port 2. Therefore, the reflection coefficients are −5.59 dB and

Table 1: The scattering parameters and the electric field distributions at the reactive mode region.

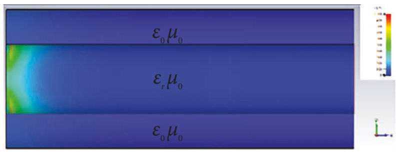
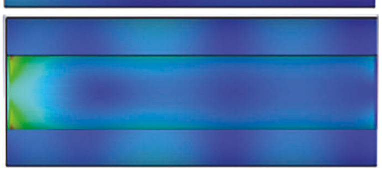
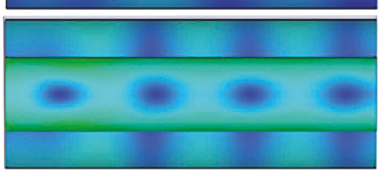
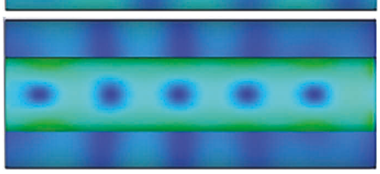
f (GHz)	E-Field distribution	S-parameters
1		$S_{11} = -5.59$ dB $S_{21} = -49.31$ dB
2		$S_{11} = -10.17$ dB $S_{21} = -38.34$ dB

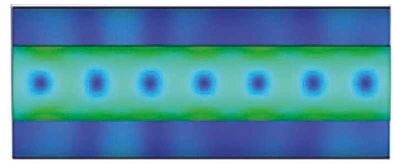
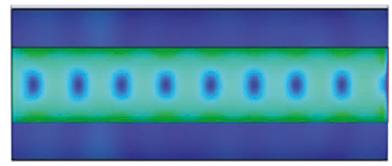
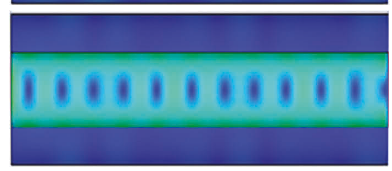
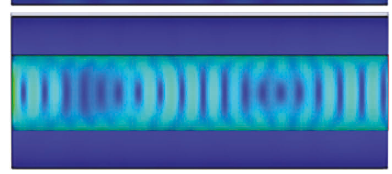
Table 2: The scattering parameters and the electric field distributions at the antenna mode region.

f (GHz)	E-Field distribution	S-parameters
3		$S_{11} = -10.74$ dB $S_{21} = -26.68$ dB
4		$S_{11} = -9.53$ dB $S_{21} = -14.46$ dB
5		$S_{11} = -13.56$ dB $S_{21} = -5.36$ dB
6		$S_{11} = -11.53$ dB $S_{21} = -1.93$ dB

−10.17 dB for 1 GHz and 2 GHz, respectively, while the transmission coefficients are −49.31 dB and -38.34 dB. It is clearly seen that the simulation results totally match the analytical solutions in this region. The scattering parameters and the electric field patterns in the antenna mode region (2.31–6.63 GHz) where the phase constant is higher than the attenuation constant and the electromagnetic energy mostly leaks and also propagates along the waveguide as a surface wave are given in Table 2 for the frequencies 3 GHz, 4 GHz, 5 GHz, and 6 GHz.

The electromagnetic energy starts to leak outside the dielectric in the antenna region as seen in E-Field distributions. The certain frequency at where leakage starts were discussed in the literature. After the studies of Lin et al. in references [14, 15], the theoretical point where the leakage starts and the leaky-wave/antenna region begins was

Table 3: The scattering parameters and the electric field distributions at the guided mode region.

f (GHz)	E-Field distribution	S-parameters
7		$S_{11} = -23.38$ dB $S_{21} = -1.17$ dB
8		$S_{11} = -16.34$ dB $S_{21} = -0.65$ dB
10		$S_{11} = -19.14$ dB $S_{21} = -0.38$ dB
15		$S_{11} = -24.57$ dB $S_{21} = -0.24$ dB

accepted as the point in which the normalized phase constant is higher than the normalized attenuation constant and both are less than unity. The simulation results support the theoretical acceptance as seen in Figure 5 and Table 2. The leakage is low at 3 and 4 GHz for TM_{01} mode for the structure because the reflection constant is about −10 dB, so the energy remains mostly to reflect the feed line instead of leaking. The higher frequencies that 4.8 GHz in the antenna mode region is more suitable for antenna applications because the attenuation constant is lower than −10 dB at all frequency points with a low phase constant. For example, the S_{11} and S_{21} are -13 dB and −5.36 dB, respectively, for 5 GHz or −18.5 dB and −2.48 dB for 5.69 GHz. The leaky-wave characteristic is useful to design dielectric resonator antennae. The scattering parameters and the electric field distributions in the guided mode region (frequencies higher than 6.63 GHz) where the attenuation constant is equal to zero and the propagation constant is pure real and the electromagnetic energy propagates along the dielectric waveguide as guided mode and surface mode are given Table 3 for the frequencies 7 GHz, 8 GHz, 10 GHz, and 15 GHz.

The cutoff frequency and the starting point of the guided modes region are clear in the literature as distinct from the leak-wave/antenna region. In an open dielectric waveguide, the leakage outside of the structures continues at higher frequencies than the cutoff frequency due to the open boundary. However, the amount of this leakage is much lower compared to the transmitted energy to the receiver/load port. The simulation results show that the transmission coefficient increased from −1.37 dB at the cutoff frequency to −0.24 dB at 15 GHz in the guided mode region. As a result, the electromagnetic energy propagates mostly along the dielectric rod and the leakage gets weaker and weaker as much as neglected in the guided mode region while the frequency is increasing.

5. Conclusion

Numerical methods and simulation software may be the best ways to obtain the solution of the geometrically complex structures and the structures having no exact analytic solution. Because it is hard to obtain the results experimentally and it most probably takes a long time and costs much. However, deriving analytical solutions to an electromagnetic problem is still important to comprehend the basis or essence of electromagnetic phenomenon and the behavior of electromagnetic waves. In other words, the correct interpretation of numerical results depends on knowing the essence of guided waves on specific structures. In the study, it is aimed to obtain simulation results for TM-guided modes and leaky-wave modes of circular dielectric rod previously calculated analytical solutions and determine the relationship between the simulation results and analytic results. The circular dielectric rod with $a = 10$ mm and $\varepsilon_r = 4$ has been simulated in CST Microwave Studio software. The dispersion curve of the structure obtained from the analytic solution is presented and the different frequency regions such as the reactive mode, leaky-wave/antenna mode, and guided mode, are emphasized on the curve. The scattering parameters and the E-Field distributions obtained from the simulation for different frequencies are presented and matched with the analytic solution. The simulation results are totally matched with the analytic solutions for the reactive mode region and the guided mode region for higher frequencies. However, there is mostly consonance between simulation and analytic results for the leaky-wave/antenna mode region. The protracted discussion in the literature is to fix the boundary of the regions and the transition point from one region to another region. The point where the phase constant gets higher than the attenuation constant and both are lower than unity is accepted as the transition point from the reactive mode region to the leaky-wave/antenna mode region. The simulation results show the leakage starts in the vicinity of the point and the electromagnetic energy leaks at the leaky-wave/antenna mode region and the leakage decrease at the guided mode region as much as it can be neglected while the frequency increases. Figure 3 presents a good brief for the results and shows the harmony between the analytical results and the simulations.

The leaky-wave characteristic and guided mode characteristic are useful to design dielectric resonator antennae or use the structure as a waveguide with minimum leakage in integrated circuits. The presented results in the study enable the antenna or integrated circuit designer to have information about the leakage and the guiding regions of the circular rod before making an application. The study shows the correspondence between the theoretical results and simulation and presents effectual harmony between both. Thus, visualizing electromagnetic behavior can be possible for leakage and guidance obtained from theoretical calculations. An experimental study to confirm the presented results in the study can be performed as a future work [30].

Data Availability

MATLAB codes used to obtain numerical results for Davidenko's method are shared as supplemental documents. The MATLAB code "TM_leaky modes" can be executed to calculate the propagation constant (the phase const. and the attenuation const.) for a certain frequency. The obtained results are presented in the manuscript. The document, "Description of Supplementary Files," has been attached as a separate file, and has also been added at the end of the manuscript as information.

Disclosure

The manuscript was shared in the preprint server SSRN.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Supplementary Materials

Four *m* files, "TM_leaky modes, Davidenko, *f*TM, *f*TMt," have been added as supplementary files. The leaky modes for circular dielectric rod can be obtained from TM_leaky modes on MATLAB software. The leaky modes for a circular dielectric rod with $a = 10$ mm and $\varepsilon_r = 4$ have been presented online 216 in the manuscript. Anyone can compute the leaky modes in the frequency region between zero and the cutoff frequency, 6.63 GHz, by using the TM_leaky modes. In order to compute the modes, it is firstly necessary to write an initial guess for the first mode at a certain frequency in the line "TM01 = (5e9 −0.98 0.05); % (The operating fr. the phase const. the attenuation const.)." For example, the initial guess for the propagation constant is $\gamma = 0.98 + 0.05i$ at $f = 5$ GHz, so the initial guess for the phase constant is 0.98, and the initial guess for the attenuation constant is 0.05. The line "for fr = 5e9 : −0.1e9 : 0.1e9 % the operating frequency" determines the frequency region where the leaky modes will be obtained. For example, the frequency starts at 5 GHz and finishes 100 MHz to the description, "for fr = 5e9 : −0.1e9 : 0.1e9." The guided modes that exist above the cutoff frequency can be obtained from the "Open_Boundary_ChaEq_TM." In order to compute the guided modes, it is only necessary to define the frequency range in the line "for f = 6.7e9 : 0.1e9 : 15e9." For example, the description "for f = 6.7e9 : 1e9 : 15e9" computes the guided modes in the frequency region between 6.7 GHz and 15 GHz. The CST data could not include as supplementary files because the system cannot support the file format. However, they will share if they are required by the reviewers. (*Supplementary Materials*)

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