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Research Article

Simple Matrix Equation (SME) Method for Pattern Synthesis of Conformal Antenna Array with Arbitrary Architecture

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A novel noniterative method for pattern synthesis of conformal antenna array is put forward. First, a new pattern function formula for general conformal antenna array is derived. Then, according to the new pattern function formula, a new block matrix equation (BME) and a simple matrix equation (SME) are obtained. SME has the same form as the equation of linear array pattern synthesis. The new method can be applied to pattern synthesis of any conformal array. Moreover, due to SME having the same form as the matrix equation of linear array pattern synthesis, the new simple matrix equation can be dealt with by the approaches for linear array pattern synthesis, which significantly expands the existing approaches for conformal antenna array pattern synthesis. Two different conformal array configurations are taken as the examples to demonstrate the advantages of the new method. Results of the simulations show that the new method can flexibly and effectively be applied to synthesize patterns for various conformal array architectures.

1. Introduction

Design and analysis of conformal array have always been a difficult work since conformal array was presented several decades ago for its complicated architecture. Some special and frequently used conformal array configurations have been mainly explored by researchers. The cylindrical conformal antenna arrays were studied in references [1, 2]. In references [3, 4], the conical conformal antenna arrays were investigated. A method of radiation pattern computation of pyramidal conformal antenna array was developed in reference [5]. In references [6–9], the spherical conformal antenna arrays were researched.

The antenna elements of conformal array are mounted on the surface of the host and the shape of the surface was not allowed to be changed, which can make conformal array meet the requirements of a lot of special individual applications and also make every antenna element axis have different orientation. Hence, the conformal array's radiated beam pattern cannot be calculated from array beam pattern multiplication theorem—array element directivity function multiplying array factor.

Pattern synthesis of conformal antenna array is much more complex than linear antenna array, and the methods of conformal array pattern synthesis are much less than those for linear arrays. In references [10, 11], Euler's rotation approach was utilized to generate conformal beam pattern. Based on Euler's rotation approach, rectangular microstrip patch antenna on spherical structure was analyzed [6], frequency-invariant pattern of conformal array antenna with low cross-polarisation was synthesized [12], an adaptive wideband beam forming algorithm was presented to form beam for conformal array [13], and optimum pattern synthesis of nonuniform spherical array was discussed in reference [14]. Three smart approaches were presented in the article [15–17] to synthesize pattern for conformal array. In reference [18], the constructive analytical phasing (CAP) method based on a new coordinate rotation formula was presented to calculate the pattern of conformal array. In references [19–21], the geometric algebra, finite-difference time-domain (FDTD), and the moment method were developed for conformal array pattern synthesis. In reference [22], a multidimension to one-dimension transformation approach was developed for the similar antenna array pattern synthesis.

Most of these methods mentioned above for conformal array pattern synthesis were based on Euler's rotation approach, in which the antenna element pattern was derived through its coordinate rotation operation and the process was complicated. All the solutions of these methods for conformal array were acquired by iterative computation formulas. These computation formulas for conformal array were different from those for linear array, and their computation processes were much more complicated. Moreover, lots of methods for linear array cannot be applied in the process for conformal array pattern synthesis. This situation significantly impacts the effectiveness of these methods for conformal array. Moreover, there is convergence problem in smart approaches. The reported general success rate of smart methods is not more than 60%, and the success case of smart methods for the element number of array more than 50 has not been reported due to the huge complexity and a significant drop in the success rate. The success rate drops very fast while the element number of array increases.

To overcome these problems, this article presents a new simple analytical pattern formula for conformal array pattern synthesis. A novel pattern synthesis block matrix formula for conformal array is put forward first. Then, a new method is presented to transform the matrix formula into a form similar to the matrix equation of the linear array, so all the abundant methods for linear array can be utilized to synthesize the beam pattern for conformal array, and in particular, many noniterative computation formulas, such as the least square method and the Fourier transform (FT) method, can be applied for conformal array pattern synthesis, which can significantly improve the diversity of the method of pattern synthesis for conformal array and reduce the amount of the computation. In addition, the new method is a general approach that can be applied to any conformal array architecture.

The rest of the contents of this paper are organized as follows: in Section 2, new mathematical model and analytical formula for pattern synthesis of conformal array are presented; in Section 3, a new block matrix equation for pattern synthesis of conformal array is developed; in Section 4, a new simple matrix equation for pattern synthesis of conformal arrays is presented. It can be easily solved by using methods as similar as those for linear array pattern synthesis; examples are taken to demonstrate the effect of the new method in Section 5; and Section 6 draws conclusions.

2. Conformal Array Pattern Function Formula

The configuration of an arbitrary conformal antenna array is considered, as shown in Figure 1. Assuming the element number of the array to be M, the array's m^{th} antenna element location is (x_m, y_m, z_m), and the current excitation of the m^{th} element is I_m. Then, the beam pattern function of the conformal array can be expressed as follows:

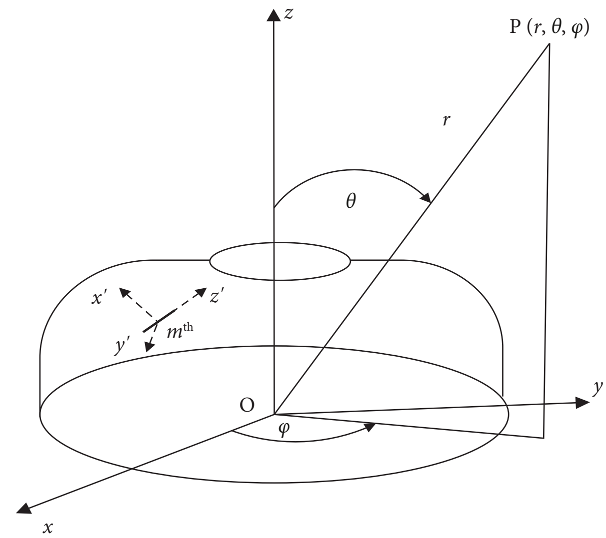


FIGURE 1: The considered arbitrary conformal antenna array.

$$F(\theta, \varphi) = \sum_{m=1}^{M} I_m S(m, \theta, \varphi), \quad m = 1, 2, \ldots, M, \tag{1}$$

wherein $S(m, \theta, \varphi)$ is the m^{th} antenna element's pattern function, φ is the azimuth angle of a far field point P(r, θ, φ), θ is the elevation angle of point P, r is the distance of point P, and $F(\theta, \varphi)$ is the beam pattern function of the conformal array.

Assuming that all antenna elements of the conformal antenna array are homogeneous but with different axis orientations and locations and that the m^{th} antenna element's axis orientation is (β_m, γ_m), we set up a new local coordinate system for the m^{th} antenna element with z' coordinate along its axis orientation (β_m, γ_m). In the m^{th} antenna element's new local coordinate system, the space direction coordinate can be written as (θ'_m, φ'_m). The m^{th} antenna element's pattern function is denoted as $f(\theta'_m, \varphi'_m)$. According to Euler's right-handed rotation method in the x-y-z rotation order with the rotation angles $\alpha_m = 0$, β_m, and γ_m, in turn [10], θ'_m and φ'_m can be derived as follows:

$$\theta'_m = \arccos\left(\cos\theta\cos\beta_m + \sin\theta\sin\beta_m\cos(\varphi - \gamma_m)\right), \tag{2}$$

$$\varphi'_m = \arctan\left(\frac{\sin\theta\sin(\varphi - \gamma_m)}{\sin\theta\cos\beta_m\cos(\varphi - \gamma_m) - \cos\theta\sin\beta_m}\right) - \alpha_m. \tag{3}$$

In far field,

$$S(m, \theta, \varphi) = f(\theta'_m, \varphi'_m) e^{j2\pi/\lambda\,(x_m \sin\theta\cos\varphi + y_m \sin\theta\sin\varphi + z_m\cos\theta)}, \quad m = 1, 2, \ldots, M, \tag{4}$$

wherein λ is the wavelength of the radiated signal. Therefore, equation (1) can be re-expressed as follows:

$$F(\theta,\varphi) = \sum_{m=1}^{M} I_m f(\theta'_m, \varphi'_m) e^{j2\pi/\lambda\,(x_m \sin\theta\cos\varphi + y_m \sin\theta\sin\varphi + z_m\cos\theta)}, \quad m = 1, 2, \ldots, M. \tag{5}$$

Let

$$\mathbf{I} = [I_1\, I_2 \cdots I_m \cdots I_M]^{\mathrm{T}}, \tag{6}$$

$$\mathbf{S}(\theta,\varphi) = [S(1,\theta,\varphi)\, S(2,\theta,\varphi) \cdots S(m,\theta,\varphi) \cdots S(M,\theta,\varphi)], \tag{7}$$

where in equation (6), the superscript T denotes transpose operation. Hence, equation (1) can be expressed as follows:

$$F(\theta,\varphi) = \mathbf{S}(\theta,\varphi)\mathbf{I}. \tag{8}$$

Equation (8) is the pattern function of the conformal array. It can be transformed into a simple matrix equation to solve.

3. Block Matrix Equation of Conformal Array Pattern Synthesis

We set θ in the range of [0°, 180°] and φ in [0°, 180°]. Their discrete values are $\theta_1, \theta_2, \ldots, \theta_{k1}, \ldots, \theta_{K1}$ and $\varphi_1, \varphi_2, \ldots, \varphi_{k2}, \ldots, \varphi_{K2}$, respectively.

Let

$$\mathbf{F} = \begin{bmatrix} F(\theta_1,\varphi_1) & F(\theta_1,\varphi_2) & \ldots & F(\theta_1,\varphi_{K_2}) \\ F(\theta_2,\varphi_1) & F(\theta_2,\varphi_2) & \ldots & F(\theta_1,\varphi_{K_2}) \\ \vdots & \vdots & \vdots & \vdots \\ F(\theta_{K_1},\varphi_1) & F(\theta_{K_1},\varphi_2) & \ldots & F(\theta_{K_1},\varphi_{K_2}) \end{bmatrix}, \tag{9}$$

wherein $\mathbf{F}$ is the discrete matrix of the pattern function $F(\theta,\varphi)$ in far field.

Let

$$\mathbf{S}_m = \begin{bmatrix} S(m,\theta_1,\varphi_1) & S(m,\theta_1,\varphi_2) & \ldots & S(m,\theta_1,\varphi_{K_2}) \\ S(m,\theta_2,\varphi_1) & S(m,\theta_2,\varphi_2) & \ldots & S(m,\theta_1,\varphi_{K_2}) \\ \vdots & \vdots & \vdots & \vdots \\ S(m,\theta_{K_1},\varphi_1) & S(m,\theta_{K_1},\varphi_2) & \ldots & S(m,\theta_{K_1},\varphi_{K_2}) \end{bmatrix}, \quad m = 1, 2, \ldots, M, \tag{10}$$

wherein $\mathbf{S}_m$ is the discrete matrix of the function $S(m,\theta,\varphi)$.

Let

$$\mathbf{S} = [\,\mathbf{S}_1 \;\; \mathbf{S}_2 \;\; \cdots \;\; \mathbf{S}_M]. \tag{11}$$

So, equation (8) can be re-expressed as follows:

$$\mathbf{F} = \sum_{m=1}^{M} \mathbf{S}_m I_m = \mathbf{SI}. \tag{12}$$

This is the matrix equation of conformal array pattern synthesis. But this equation is a block matrix equation (BME).

4. Simple Matrix Equation of Conformal Array Pattern Synthesis

Directly solving equation (12) is difficult; we transform this equation into an easily solved new equation and then solve it.

Let

$$\mathbf{F}(:,i) = \begin{bmatrix} F(\theta_1,\varphi_i) \\ F(\theta_2,\varphi_i) \\ \vdots \\ F(\theta_{K_1},\varphi_i) \end{bmatrix}, \quad i = 1, 2, \cdots, K_2, \tag{13}$$

therefore

$$\mathbf{F} = [\,\mathbf{F}(:,1) \;\; \mathbf{F}(:,2) \;\; \cdots \;\; \mathbf{F}(:,K_2)\,]. \tag{14}$$

Similarly, let

$$\mathbf{S}_m(:,i) = \begin{bmatrix} S(m,\theta_1,\varphi_i) \\ S(m,\theta_2,\varphi_i) \\ \vdots \\ S(m,\theta_{K_1},\varphi_i) \end{bmatrix}, \quad m = 1, 2, \cdots, M;\; i = 1, 2, \cdots, K_2, \tag{15}$$

therefore

$$\mathbf{S}_m = [\,\mathbf{S}_m(:,1) \;\; \mathbf{S}_m(:,2) \;\; \cdots \;\; \mathbf{S}_m(:,K_2)], \quad m = 1, 2, \cdots, M. \tag{16}$$

According to equation (12), we can get

$$\mathbf{F}(:,i) = \sum_{m=1}^{M} \mathbf{S}_m(:,i)\, I_m, \quad i = 1, 2, \cdots, K_2;\ m = 1, 2, \cdots, M. \tag{17}$$

Let

$$\mathbf{b} = \begin{bmatrix} \mathbf{F}(:,1) \\ \mathbf{F}(:,2) \\ \vdots \\ \mathbf{F}(:,K_2) \end{bmatrix}. \tag{18}$$

Similarly, let

$$\mathbf{A}_m = \begin{bmatrix} \mathbf{S}_m(:,1) \\ \mathbf{S}_m(:,2) \\ \vdots \\ \mathbf{S}_m(:,K_2) \end{bmatrix}, \quad m = 1, 2, \cdots, M, \qquad \mathbf{A} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 & \cdots & \mathbf{A}_M \end{bmatrix}. \tag{19}$$

Therefore, equation (17) can be re-expressed as follows:

$$\mathbf{b} = \sum_{m=1}^{M} \mathbf{A}_m I_m = \mathbf{AI}. \tag{20}$$

It is apparent that equation (20) is a general simple matrix equation, not a block matrix equation, and $\boldsymbol{b}$ is a column vector. Equation (20) has the similar form as the matrix equation of the linear array pattern synthesis. So, equation (20) can be solved by using the existing methods for linear array pattern synthesis. On the one hand, equation (20) can be used to calculate the beam pattern of conformal array from $\boldsymbol{A}$ and $\boldsymbol{I}$. On the other hand, when the expected pattern $\boldsymbol{b}$ is prescribed, the antenna element excitation vector $\boldsymbol{I}$ can be solved from equation (20). Equation (20) is named as the simple matrix equation (SME) method for conformal array pattern synthesis.

5. Examples of the New Method

Two examples are taken to demonstrate the performance of the new method and its effect in this part.

Example 1. A cylinder conformal array configuration is considered in Figure 2. There are $N = 30$ elements equidistantly distributed as the distance $\lambda/2$ along a circle centered at the origin point O on the x-o-y plane with radius $a = 1$ m. This circle is the first layer of the array. The first layer is parallelly moved up along $+z$ coordinate axis direction to get other 9 layers of the cylinder array by $(k-1)\lambda/2$ distance with k referring to the k^{th} layer. It is worth mentioning that every other layer is a copy of the first layer in every aspect aside from having different z coordinates. λ is the central wavelength of the signal transmitted by the array. φ is the azimuth angle of a far field point $P(r, \theta, \varphi)$, θ is the elevation angle of point P, and r is the distance of point P from the origin point O. In the first layer, it is easy to get the spherical coordinate location of

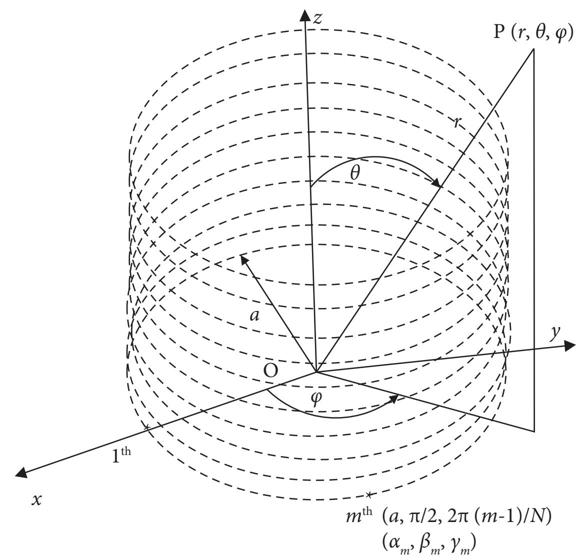


Figure 2: Architecture of the cylinder conformal array.

the m^{th} antenna element, that is, $(a,\ \pi/2,\ 2\pi(m-1)/N)$. Assuming that all antenna elements are homogeneous half wavelength patch dipole antennae but with different axis orientations and locations, let the m^{th} antenna element of the first layer axis orientation be (β_m, γ_m). We set up a new local coordinate system x'-y'-z' for the m^{th} antenna element of the first layer with z' coordinate along its axis orientation (β_m, γ_m). In the first layer m^{th} antenna element's new local coordinate system, the space direction coordinate can be written as (θ_m', φ_m'). According to Euler's right-handed rotation method in the x-y-z rotation order with the rotation angles $\alpha_m = 0$, $\beta_m = \pi/6$, and $\gamma_m = 2\pi(m-1)/N$ in turn [10], θ_m' and φ_m' can be derived from equations (2) and (3). The first layer m^{th} antenna element pattern function $f(\theta_m', \varphi_m')$ can be expressed as follows:

$$f(\theta_m', \varphi_m') = \frac{\cos(\pi/2 \cos\theta_m')}{\sin(\theta_m')}. \tag{21}$$

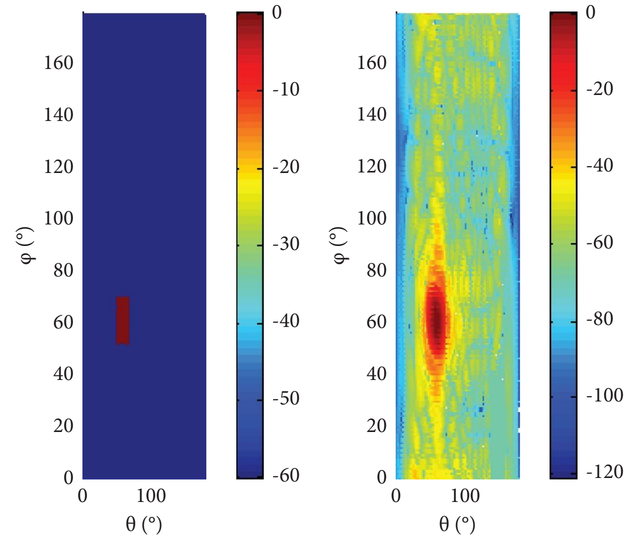
Because every other layer is a copy of the first layer in every aspect aside from having different z coordinates, for m^{th} antenna element of every layer in every layer's local coordinate system, the pattern function of the element is the same as equation (21).

The new method is applied to synthesize the beam pattern for the cylinder array. We set θ in the range of [0°, 180°] and φ in [0°, 180°]. Their discrete values $\theta_1, \theta_2, \ldots, \theta_{k1}, \ldots, \theta_{K1}$ and $\varphi_1, \varphi_2, \ldots, \varphi_{k2}, \ldots, \varphi_{K2}$ all are equidistantly distributed with the adjacent distance 1°. When the target pattern is given, the antenna excitation vector can be obtained from equation (20) [22] as follows:

$$\mathbf{I} = \left(\mathbf{A}^{\mathrm{H}}\left(\mathbf{D}^{-1}\right)^{3}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{H}}\mathbf{b}, \tag{22}$$

where the superscript H denotes conjugation transpose operation, and

$$\mathbf{D} = \operatorname{diag}(\operatorname{abs}(\mathbf{b})), \tag{23}$$



(a) (b)

Figure 3: 3D beam patterns of simulation results of cylinder array are as follows: (a) the target pattern and (b) the synthesized pattern of the new method.

where diag denotes diagonalization operation, and abs denotes getting the absolute value operation.

Here, the target pattern **b** is generated by using MATLAB; the matrix **A** is obtained according to the new method mentioned above. After **I** has been obtained from equation (22), the pattern formed by **I** can be obtained from equation (20).

MATLAB software is applied to simulate this example. The three-dimension (3D) figures of the target pattern and the synthesized pattern using the new method are shown in Figures 3(a) and 3(b), respectively.

From Figure 3, it can be seen that the new method well accomplishes the beam pattern synthesis mission of the cylinder conformal array. The location and the attenuation of the synthesized beam well accord with the target beam.

The two-dimension (2D) figures of simulation results are shown in Figures 4(a) and 4(b), where SME denotes the result of the simple matrix equation method, TARG refers to the target beam pattern, and CAP refers to the result of constructive analytical phasing (CAP) method in reference [18] as a comparison object. In order to further verify the new method, ANSYS HFSS software is applied to simulate the example. HFSS simulation results are also shown in Figures 4(a) and 4(b) with the data marked by HFSS. Figure 4(a) is the results of $\varphi = 60^{\circ}$. Figure 4(b) is the results of $\theta = 60^{\circ}$.

ANSYS HFSS software is used to simulate this example with the signal wavelength $\lambda = 4\sin(\pi/N) = 7.3$ mm. The rogers5880 material substrate with thickness $d_1 = 0.508$ mm is used to bear the half wavelength dipole microwave patch antenna with the copper thickness $d_2 = 35\,\mu$m. The dielectric permittivity ε_r of rogers5880 material is 2.2, and its loss tangent $\tan\delta$ is 0.001.

From Figure 4, it can be seen that the result of the new method marked by SME has the higher attenuation than CAP and HFSS simulation results. The difference between SME and HFSS should be due to the near field mutual coupling among the antenna elements of the array. The near field mutual coupling will not be discussed here in order to focus on the new method.

Example 2. A semispherical conformal array configuration is considered and shown in Figure 5. The semisphere is centered at the origin point O with radius $a = 1$ m. Let φ be the azimuth angle and θ be the elevation angle of a space direction. As shown in Figure 5, the benchmark line is the arc on the semisphere when θ from 0 to 90° and $\varphi = 0^{\circ}$. There are $N = 11$ elements equidistantly distributed as the distance $\lambda/2$ along the benchmark line shown as 1^{th} to N^{th} in Figure 5. The equator circle is the circle on the semisphere and x-o-y plane. It is easy to see that the N^{th} antenna element is on the equator circle. The m^{th} latitude circle on the semisphere parallels to the equator circle and goes through the m^{th} antenna element on the benchmark line with m from 1 to $N-1$. On the equator circle and all the latitude circles, there are antenna elements equidistantly distributed as the distance $\lambda/2$ along each circle. λ is the central wavelength of the signal transmitted by the array. Assuming all antenna elements are homogeneous half wavelength patch dipole antennae but with different axis orientations and locations, let the k^{th} antenna element's spherical coordinates be (a, θ_k, and φ_k). Assuming its axis orientation being (β_k, γ_k) and $\beta_k = \theta_k$, $\gamma_k = \varphi_k + \pi/2$. According to Euler's right-handed rotation method in the x-y-z rotation order with the rotation angles $\alpha_k = 0$, β_k, and γ_k in turn, the array k^{th} antenna element pattern function can be obtained from equation (22).

Similar to Example 1, the target pattern **b** is generated by using MATLAB; the matrix **A** is obtained according to the new method mentioned above. After **I** has been obtained from equation (22), the pattern formed by **I** can be obtained from equation (20).

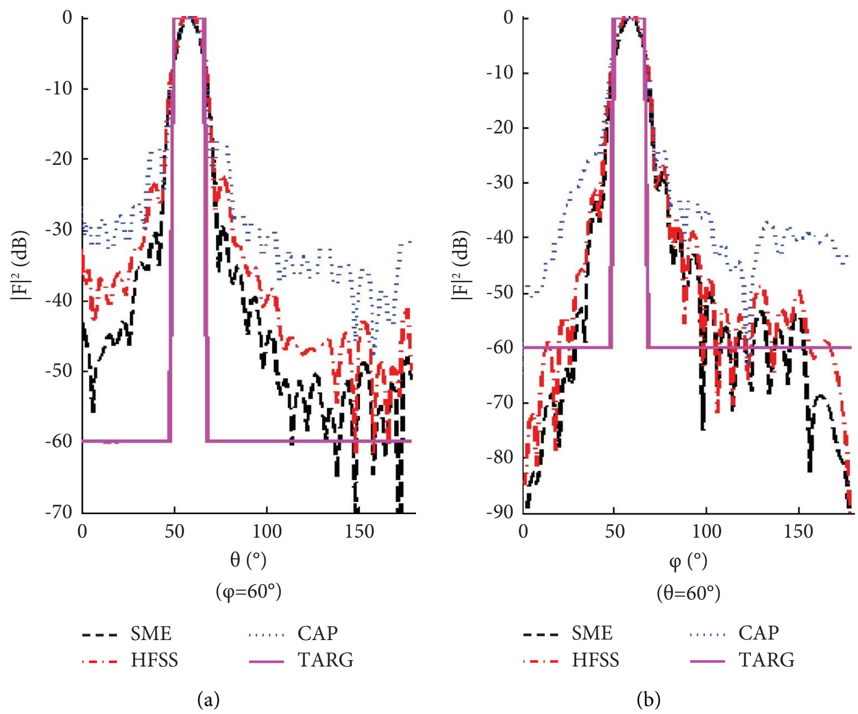


Figure 4: 2D beam patterns of simulation results of cylinder array and the comparison with CAP and HFSS simulation results are as follows: (a) $\varphi = 60^{\circ}$ and (b) $\theta = 60^{\circ}$.

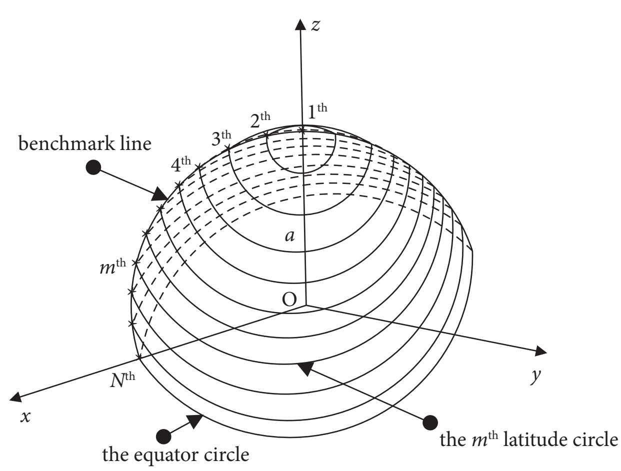


Figure 5: Arrangement of the semispherical conformal array.

The 3D figures of the target pattern and the synthesized pattern using the new method are shown in Figures 6(a) and 6(b), respectively. From Figure 6, it can be learned that the new method well accomplishes the beam pattern synthesis mission of the semisphere conformal array. The location and the attenuation of the synthesized beam well accord with the target beam.

The 2D figures of simulation results are shown in Figures 7(a) and 7(b), where SME denotes the result of the simple matrix equation method, TARG refers to the target beam pattern, and CAP refers to the result of constructive analytical phasing (CAP) method in reference [18] as a comparison object. ANSYS HFSS software simulation results are also shown in Figures 7(a) and 7(b) with the data marked by HFSS. Figure 7(a) is the results of $\varphi = 60^{\circ}$. Figure 7(b) is the results of $\theta = 60^{\circ}$.

ANSYS HFSS software is still used to simulate this example with the signal wavelength $\lambda = 4\sin(\pi/4/N) = 5.4$ mm. The rogers5880 material substrate with thickness $d_1 = 0.508$ mm is used to bear the half wavelength dipole microwave patch antenna with the copper thickness $d_2 = 35\,\mu$m.

From Figure 7, it can be seen that the results of the new method marked by SME have higher attenuation than CAP and HFSS simulation results. The difference between SME and HFSS should be due to the near-field mutual coupling

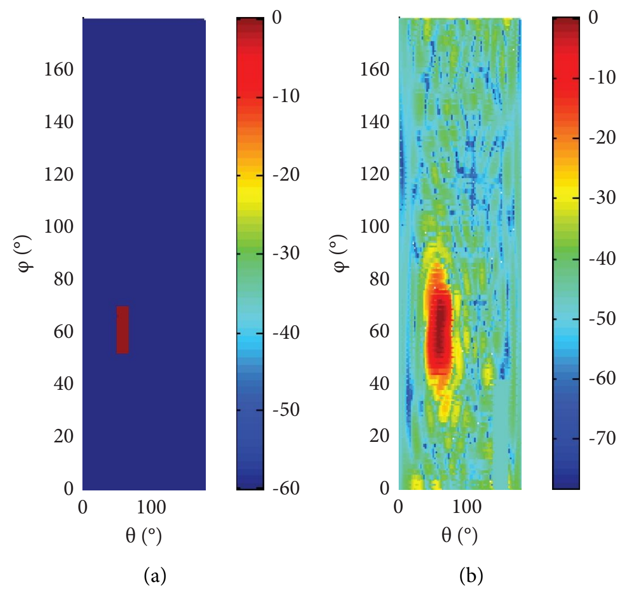


Figure 6: 3D beam patterns of simulation results of semisphere array are as follows: (a) the target pattern and (b) the synthesized pattern of the new method.

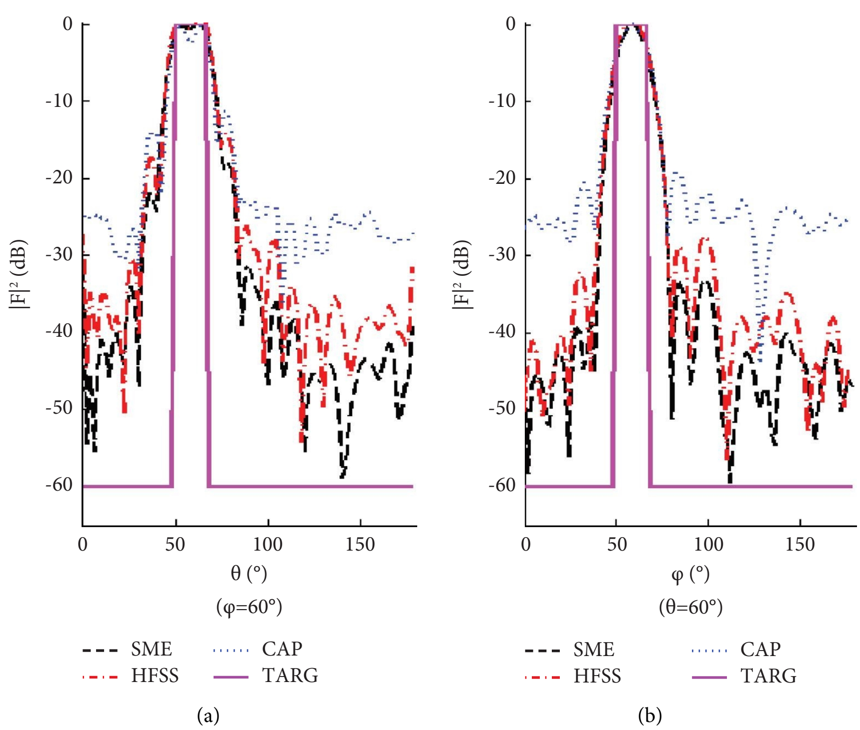


Figure 7: 2D beam patterns of simulation results of semisphere array and the comparison with CAP and HFSS simulation results as follows: (a) $\varphi = 60^{\circ}$ and (b) $\theta = 60^{\circ}$.

among the antenna elements of the array. The near field mutual coupling will not be discussed here in order to focus on the new method.

The SME method is a noniterative method. The results of SME can be obtained through equation (22) straightly. Therefore, the new method has much less computation complexity than published conformal array pattern synthesis approaches that all are iterative methods. Both of the abovementioned examples are simulated on MATLAB software platform on a laptop with Intel i7-7700HQ and 16G memory. It takes no more than one minute to complete the simulation process in each example. The computing process is swift.

The results of the examples show that the new method has excellent effect for conformal array pattern synthesis with little amount of computation. In addition, the adjacent spacing between the discrete values of θ and φ can be arbitrarily set wider to further reduce the amount of computation, and satisfactory results can also be achieved in the light of our experiments.

6. Conclusions

An analytical function formula is developed for conformal antenna array pattern synthesis by using Euler's rotation approach. By a new way, the function is discretized to a block matrix equation (BME). Based on the BME, a simple matrix equation (SME) method is developed for conformal array pattern synthesis. Due to the similar form with the matrix equation of the linear array pattern synthesis, SME can be solved by many approaches for linear array pattern synthesis. The result of SME can be straightly obtained through a noniterative matrix equation. Therefore, the new method has much less computation complexity than published conformal array pattern synthesis approaches that all are iterative methods. Two examples are taken to demonstrate the effectiveness and flexibility of the new method. The results of the simulations show that the new method is highly efficient with less amount of computation.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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