Hindawi
International Journal of Antennas and Propagation
Volume 2023, Article ID 9315869, 6 pages
https://doi.org/10.1155/2023/9315869

Research Article

Off-Grid DOA Estimation Based on Compressed Sensing on Multipath Environment

Bin Hu, Xueyong Shen, Min Jiang, and Kaiheng Zhao

Nanjing Research Institute of Electronic Technology, Nanjing, China

Correspondence should be addressed to Bin Hu; hubin_1147@163.com

Received 9 July 2022; Revised 27 December 2022; Accepted 13 March 2023; Published 10 April 2023

Academic Editor: Atsushi Mase

Due to the existence of multipath propagation, the array will receive the multipath propagation signals at the same time while receiving the target signal and the performance of directional of arrival (DOA) estimation will be influenced. In this paper, an off-grid DOA estimation technique based on compressed sensing (CS) on multipath environment is proposed. To deal with the off-grid problem and the multipath propagation problem, we present a compressed sensing based method. This method regards the multipath propagation coefficients vector as a superparameter and the grid as an adjustable parameter. Then, the problem of multipath propagation coefficients estimation is converted to the estimation of an error matrix and the grid is refined iteratively. The simulation results show that the method can achieve off-grid DOA estimation in the presence of the multipath propagation signals.

1. Introduction

In radar signal processing, direction of arrival (DOA) estimation has wide applications and many algorithms were proposed [1, 2]. MUSIC algorithm [3] is one of the classical algorithms, which has high resolution and can effectively distinguish adjacent targets. However, it is difficult to effectively estimate the DOA of coherent signals. According to the paper [4, 5], compressed sensing (CS) theory based methods are proposed. These methods make use of the sparsity of the signal in the spatial domain and use the steering vector corresponding to the spatial angle grid points of the array as the atoms of the dictionary matrix to construct the sparse representation model of the received signal. One limitation of compressed sensing based DOA estimation is that the angle of the target must lie on the predivided grid. The probability of target DOA falling on the predivided grid is very low. When the target is not on the grid, the recovered signal will have energy leakage and grid mismatch, which is not conducive to the detection of the targets. This problem is called off-grid problem. In order to solve this problem, many methods are proposed. In [6], researchers carried out the first-order Taylor expansion of the steering vector at the grid point closest to the target and proposed a ℓ_1 norm method, which reduced the error of DOA estimation. An off-grid DOA estimation method based on sparse Bayesian learning (SBL) is proposed by combining the first-order Taylor expansion model and SBL theory in [7]. This method reduces the grid size of the model through the Taylor expansion of grid points near the target. The mismatch error is divided to improve the accuracy of DOA estimation, but this method increases the amount of computation due to the introduction of new estimation parameters.

In real array signal processing, due to the complexity of the actual environment, the array will also receive multipath signals from the same transmitting source while receiving signals of interest. These signals are strongly coherent with the signals that we are interested in, and the azimuth is very close. The DOAs of the signals may not be estimated accurately. In order to eliminate the influence of the multipath propagation signals on the accuracy of DOAs estimation, many algorithms have been proposed for the calibration of the gain/phase uncertainties. The most common multipath suppression methods are based on parameter estimation. This kind of methods uses the multipath signal as the amount to be estimated to estimate the amplitude, delay, and phase of the multipath signal, and then in the subsequent processing according to the estimation result, the multipath error is extracted from the receiver and culled from the signal. One of the most conventional methods is the forward-backward spatial smoothing (FBSS) method [8]. This method can effectively realize DOA estimation of coherent signals, but it also sacrifices the degree of freedom. In [9], maximum likelihood (ML) method is proposed. This method performs well in the case of less snapshots and low signal-to-noise ratio. However, this method needs multidimensional scanning, and the amount of computation is huge, so it is not suitable for real-time processing. In addition, many improved methods based on maximum likelihood estimation were also proposed. Reference [10] directly performs maximum likelihood estimation on the received signal, which is called the sequential maximum likelihood estimation method (sequential maximum likelihood, SML). The method is based on the maximum likelihood estimation of the received signal itself during the iterative initialization and iterative process of each path. Reference [11] uses the sequential importance sampling (sequential importance sampling, SIS) technique, that is, the method of particle filtering, to estimate the multipath parameters. This method can utilize the prior information of the channel, but it also limits its use without prior information. In [12], CS based method is proposed and performs well, but the off-grid problem was not considered.

In this paper, in order to reduce the influence of multipath propagation and accurately estimate the DOA of the signal, an off-grid DOA estimation method based on compressed sensing is proposed. Firstly, the multipath signal receiving model with the off-grid problem is converted to an error in variables (EIVs) model based on the CS theory. At this point, the estimation of multipath coefficients is transformed into the estimation of an error matrix which is related to the multipath coefficients. Then, an efficient sparse least square (TLS) framework-based method for the estimation of the DOAs and the error matrix is proposed. And, a grid updating method for the off-grid refinement is presented.

The rest of this paper is organized as follows. In Section 2, the sparse array signal receiving model with the off-grid problem is presented. In Section 3, the TLS based method for sparse coefficients estimation and a grid updating method for the grid refinement are presented. The simulation results are shown to prove the correctness of the proposed algorithms in Section 4. Finally, conclusions are drawn.

2. Array Model

2.1. Signal Model. Supposing that a uniform linear array (ULA) with M array elements receives K far field narrowband signals from K directions of $\theta = [\theta_1, \theta_2, \cdots, \theta_K]^{\mathrm{T}}$. t denotes the tth snapshot, the output of this ULA $\mathbf{X}(t) = [x_1(t), x_2(t), \cdots, x_M(t)]$ can be expressed as

$$\mathbf{X}(t) = \sum_{k=1}^{K} s_k(t)\,\mathbf{a}_0(\theta_k) = \mathbf{A}(\boldsymbol{\theta})\mathbf{s}(t), \tag{1}$$

where $\mathbf{A}(\theta) = [\mathbf{a}_0(\theta_1), \mathbf{a}_0(\theta_2), \cdots, \mathbf{a}_0(\theta_K)]$ is the array manifold matrix, $\theta = [\theta_1, \theta_2, \cdots, \theta_K]$ and $\mathbf{a}_0(\theta_k)$ is the array steering vector of the k th signal

$$\mathbf{a}_0(\theta_k) = \left[1, e^{j2\pi d/\lambda \sin\theta_k}, \cdots, e^{j2\pi(M-1)d/\lambda \sin\theta_k}\right]^T, \tag{2}$$

where λ is the wavelength, $k = 1, 2, \cdots, K$, d is the array spacing and $d/\lambda = 1/2$.

The whole space is discretized into $N\,(N \gg K)$ equal parts and $\boldsymbol{\theta} = [\vartheta_1, \vartheta_2, \cdots, \vartheta_N]$ denotes the grid we discretized. We get a new array manifold matrix:

$$\mathbf{A}(\boldsymbol{\theta}) = [\mathbf{a}(\vartheta_1), \mathbf{a}(\vartheta_2), \cdots, \mathbf{a}(\vartheta_N)]. \tag{3}$$

The observation vector can be rewritten as follows [13]:

$$\mathbf{X}(t) = \mathbf{A}(\boldsymbol{\theta})\mathbf{S}(t). \tag{4}$$

Obviously, only K elements in S are nonzero, and the values of other elements are all zero. Therefore, S is sparse.

Considering the received white Gaussian noise $\mathbf{N}(t)$ can be rewritten as

$$\mathbf{X}(t) = \mathbf{A}(\boldsymbol{\theta})\mathbf{S}(t) + \mathbf{N}(t). \tag{5}$$

2.2. Multipath Propagation Model. When the radar detects a low elevation target, the transmitted signal and the multipath signal reflected from the ground will form an interference effect in the airspace. If the transmitted direct signal at the elevation of the target is exactly in phase with the multipath signals, the radar power will be effectively improved. On the contrary, if the transmitted direct signal is reversed from the multipath signals, the radar power will be weakened.

Under the condition of multipath propagation, the signal received by radar consists of two parts: one part is the direct signal, which is directly received by the array, and the other part is the multipath signal, which is received by the array after being reflected by the ground. The spatial angle of the two kinds of signals is relatively close and they are coherent signals.

Assuming that the signal with an angle of θ_1 in (1) is the direct signal, and other signals are multipath signals. The direct signal can be expressed as

$$\mathbf{X}_d(t) = \mathbf{A}(t)\mathbf{S}(t) + \mathbf{N}(t). \tag{6}$$

And, the multipath signals can be expressed as

$$\begin{aligned}\mathbf{X}_i(t) &= \Gamma_i \mathbf{X}_d(t),\\ \Gamma_i &= \rho_i e^{j\varphi_i},\end{aligned} \tag{7}$$

where Γ_i is the multipath coefficient of ith multipath signal, ρ_i is the reflection coefficient, and φ_i is the phase difference. The array receives the composite signal of direct signal and multipath signals, which is expressed as

$$\mathbf{X}_r(t) = \mathbf{X}_d(t) + \sum_{i=2}^{K} \mathbf{X}_i(t) + \mathbf{N}(t) = \mathbf{A}\Gamma\mathbf{S}(t) + \mathbf{N}(t), \tag{8}$$

where $\Gamma = \mathrm{diag}[1, \Gamma_2, \cdots, \Gamma_K]$ is the multipath coefficients matrix.

2.3. Off-Grid Model. Compressed sensing theory uses the sparsity of signals in the spatial domain to divide the spatial domain at equal intervals. However, the possibility that the target in the actual environment just falls on the divided grid point is very low, which will lead to the off-grid problem.

$\Delta\theta_i$ represents the deviation between the angle of the ith signal and the nearest grid point. $\Delta\boldsymbol{\theta}$ represents the vector composed of deviation values. $\boldsymbol{\theta}_0$ denotes meshing points. At this time, the angles of the signals can be expressed as

$$\widehat{\boldsymbol{\theta}} = \boldsymbol{\theta}_0 + \Delta\boldsymbol{\theta}. \tag{9}$$

The observation vector can be rewritten as

$$\mathbf{X}(t) = \mathbf{A}(\widehat{\boldsymbol{\theta}})\Gamma\mathbf{S}(t) + \mathbf{N}(t). \tag{10}$$

This paper focuses on achieving the estimation of sparse coefficients $\mathbf{S}(t)$ and grid division deviation $\Delta\boldsymbol{\theta}$ in the multipath environment Γ and obtaining the accurate estimation of the target angle.

3. Proposed Methods

In order to effectively estimate the target DOA in the multipath environment, the multipath coefficients matrix is firstly assumed to be a disturbance related to the array manifold matrix. At this time, the observation vector can be rewritten as

$$\begin{aligned}\mathbf{X} &= \mathbf{A}\Gamma\mathbf{S} + \mathbf{N},\\ &= \mathbf{AS} + \mathbf{A}(\Gamma - \mathbf{I})\mathbf{S} + \mathbf{N},\\ &= (\mathbf{A} + \mathbf{E})\mathbf{S} + \mathbf{N},\end{aligned} \tag{11}$$

where $\mathbf{E} = \mathbf{A}(\Gamma - \mathbf{I})$ is an error matrix which is related to the multipath coefficients matrix.

Using CS data reconstruction algorithm, the sparse coefficients in (3) can be estimated. However, due to the existence of the unknown multipath signals, the RIP of the observation matrix has changed. If the influence of the multipath signals is ignored and the signal is reconstructed directly, the estimation accuracy of the sparse coefficients will be affected.

The problem in (11) can be estimated using a sparse least squares framework. This sparse least squares framework can be described as follows [13]:

$$\begin{aligned}&\underset{\mathbf{S},\Delta\boldsymbol{\theta},\mathbf{E},\mathbf{N}}{\mathrm{argmin}} \left\| [\,\mathbf{E}\ \ \mathbf{N}\,] \right\|_F^2 + \lambda\|\mathbf{S}\|_1,\\ &s.t.\quad \mathbf{X} = (\mathbf{A}(\widehat{\boldsymbol{\theta}}) + \mathbf{E})\mathbf{S} + \mathbf{N}.\end{aligned} \tag{12}$$

Eliminating the noise and the problem is transformed into an unconstrained optimization problem:

$$\min_{\mathbf{S},\Delta\boldsymbol{\theta},\mathbf{E}} \|\mathbf{E}\|_2^2 + \|\mathbf{X} - [\mathbf{A}(\widehat{\boldsymbol{\theta}}) + \mathbf{E}]\mathbf{S}\|_F^2 + \lambda\|\mathbf{S}\|_{1,2}. \tag{13}$$

It can be seen that this problem is a nonconvex optimization problem, and a very effective method is to use gradient descent algorithm. The characteristic of this algorithm is to estimate the parameters in an iterative way. Firstly, assuming that one of the parameters is known, then using the known parameter to estimate the other parameter, and then using the estimated parameter to estimate the other parameter until the iteration converges.

Firstly, assuming that the error matrix $\mathbf{E}$ and the angle $\widehat{\theta}$ are known, i denotes the ith iteration, the problem of estimating the sparse coefficient matrix $\mathbf{S}$ in (13) can be expressed as

$$\min_{\mathbf{S}^i,\widehat{\boldsymbol{\theta}}^i} \|\mathbf{S}^i\|_{1,2} \quad s.t. \quad \left\|\mathbf{X} - \left[\mathbf{A}\left(\widehat{\boldsymbol{\theta}}^i\right) + \mathbf{E}^{i-1}\right]\mathbf{S}^i\right\|_F \le \varepsilon. \tag{14}$$

The sparsity coefficient can be solved by the following equation:

$$\mathbf{S}^i = \underset{\mathbf{S}^i}{\mathrm{argmin}} \left\|\mathbf{X} - \left[\mathbf{A}\left(\widehat{\boldsymbol{\theta}}^{i-1}\right) + \mathbf{E}^{i-1}\right]\mathbf{S}^{i-1}\right\|_2. \tag{15}$$

The problem in (11) can be solved by common compressed sensing signal reconstruction algorithms, including the greedy algorithm and the convex optimization algorithm. This paper chooses greedy algorithm to solve this problem, which is characterized by low computational complexity and simple operation. Getting the sparse coefficients $\mathbf{S}^i$, the grid needs to be updated with the estimated $\mathbf{S}^i$. At this time, the grid updating problem is

$$\min_{\widehat{\boldsymbol{\theta}}^i} \left\|\mathbf{X} - \left[\mathbf{A}\left(\widehat{\boldsymbol{\theta}}^{i-1}\right) + \mathbf{E}^{i-1}\right]\mathbf{S}^i\right\|_2. \tag{16}$$

This problem is also a constrained nonlinear optimization problem, which can be solved by gradient descent method. At this point, the solution of the problem (15) can be transformed into the following equation [14]:

$$\underset{\boldsymbol{\Psi}}{\mathrm{argmin}} \left\|\mathbf{r} - \mathbf{B}_l\boldsymbol{\Psi}\right\|_2^2, \tag{17}$$

where $\mathbf{r} = \mathbf{X} - [\mathbf{A}(\widehat{\boldsymbol{\theta}}^{i-1}) + \mathbf{E}^{i-1}]\mathbf{S}^i$, $\mathbf{B}_l = [\mathbf{A}'(\widehat{\boldsymbol{\theta}}^{i-1})]$, and $\mathbf{A}'(\widehat{\boldsymbol{\theta}}^{i-1})$ means $\mathbf{A}(\widehat{\boldsymbol{\theta}}^{i-1})$ taking the first-order derivative w.r.t. $\widehat{\boldsymbol{\theta}}$. $\Psi = [\widehat{\boldsymbol{\theta}}^i - \widehat{\boldsymbol{\theta}}^{i-1}]^T$.

At this point, the solution of the grid updating problem can be transformed into the following equation:

$$\widehat{\boldsymbol{\theta}}^i = \widehat{\boldsymbol{\theta}}^{i-1} + \gamma\cdot\Delta\mathrm{u}\cdot\mathbf{p}(1\colon N), \tag{18}$$

where γ is the step size parameter, $\Delta\mathrm{u}$ is the meshing interval, and $\mathbf{p} = \mathrm{real}(\mathbf{B}_l\cdot\mathbf{r})$.

Using the estimated sparse coefficients $\mathbf{S}^i$ and grid $\widehat{\theta}^i$, the error matrix is estimated:

$$\mathbf{E}^i = \left[\mathbf{X} - \mathbf{A}\left(\widehat{\boldsymbol{\theta}}^i\right)\mathbf{S}^i\right]\left(\mathbf{S}^i\right)^T\left[\mathbf{I} + \mathbf{S}^i\left(\mathbf{S}^i\right)^T\right]^{-1}. \tag{19}$$

The estimation of multipath coefficient matrix can be obtained from the estimation results of error matrix:

$$\Gamma^i = \left(\mathbf{A}\left(\widehat{\boldsymbol{\theta}}^i\right)\right)^+\mathbf{E}^i + \mathbf{I}. \tag{20}$$

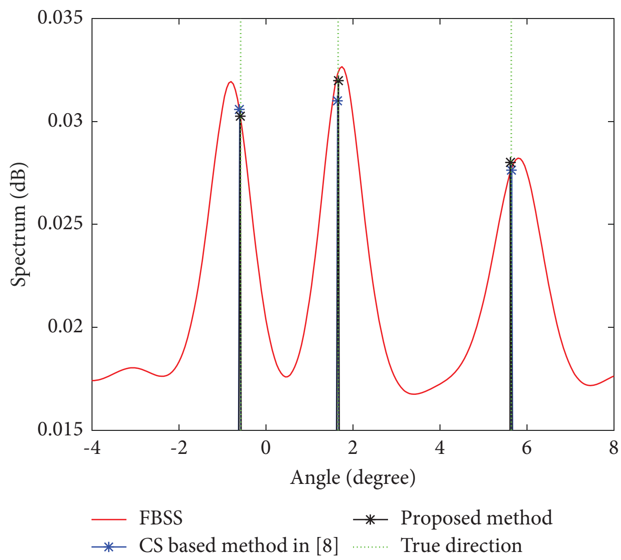

Figure 1: Beam patterns indifferent conditions.

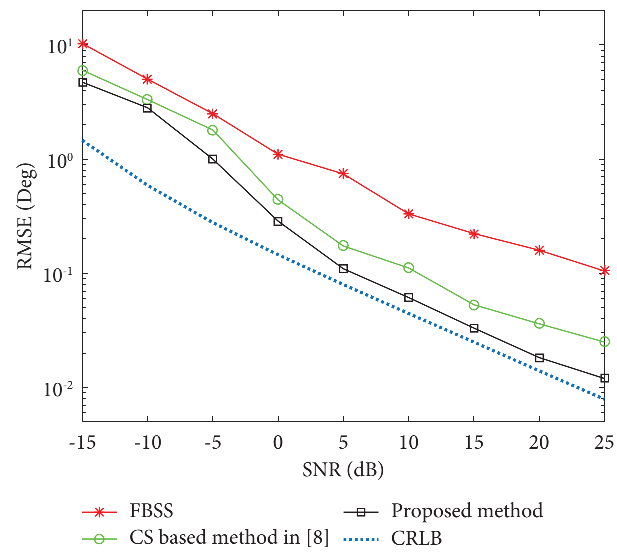

Figure 2: The DOA estimation RMSE vs SNR (dB).

When the iteration process satisfies the iteration termination condition, the iteration terminates. Through the estimated sparse coefficients and the updated grid, we can get the DOA estimation in multipath environment.

4. Simulation

In the sections above, the problem of off-grid DOA estimation in multipath environment was analyzed, the basic model was established, and TLS based methods were proposed to solve the problem. In this section, simulation experiments are conducted and the results are shown. First, the simulation results show the DOA estimation results in multipath environment with different methods. Then, the Monte Carlo analysis on the root mean square error (RMSE) of the DOA estimation results are given. In order to verify the effectiveness of the proposed method, the method in this paper is compared with several other methods, including the TLS based algorithm proposed in this paper, the FBSS method in [6], and the CS based method in [8].

Firstly, consider a ULA with M ($M = 80$) array antennas which is spaced with half wavelength. The center frequency of the signal is 9 GHz. Three ($K = 3$) coherent far field narrow-band signals are received from different directions and the DOAs are $\theta_1 = -0.58^\circ$, $\theta_2 = 1.66^\circ$, and $\theta_3 = 5.64^\circ$. The second signal is the target signal, and the other two signals are multipath signals that are coherent with the target signal. The multipath coefficients of the two multipath signals are $0.91e^{j0.8\pi}$ and $0.82e^{j1.5\pi}$. The space is divided at 0.1° intervals from -10° to 10°.

The first simulation shows the single DOA estimation results of several different methods in multipath environment. The SNR is 25 dB. 10 snapshots are collected. And, the results are shown in Figure 1.

It can be seen from the results in Figure 1 that although the FBSS method can achieve DOA estimation of coherent signals, there is still a large error comparing with the actual angle. Because the CS based method does not consider the off-grid problem, the estimated DOAs falls on the grid point closest to the actual angle of the target. The method proposed in this paper can solve the off-grid problem and achieve DOA estimation of adjacent targets at the same time.

In order to further verify the algorithm proposed in this paper, a Monte Carlo test on the RMSE is conducted in the following simulation。The derivation of the CRLB can be seen in [15].

In simulation 2, Monte Carlo is analyzed 500 times on the RMSE. Figure 2 shows the RMSE vs. different SNR, the SNR changes from −15 dB to 25 dB at 5 dB intervals, the number of snapshots is 10.

Figure 2 shows that under the condition of low SNR, the DOA estimation errors of these methods are large due to the influence of noise. With the change of SNR, the RMSE of the method proposed in this paper is lower than other methods, thus verifying the effectiveness of this method under the condition of different SNR.

In simulation 3, Monte Carlo is analyzed 500 times again on the RMSE. Figure 3 presents the RMSE vs. different number of snapshots; the number of snapshots changes from 10 to 100 at 10 intervals, the SNR is 25 dB.

The simulation results in Figure 3 illustrate that the RMSE of these methods changes slightly with the change of snapshot number, but decreases to a certain extent with the increase of snapshot number and sample number. At the same time, the RMSE of the method proposed in this paper is smaller under different snapshot numbers comparing with other methods,.

In simulation 4, we still do Monte Carlo analysis 500 times on the RMSE. Figure 4 presents the RMSE vs different angular intervals. Considering two signals, one is the target signal, the other is the multipath signal, and the angle of the target signal is, 1.66° the angle of the multipath signal is $(1.66 + d)^\circ$, d changes from 1.2 degrees to 3 degrees in 0.2 degree interval, the SNR is 25 dB and the number of snapshots is 50.

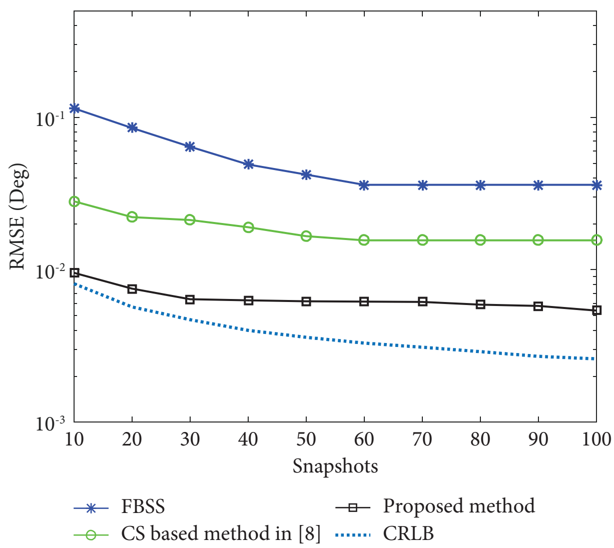

Figure 3: RMSE of the DOA estimation vs the snapshots.

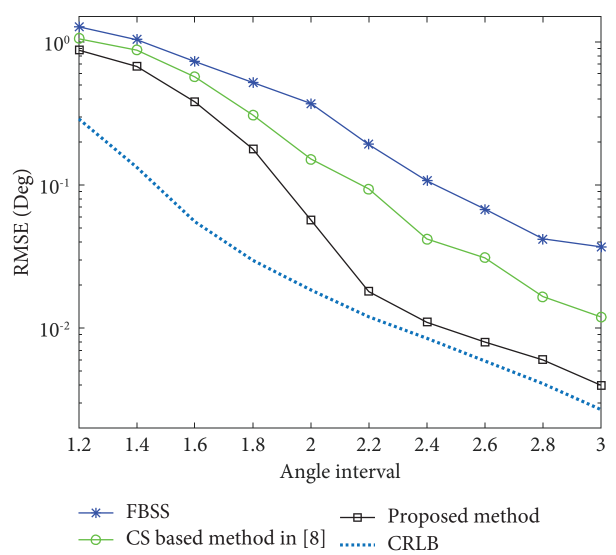

Figure 4: RMSE of the DOA estimation vs different angular intervals.

It can be seen from the simulation results in Figure 4 that when the angles of the multipath signal and the target signal are very close, the angle estimation deviation of the target is relatively large. Compared with the method proposed in this paper, the error is smaller.

5. Conclusions

In this paper, a compressed sensing based method is proposed for the off-grid DOA estimation in the multipath environment. The most important step in the method is transforming the signal receiving model into an EIV model and then estimating the sparse coefficients by total least square method. The estimated sparse coefficients are used to update the grid, and then the updated grid and the estimated sparse coefficients are used to estimate the multipath coefficients. Since the problem to be solved is a nonconvex optimization problem, this paper estimates multiple position parameters by iteration. When the iteration ends, the estimation of parameters is obtained. The simulation results show that it has a better performance under different conditions. In addition, only the most commonly greedy algorithm is used in the reconstruction of sparse coefficients in this paper, and there are many compressed sensing signal reconstruction algorithms. How to use the compressed sensing algorithm to achieve more accurate signal reconstruction under different conditions is worthy of further study.

Data Availability

The results of this paper are all simulation results, no data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

The research in this paper is supported by Nanjing Research Institute of Electronic Technology.

References

[1] J. Capon, "High-resolutionfrequency-wavenumber spectrum analysis," *Proceedings of the IEEE*, vol. 57, no. 8, pp. 1408–1418, 1969.

[2] M. Wax and Y. Anu, "Performance analysis of the minimum variance beamformer," *IEEE Transactions on Signal Processing*, vol. 44, no. 4, pp. 928–937, 1996.

[3] R. O. Schmidt, "Multiple emitter location and signal parameter-estimation," *IEEE Transactions on Antennas and Propagation*, vol. 34, no. 3, pp. 276–280, 1986.

[4] L. D. David, "Compressed sensing," *IEEE Transactions on Information Theory*, vol. 52, pp. 1289–1306, 2006.

[5] S. Fortunati, R. Grasso, F. Gini, and M. S. Greco, "Single-snapshot DOA estimation by using compressed sensing," *EURASIP Journal on Applied Signal Processing*, vol. 120, pp. 1–17, 2014.

[6] R. Jagannath and K. V. S. Hari, "Block sparse estimator for grid matching in single snapshot DOA estimation," *IEEE Signal Processing Letters*, vol. 20, no. 11, pp. 1038–1041, 2013.

[7] Z. Yang, L. Xie, and C. Zhang, "Off-grid direction of arrival estimation using sparse bayesian inference," *IEEE Transactions on Signal Processing*, vol. 61, no. 1, pp. 38–43, 2013.

[8] S. U. Pillai and B. H. Kwon, "Forward/backward spatial smoothing techniques for coherent signal identification," *IEEE Transactions on Acoustics, Speech, & Signal Processing*, vol. 37, no. 1, pp. 8–15, 1989.

[9] M. Wax, "Detection and localization of multiple sources in noise with unknown covariance," *IEEE Transactions on Signal Processing*, vol. 40, no. 1, pp. 245–249, 1992.

[10] N. Sokhandan, "A novel multipath estimation and tracking algorithm for urban GNSS navigation applications," in *Proceedings of the 26th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2013)*, pp. 16–20, Nashville, TN, USA, September 2013.

[11] P. Closas, C. Fernandez-Prades, J. A. Fernandez, and A. Ramírez-González, "Multipath mitigation using particle filtering," in *Proceedings of the 19th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS 2006)*, Fort Worth, TX, USA, September 2006.

[12] A. Das, W. S. Hodgkiss, and P. Gerstoft, "Peer-reviewed technical communication-coherent multipath direction-of-arrival resolution using compressed sensing," *IEEE Journal of Oceanic Engineering*, vol. 42, no. 2, pp. 494–505, 2017.

[13] B. Hu, X. Wu, X. Zhang, Q. Yang, and W. Deng, "DOA estimation based on compressed sensing with gain/phase uncertainties," *IET Radar, Sonar & Navigation*, vol. 12, no. 11, pp. 1346–1352, 2018.

[14] S. Camlica, C. Ali, and O. Arikan, "Autofocused spotlight SAR image reconstruction of off-grid sparse scenes," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 53, no. 4, pp. 1880–1882, 2017.

[15] Y. Zheng, *Study on Some Issues of Low-Angle Altitude Measurement for VHF Array Radar*, Xidian University, Xi'an, China, 2017.