

Research Article

A Probabilistic Physics of Failure Approach for Structure Corrosion Reliability Analysis

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Corrosion is recognized as one of the most important degradation mechanisms that affect the long-term reliability and integrity of metallic structures. Studying the structural reliability with pitting corrosion damage is useful for risk control and safety operation for the corroded structure. This paper proposed a structure corrosion reliability analysis approach based on the physics-based failure model of pitting corrosion, where the states of pitting growth, pit-to-crack, and cracking propagation are included in failure model. Then different probabilistic analysis methods such as Monte-Carlo Simulation (MCS), First-Order Reliability Method (FORM), Second-Order Reliability Method (SORM), and response surface method are employed to calculate the reliability. At last, an example is presented to demonstrate the capability of the proposed structural reliability model and calculating methods for structural corrosion failure analysis.

1. Introduction

Corrosion is recognized as one of the most important degradation mechanisms that affect the long-term reliability and integrity of metallic structures [1]. Corrosion can cause several kinds of defect on structure with the stress load and different corrosion environment, such as uniform corrosion, corrosion fatigue, pitting corrosion, and hydrogen embrittlement. As the increase of service time for metallic structure, the corrosion damage will grow, and the pit will change to a crack especially with static or circle loads. So the reliability and lifetime predictions of structure with pitting corrosion damage are significant for structure safety and risk mitigation.

The development of effective localized corrosion damage model is essential for reliability assessment. A simple empirical model was developed to describe the relationship between corrosion time and pitting depth based experimental data. In recent years, some stochastic method has been made in modeling pitting corrosion through Markov chains. Caleyó et al. used a nonhomogenous Markov process to model pit depth growth [2]. Valor et al. proposed a new stochastic

model in which pitting initiation is modeled as a Weibull process [3]. Another type of corrosion damage model has been made based on electrochemical and mechanical process. Goswami and Hoepfner identified a seven-stage conceptual model for corrosion fatigue. The electrochemical effects in pit formation and the role of pitting in fatigue and corrosion fatigue crack nucleation behavior were considered in that model. Shi and Mahadevan represented a computational implementation approach based on the seven-stage conceptual model for corrosion fatigue life prediction [4]. Harlow and Wei proposed a three-stage probabilistic model including crack initiation, surface crack to grow into a through crack, and crack fracture. But all the models focus on structure with circle loads and the stress effect was not included in the pit growth phase. Stress corrosion crack is another important failure model for structure with static loads. Wu [5] proposed a probabilistic-mechanistic approach focused on modeling SCC propagation of Alloy 600 SG tubes with uncertainty. But the pit growth process was not contained in this model. A transition model for pitting to corrosion fatigue crack nucleation was first proposed by Kondo and further discussed by Harlow and Wei [6].

This paper proposed a physics-based failure model for structure with pitting corrosion damage. The mechanical stress effects for pitting growth are coupled in the integrated corrosion process. The three stages of pitting corrosion damage, pitting growth, pit-to-crack, and cracking propagation, are considered in the developed physics-based failure model. Then the time-dependent limit state function of corrosion structure is defined by fracture theory. The Monte-Carlo Simulation, FORM, and SORM are employed to calculate the reliability.

2. Reviews of Structure Reliability Analysis Method

2.1. MCS Method. Consider a performance model $G(\mathbf{X})$ under the existence of uncertainties, where the system fails if $G(\mathbf{X}) < 0$. If the joint probability density function of random variables \mathbf{X} is $f_{\mathbf{X}}$, the statistical description of a probabilistic performance that fails is then completely characterized by the cumulative density function (CDF) as

$$P_f = \Pr(G < 0) = \int_{G < 0} f_{\mathbf{X}}(\mathbf{X}) d\mathbf{X}. \quad (1)$$

In practice, the integration boundary $G(\mathbf{X}) = 0$ and the high dimensionality make it difficult or even impossible to

$$\mathbf{U}_i = \Phi^{-1} [F_{X_i}(\mathbf{X}_i)], \quad (2)$$

$$R = \int_{g(u_1, u_2, \dots, u_n) \geq 0} \dots \int_{g(u_1, u_2, \dots, u_n) \geq 0} \prod_{i=1}^n \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}u_i^2\right) du_1 du_2 \dots du_n. \quad (3)$$

Step 2. Search for the Most Probable Point (MPP):

$$\begin{aligned} g(\mathbf{U}) &\approx L(\mathbf{U}) = g(\mathbf{u}^*) + \nabla g(\mathbf{u}^*)(\mathbf{U} - \mathbf{u}^*)^T, \\ \nabla g(\mathbf{u}^*) &= \left(\frac{\partial g(\mathbf{U})}{\partial \mathbf{U}_1}, \frac{\partial g(\mathbf{U})}{\partial \mathbf{U}_2}, \dots, \frac{\partial g(\mathbf{U})}{\partial \mathbf{U}_n} \right) \Big|_{\mathbf{u}^*}, \\ \min_{\mathbf{u}} \quad &\beta = \|\mathbf{u}\| \\ \text{s.t.} \quad &G(\mathbf{u}) = 0. \end{aligned} \quad (4)$$

Step 3. After calculating, the reliability index β is obtained by an optimization problem. The probability in (1) is then computed analytically by the following equation:

$$P_f = \Pr(G < 0) \cong \Phi(-\beta). \quad (5)$$

2.3. SORM. In SORM [13–15], the performance function is approximated by the second-order Taylor series at MPP point. The approximation is given by

$$g(\mathbf{U}) \approx Q(\mathbf{U})$$

obtain an analytical solution to the probability integration in (1).

The MCS method is showed as follows [7, 8]:

$$\begin{aligned} P_f &= \frac{1}{N} \sum_{i=1}^n I(G), \\ I(G) &= \begin{cases} 1, & G < 0, \\ 0, & G > 0, \end{cases} \end{aligned} \quad (6)$$

where N is the total number of simulations. $I(G)$ is an indicator function. However the MCS method needs expensive computational cost.

2.2. FORM. The approximation methods are therefore used for reliability analysis in order to reduce the computational cost. FORM [9, 10] solves the probability integral by simplifying the performance function $G(\mathbf{X})$ using the first-order Taylor series expansion at the Most Probable Point (MPP). The flowchart of the FORM for reliability analysis is shown in Figure 1. FORM involves three steps to approximate the integral [11, 12].

Step 1. Transform original random variables \mathbf{X} in \mathbf{X} -space to standard normal random variables \mathbf{U} in \mathbf{U} -space, and the performance function is expressed as $G(\mathbf{U})$:

$$\begin{aligned} &= g(\mathbf{u}^*) + \nabla g(\mathbf{u}^*)(\mathbf{U} - \mathbf{u}^*)^T \\ &\quad + \frac{1}{2}(\mathbf{U} - \mathbf{u}^*)^T \mathbf{H}(\mathbf{u}^*)(\mathbf{U} - \mathbf{u}^*), \end{aligned} \quad (7)$$

where $\mathbf{H}(\mathbf{u}^*)$ is the Hessian matrix evaluated at the MPP; the gradient vector ∇g at MPP and the vector from MPP to the origin are parallel as

$$\frac{\mathbf{u}^*}{\beta} = \frac{\nabla g}{\|\nabla g\|} = \boldsymbol{\alpha}. \quad (8)$$

After a set of linear transformations, such as coordinate rotation and orthogonal diagonalization, the performance function is further simplified as

$$Q(\mathbf{U}) = \mathbf{U}_n - \left(\beta + \frac{1}{2} \mathbf{U}'^T \mathbf{D} \mathbf{U}' \right). \quad (9)$$

When β is large enough, an asymptotic solution of the probability of failure can be then derived as

$$P_f = \Pr(G < 0) \cong \Phi(-\beta) \prod_{i=1}^{n-1} (1 + \beta \kappa_i)^{1/2}, \quad (10)$$

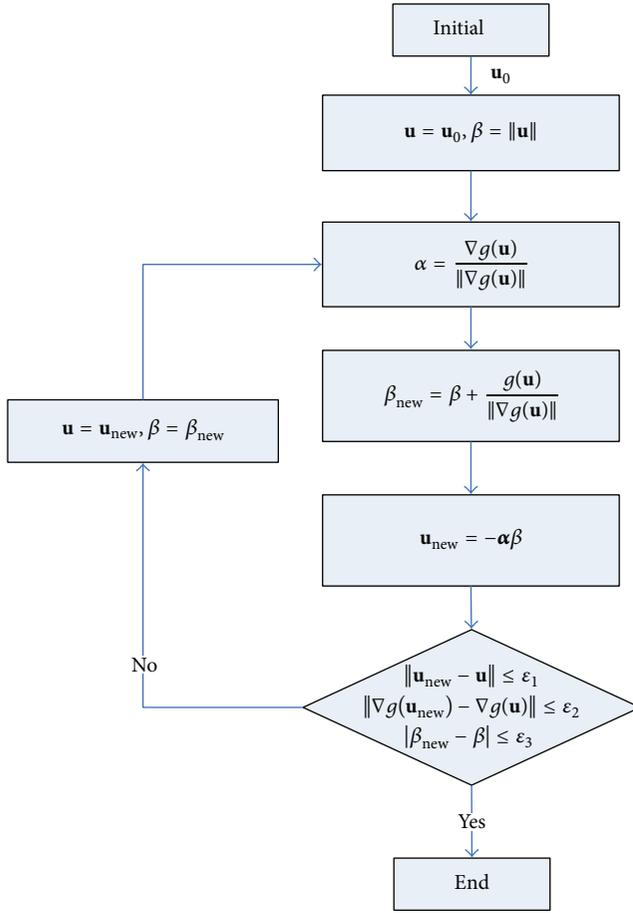


FIGURE 1: The flowchart of the FORM for reliability analysis.

where κ_i denotes the i th main curvature of the performance function $g(\mathbf{U})$. The approximation of the performance function in FORM and SORM is shown in Figure 2.

3. Physics of Failure Modeling for Pitting Corrosion

The degradation (damage) process for the crack development and propagation has been studied and modeled in many different ways [16]. In the proposed method, the pitting corrosion crack consists of four stages as shown in Figure 3.

The first stage in the corrosion damage process is pitting nucleation. It is related to the electrochemical process during corrosion, and the pit initial size and nucleation time depend on factors such as materials, corrosion environment, loads, and electrolytes. This process is very complex and the physics model of pit nucleation is not well understood yet. In this paper, we therefore assume the initial size of pit as a random variable. This distribution can be obtained by experimental data.

A simplified model for pit growth proposed by Harlow and Wei is used [6, 17]. The model assumes a pit of hemispherical shape growing at a constant volumetric rate in accordance

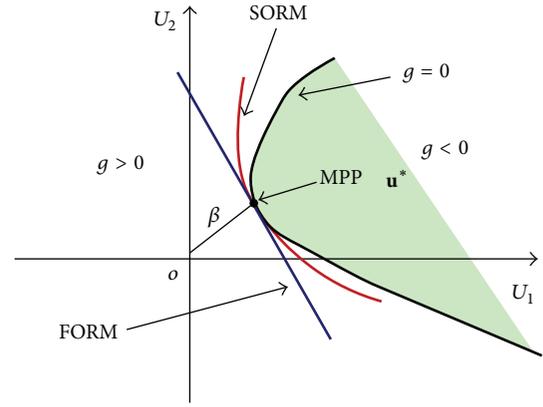


FIGURE 2: Comparison of FORM and SORM.

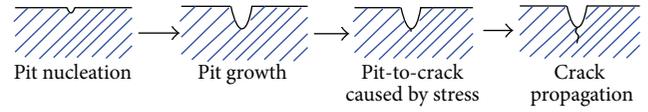


FIGURE 3: Pitting corrosion growth process with stress.

with Faraday's law from an initial radius size. The rate of pit growth is given as

$$\frac{dV}{dt} = \frac{Mi_0}{zF\rho} \exp\left(-\frac{\Delta H}{RT}\right), \quad (11)$$

where V is the hemispherical pit volume, $V = 2\pi a^3/3$, a is the pit size, M is the molecular weight of the material, i_0 is the pitting current coefficient, z is the valence, F is Faraday's constant, ρ is the material density, ΔH is the activation energy, R is the gas constant, and T is the absolute temperature.

Based on the electrochemical theory, the electrical current on an electrode depends on the electrode potential. A general representation of the polarization of an electrode is described in the Butler-Volmer equation:

$$J = j_0 \exp\left(\frac{zF(\varphi - \varphi_{eq})}{RT}\right), \quad (12)$$

where J is electrode current density, A/m^2 , j_0 is exchange current density, A/m^2 , φ is electrode potential, V, and φ_{eq} is equilibrium potential, V. When the stress is applied on the metal materials with elastic deformation, the equilibrium potential will be varied according to Gutman's theory [18, 19]:

$$\Delta\varphi_{eq} = -\frac{\Delta PV_m}{zF}, \quad (13)$$

where ΔP is the spherical part of macroscopic stress tensor excess pressure (Pa) and V_m is the molar volume of the metal. The current density with the stress effects will be changed as

$$\begin{aligned} J &= j_0 \exp\left(\frac{zF(\varphi - \varphi_{eq} - \Delta\varphi_{eq})}{RT}\right) \\ &= j_0 \exp\left(\frac{zF(\varphi - \varphi_{eq})}{RT}\right) \exp\left(\frac{\Delta PV_m}{RT}\right). \end{aligned} \quad (14)$$

The coefficient- $\exp(\Delta PV_m/RT)$ is used for depicting the variance of corrosion current with stress effects compared without stress situation.

In this paper, the effects of stress applied are considered based on (14); then the pit growing depth at the time t can be changed to take into account the coupled effects of stress load and corrosion environment as

$$\begin{aligned} a(t) &= \left(\frac{3Mi_0}{2\pi zF\rho} \exp\left(-\frac{\Delta H}{RT}\right) \exp\left(\frac{V_m \Delta P}{RT}\right) * t + a_0^3\right)^{1/3}, \end{aligned} \quad (15)$$

where I_0 is the pitting current, V_m is the molar volume of the material, and ΔP is the bulk component of stress tensor. As the pit depth grows, the stress intensity factor at the pit tip will increase correspondingly and when it reaches a value beyond the threshold value of stress corrosion crack (SCC), the pit will transform into a crack and the SCC will thus occur. The criteria of a pit transforming into a crack can be obtained based on the thresholds of SCC and the fracture toughness as [6, 7]

$$K_{pit} \geq K_{ISCC}, \quad (16)$$

where K_{ISCC} is the threshold of stress corrosion cracking and K_{pit} is the stress intensity factor for the surface of the pit. From

$$G(\mathbf{x}, t) = K_{IC} - \beta\sigma\sqrt{\pi a(t)} = \begin{cases} K_{IC} - \beta\sigma\sqrt{\pi\left(\frac{3Mi_0}{2\pi zF\rho} \exp\left(-\frac{\Delta H}{RT}\right) \exp\left(\frac{V_m \Delta P}{RT}\right) * t + a_0^3\right)^{1/3}}, & \text{(Pit stage),} \\ K_{IC} - \beta\sigma\sqrt{\pi\left(a_{ci} + \int_{t_{ci}}^t C(K_I - K_{ISCC})^m dt\right)}, & \text{(Crack stage),} \end{cases} \quad (19)$$

where β is the shape parameter for the crake, σ is the static stress load, and $a(t)$ is the pit depth in pitting growth stage

the fracture mechanics theory, the critical size of a pit at the transition to a crack can be calculated as

$$a_{ci} = \frac{1}{\pi} \left(\frac{K_{IC}\Phi}{1.12k_t\Delta\sigma}\right)^2 \left(1 - \eta\frac{\rho zF}{\alpha M\sigma_s}\right), \quad (17)$$

where K_{ISCC} is the threshold stress intensity factor of SCC, which means the minimum stress required for SCC propagation, K_{IC} is the fracture toughness of the material, η is the anodic polarization potential, α is the coefficient constant, σ_s is the yield stress, Φ is shape factor, and k_t is the stress concentration factor of the pit hole.

After the transition from a pit to a crack, the corrosion damage process will turn into the crack propagation stage. Unlike corrosion fatigue crack which is usually caused by the combination of cyclic load and a corrosive environment, the stress corrosion cracking is generally induced by a static tensile or torsional load to open and sustain the crack. The stress intensity factor at the crack propagation stage is generally a function of the total stress and the crack length. Similarly, the corrosion fatigue failure usually occurs once the stress intensity factor reaches a value beyond the threshold value K_{IC} . Following studies reported in the literature [12–14], in this study it is also assumed that the empirical model, as shown below, can be used for stress corrosion crack propagation, which is similar to the fatigue crack propagation modeled by Paris' Law:

$$\frac{da}{dt} = C(K_I - K_{ISCC})^m, \quad (18)$$

where C and m are the model constants of crack propagation. Note that the developed pit growth model takes into account the coupled effects of corrosion environment and the mechanical stresses at the pit growth stage of the corrosion damage process.

Following the terminology of structure reliability methods and stress-strength interference theory, corrosion failure can generally be defined as the stress intensity factor exceeding the fracture toughness based on fracture mechanics. Accordingly, with the assumptions that (1) the corrosion pit of hemispherical shape grows at a constant volumetric rate, (2) the corrosion current density with the stress effects will be changed as (3), and (3) the pit will transform to crack when the stress effect dominates the corrosion pit growth, the corresponding time-dependent limit state function for corrosion reliability analysis can be written as

or the crack size in crack propagation stage at a given time t . Based on the time-dependent limit state function expressed

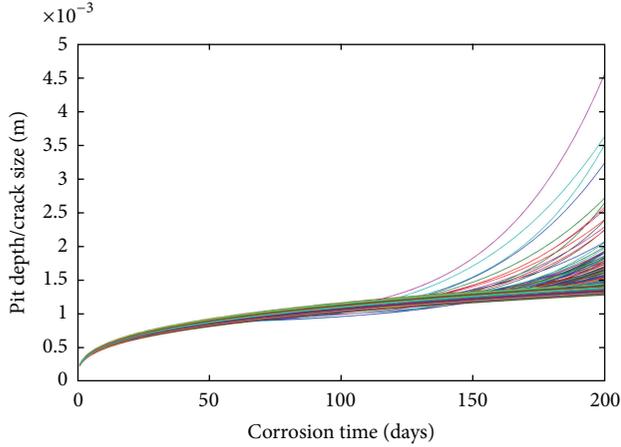


FIGURE 4: Pit depth growth curve with different corrosion time.

in (10), $G(\mathbf{x}, t) > 0$ denotes the safe state whereas $G(\mathbf{x}, t) < 0$ represents the corrosion failure state.

4. Numerical Example

A case study is employed in this section to demonstrate the proposed physics of failure based corrosion model and reliability analysis approach. The case study structure is idealized as an infinite plate with a pitting corrosion defect while the pit corrosion occurs on the surface of the structure material, and the corrosive environment is assumed to be known. The structure material considered in this case study is the aluminum alloy, since it has been widely used in aerospace structures, energy engineering, and marine engineering applications. The uncertainties involved in material properties as well as the corrosion model parameters are considered and modeled with Gaussian random variables. The random and deterministic parameters used in this case study are listed in Tables 1 and 2, respectively [4, 6]. As shown in Table 1, seven different random variables are employed in this case study, as specified by the mean values and standard deviations, where the standard deviations for all random variables have been taken as 5% of the given mean values. To demonstrate the proposed corrosion model at the pit growth stage considering the coupled effects of corrosion environment and the mechanical stresses, the corrosion time has been assumed to begin right after the pit nucleation, and without losing the generosity the random pit nucleation has not been considered in this example.

Figure 4 provides 100 random realizations of the pit growth curve over time considering the random inputs as shown in Table 1. It is clear from the figure that the random inputs yield a large deviation for the pit growth curve over time and a few curves rising obviously after the pit transforms to crack propagation. Figure 5 shows that the G value at the corrosion time equals 200 days. Note that the corrosion time does not include the pit nucleation time.

The structure reliability of pitting corrosion damage is calculated using MCS and FORM. The MCS is used with a large number of samples ($N = 100,000$) as the benchmark solution

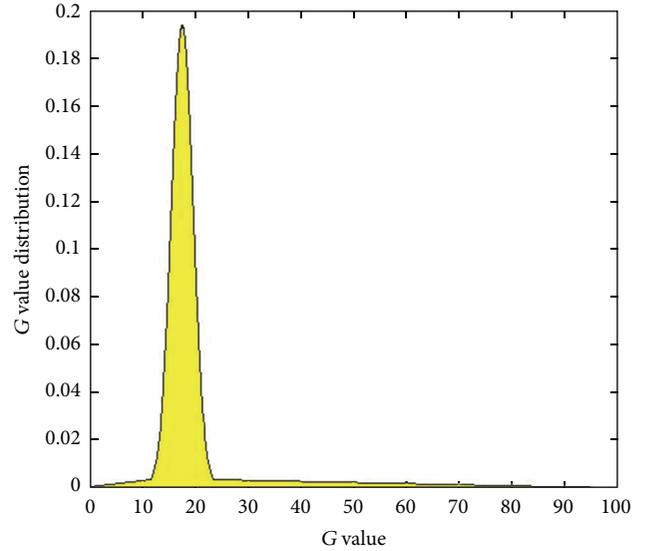
FIGURE 5: G value distribution at $t = 200$ days.

TABLE 1: Random variables for case study.

Random variables	Unit	Mean	STD (mean*%)
a_0	m	$1.98e - 6$	5%/10%
I_0	c/s	$6.5e - 6$	5%/10%
K_{IC}	MPa/m ²	35	5%/10%
σ_s	MPa	470	5%/10%
$\Delta\sigma$	MPa	90	5%/10%
C	—	$2.8e - 11$	5%/10%
m	—	1.16	5%/10%

TABLE 2: Deterministic model parameters for the case study.

Variables	Value
k_t	2.6
α	1.15
η	0.12 V
β	2.6
M	27 g/mol
R	8.314 J/mol K
z	3
ρ	$2.7e6$ g/m ³
F	96514 C/mol
ΔH	50 KJ/mol
T	293 K
V_m	10 cm ³ /mol

to check the accuracy of the FORM method. With the random variables shown in Table 1, the corrosion reliability analysis is then carried out with different corrosion times using two different approaches as mentioned earlier and the analysis results are summarized in Tables 3 and 4. The comparison of the reliability estimations and the absolute percentage errors obtained using the FORM and SORM compared with the MCS results are also shown in Figures 6 and 7. It is

TABLE 3: The reliability results with different corrosion time.

Corrosion time (days)	Reliability				
	MCS	FORM	FORM error (%)	SORM	SORM error (%)
250	0.99815	0.99868	0.05310	0.99834	0.01903
300	0.99113	0.99496	0.38643	0.99302	0.02
350	0.97036	0.98297	1.29952	0.97843	0.083
400	0.93395	0.95372	2.11682	0.94532	1.2174
450	0.88074	0.90589	2.85555	0.89740	1.8916
500	0.81476	0.84112	3.23531	0.83059	1.943

TABLE 4: Comparison of computational efficiencies for FORM and SORM.

Corrosion time (days)	250	300	350	400	450	500
Number of G functions evaluated (FORM)	800	800	136	232	288	128
Number of G functions evaluated (SORM)	800	800	347	489	548	571

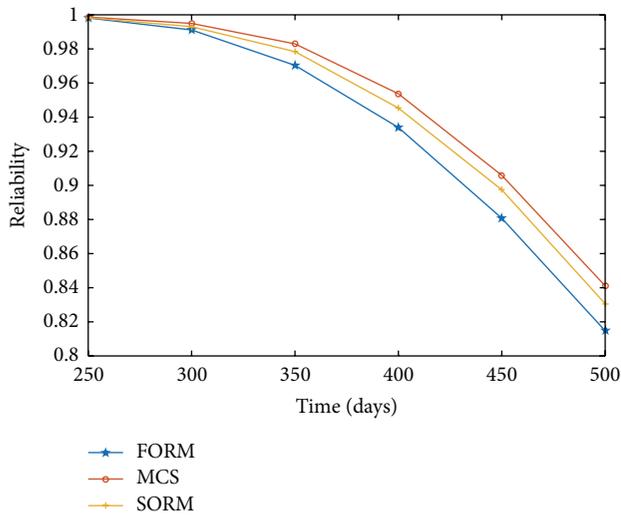


FIGURE 6: Reliability analysis results at different corrosion times.

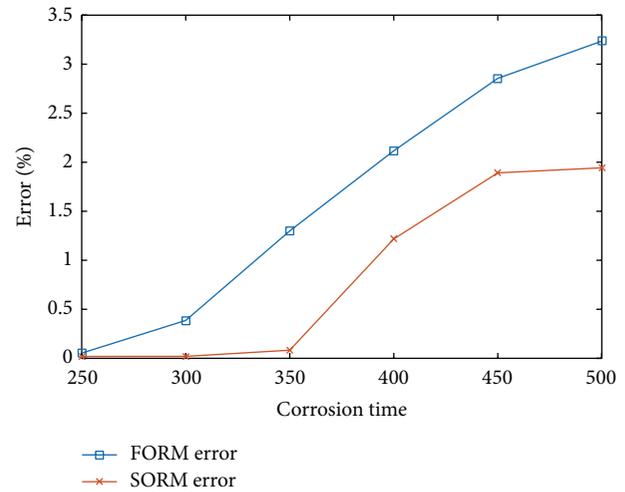


FIGURE 7: The computational error of FORM and SORM.

clear from the figures that the reliability estimation error for the FORM tends to increase with longer corrosion time, mainly due to the linearization of the limit state the FORM used. Besides the accuracy performance, the efficiencies of corrosion reliability analysis using the FORM, SORM, and MCS are also compared, as the results shown in Table 4, in which the number of sample points being evaluated for the limit state function is employed as the accuracy measure. As shown in the table, the FORM generally requires less sample points to be evaluated to conduct reliability analysis, compared with the MCS method. The results also show that the FORM is more efficient than SORM, but its calculation accuracy is lower than SORM. Moreover, due to the gradient-based searching process employed by the FORM to find the MPP, the gradient information must be provided, where in this study the finite difference method has been employed for the FORM to provide the required information. As the gradient-based method is used for MPP search, the searching process may not converge to the true MPP efficiently in some

scenarios, as shown in the table for corrosion time of 250 days and 300 days in which the total numbers of sample evaluations for both have reached the upper limit of 800.

5. Conclusion and Future Work

The pitting corrosion growth model considering the coupled effects of stress and corrosion environment is proposed based on mechanical theory. And the time-dependent limit state function is discussed for stress corrosion crack failure based on the corrosion growth model. MCS and FORM are used for reliability assessment approaches. The example shows that the model is useful for structure reliability analysis, but the model parameters need verification by more experimental data. Apply this mechanical model for structure corrosion failure prognostics. Integrate the Bayesian inference method for parameter estimation.

Competing Interests

The authors declare that they have no competing interests.

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