Hindawi Publishing Corporation
International Journal of Chemical Engineering
Volume 2013, Article ID 380679, 8 pages
http://dx.doi.org/10.1155/2013/380679

Research Article

Heat and Mass Transfer Effect on MHD Flow of a Viscoelastic Fluid through a Porous Medium Bounded by an Oscillating Porous Plate in Slip Flow Regime

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Received 30 March 2013; Accepted 18 June 2013

Academic Editor: Jose C. Merchuk

Unsteady flow of an eclectically conducting and incompressible viscoelastic liquid of the Walter B' model with simultaneous heat and mass transfer near an oscillating porous plate in slip flow regime under the influence of a transverse magnetic field of uniform strength is presented. The governing equations of the flow field are solved by a regular perturbation method for small elastic parameter, and the expressions for the velocity, temperature, concentration, skin friction C_f, the heat flux in terms of the Nusselt number N_u , and the rate of mass transfer in terms of the Sherwood number S_h are obtained. The effects of the important flow parameters on the dynamics are discussed. Findings of the study reveal that the rarefaction parameter accelerates the fluid particles in the flow domain. Elastic parameter contributes to sudden fall of the velocity near the plate. Magnetic force contributes to greater skin friction as the time elapses. Destructive reaction reduces, whereas generative reaction enhances the concentration distribution.

1. Introduction

Free convective flow in presence of heat source has been a subject of interest of many researchers because of its possible application to geophysical sciences, astrophysical sciences, and in cosmical studies. Such flows arise either due to unsteady motion of the boundary or the boundary temperature. The study of fluctuating flow is important in the paper industry and many other technological fields. Therefore, many researchers have paid their attention towards the fluctuating flow of viscous incompressible fluid past an infinite plate. Singh et al. [1] have analyzed the heat and mass transfer in MHD flow of a viscous fluids past a vertical plate under oscillatory suction velocity. Sharma and Singh [2] have reported the unsteady MHD-free convective flow and heat transfer along a vertical porous plate with variable suction and internal heat generation. The problem of slip flow regime is very important in this era of modern science, technology, and vast ranging industrialization. In many practical applications, the particle adjacent to a solid surface no longer takes the velocity of the surface. The particle at the surface has a finite tangential velocity; it slips along the surface. The flow regime is called the slip flow regime, and its effect cannot be neglected. The fluid slippage phenomenon at the solid boundaries appear in many applications such as microchannels or nanochannels and in application where a thin film of light oils is attached to the moving plates or when the surface is coated with special coating such as thick monolayer of hydrophobic octadecyltrichlosilane, that is, lubrication of mechanical device where a thin film of lubricant is attached to the surface slipping over one another or when the surfaces are coated with special coating to minimize the friction between them. Singh and Gupta [3] have discussed the MHD-free convective flow of a viscous fluid through a porous medium bounded by an oscillating porous plate in slip flow regime with mass transfer. Khandelwal and Jain [4] have analyzed the unsteady MHD flow of a stratified fluid through porous medium over a moving plate in slip flow regime. Das et al. [5] have studied the magnetohydrodynamic unsteady flow of a viscous stratified fluid through a porous medium past a porous flat moving plate in the slip flow regime with

heat source. The study of heat and mass transfer problems with chemical reaction is of great practical importance to engineers and scientists because of their almost universal occurrence in many branches of science and engineering. A few representative fields of interest in which combined heat and mass transfer along with chemical reaction play an important role are chemical process industries such as food processing and polymer production. Mahapatra et al. [6] have studied effects of chemical reaction on free convection flow through a porous medium bounded by a vertical surface. Muthucumaraswamy [7] has studied effects of chemical reaction on a moving isothermal vertical surface with suction. Al-Odat and Al-Azab [8] have studied influence of chemical reaction on a transient MHD-free convection flow over a moving vertical plate.

Viscoelastic fluid flow through porous media has attracted the attention of scientists and engineers because of its importance notably in the flow of the oil through porous rocks, the extraction of energy from geothermal region, and the filtration of solids from liquids and drug permeation through human skin. The flow through porous media also occurs in the ground water hydrology, irrigation and drainage problems, absorption and filtration processes in the ground water hydrology, irrigation and drainage problems, absorption and filtration processes in chemical engineering, and soil erosion and tile drainage. Chaudhary and Jain [9] have investigated the effects of the Hall current and radiation on MHD mixed convection flow of a viscoelastic fluid past and infinite vertical plate. Sahoo et al. [10] have discussed the unsteady two-dimensional MHD flow and heat transfer of an elastic-viscous liquid past an infinite hot vertical porous surface bounded by porous medium with source/sink. Kumar and Chand [11] have studied the effect of slip conditions and the Hall current on unsteady MHD flow of a viscoelastic fluid past an infinite vertical porous plate through porous medium.

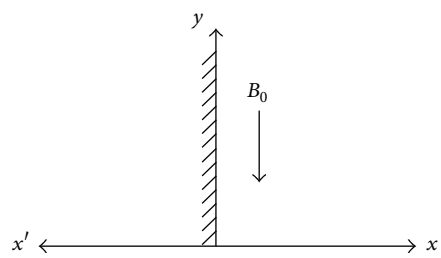
The objective of the present study is to consider the viscoelastic fluid past a vertical plate in the slip flow regime packed with uniform porous matrix in the presence of a transverse magnetic field. In this paper, we consider the problem of Singh and Gupta [3] in viscoelastic fluid of Walter's B' model taking into account the effect of heat source parameter and chemical reaction parameter.

2. Formulation and Solution of the Problem

The physical configuration consists of an unsteady flow of an electrically conducting and incompressible viscoelastic liquid of Walter's B' model with simultaneous heat and mass transfer near an oscillating infinite porous plate in slip flow regime with heat source and chemical reaction under the influence of a transverse magnetic field of uniform strength. The y-axis is taken along the plate in vertical direction, and x-axis is perpendicular to it. A uniform magnetic field of strength B_0 is applied in the direction of y-axis. Let u and v be the velocity components in x- and y-directions, respectively. As the plate is of infinite length, all the variables in the problem are functions of y and t. Initially, the plate and fluid are at rest, and then the plate is set to an oscillatory motion. The Reynolds number is assumed to be very small, and the induced magnetic field due to the flow is neglected with respect to the applied magnetic field. The pressure p in the fluid is assumed to be constant. If v_0 represents the suction or injection velocity at the plate, the equation of continuity is

$$\frac{\partial v}{\partial y} = 0. \tag{1}$$

Under the condition $y = 0$, $v = -v_0$ everywhere. We have



Flow Diagram

Now, the governing boundary layer equations of the flow field are

$$\begin{aligned}
\frac{\partial u}{\partial t} - v_0 \frac{\partial u}{\partial y} &= \upsilon \frac{\partial^2 u}{\partial y^2} - \sigma \frac{B_0^2 u}{\rho} - \frac{\upsilon u}{K} + g\beta (T - T_\infty) \\
&\quad + g\widehat{\beta} (C - C_\infty) - \frac{K_0}{\rho} \left(\frac{\partial^3 u}{\partial t \partial y} + v_0 \frac{\partial^3 u}{\partial y^3} \right), \\
\frac{\partial T}{\partial t} - v_0 \frac{\partial T}{\partial y} &= \alpha \frac{\partial^2 T}{\partial y^2} - S (T - T_\infty), \\
\frac{\partial C}{\partial t} - v_0 \frac{\partial C}{\partial y} &= D \frac{\partial^2 C}{\partial y^2} - K_l (C - C_\infty).
\end{aligned} \tag{2}$$

The first order velocity slip boundary conditions of the problem when the plate executes linear harmonic oscillations in its own plane are given by

$$\begin{aligned}
&y = 0 : u = U_0 e^{\text{int}} + L_1 \frac{\partial u}{\partial y}, \qquad T = T_\infty, \qquad C = C_\infty, \\
&y \longrightarrow \infty : u \longrightarrow 0, \qquad T \longrightarrow T_\infty, \qquad C \longrightarrow C_\infty,
\end{aligned} \tag{3}$$

where $L_1 = (2 - m_1)(L/m_1)$, $L = \mu(\pi/2p\rho)^{1/2}$ is the mean free path, and m_1 is Maxwell's reflection coefficient.

On introducing the following nondimensional quantities:

$$y^* = \frac{U_0}{v} y, \qquad u^* = \frac{u}{U_0}, \qquad t^* = \frac{U_0^2}{v} t,$$
$$\theta^* = \frac{T - T_\infty}{T_w - T_\infty}, \qquad \phi^* = \frac{C - C_\infty}{C_w - C_\infty},$$
$$v_0{}^* = \frac{v_0}{U_0}, \qquad n^* = \frac{v}{U_0^2} n,$$
$$K_p = \frac{K U_0^2}{v}, \qquad R = \frac{L_1 U_0}{v}, \qquad S^* = \frac{vS}{U_0^2}, \tag{4}$$
$$K_c = \frac{v K_l}{U_0^2}, \qquad M = \frac{\sigma B_0^2 v}{\rho U_0^2}, \qquad G_r = \frac{g\beta v (T_w - T_\infty)}{U_0^3},$$
$$G_m = \frac{g\widehat{\beta} v (C_w - C_\infty)}{U_0^3}, \qquad R_c = \frac{U_0^2 K_0}{\rho v^2},$$
$$\Pr = \frac{v}{\alpha}, \qquad S_c = \frac{v}{D},$$

in (2) and dropping the asterisks, we have

$$\frac{\partial u}{\partial t} - v_0 \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - R_c \left(\frac{\partial^3 u}{\partial t \partial y^3} + v_0 \frac{\partial^3 u}{\partial y^3} \right) - \left(M^2 + \frac{1}{K_p} \right) u + G_r \theta + G_m \phi, \tag{5}$$

$$\Pr \left(\frac{\partial \theta}{\partial t} - v_0 \frac{\partial \theta}{\partial y} \right) = \frac{\partial^2 \theta}{\partial y^2} - \Pr S \theta, \tag{6}$$

$$S_c \left(\frac{\partial \phi}{\partial t} - v_0 \frac{\partial \phi}{\partial y} \right) = \frac{\partial^2 \phi}{\partial y^2} - K_c S_c \phi, \tag{7}$$

with boundary conditions

$$y = 0 : u = e^{\text{int}} + R \frac{\partial u}{\partial y}, \qquad \theta = 1, \qquad \phi = 1,$$
$$y \longrightarrow \infty : u \longrightarrow 0, \qquad \theta \longrightarrow 0, \qquad \phi \longrightarrow 0. \tag{8}$$

Equation (5) is of third order, and two boundary conditions are available. Due to inadequate boundary condition, a perturbation method has been applied with $R_c < 1$ as the perturbation parameter. This assumption is quite consistent as the model under consideration is valid only for slightly elastic fluid.

Consider the following:

$$u = u_0 + R_c u_1 + O(R_c)^2,$$
$$\theta = \theta_0 + R_c \theta_1 + O(R_c)^2, \tag{9}$$
$$\phi = \phi_0 + R_c \phi_1 + O(R_c)^2.$$

Substituting (9) in (5)–(7) and equating like powers of R_c, we get the following.

Zeroth order equations:

$$\frac{\partial u_0}{\partial t} - v_0 \frac{\partial u_0}{\partial y} = \frac{\partial^2 u_0}{\partial y^2} - \left(M^2 + \frac{1}{K_p} \right) u_0 + G_r \theta_0 + G_m \phi_0,$$
$$\Pr \left(\frac{\partial \theta_0}{\partial t} - v_0 \frac{\partial \theta_0}{\partial y} \right) = \frac{\partial^2 \theta_0}{\partial y^2} - \Pr S \theta_0,$$
$$S_c \left(\frac{\partial \phi_0}{\partial t} - v_0 \frac{\partial \phi_0}{\partial y} \right) = \frac{\partial^2 \phi_0}{\partial y^2} - K_c S_c \phi_0, \tag{10}$$

first order equations:

$$\frac{\partial u_1}{\partial t} - v_0 \frac{\partial u_1}{\partial y} = \frac{\partial^2 u_1}{\partial y^2} - \left(M^2 + \frac{1}{K_p} \right) u_1 + G_r \theta_1 + G_m \phi_1 - v_0 \frac{\partial^3 u_1}{\partial y^3} - \frac{\partial^3 u_0}{\partial t \partial y^2},$$
$$\Pr \left(\frac{\partial \theta_1}{\partial t} - v_0 \frac{\partial \theta_1}{\partial y} \right) = \frac{\partial^2 \theta_1}{\partial y^2} - \Pr S \theta_1, \tag{11}$$
$$S_c \left(\frac{\partial \phi_1}{\partial t} - v_0 \frac{\partial \phi_1}{\partial y} \right) = \frac{\partial^2 \phi_1}{\partial y^2} - K_c S_c \phi_1.$$

The corresponding boundary conditions are

$$y = 0 : u_0 = e^{\text{int}} + R \frac{\partial u_0}{\partial y}, \qquad u_1 = 0, \qquad \theta_0 = 1,$$
$$\theta_1 = 1, \qquad \phi_0 = 1, \qquad \phi_1 = 1,$$
$$y \longrightarrow \infty : u_0 \longrightarrow 0, \qquad u_1 \longrightarrow 0, \qquad \theta_0 \longrightarrow 0, \tag{12}$$
$$\theta_1 \longrightarrow 0, \qquad \phi_0 \longrightarrow 0, \qquad \phi_1 \longrightarrow 0.$$

In order to reduce the system of partial differential equations (10)–(11) to a system of ordinary differential equations, we further introduce

$$u_0 (y, t) = u_{00} (y) + u_{01} (y) e^{\text{int}},$$
$$u_1 (y, t) = u_{10} (y) + u_{11} (y) e^{\text{int}},$$
$$\theta_0 (y, t) = \theta_{00} (y) + \theta_{01} (y) e^{\text{int}},$$
$$\theta_1 (y, t) = \theta_{10} (y) + \theta_{11} (y) e^{\text{int}},$$
$$\phi_0 (y, t) = \phi_{00} (y) + \phi_{01} (y) e^{\text{int}},$$
$$\phi_1 (y, t) = \phi_{10} (y) + \phi_{11} (y) e^{\text{int}}. \tag{13}$$

Substituting (13) into (10)–(11) and equating the harmonic and nonharmonic terms, we obtain

$$\begin{aligned}
&u_{00}''+ v_0u_{00}' - \left(M^2 + \frac{1}{K_p}\right)u_{00} = -G_r\theta_{00} - G_m\phi_{00},\\
&u_{01}''+ v_0u_{01}' - \left(M^2 + \frac{1}{K_p} + in\right)u_{01} = -G_r\theta_{01} - G_m\phi_{01},\\
&u_{10}''+ v_0u_{10}' - \left(M^2 + \frac{1}{K_p}\right)u_{10} = -G_r\theta_{10} - G_m\phi_{10} + v_0u_{00}''',\\
&u_{11}'' + v_0u_{11}' - \left(M^2 + \frac{1}{K_p} + in\right)u_{11},\\
&\qquad = -G_r\theta_{11} - G_m\phi_{11} + v_0u_{01}''' + inu_{01}'',\\
&\theta_{00}''+ v_0\mathrm{Pr}\theta_{00}' - \mathrm{Pr}S\theta_{00} = 0,\\
&\theta_{01}''+ v_0\mathrm{Pr}\theta_{01}' - (in\mathrm{Pr} + \mathrm{Pr}S)\,\theta_{01} = 0,\\
&\theta_{10}''+ v_0\mathrm{Pr}\theta_{10}' - \mathrm{Pr}S\theta_{10} = 0,\\
&\theta_{11}''+ v_0\mathrm{Pr}\theta_{11}' - (in\mathrm{Pr} + \mathrm{Pr}S)\,\theta_{11} = 0,\\
&\phi_{00}''+ v_0S_c\phi_{00}' - K_cS_c\phi_{00} = 0,\\
&\phi_{01}''+ v_0S_c\phi_{01}' - (inS_c + K_cS_c)\,\phi_{01} = 0,\\
&\phi_{10}''+ v_0S_c\phi_{10}' - K_cS_c\phi_{10} = 0,\\
&\phi_{11}''+ v_0S_c\phi_{11}' - (inS_c + K_cS_c)\,\phi_{11} = 0,
\end{aligned} \tag{14}$$

with boundary conditions

$$\begin{aligned}
&y = 0 : u_{00} = R\frac{\partial u_{00}}{\partial y}, \qquad u_{01} = 1 + R\frac{\partial u_{01}}{\partial y},\\
&u_{10} = 0, \qquad u_{11} = 0, \qquad \theta_{00} = 1,\\
&\theta_{01} = 0, \qquad \theta_{10} = 1, \qquad \theta_{11} = 0,\\
&\phi_{00} = 1, \qquad \phi_{01} = 0, \qquad \phi_{10} = 0, \qquad \phi_{11} = 0,\\
&y \longrightarrow \infty : u_{00} \longrightarrow 0, \qquad u_{01} \longrightarrow 0, \qquad u_{10} \longrightarrow 0,\\
&u_{11} \longrightarrow 0, \qquad \theta_{00} \longrightarrow 0, \qquad \theta_{01} \longrightarrow 0,\\
&\theta_{10} \longrightarrow 0, \qquad \theta_{11} \longrightarrow 0, \qquad \phi_{00} \longrightarrow 0,\\
&\phi_{01} \longrightarrow 0, \qquad \phi_{10} \longrightarrow 0, \qquad \phi_{11} \longrightarrow 0.
\end{aligned} \tag{15}$$

The solutions of (14) applying boundary conditions (15) are

$$\begin{aligned}
u_{00} &= A_1e^{-a_1y} + A_2e^{-a_2y} + A_3e^{-a_3y},\\
u_{01} &= A_4e^{-a_4y},\\
u_{10} &= A_5e^{-a_1y} + A_6e^{-a_2y} + A_6e^{-a_3y},\\
u_{11} &= A_8e^{-a_4y},\\
\theta_{00} &= e^{-a_1y},\\
\theta_{01} &= \theta_{10} = \theta_{11} = 0,\\
\phi_{00} &= e^{-a_2y},\\
\phi_{01} &= \phi_{10} = \phi_{11} = 0.
\end{aligned} \tag{16}$$

Hence, the velocity, temperature, and concentration of the flow field are

$$\begin{aligned}
u = {} & A_1e^{-a_1y} + A_2e^{-a_2y} + A_3e^{-a_3y}\\
&+ A_4e^{-a_4y}(\cos nt + i\sin nt)\\
&+ R_c\left\{A_5e^{-a_1y} + A_6e^{-a_2y} + A_7e^{-a_3y}\right.\\
&\qquad \left. + A_8e^{-a_4y}(\cos nt + i\sin nt)\right\},\\
\theta = {} & e^{-a_1y},\\
\phi = {} & e^{-a_2y}.
\end{aligned} \tag{17}$$

The skin friction at the plate is given by

$$C_f = \frac{\tau_{xy}}{U_0^2} = \frac{\partial u}{\partial y} - R_c\left(\frac{\partial^2 u}{\partial t\partial y} + v_0\frac{\partial^2 u}{\partial y^2}\right)\bigg|_{y=0}, \tag{18}$$

where

$$\tau_{xy} = \frac{\partial u}{\partial y} - \frac{K_0}{\rho}\left(\frac{\partial^2 u}{\partial t\partial y} + v_0\frac{\partial^2 u}{\partial y^2}\right). \tag{19}$$

The rate of heat transfer, that is, the heat flux at the plate in terms of the Nusselt number, is given by

$$N_u = -\frac{\partial\theta}{\partial y}\bigg|_{y=0} = a_1. \tag{20}$$

The rate of mass transfer at the plate in terms of the Sherwood number is given by

$$S_h = -\frac{\partial\phi}{\partial y}\bigg|_{y=0} = a_2. \tag{21}$$

3. Results and Discussion

The problem of unsteady flow of an electrically conducting and incompressible viscoelastic liquid of Walter's B' model with heat and mass transfer near an oscillating infinite porous

plate in slip flow regime with heat source and chemical reaction parameter under the influence of a transverse magnetic field of uniform strength has been considered. The effects of the flow parameters such as the Prandtl number Pr, porosity parameter K_p, magnetic parameter M, elastic parameter R_c, heat source parameter S, chemical reaction parameter K_c, thermal Grashof number G_r, the mass Grashof number G_m, the Scmidt number S_c, suction parameter v_0, and rarefaction parameter R on the velocity field have been studied analytically and presented with the help of Figures 1–4. The effects of the flow parameters on the temperature field and concentration distribution have been presented in Figures 5 and 6, respectively. Further, the effects of the flow parameters on the skin friction, heat flux, and rate of mass transfer have been discussed with the help of Tables 1–3. For numerical computation, the values of G_r are taken positive. This indicates that the study has been carried out under the influence of the cooling of the plate. Also, we have taken $nt = \pi/2$. The interesting aspect of the problem is to study the combined effect of the flow parameters with that of the first order velocity slip boundary condition when the plate executes linear harmonic oscillation in its own plane.

Figure 1 shows the effect of the Prandtl number (Pr), permeability parameter (K_p) and magnetic parameter (M) on velocity profile. For this figure, we have taken that $S = 1$, $G_r = 5$, $G_m = 5$, $S_c = 0.24$, $R_c = 0.5$, $K_c = 2$, $R = 0.2$, and $v_0 = 2$. It is observed that the increase in the Prandtl number as well as permeability parameter decreases the velocity of the flow field, whereas increase in magnetic parameter increases it. Since Prandtl number is the ratio of kinematic viscosity to thermal diffusivity, so as Pr increases, the kinematic viscosity of the fluid dominates the thermal diffusivity of the fluid which leads to decreasing the velocity of the flow field. The application of transverse magnetic field sets up the Lorentz force, which enhances the fluid velocity.

Figure 2 shows the effect of elastic parameter (R_c), heat source parameter (S), and chemical reaction (K_c) parameter on velocity frofile. For this figure, we have taken that Pr = 0.71, $K_p = 1$, $M = 2$, $G_r = 5$, G_m, $= 5$ $S_c = 0.24$, $R = 0.2$, and $v_0 = 2$. It is observed that the velocity of the flow field decreases due to the presence of elastic parameter, chemical reaction parameter, and heat source parameter. For $R_c = S = K_c = 0$, the present work agrees with the work of Singh and Gupta [3].

Figure 3 depicts the effect of the thermal Grashof number (G_r), the mass Grashof number (G_m), and the Schmidt number (S_c), on velocity frofile. For this figure, we have taken that Pr = 0.71, $S = 1$, $K_p = 1$, $M = 2$, $R_c = 0.5$, $K_c = 2$, $R = 0.2$, and $v_0 = 2$. It is observed that for the heavier species, that is, with increasing S_c, the velocity decreases. The velocity of the flow field decreases due to the increase in the thermal Grashof number. Moreover, buoyancy effect (G_m) due to mass transfer enhances the velocity.

Figure 4 depicts the effect of suction parameter (v_0) and rarefaction parameter (R) on velocity frofile. For this figure, we have taken that Pr = 0.71, $S = 1$, $K_p = 1$, $M = 2$, $R_c = 0.5$, $K_c = 2$, $G_r = 5$, $G_m = 5$, and $S_c = 0.24$. It is observed that the velocity of the flow field decreases due the presence of suction

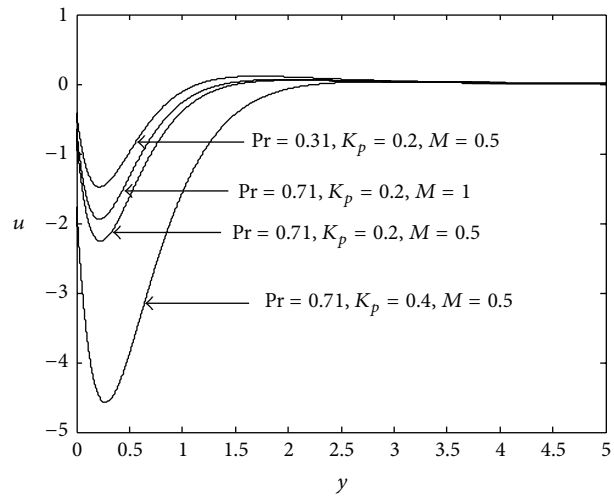


Figure 1: Effect of Pr, K_p and M on velocity profile.

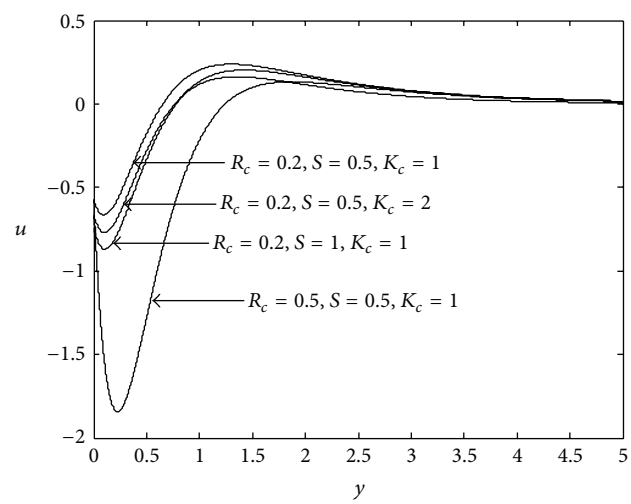


Figure 2: Effect of R_c, S, and K_c on velocity profile.

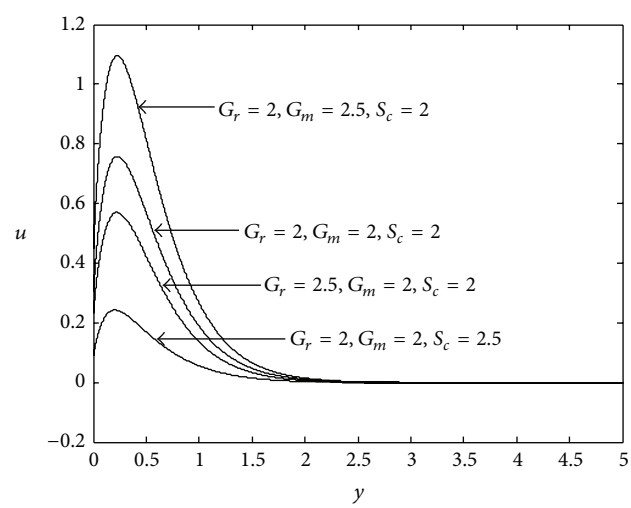


Figure 3: Effect of G_r, G_m, and S_c on velocity profile.

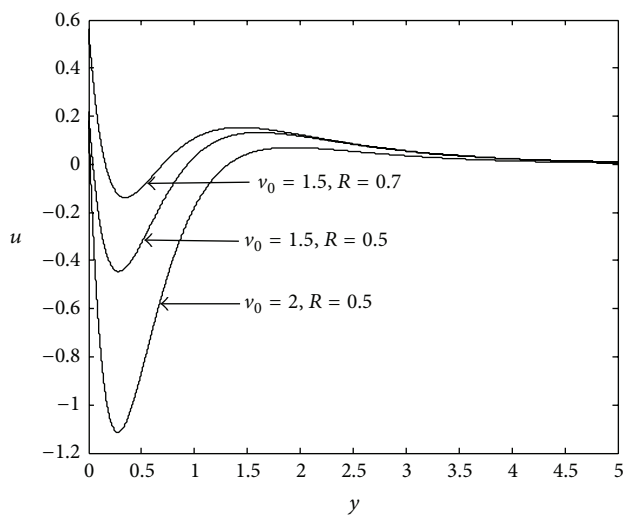


Figure 4: Effect of v_0 and R on velocity profile.

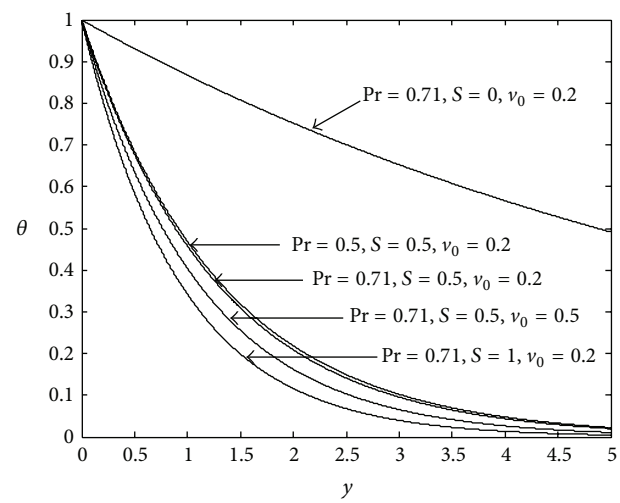


Figure 5: Effect of Pr, S, and v_0 on temperature profile.

parameter, but the reverse effect is observed due the presence of the rarefaction parameter.

Figure 5 shows the effect of the Prandtl number, heat source parameter, and suction parameter on the temperature of the flow field. It is observed that the temperature of the flow field diminishes as the Prandtl number increases. This is consistent with the fact that the thermal boundary layer thickness decreases with increasing Prandtl number. Presence of heat source reduces the temperature of the flow field. This may happen due the elastic property of the fluid. It is observed that temperature of the flow field diminishes as the suction parameter increases.

Figure 6 depicts the effect of the Schmidt number, chemical reaction parameter, and suction parameter on concentration distribution. The concentration distribution decreases at all points of the flow field with the increase in the Schmidt number. This shows that the heavier diffusing species have a greater retarding effect on the concentration distribution of the flow field. It is observed that a destructive reaction ($K_c > 0$) reduces the concentration distribution, whereas a generative reaction ($K_c = 0$) enhances it. Also, it is observed that presence of suction parameter diminishes the concentration distribution.

The skin friction is an important phenomenon which characterizes the frictional drag at the solid surface. From Table 1, it is observed that the skin friction decreases with the increase in all the forcing forces, but it is interesting to note that the skin friction increases with the increase in magnetic parameter.

From Table 2, it is to note that all the entries are positive. It is seen that the Prandtl number (Pr), heat source (S) and suction parameter (v_0) increase the rate of heat transfer at the surface of the plate.

From Table 3 it is to note that all the entries are positive. It is observed that Schmidt number (S_c), chemical reaction parameter (K_c), and suction parameter (v_0) increase the rate of mass transfer at the surface of the plate.

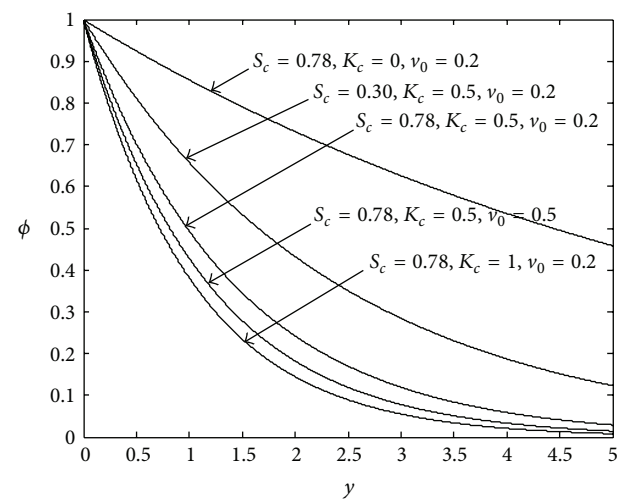


Figure 6: Effect of S_c, K_c, and v_0 on concentration profile.

4. Conclusion

A theoretical study of unsteady MHD incompressible viscoelastic liquid of Walter's B' model with heat and mass transfer near an oscillating infinite porous plate in slip flow regime under the influence of a transverse magnetic field of uniform strength is considered. Some of the important findings of the problem are given in the following.

(i) Presence of the Prandtl number decreases the velocity of the flow field, whereas presence magnetic field increases it.

(ii) The velocity of the flow field decreases suddenly near the plate due to the presence of elastic parameter.

(iii) The velocity of the flow field decreases due to the increase in the thermal Grashof number.

Table 1: Skin friction (C_f).

Pr	K_p	M	R_c	S	K_c	G_r	G_m	S_c	R	v_0	C_f
0.30	1	2	0.5	1	2	5	5	0.24	0.2	2	18.3921
0.50	1	2	0.5	1	2	5	5	0.24	0.2	2	10.8793
0.50	1.5	2	0.5	1	2	5	5	0.24	0.2	2	7.1877
0.50	1	2.5	0.5	1	2	5	5	0.24	0.2	2	30.4390
0.50	1	2	0.2	1	2	5	5	0.24	0.2	2	13.0904
0.50	1	2	0.5	0.5	2	5	5	0.24	0.2	2	17.9390
0.50	1	2	0.5	1	1.5	5	5	0.24	0.2	2	11.6350
0.50	1	2	0.5	1	2	4.5	5	0.24	0.2	2	13.1659
0.50	1	2	0.5	1	2	5	4.5	0.24	0.2	2	11.8927
0.50	1	2	0.5	1	2	5	5	0.30	0.2	2	9.1301
0.50	1	2	0.5	1	2	5	5	0.24	0.5	2	2.75040
0.50	1	2	0.5	1	2	5	5	0.24	0.2	1.5	14.0484

Table 2: The Nusselt number (N_u).

Pr	S	v_0	N_u
0.50	0.5	0.2	0.75887
0.71	0.5	0.2	0.78166
0.71	1.0	0.2	1.07351
0.71	0.5	0.5	0.90654

Table 3: The Sherwood number (S_h).

S_c	K_c	v_0	S_h
0.30	0.5	0.2	0.41845
0.78	0.5	0.2	0.70735
0.78	1.0	0.2	0.96461
0.78	0.5	0.5	0.84923

(iv) Thermal boundary layer thickness decreases with increasing the Prandtl number.

(v) Heavier diffusing species have a greater retarding effect on the concentration distribution.

Appendix

Consider the following:

$$a_1 = \frac{1}{2}\left[v_0\mathrm{Pr} + \sqrt{(v_0\mathrm{Pr})^2 + 4S\mathrm{Pr}}\right],$$

$$a_2 = \frac{1}{2}\left[v_0 S_c + \sqrt{(v_0 S_c)^2 + 4K_c S_c}\right],$$

$$a_3 = \frac{1}{2}\left[v_0^2 + \sqrt{v_0^2 + 4Q}\right], \quad Q = M^2 + \frac{1}{K_p},$$

$$a_4 = \frac{1}{2}\left[v_0^2 + \sqrt{v_0^2 + 4(Q + in)}\right],$$

$$A_1 = \frac{-G_r}{a_1^2 - a_1 v_0 - Q}, \qquad A_2 = \frac{-G_r}{a_2^2 - a_2 v_0 - Q},$$

$$A_3 = \frac{-1}{1 + a_3 R}\left[(a_1 R + 1)A_1 + (a_2 R + 1)A_2\right],$$

$$A_4 = \frac{1}{1 + Ra_4}, \qquad A_5 = \frac{a_1^3 A_1 v_0}{a_1^2 - a_1 v_0 - Q},$$

$$A_6 = \frac{a_2^3 A_2 v_0}{a_2^2 - a_2 v_0 - Q}, \qquad A_7 = A_5 + A_6 - \frac{v_0 a_3^3 A_3 y}{v_0 - 2a_3},$$

$$A_8 = \frac{\left(-v_0 a_4^3 A_4 + ina_4^2 A_4\right) y}{v_0 - 2a_4}. \tag{A.1}$$

Nomenclature

x, y: Coordinate axes
u, v: Velocity components in x- and y-directions
t: Time variable
μ: Dynamic viscosity
υ: Kinematic viscosity
α: Thermal conductivity
p: Pressure
g: Acceleration due to gravity
β: Coefficient of volume expansion
$\widehat{\beta}$: Coefficient of volume expansion with concentration
U_0: Reference velocity
T: Dimensional temperature
θ: Nondimensional temperature
C: Dimensional concentration
ϕ: Nondimensional concentration
Pr: The Prandtl number
G_r: The thermal Grashof number
G_m: The mass Grashof number
S_c: The Schmidt number
M: Magnetic parameter
S: Heat source/sink parameter
K_l: Dimensional chemical reaction parameter
K_c: Nondimensional chemical reaction parameter
T_w: Temperature at the wall
T_∞: Temperature far away from the wall
C_w: Concentration at the wall
C_∞: Concentration far away from the wall
σ: Electric conductivity
B_0: Uniform magnetic field
v_0: Suction/injection velocity
D: Mass diffusion
ρ: Density
K: Dimensional porosity parameter
K_p: Nondimensional porosity parameter
K_0: Dimensional elastic parameter
R_c: Nondimensional elastic parameter
n: Frequency of oscillation.

Acknowledgment

The author wish to express his special thanks to Professor G. C. Dash, S "O" A University, Bhubaneswar, Odisha, India for his valuable suggesion and constant encouragement to complete the work.

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