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Research Article

Stability, Bifurcation, and Chaos Control of Two-Sided Market Competition

Jianli Xiao,[1,2] **Hanli Xiao**,[3] **Xinchang Zhang**,[4] **and Xiang You**[5]

[1]*Management School, Fudan University, Shanghai 200433, China*
[2]*School of Business Administration, Nanchang Institute of Technology, Nanchang 330099, China*
[3]*School of Tourism and Resources Environment, Qiannan Normal University for Nationalities, Duyun 55900, China*
[4]*School of Foreign Languages and Literature, Nanchang Institute of Technology, Nanchang 330099, China*
[5]*Department of Finance and Economics, Taiyuan University, Taiyuan 030000, China*

Correspondence should be addressed to Jianli Xiao; 17110690001@fudan.edu.cn

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Benefitting from the popular uses of internet technologies, two-sided market has been playing an increasing prominent role in modern times. Users and developers can interact with each other through two-sided platforms. The two-sided market structure has been investigated profoundly. Through building a dynamics two-sided market model with bounded rational, stability conditions of the two-sided market competition system are presented. With the help of bifurcation diagram, Lyapunov exponent, and strange attractor, the stability of the two-sided market competition model is simulated. At last, we use the time-delayed feedback control (TDFC) method to control the chaos. Our main results are as follows: (1) when the adjustment speed of two-sided increases, the system becomes bifurcation, and chaos state happens finally. When the system is stable, the consumer fee is positive while developer fee is negative. (2) When the user externality increases, the stable area of the system increases, and the difference in user externality leads the whole system more stable. When the system is stable, the developer fee decreases. (3) The stable area becomes larger when developer externality increases; when the system is stable, the user fee becomes lower and developer fee becomes higher when developer externality increases. (4) The TDFC method is presented for controlling the chaos; we find that the system becomes more stable under the TDFC method.

1. Introduction

With the rapid development of internet technologies, platforms are becoming a common form in real-world business. There are different examples of two-sided markets in the today market [1, 2], such as newspaper, ride-sharing, and TV channels. Different agents have the cross-group relationship through the platforms, and two-sided market structure makes the competition between platforms more complex. Internet platform decreases the cost of communication between different agents [3]; however, there are lots of problems emerges as the different characters of platforms, for example, the develop of ride-sharing platform brings lots of challenge for government [4]. The reason of the chaos state is because the agents are bounded rational and the network externality between agents.

This paper related to the two-sided market theory. The earlier two-sided market articles are mostly about the horizontal competition, such as [5, 6]; two-sided markets compete for their network externality; when other sides have more agents, the consumer would has more utility. Affeldt et al. [7] extend the upward pricing pressure from the one-sided market to two-sided markets. Lam [8] shows switching costs have different effects in a dynamic two-sided model from the traditional market. The two-sided platform uses price decision as a competition strategy. Caillaud and Jullien [9] analyze the pricing strategy in the two-sided market. Choi [10] analyzes the pricing strategy in multihoming. Belleflamme and Toulemonde [11] study intragroup externality in the two-sided market.

The second branch of literature is dynamics competition. The second branch of literature is dynamics competition. A nonlinear behavior was widely studied in different fields, such as Bao et al. [12, 13], and Li et al. [14] investigate hyperchaos in memristor. The economy complexity behavior was discovered by many researchers. Onozaki et al. [15] analyze a dynamic of a cobweb market. Du et al. [16] find that the control of chaos improve performance of system. Dubiel-Teleszynski [17] studies nonlinear dynamics in a duopoly game with diseconomies of scale. Elsadany [18] investigates bounded rationality in dymamics of a Cournot duopoly game. Fanti and Gori [19] study quantity competition in the duopoly game. Jayanthi and Sinha [20] investigate the chaotic characteristics of innovation in high technology manufacturing. Guo et al. [21] show the information intermediary and the end users have the chaos behavior. Chaos theory has a close relationship with the industrial organization [12–23].

The two-sided market, such as TV channels, has a complex behavior in competition [24]. In order to deal with the complexity, there are several mechanisms related to the control of chaotic systems, such as feed-forward control, OGY method, and time-delayed feedback control (TDFC) [25]. TDFC becomes increasingly popular for its advantages that the feedback does not need fine understanding of the system [26]. Sukono et al. [27] and Vaidyanathan et al. [28] investigate the bifurcation and control in the financial risk system.

The rest of the paper is organized as follows: Section 2 describes the two-sided market competition model. Section 3 presents the fixed points of the system and stability of the fixed points. Section 4 shows the simulation of the two-sided market model and gives the time-delayed feedback control of system. Section 5 draws the conclusions.

2. The Model

Our two-sided market is shown in Figure 1, the two two-sided platforms make a fee decision about the user and developer, the developer is independent to each other, and the user and developer interacted with each other through two-sided platforms.

In our model, our two-sided market competition model is similar to Armstrong [5], but our model have new features. Our model is different from Armstrong's model in that our model in the developer side is independent; in the real world, there are most common that the two-sided platform competing with each other in one side, such as different newspapers competes only in readers, but in the writer side, they have no competition.

There are two two-sided platforms who connect users and developers. Suppose that users are heterogeneous, whose preference distribute between a unit interval$[0, 1]$, two-sided platforms maximize their profits through charging fees on developers and users. Set$\alpha$is a user externality parameter which is from developers to users, as Hagiu and Halaburda [29], we set $0 < \alpha < 1$, $p_{ui}, i = 1, 2$ is the fee that the two-sided platform charged on users, andq_{di}, i = 1, 2 is the developers quantity who join platform i.

Users whose position in x join platform 1 utility function is

$$\mathrm{u}_{\mathrm{u}1} = \alpha q_{d1} - p_{u1} + x. \tag{1}$$

In platform 2, the users' utility function is denoted:

$$u_{u2} = \alpha q_{d2} - p_{u2} + 1 - x, \tag{2}$$

Set $\beta$is a developer externality parameter which is from users to developers, for more consumer leads to more developers, as Hagiu and Halaburda [29], we assume $1 > \beta > 0$. $p_{di}, i = 1, 2$ is the fee that the two-sided platform charged on developers, and $q_{ui}, i = 1, 2$ is the users' quantity who join platform i.

The number of developers who joins platform 1 is

$$q_{d1} = \beta q_{u1} - p_{d1}. \tag{3}$$

The number of developers who joins platform 2 is

$$q_{d2} = \beta q_{u2} - p_{d2}. \tag{4}$$

The externality between two groups is asymmetric. For simplicity, we assume marginal costs is 0. Then, platform 1's profit function is

$$\pi_1 = p_{u1} q_{u1} + p_{d1} q_{d1}. \tag{5}$$

Platform 2's profit function is

$$\pi_2 = p_{u2} q_{u2} + p_{d2} q_{d2}. \tag{6}$$

Let $u_{u1} = u_{u2}$, then the users quantity is

$$\begin{cases} q_{u1} = \dfrac{\alpha(q_{d1} - q_{d2}) + p_{u2} - p_{u1} + 1}{2}, \\ q_{u2} = \dfrac{\alpha(q_{d2} - q_{d1}) + p_{u1} - p_{u2} + 1}{2}. \end{cases} \tag{7}$$

Inserting equation (7) into equations (3) and (4), we get the number of users and developers who join the platform 1 using

$$\begin{cases} q_{u1} = \dfrac{1 - \alpha\beta + \alpha(p_{d2} - p_{d1}) + p_{u2} - p_{u1}}{2(1 - \alpha\beta)}, \\ q_{d1} = \dfrac{\beta - \alpha\beta^2 + \alpha\beta(p_{d2} + p_{d1}) + \beta(p_{u2} - p_{u1}) - 2p_{d2}}{2(1 - \alpha\beta)}. \end{cases} \tag{8}$$

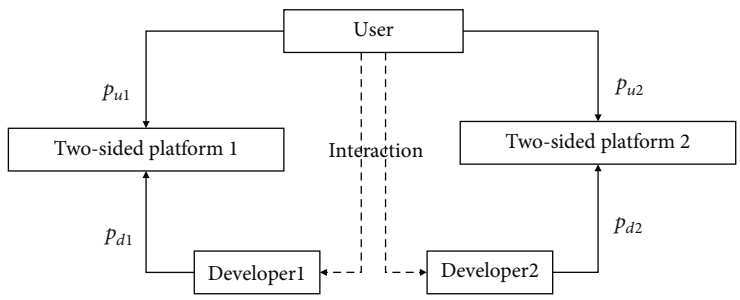

Figure 1: Two-sided market structure.

The number of users and developers who join the platform 1 is

$$\begin{cases} q_{u2} = \dfrac{1-\alpha\beta+\alpha(p_{d1}-p_{d2})+p_{u1}-p_{u2}}{2(1-\alpha\beta)}, \\ q_{d2} = \dfrac{\beta-\alpha\beta^2+\alpha\beta(p_{d2}+p_{d1})+\beta(p_{u1}-p_{u2})-2p_{d1}}{2(1-\alpha\beta)}. \end{cases} \tag{9}$$

The platform makes its price decision to maximize its profit; inserting equation (8) in equation (5), the platform 1's profit function can be written as

$$\pi_1 = \frac{p_{u1}(1-\alpha\beta+\alpha(p_{d2}-p_{d1})+p_{u2}-p_{u1})}{2(1-\alpha\beta)} + \frac{p_{d1}\left(\beta-\alpha\beta^2+\alpha\beta(p_{d2}+p_{d1})+\beta(p_{u2}-p_{u1})-2p_{d2}\right)}{2(1-\alpha\beta)}. \tag{10}$$

Similarly, inserting equation (9) into equation (6), the platform 2's profit function can be written as

$$\pi_2 = \frac{p_{u2}(1-\alpha\beta+\alpha(p_{d1}-p_{d2})+p_{u1}-p_{u2})}{2(1-\alpha\beta)} + \frac{p_{d2}\left(\beta-\alpha\beta^2+\alpha\beta(p_{d2}+p_{d1})+\beta(p_{u1}-p_{u2})-2p_{d1}\right)}{2(1-\alpha\beta)}. \tag{11}$$

Taking the first-order derivative of (10) with respect to p_{d1}, we obtain the following FOC:

$$\frac{\partial\pi_1}{\partial p_{d1}} = \frac{\beta}{2} + \frac{2(\alpha\beta-2)p_{d1}+\alpha\beta p_{d2}+\beta(p_{u2}-p_{u1})-\alpha p_{u1}}{2(1-\alpha\beta)} = 0. \tag{12}$$

Similarly, we take the first-order derivative of (11) with respect to p_{d2}, we obtain the following FOC:

$$\frac{\partial\pi_2}{\partial p_{d2}} = \frac{\beta}{2} + \frac{2(\alpha\beta-2)p_{d2}+\alpha\beta p_{d1}+\beta(p_{u1}-p_{u2})-\alpha p_{u2}}{2(1-\alpha\beta)} = 0. \tag{13}$$

From equation (12), platform 1 makes developer fee decision

$$p_{d1} = \frac{\beta-\alpha\beta^2+\alpha\beta p_{d2}-(\alpha+\beta)p_{u1}+\beta p_{u2}}{2(2-\alpha\beta)}. \tag{14}$$

From equation (13), platform 2 makes developer fee decision

$$p_{d2} = \frac{\beta-\alpha\beta^2+\alpha\beta p_{d1}-(\alpha+\beta)p_{u2}+\beta p_{u1}}{2(2-\alpha\beta)}. \tag{15}$$

Substitute equation (14) into platform 1 profit function (10), then we get platform 1's profit function

$$\pi_1 = \frac{p_{u1}\left(1-\alpha\beta+\alpha\left(p_{d2}-\beta-\alpha\beta^2+\alpha\beta p_{d2}-(\alpha+\beta)p_{u1}+\beta p_{u2}/2(2-\alpha\beta)\right)+p_{u2}-p_{u1}\right)}{2(1-\alpha\beta)} + \frac{\left(\left(\beta-\alpha\beta^2+\alpha\beta p_{d2}-(\alpha+\beta)p_{u1}+\beta p_{u2}\right)\beta-\alpha\beta^2+\alpha\beta p_{d2}-(\alpha+\beta)p_{u1}+\beta p_{u2}/(2(2-\alpha\beta))\right)\left(\beta-\alpha\beta^2+\alpha\beta\left(p_{d2}+\left(\left(\beta-\alpha\beta^2+\alpha\beta p_{d2}-(\alpha+\beta)p_{u1}+\beta p_{u2}\right)/(2(2-\alpha\beta))\right)\right)+\beta(p_{u2}-p_{u1})-2p_{d2}\right)}{2(1-\alpha\beta)}. \tag{16}$$

Substitute equation (15) into platform 2 profit function (11), then we get platform 2's profit function

$$\pi_2 = \frac{p_{u2}\left(1-\alpha\beta+\alpha\left(p_{d1}-\left(\left(\beta-\alpha\beta^2+\alpha\beta p_{d1}-(\alpha+\beta)p_{u2}+\beta p_{u1}\right)\beta-\alpha\beta^2+\alpha\beta p_{d1}-(\alpha+\beta)p_{u2}+\beta p_{u1}/(2(2-\alpha\beta))2(2-\alpha\beta)\right)\right)+p_{u1}-p_{u2}\right)}{2(1-\alpha\beta)} + \frac{\left(\left(\beta-\alpha\beta^2+\alpha\beta p_{d1}-(\alpha+\beta)p_{u2}+\beta p_{u1}\right)/(2(2-\alpha\beta))\right)\left(\beta-\alpha\beta^2+\alpha\beta\left(\left(\left(\beta-\alpha\beta^2+\alpha\beta p_{d1}-(\alpha+\beta)p_{u2}+\beta p_{u1}\right)/(2(2-\alpha\beta))\right)+p_{d1}\right)+\beta(p_{u1}-p_{u2})-2p_{d1}\right)}{2(1-\alpha\beta)}. \tag{17}$$

Differentiating equation (16) with respect to p_{u1}, we get

$$\frac{\partial\pi_1}{\partial p_{u1}} = \frac{-\alpha\left(4-3\alpha\beta-\beta^2\right)p_{d2}-\left(4-3\alpha\beta-\beta^2\right)p_{u2}+\left(8-\alpha^2-6\alpha\beta-\beta^2\right)p_{u1}-4+7\alpha\beta-3\alpha^2\beta^2+\beta^2-\alpha\beta^3}{4(1-\alpha\beta)(2-\alpha\beta)}. \tag{18}$$

Differentiating equation (17) with respect to p_{u2}, we get

$$\frac{\partial\pi_2}{\partial p_{u2}} = \frac{-\alpha\left(4-3\alpha\beta-\beta^2\right)p_{d1}-\left(4-3\alpha\beta-\beta^2\right)p_{u1}+\left(8-\alpha^2-6\alpha\beta-\beta^2\right)p_{u2}-4+7\alpha\beta-3\alpha^2\beta^2+\beta^2-\alpha\beta^3}{4(1-\alpha\beta)(2-\alpha\beta)}. \tag{19}$$

When the former equations (18) and (19) become equilibrium, then from (10) and (11), we can get

$$\begin{cases} p_{d1} = \dfrac{\beta\left(4-\alpha^2-3\alpha\beta\right)p_{u2}+\left(2\alpha^2\beta+3\alpha\beta^2-4\alpha-4\beta\right)p_{u1}+\beta(\alpha\beta-4)(\alpha\beta-1)}{(\alpha\beta-4)(3\alpha\beta-4)}, \\ p_{d2} = \dfrac{\beta\left(4-\alpha^2-3\alpha\beta\right)p_{u1}+\left(2\alpha^2\beta+3\alpha\beta^2-4\alpha-4\beta\right)p_{u2}+\beta(\alpha\beta-4)(\alpha\beta-1)}{(\alpha\beta-4)(3\alpha\beta-4)}. \end{cases} \tag{20}$$

Insert equation (20) into equation (18), then we can get platform 1's profit maximum condition:

$$\frac{\partial\pi_1}{\partial p_{u1}} = \frac{4-3\alpha\beta-\beta^2}{8-6\alpha\beta} + \frac{\left(4-3\alpha\beta-\beta^2\right)\left(4-3\alpha\beta-\alpha^2\right)p_{u2}}{2(1-\alpha\beta)(4-\alpha\beta)(4-3\alpha\beta)} + \frac{\left((4-3\alpha\beta)\left(\alpha^2+\beta^2\right)-(4-2\alpha\beta)(8-7\alpha\beta)\right)p_{u1}}{2(1-\alpha\beta)(4-\alpha\beta)(4-3\alpha\beta)} = 0. \tag{21}$$

Insert equation (20) into equation (19), then we can get platform 2's profit maximum condition

$$\frac{\partial\pi_2}{\partial p_{u2}} = \frac{4-3\alpha\beta-\beta^2}{8-6\alpha\beta} + \frac{\left(4-3\alpha\beta-\beta^2\right)\left(4-3\alpha\beta-\alpha^2\right)p_{u1}}{2(1-\alpha\beta)(4-\alpha\beta)(4-3\alpha\beta)} + \frac{\left((4-3\alpha\beta)\left(\alpha^2+\beta^2\right)-(4-2\alpha\beta)(8-7\alpha\beta)\right)p_{u2}}{2(1-\alpha\beta)(4-\alpha\beta)(4-3\alpha\beta)} = 0. \tag{22}$$

Equations (21) and (22) are the condition that platform 1 and platform 2's maximize their profits.

We assume the platforms use bounded rational expectation, and each platform adopts the myopic adjustment [18]; then, the competition system can be written as

$$\begin{cases} p_{u1}(t+1) = p_{u1}(t) + \lambda_1 p_{u1} \dfrac{\partial \pi_1}{\partial p_{u1}}, \\ p_{u2}(t+1) = p_{u2}(t) + \lambda_2 p_{u2} \dfrac{\partial \pi_2}{\partial p_{u2}}. \end{cases} \tag{23}$$

Following, we analyze the equilibrium of system (23).

3. Equilibrium Analysis

Now, we turn to the two-sided market competition equilibrium, when $p_{u1}(t+1) = p_{u1}(t)$, $p_{u2}(t+1) = p_{u2}(t)$; then, we combined equations (21)–(23); then, we can conclude that there are four fixed points in system (23): $E1(0,0)$, $E2(0, ((1-\alpha\beta)(4-\alpha\beta)(4-3\alpha\beta-\beta^2))/((4-2\alpha\beta)(8-7\alpha\beta)-(4-3\alpha\beta)(\alpha^2+\beta^2)))$, $E3(((1-\alpha\beta)(4-\alpha\beta)(4-3\alpha\beta-\beta^2))/((4-2\alpha\beta)(8-7\alpha\beta)-(4-3\alpha\beta)(\alpha^2+\beta^2)), 0)$, and $E4(1-(3/4)\alpha\beta-(1/4)\beta^2, 1-(3/4)\alpha\beta-(1/4)\beta^2)$. When $0<\alpha<1$ and $0<\beta<1$, it is obvious that four points are positive.

The point $E1(0,0)$ Jacobian matrix is

$$\begin{pmatrix} 1+\lambda_1 \dfrac{4-3\alpha\beta-\beta^2}{8-6\alpha\beta} & 0 \\ 0 & 1+\lambda_2 \dfrac{4-3\alpha\beta-\beta^2}{8-6\alpha\beta} \end{pmatrix}. \tag{24}$$

It is easy to see that the root of the determinant, $r_1 = 1 + \lambda_1((\beta/2) + (-\beta/2(4-3\alpha\beta))) > 1$, $r_2 = 1 + \lambda_2(4-3\alpha\beta-\beta^2/8-6\alpha\beta) > 1$, $E1$ is a repelling point, and $E1$ is unstable [18].

$E2(0, ((1-\alpha\beta)(4-\alpha\beta)\ (4-3\alpha\beta-\beta\ ^2)/(4-2\alpha\beta)\ (8-7\alpha\beta)-(4-3\alpha\beta)\ (\alpha^2+\beta\ 2)))$ Jacobian matrix is

$$\begin{pmatrix} 1+\lambda_1 a_{11} & 0 \\ \lambda_2 p_{u2} a_{21} & 1+\lambda_2 p_{u2} a_{22} \end{pmatrix}, \tag{25}$$

where $a_{11} = ((4-3\alpha\beta-\beta^2)/(8-6\alpha\beta)) + (((4-3\alpha\beta-\beta^2)(4-3\alpha\beta-\alpha^2)p_{u2})(4-3\alpha\beta-\beta^2)(4-3\alpha\beta-\alpha^2)p_{u2}/(2(1-\alpha\beta)(4-\alpha\beta)(4-3\alpha\beta))2(1-\alpha\beta)(4-\alpha\beta)(4-3\alpha\beta)) = ((4-3\alpha\beta-\beta^2)4-3\alpha\beta-\beta^2/(8-6\alpha\beta)8-6\alpha\beta) + (((4-3\alpha\beta-\beta^2)^2(4-3\alpha\beta-\alpha^2))(4-3\alpha\beta-\beta^2)^2(4-3\alpha\beta-\alpha^2)/(2(4-3\alpha\beta)((4-2\alpha\beta)(8-7\alpha\beta)-(4-3\alpha\beta)(\alpha^2+\beta^2)))2(4-3\alpha\beta)((4-2\alpha\beta)(8-7\alpha\beta)-(4-3\alpha\beta)(\alpha^2+\beta^2))) > 0$, so $1+\lambda_1 a_{11} > 1$.

$a_{22} = ((4-3\alpha\beta)(\alpha^2+\beta\ ^2)-(4-2\alpha\beta)\ (8-7\alpha\beta))/(2(1-\alpha\beta)\ (4-\alpha\beta)(4-3\ \alpha\beta)) < 0$, so $1+\lambda_2 p_{u2} a_{22} < 1$, then the $E2$ point is a saddle point [18].

Similarly, $E3(((1-\alpha\beta)(4-\alpha\beta)(4-3\alpha\beta-\beta^2))/((4-2\alpha\beta)(8-7\alpha\beta)-(4-3\alpha\beta)(\alpha^2+\beta^2)), 0)$ is a saddle point.

$E4(-(3/4)\ \alpha\beta-(1/4)\beta\ 2+1, -\ (3/4)\alpha\beta-(1/4)\ \beta^2+1)$ Jacobian matrix is

$$\begin{pmatrix} 1+\lambda_1 p_{u1}^* b_{11} & \lambda_1 p_{u1}^* b_{12} \\ \lambda_2 p_{u2}^* b_{21} & 1+\lambda_2 p_{u2}^* b_{22} \end{pmatrix}, \tag{26}$$

where $p_{u1}^* = -(3/4)\alpha\beta-(1/4)\beta^2+1$, $p_{u2}^* = -(3/4)\alpha\beta-(1/4)\beta^2+1$.

$$\begin{aligned} b_{11} &= \frac{(4-3\alpha\beta)(\alpha^2+\beta^2)-(4-2\alpha\beta)(8-7\alpha\beta)}{2(1-\alpha\beta)(4-\alpha\beta)(4-3\alpha\beta)}, \\ b_{12} &= \frac{(4-3\alpha\beta-\beta^2)(4-3\alpha\beta-\alpha^2)}{2(1-\alpha\beta)(4-\alpha\beta)(4-3\alpha\beta)}, \\ b_{21} &= \frac{(4-3\alpha\beta-\beta^2)(4-3\alpha\beta-\alpha^2)}{2(1-\alpha\beta)(4-\alpha\beta)(4-3\alpha\beta)}, \\ b_{22} &= \frac{(4-3\alpha\beta)(\alpha^2+\beta^2)-(4-2\alpha\beta)(8-7\alpha\beta)}{2(1-\alpha\beta)(4-\alpha\beta)(4-3\alpha\beta)}. \end{aligned} \tag{27}$$

It is easy to conclude that $b_{11} = b_{22}$, $b_{21} = b_{21}$.

The characteristic equation of Jacobian matrix is $x^2 - \mathrm{tr}\, x + \mathrm{Det}$.

The trace of Jacobian matrix is $\mathrm{Tr} = 2 + \lambda_1 p_{u1}^* b_{11} + \lambda_2 p_{u2}^* b_{22}$.

The determinant is $\mathrm{Det} = (1+\lambda_1 p_{u1}^* b_{11})(1+\lambda_2 p_{u2}^* b_{22}) - \lambda_1\lambda_2 p_{u1}^* p_{u2}^* b_{12} b_{21}$.

$$\begin{aligned} \mathrm{Tr}^2 - 4\mathrm{Det} &= (2+\lambda_1 p_{u1}^* b_{11} + \lambda_2 p_{u2}^* b_{22})^2 \\ &\quad - 4((1+\lambda_1 p_{u1}^* b_{11})(1+\lambda_2 p_{u2}^* b_{22}) - \lambda_1\lambda_2 p_{u1}^* p_{u2}^* b_{12} b_{21}) \\ &= (\lambda_1 p_{u1}^* b_{11} - \lambda_2 p_{u2}^* b_{22})^2 + 4\lambda_1\lambda_2 p_{u1}^* p_{u2}^* b_{12} b_{21} > 0. \end{aligned} \tag{28}$$

So, the characteristic equation's two roots are real [17, 19].

Following Jury rule [18, 19], the stability conditions for system (23) are

$$\begin{cases} \text{(a)} 1 + \mathrm{Tr} + \mathrm{Det} > 0, \\ \text{(b)} 1 - \mathrm{Tr} + \mathrm{Det} > 0, \\ \text{(c)}\ 1 - \mathrm{Det} > 0. \end{cases} \tag{29}$$

The (a) condition is

$$\begin{aligned} 1 + \mathrm{Tr} + \mathrm{Det} &= 1 + 2 + \lambda_1 p_{u1}^* b_{11} + \lambda_2 p_{u2}^* b_{22} \\ &\quad + (1+\lambda_1 p_{u1}^* b_{11})(1+\lambda_2 p_{u2}^* b_{22}) - \lambda_1\lambda_2 p_{u1}^* p_{u2}^* b_{12} b_{21} \\ &= 4 + 2\lambda_1 p_{u1}^* b_{11} + 2\lambda_2 p_{u2}^* b_{22} + \lambda_1\lambda_2 p_{u1}^* p_{u2}^* (b_{11} b_{22} - b_{12} b_{21}) \\ &= 4 + 2 b_{11} p_{u1} (\lambda_1 + \lambda_2) + \lambda_1\lambda_2 p_{u1}^2 (b_{11}^2 - b_{12}^2) > 0. \end{aligned} \tag{30}$$

We can conclude that $\lambda_1 < (-4 - 2b_{11} p_{u1}^* \lambda_2)/(2b_{11} p_{u1}^* + \lambda_2 p_{u1}^{*2}(b_{11}^{*2} - b_{12}^{*2}))$.

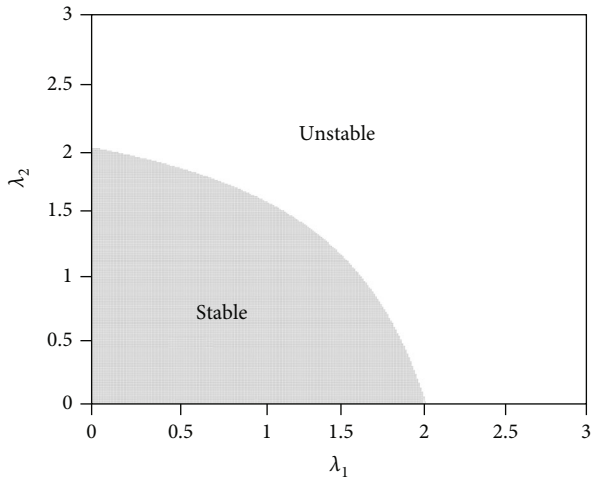

Figure 2: Stable area of system (23).

The (b) condition is

$$\begin{aligned} 1 - \mathrm{Tr} + \mathrm{Det} &= 1 - 2 - \lambda_1 p_{u1}^* b_{11} - \lambda_2 p_{u2}^* b_{22} \\ &+ (1 + \lambda_1 p_{u1}^* b_{11})(1 + \lambda_2 p_{u2}^* b_{22}) - \lambda_1 \lambda_2 p_{u1}^* p_{u2}^* b_{12} b_{21} \\ &= \lambda_1 \lambda_2 p_{u1}^* p_{u2}^* (b_{11} b_{22} - b_{12} b_{21}) > 0. \end{aligned} \tag{31}$$

For $b_{11} = b_{22}$ and $p_{12} = p_{21}$, so $1 - \mathrm{Tr} + \mathrm{Det} = \lambda_1 \lambda_2 p_{u1}^* p_{u2}^* (b_{11} - b_{12})(b_{11} + b_{12}) > 0$.

The (b) condition is satisfied.

The (c) condition is

$$\begin{aligned} 1 - \mathrm{Det} &= 1 - (1 + \lambda_1 p_{u1}^* b_{11})(1 + \lambda_2 p_{u2}^* b_{22}) \\ &+ \lambda_1 \lambda_2 p_{u1}^* p_{u2}^* b_{12} b_{21} = -\lambda_1 p_{u1}^* b_{11} \\ &- \lambda_2 p_{u2}^* b_{22} + \lambda_1 \lambda_2 p_{u1}^* p_{u2}^* (b_{12} b_{21} - b_{11} b_{22}) > 0. \end{aligned} \tag{32}$$

We can conclude that $\lambda_1 < (\lambda_2 p_{u2}^* b_{22}) / (\lambda_2 p_{u1}^* p_{u2}^* (b_{12} b_{21} - b_{11} b_{22}) - p_{u1}^* b_{11})$.

The stable area can be illustrated in Figure 2. The dark area is the stable area, and the white area is the unstable area. We can see that when λ_1, λ_2 is small in an extent, system (23) is stable; when λ_1, λ_2 is large, the system would become bifurcation and finally chaos.

4. Simulation

Numerical examples such as bifurcation diagram, maximum Lyapunov exponents, and strange attractor can help us illustrate dynamics of system (23). In this section, our purpose is to show the adjustment speed, buyers' externality parameter, and sellers' externality parameter effects on stability of system (23).

4.1. Adjustment Speed Effect. We first fix the parameters set $\alpha = 0.2, \beta = 0.1, \lambda_2 = 1.2$, the initial state is $p_{u1}(1) = 0.1, p_{u2}(1) = 0.3$, after take iteration 700 times, and the bifurcation diagram is illustrated in Figure 3. We find that when $\lambda_1 < 1.511$, the Nash equilibrium point, $E4$ is stable, when λ_1 larger than 1.511, system (23) becomes bifurcation, and after

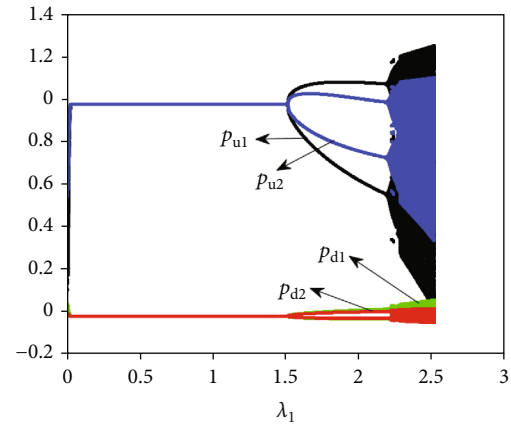

Figure 3: Bifurcation diagram with respect to λ_1 when $\alpha = 0.2, \beta = 0.1, \lambda_2 = 1.2, p_{u1}(1) = 0.1, p_{u2}(1) = 0.3$, and iteration = 700.

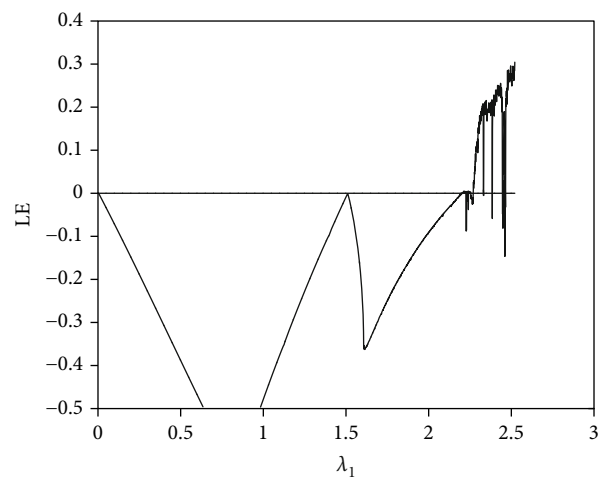

Figure 4: The maximum Lyapunov exponent with respect to λ_1 corresponding to Figure 3.

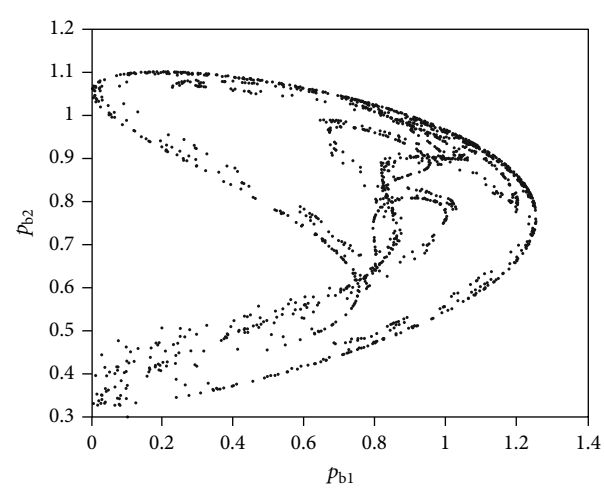

Figure 5: The corresponding chaotic strange attractor of system (23) when $\lambda_1 = 2.522$.

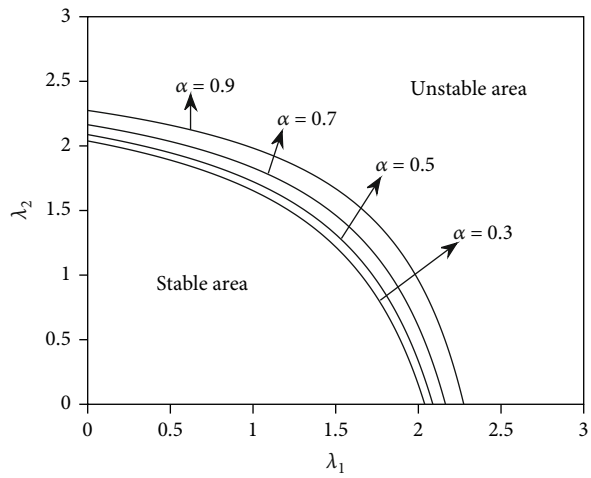

Figure 6: Stable area of (λ_1, λ_2) with different values of α.

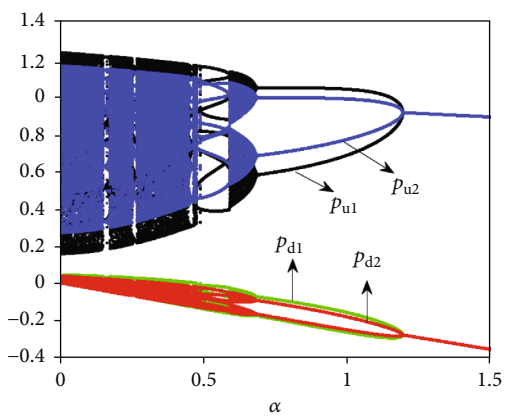

Figure 7: Bifurcation diagram with respect to α when $\beta = 0.1, \lambda_1 = 2.1, \lambda_2 = 1.5, p_{u1}(1) = 0.1, p_{u2}(1) = 0.3$, and iteration = 700.

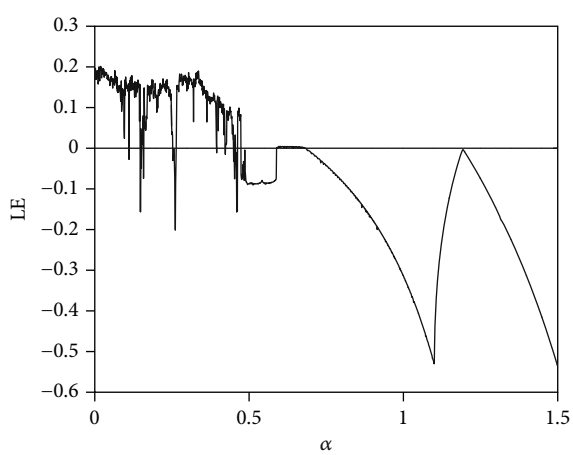

Figure 8: The maximum Lyapunov exponent with respect to α corresponding to Figure 7.

$\lambda_1 > 2.200$, the chaos state happens. We can see that when system (23) is stable, the two platform's equilibrium are exactly symmetric, $p_{u1} = p_{u2}$ and $p_{d1} = p_{d2}$. The users' fee is larger than developers' fee, and the developers' fee is lower

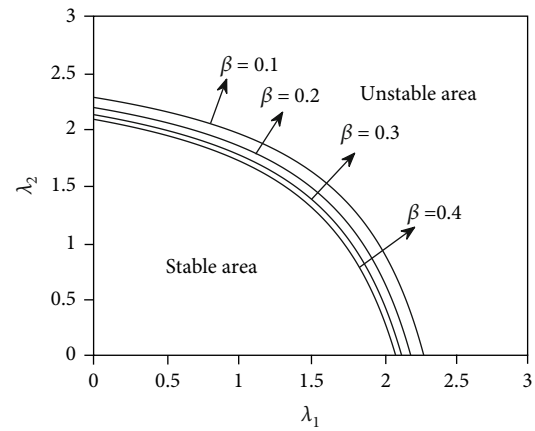

Figure 9: Stable area of (λ_1, λ_2) with different values of β.

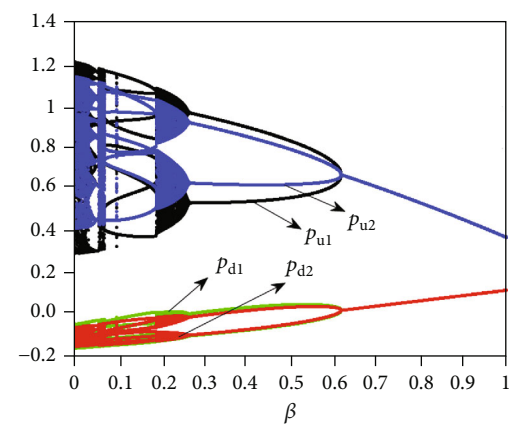

Figure 10: Bifurcation diagram with respect to β changes when $\alpha = 0.5$, d = 1, $\lambda_1 = 2.1, \lambda_2 = 1.5, p_{u1}(1) = 0.1, p_{u2}(1) = 0.3$, and iteration = 700.

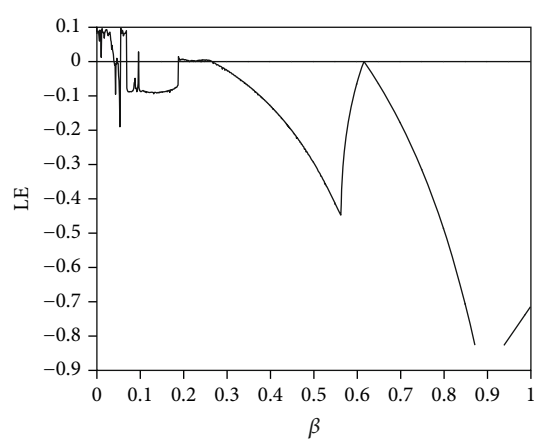

Figure 11: The maximum Lyapunov exponent with respect to β corresponding to Figure 10.

than zero; this is a common phenomenon in the two-sided market pricing strategy.

As is shown in Figure 4, the maximum Lyapunov exponent (LE) changes corresponding to Figure 3, as [13], we calculate LE with Wolf's Jacobian algorithm. The LE returns to

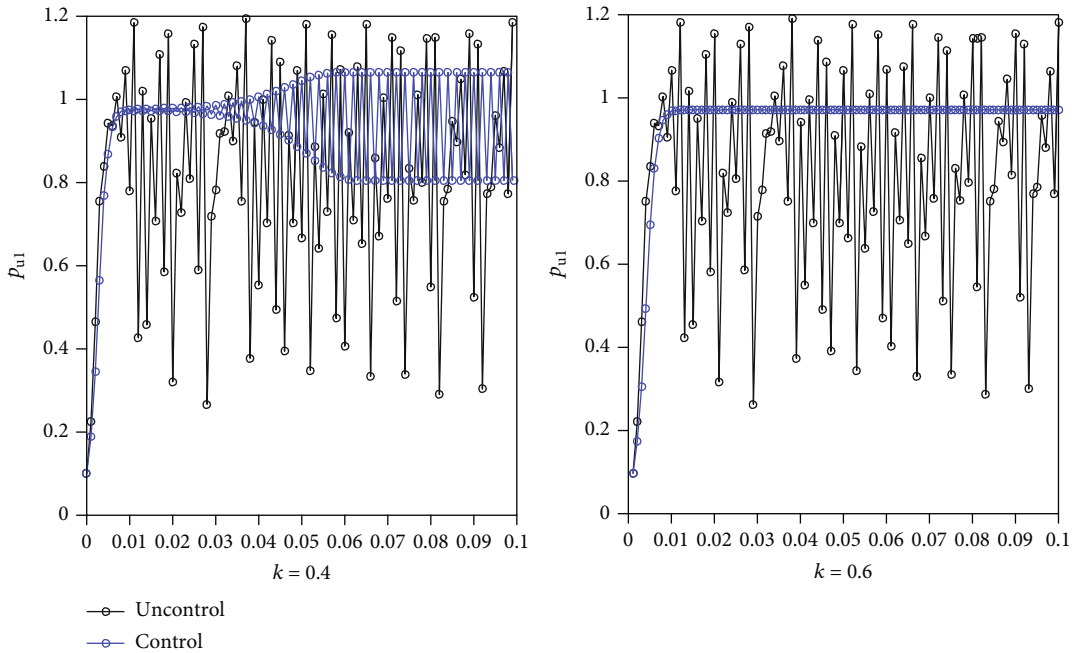

Figure 12: The time series difference between system (23) and under TDFC when $\alpha = 0.2$, $\beta = 0.1$, $\lambda_1 = 2.522$, and $\lambda_2 = 1.2$.

zero when $\lambda_1 \approx 1.511$, which means system (23) becomes bifurcation; when $\lambda_1 \approx 2.200$, the LE becomes larger than zero, meaning system (23) enters the chaos state. And Figure 5 shows the strange attractor when $\lambda_1 = 2.522$. We can see that when system (23) is chaotic state, the p_{u1} and p_{u2} are in a certain interval.

4.2. α Effects on Stable. The externality parameter that developers bring to users affects the stability of system (23); we fixed parameter set $\beta = 0.1$, $\lambda_1 = 2.1$, and $\lambda_2 = 1.5$.

Figure 6 shows the stable area with different externality parameters α. Compare the stable area in different values of α, 0.3, 0.5, 0.7, and 0.9. We can see that when α is larger, the stable area of system (23) increases. The externality in consumer side can be seen as the difference between two platforms; the difference in externality leads the whole system more stable.

Figure 7 shows fee bifurcation with the change of user externality parameter α. When externality parameter increases $\alpha \approx 1.194$, p_{u1} and p_{u2} change from the chaos state to thr two-period bifurcation, and after $\alpha \approx 0.682$, system (23) becomes stable, and in the stable state, p_{d1} and p_{d2} decrease, and below zero, and platform's profit mainly from the fee that consumer pays to the platform. This strategy is also known as the divide and conquer strategy [25].

Figure 8 illustrates the maximum Lyapunov exponents with change of α; when the $\alpha < 0.682$, the LE is larger than zero, and system (23) is in chaos state; when $0.682 < \alpha < 1.1194$, the system becomes bifurcation; when $\alpha > 1.1194$, the system becomes stable.

4.3. β Effects on Stable. We first fix system (23) parameter set $\alpha = 0.5$, d = 1, $\lambda_1 = 2.1$, $\lambda_2 = 1.5$. Figure 9 shows stable area changers with respect to β when other parameters are given. The stable area becomes larger when β increasing; this means system (23) when β is larger.

Figures 10 and 11 show the bifurcation diagram and maximum Lyapunov exponent for β changes when other parameters are given. We can see that when $\beta < 0.267$, system (23) is in chaos state; after $\beta > 0.267$, system (23) becomes bifurcation. After $\beta > 0.617$, the system becomes stable and user fee becomes lower while developer fee becomes higher.

From Figure 11, we can see that when $\beta < 0.267$, the system is in a chaotic state, when $\beta \approx 0.617$, the system is in bifurcation, while $\beta > 0.617$, the system becomes stable.

From simulation, we can infer that the externality parameters in the developer side and user side make the system more chaotic; this means when the externality parameters are large enough, the system becomes out of control.

4.4. Time-Delayed Feedback Control (TDFC). From first simulation in Section 5.1, given when $\alpha = 0.2$, $\beta = 0.1$, $\lambda_2 = 1.2$ are fixed, when λ_1 large enough, system (23) become chaos state.

We use the TDFC to alleviate the chaos state. The TDFC method becomes a popular method to alleviate chaos; as [30], we propose the delay feedback control parameter k, we assume a TDFC method $F(t+1) = k(p_{u1}(t) - p_{u1}(t+1))$, we insert this mechanism into system (23), and then, we can get

$$\begin{cases} p_{u1}(t+1) = p_{u1}(t) + \lambda_1 p_{u1} \dfrac{\partial \pi_1}{\partial p_{u1}} + F(t+1), \\ p_{u2}(t+1) = p_{u2}(t) + \lambda_2 p_{u2} \dfrac{\partial \pi_2}{\partial p_{u2}}. \end{cases} \tag{33}$$

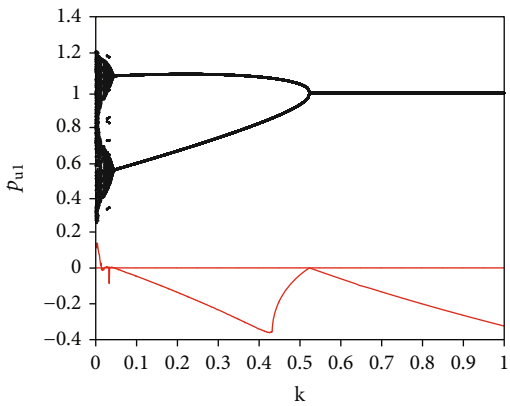

Figure 13: Bifurcation diagram with respect to k and the maximum Lyapunov exponent with respect to k when $\alpha = 0.2$, $\beta = 0.1$, $\lambda_2 = 1.2$, and $\lambda_2 = 1.2$.

So, we get the TDFC system

$$\begin{cases} p_{u1}(t+1) = p_{u1}(t) + \dfrac{\lambda_1}{1+k} p_{u1} \dfrac{\partial \pi_1}{\partial p_{u1}}, \\ p_{u2}(t+1) = p_{u2}(t) + \lambda_2 p_{u2} \dfrac{\partial \pi_2}{\partial p_{u2}}. \end{cases} \tag{34}$$

Then, we fixed the parameter set $\alpha = 0.2, \beta = 0.1, \lambda_1 = 2.522, \lambda_2 = 1.2$, From Figure 5, we know the system is in a chaotic state. Figure 12 shows p_{b1} in this chaotic state; then, we let $k = 0.4$ and $k = 0.6$; we can see that when $k = 0.4$, system (23) goes into 2-period bifurcation; when $k = 0.6$, system (23) goes into a stable state.

Figure 13 shows that when $k = 0$, the original system is chaos; when k becomes large, until $k > 0.045$, system (23) becomes change to bifurcation; and when $k > 0.525$ system (23) becomes stable in the NE point. The simulation shows that the TDFC can alleviate the stability of the two-sided chaos system.

5. Conclusion and Limitations

5.1. Discussion and Conclusions. Two-sided platforms are used by different agents to interact with each other; however, the two-sided markets challenges arise as a result of the externality in different sides. We analyze the complexity in two-sided market competition.

We build a two-sided platform competition game model and then investigate the dynamics of competition between two platforms when two platforms are bounded rational; we find that the stability of system (23) is influenced by adjustment speed and users' and developers' externality parameters. When the adjustment speed of two-sided markets is larger, the systems becomes more instable. Both users' and developers' externality parameters make the system more stable and enhance the fee in its own side, while lowering the other side price. We then introduce the TDFC mechanism and find that TDFC alleviates the chaos state.

5.2. Theoretical Contributions. Our research results have several contributions. Our research focus on the chaotic phenomenon in the two-sided market. Compared with other previous studies, we combined the two-sided market theory and nonlinear dynamics. We assume that platform is bounded rational, and they can only achieve limit information; this is more in line with reality.

5.3. Managerial Implications. Our research has three strategic implications for platform decision. The platform might presents complexity behaviors. From a strategy perspective, our results imply that platforms have incentives to affect the consumer heterogeneous to alleviate chaotic state; however, this may reduce platform profit. Platform makes their decisions compare its profit target and management control.

5.4. Limitations and Further Research. There are several limitations to this research. For the theoretical study, the relevant assumptions were too strict, such as externality parameters which we assume are small; in fact, platforms have an extensive relationship with other sides; for simplifying research, we assume the platform only compete in the consumer side; if the platform competes in two sides, the conclusion would be more general. And the multihoming and single homing are general in platform; we think this should take more attention.

Data Availability

The (data type) data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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